Pricing of electricity options

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April 6, 2009

Abstract

In this paper we generalize the approach of [12] replacing in the dynamics of the asset prices the Brownian motion by a more general Lévy process, also taking into account the occurrence of spikes. In particular, we derive a model for electricity futures markets, where we can simultaneously model evolution of futures and spot prices. At the same time we have in our model an explicit connection between electricity futures and spot price processes. Moreover, our framework contains as a special case the commonly accepted model for electricity markets, where the spot price process is an exponential of an Ornstein-Uhlenbeck process plus seasonality component. We show that the considered method combined with the Fourier transform techniques provides explicit pricing formulas for European electricity options. Furthermore, an important achievement is that the spot price dynamics in our model becomes multi-dimensional Markovian. The Markovian structure is crucial for pricing of path dependent derivatives such as electricity swing options.

1 Introduction

In the stochastic modeling of electricity markets, there are two main approaches in the literature (see e.g. [1], [12]). The first one starts with a

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stochastic model for the spot price, and from this derives the futures price dynamics by using the arbitrage theory. The second approach models directly the pricing dynamics of forward and futures contracts traded in electricity markets. We refer to [1] and [16] for an overview of literature on electricity markets.

Spot price models have two major disadvantages. Since electricity is non-storable, the spot electricity price is not a tradeable asset. This implies that it is not easy to give a precise definition of spot prices in the electricity market (see [1], [16]). For the same reason the valuation methods for traded asset prices are not adequate. The second disadvantage is that the connection between the spot and futures prices is not straightforward (see [12]). The modeled dynamics of the entire futures curve can be rarely consistent with the actually observed curves. On the contrary, futures price models attempt to systematically describe changes of the entire curve.

However, futures price models, since they normally imply a very complex non-Markovian dynamics for the spot price, are not well suited for pricing of path dependent electricity products like, for example, swing options (see e.g., [11], [15] and [25]). Markovian property of the spot price is essential for dynamic programming principle needed to find the solutions of the constrained stochastic optimal control problem of maximizing the expected profit of the path dependent electricity product (see e.g. [25]).

The significant drawback of the spot and futures models described above is the lack of flexibility to decouple spot and futures price evolution. By calibrating the futures price according to observed market data, one can no longer control the spot price and vice versa. In [12] an approach is introduced which converts the electricity market into a money market. By a currency change one gets correspondence between given electricity market and a market consisting of bonds and a risky asset. Significant benefit of this transformation is an additional source of randomness in the modeling of electricity prices. Namely, it is possible to calibrate the spot and futures processes independently including features of both electricity price processes. Furthermore, the money market in [12] is modeled under the Gaussian Heath-Jarrow-Morton (HJM) setting. Using this method, the well-established interest rate theory can be applied for pricing of electricity derivatives.

In this paper we generalize the approach of [12] replacing in the dynamics of the asset prices the Brownian motion by a more general Lévy process, also taking into account the occurrence of spikes. Advanced interest rate theory combined with change of numéraire techniques is used to find a new electricity spot price model with sufficiently flexible futures curve. The valu-
able feature of our approach is that the dynamics becomes multi-dimensional Markovian (see Section 4). As mentioned above, the Markovian structure is significant for pricing of electricity swing options. In addition, we consider valuation of electricity products in our framework. Using Fourier transform techniques, we provide analytical pricing formulas for European electricity options. Thereafter, we consider valuation of electricity swing options.

Electricity swing options are products, which hedge the electricity price risk and also the risks in the option owner’s electricity consumption process. The expression “swing options” comes from the constrain for the electricity consumption process, since the consumption “swings” between the lower and upper boundaries. We finish our paper with the stochastic optimal control problem connected to electricity swing options previously studied in [20], [15], [11], and [25]. In particular, we derive in our setting the Hamilton-Jacobi-Bellman equation associated to the swing options.

In the next section we explain a connection between electricity and fixed-income market. Then, in Section 3 we introduce an electricity market model derived by a Lévy term structure. In particular, we consider here the corresponding measure transformation. Thereafter, in Section 4 we examine the Markov property of the spot price process in our framework. Moreover, in Section 4 we show that our framework contains as a special case the commonly accepted model for an electricity market, where the spot price process is an exponential of the sum of an Ornstein-Uhlenbeck process and of a deterministic function characterising seasonality. (See f.e. [11] for more details on the spot price model of this type.) Finally, we apply the results of Sections 3 and 4 to valuation of electricity derivatives in Chapter 5.

2 Connection between electricity market and money market

Let $F(t, \tau), 0 \leq t \leq \tau$, be the futures price at time $t$ of electricity and $T$ be a finite time horizon, $\tau \leq T$. Denote the set of chronological time pairs by

$$D := \{(t, \tau) : 0 \leq t \leq \tau \leq T\}.$$ 

We model the futures market starting by the following axioms:

**C1:** For every $\tau \in [0, T]$ the futures price evolution $(F(t, \tau))_{(t,\tau) \in D}$ is a positive-valued adapted stochastic process realized on a complete filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t)_{t \in [0,T]})$.

**C2:** There exists a martingale measure $\mathbb{Q}^F$ equivalent to $\mathbb{P}$ such that for all
\[ \tau \in [0, T] \text{ the futures price process } (F(t, \tau))_{(t,\tau)\in \mathcal{D}} \text{ is a } Q^F \text{-martingale.} \]

C3: At \( t = 0 \) futures prices start at deterministic positive values \( F(0, \tau), \tau \in [0, T] \).

C4: Terminal prices form a spot price process \( S_t := F(t, t), t \in [0, T] \).

Following the approach of [12] we now convert an electricity market into a money market consisting of zero bonds \( (P(t, \tau))_{0 \leq t \leq \tau} \) equipped with an additional risky asset \( (N_t)_{t \in [0, T]} \) by using the following transformation:

\[
P(t, \tau) := \frac{F(t, \tau)}{S_t}, \quad (1) \\
N_t := \frac{1}{S_t}. \quad (2)
\]

We have that the money market defined by the currency change (1)–(2) satisfies the following axioms:

M1: \( (N_t)_{t \in [0, T]} \) and \( (P(t, \tau))_{(t,\tau)\in \mathcal{D}} \) are positive, adapted stochastic processes defined on \( (\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t)_{t \in [0, T]}) \).

M2: There exists a positive-valued, adapted numéraire process \( (C_t)_{t \in [0, T]} \) and there exists a martingale measure \( Q^M \) equivalent to \( \mathbb{P} \), such that for all \( \tau \in [0, T] \) the discounted price processes \( \hat{P}(t, \tau) := \frac{P(t, \tau)}{C_t}, (t, \tau) \in \mathcal{D} \) and \( \hat{N}_t := \frac{N_t}{C_t}, 0 \leq t \leq T \), are \( Q^M \)-martingales.

M3: Prices start at deterministic values \( N_0 \) and \( (P(0, \tau))_{\tau \in [0, T]} \).

M4: Bond prices finish at one, i.e. \( P(t, t) = 1 \), for every \( t \in [0, T] \).

We now need a slight generalization of Theorem 1 in [12].

**Theorem 2.1.** i) Suppose that the commodity market \( (F(t, \tau))_{(t,\tau)\in \mathcal{D}} \) fulfills C1 to C4 with an initial futures curve \( (F(0, \tau))_{\tau \in [0, T]} \) and a martingale measure \( Q^F \). Then the transformation (1)–(2) provides a money market satisfying M1 to M4 with the initial values

\[
P(0, \tau) := \frac{F(0, \tau)}{S_0}, \quad \forall \tau \in [0, T], \quad \text{and} \quad N_0 = \frac{1}{S_0}.
\]

ii) Suppose that the money market \( (P(t, \tau))_{(t,\tau)\in \mathcal{D}}, (N_t)_{t \in [0, T]} \) fulfills M1 to M4 with initial values \( (P(0, \tau))_{\tau \in [0, T]}, N_0 \), a discounting process \( (C_t)_{t \in [0, T]} \) and a martingale measure \( Q^M \). Then the transformation

\[
F(t, \tau) := \frac{P(t, \tau)}{N_t}, \quad (t, \tau) \in \mathcal{D},
\]
gives an electricity market with the deterministic initial futures curve \( F(0, \tau) := \frac{P(0, \tau)}{N_0} \), for all \( \tau \in [0, T] \), and the martingale measure

\[
dQ^F = \frac{N_T C_0}{C_T N_0} dQ^M. \tag{4}
\]

Note that in Theorem 1 of [12] all price processes were assumed continuous. In our proof we will only use the integrability properties of the processes involved.

**Proof.**

i) It is easy to see that the properties M1–M4 are obvious consequences of C1–C4 due to (1) and (2), if the discounting process and the martingale measure are given by

\[
C_t = \frac{1}{S_t}, \quad t \in [0, T], \quad \text{and} \quad dQ^M = dQ^F. \tag{5}
\]

ii) Define the futures price process \( F(t, \tau) \) as in (3). The process \( F(t, \tau) \) is then positive and adapted by Assumption M1. Consider the equivalent probability measure \( Q^F \) given by (4). \( F(t, \tau) \) is integrable w.r.t. \( Q^F \), since

\[
E_{Q^F}[F(t, \tau)] = E_{Q^M}\left[ \frac{P(t, \tau)}{N_t} \frac{dQ^F}{dQ^M} \right] = \frac{C_0}{N_0} E_{Q^M}\left[ \frac{P(t, \tau) N_t}{C_t} \right]
\]

by Assumption M2. Furthermore, M2 yields

\[
E_{Q^F}[F(t, \tau)|F_s] = \frac{E_{Q^M}[F(t, \tau)\frac{N_t}{C_t}|F_s]}{\frac{N_s}{C_s}} = \frac{E_{Q^M}[\frac{P(t, \tau)}{C_t}|F_s]}{\frac{N_s}{C_s}} = \frac{P(s, \tau)}{N_s} = F(s, \tau), \quad \forall \ 0 \leq s \leq t \leq \tau.
\]

Hence, \((F(t, \tau))_{0 \leq t \leq \tau}\) is a \( Q^F \)-martingale.

\[\square\]

In the following sections we apply this approach and study electricity markets derived by term structure models driven by general Lévy processes, using the HJM approach.
3 Money market construction

We follow the HJM approach and specify the term structure by modeling the (instantaneous) forward rate $f(t, \tau)$, $(t, \tau) \in \mathcal{D}$. Let $P(t, \tau)$, $(t, \tau) \in \mathcal{D}$, be the market price at moment $t$ of a bond paying 1 at the maturity time $\tau$, $\tau \leq T$. Given the forward rate curve $f(t, \tau)$ the bond prices are defined by

$$P(t, \tau) = \exp\{-\int_t^\tau f(t, s)ds\}, \quad (6)$$

while the instantaneous short rate $r$ at time $t$ is given by

$$r(t) := f(t, t). \quad (7)$$

A general introduction to fixed-income markets is given in [3].

Let $L = (L_1, \ldots, L^n)$ be an $n$-dimensional Lévy process with independent components, defined on a probability space $(\Omega, \mathcal{F}, \mathbb{Q})$ endowed with the completed canonical filtration $(\mathcal{F}_t)_{t \in [0,T]}$ associated with $L$. We denote by $(b_i, c_i, \nu_i)$ the characteristic triplet of each component $L^i$, $i = 1, \ldots, n$.

We assume that

**A1:** we are given an $\mathbb{R}$-valued and $\mathbb{R}^n$-valued stochastic process $\alpha(t, \tau)$ and $\eta(t, \tau) = (\eta^1(t, \tau), \ldots, \eta^n(t, \tau))$, $(t, \tau) \in \mathcal{D}$, respectively, such that $\alpha(t, \tau)$ and $\eta(t, \tau)$ are continuous and adapted.

**A2:** $\int_0^T \int_0^T E|\alpha(s, u)|dsdu < \infty$, $\int_0^T \int_0^T E\|\eta(s, u)\|^2dsdu < \infty$.

**A3:** $P(\tau, \tau) = 1$, $\forall \tau \in [0, T]$.

**A4:** The initial forward curve is given by a deterministic and continuously differentiable function $\tau \rightarrow f(0, \tau)$ on the interval $[0, T]$.

For the forward rate we consider a generalized HJM model, i.e. we assume that the forward rate process follows the dynamics

$$f(t, \tau) = f(0, \tau) + \int_0^t \alpha(s, \tau)ds + \sum_{i=1}^n \int_0^t \eta^i(s, \tau)dL^i_s, \quad t \leq \tau. \quad (8)$$

In terms of short rates we can rewrite (8) and (7) as

$$r(t) = f(0, t) + \int_0^t \alpha(s, t)ds + \sum_{i=1}^n \int_0^t \eta^i(s, t)dL^i_s, \quad t \leq T. \quad (9)$$

Lévy term structure models of type (8)–(9) are frequently considered in the literature (see e.g. [7], [8], [9], or [14]).
Putting (6) and (9) together and assuming that
\[ \alpha(t, \tau) = \eta(t, \tau) = 0 \text{ a.s. for } t > \tau, \] (10)
so that the forward rate (8) is defined for all \( t, \tau \in [0, T] \), we can derive the following representation for the bond price given in [6]:

\[
P(t, \tau) = P(0, \tau) \exp \left\{ \int_0^t r(u)du - \int_0^t \int_0^\tau \alpha(s,u)duds - \sum_{i=1}^n \int_0^t \int_0^\tau \eta^i(s,u)dudL^i_s \right\}. \tag{11}
\]

We now consider the bank account process as a discounting factor, i.e.

\[
C_t = \exp \{ \int_0^t r(s)ds \}. \tag{12}
\]

In order to provide a condition which ensures that \( \mathbb{Q}^M \) is a local martingale measure for
\[
\hat{P}(t, \tau) := \frac{P(t, \tau)}{C_t}, \quad t \in [0, \tau], \tag{13}
\]
we assume that there exist \( a_i < 0 \) and \( d_i > 1 \) such that the Lévy measures \( \nu_i \) of \( L^i \) satisfy

\[
\int_{\{x>1\}} e^{ux} \nu_i(dx) < \infty, \quad u \in [a_i, d_i], \quad i = 1, \ldots, n, \tag{14}
\]
(see [7] or [9]). Condition (14) ensures the existence of the cumulant generating function
\[
\Theta^i(u) := \log E[\exp(uL^i_1)] \tag{15}
\]
at least on the set \( \{u \in \mathbb{C} | \Re u \in [a_i, d_i]\} \), where \( \Re u \) denotes the real part of \( u \in \mathbb{C}, i = 1, \ldots, n \). By Lemma 26.4 in [24] \( \Theta^i \) is continuously differentiable and has the representation:

\[
\Theta^i(u) = b_i u + \frac{c_i}{2} u^2 + \int_{\mathbb{R}} (e^{ux} - 1 - ux) \nu_i(dx), \quad i = 1, \ldots, n. \tag{16}
\]

As a consequence, the Lévy processes \( L^i, i = 1, \ldots, n \), have finite moments of arbitrary order. Provided

\[
- \int_0^\tau \eta^i(s,u)du \in (a_i, d_i) \quad \text{for} \quad i = 1, \ldots, n,
\]
for any $\tau \leq T$, the HJM condition on the drift

$$
\alpha(t, x) = \sum_{i=1}^{n} \frac{\partial}{\partial x} \Theta^i \left( - \int_{0}^{x} \eta^i(t, u) du \right) \quad \text{a.s.} \quad (17)
$$

implies that $Q^M$ is a local martingale measure. The drift condition (17) is derived in [6] and [7], for an analogous drift condition in the infinite dimensional Lévy setting see [14] and [9].

Denoting by

$$
\sigma^i(t, \tau) := - \int_{0}^{\tau} \eta^i(t, u) du, \quad i = 1, \ldots, n, \quad (18)
$$

we can rewrite the HJM drift condition (17) as

$$
\int_{0}^{\tau} \alpha(s, u) du = \sum_{i=1}^{n} \int_{0}^{\tau} \frac{\partial}{\partial u} \Theta^i(\sigma^i(s, u)) du
$$

$$
= \sum_{i=1}^{n} \Theta^i(\sigma^i(s, \tau)) \quad \text{a.s.} \quad (19)
$$

Substituting (19) into (11), we get the same representation for $P(t, \tau)$ as in [7]

$$
P(t, \tau) = P(0, \tau) \exp \left\{ \int_{0}^{t} r(u) du - \sum_{i=1}^{n} \int_{0}^{t} \Theta^i(\sigma^i(s, \tau)) ds + \sum_{i=1}^{n} \int_{0}^{t} \sigma^i(s, \tau) dL^i_s \right\}.
$$

(20)

To complete the modeling of the arbitrage-free money market satisfying Assumptions M1–M4, we assume that the risky asset $N_i$ is given by

$$
N_t = \exp \left\{ \int_{0}^{t} r(u) du - \sum_{i=1}^{n} \int_{0}^{t} \Theta^i(v^i(s)) ds + \sum_{i=1}^{n} \int_{0}^{t} v^i(s) dL^i_s \right\}, \quad N_0 = 1,
$$

(21)

where $v = (v^1, \ldots, v^n)$ is a continuous and adapted process, such that

$$
\hat{N}_t = \frac{N_t}{C_t}
$$

is a well-defined local martingale under $Q^M$. 8
We consider the electricity price processes

\[ F(t, \tau) = \frac{P(t, \tau)}{N_t} \quad \text{and} \quad S(t) = \frac{1}{N_t}, \quad \forall (t, \tau) \in \mathcal{D}, \quad (22) \]

where \( P(t, \tau) \) and \( N_t \) are now given by (20) and (21).

According to Theorem 2.1 the transformation (22) gives an arbitrage-free commodity futures market with the deterministic initial futures curve \( F(0, \tau) := P(0, \tau)/N_0 = P(0, \tau) \). By the same theorem,

\[
dQ^F = \frac{N_T C_0}{C_T N_0} dQ^M = \exp \left\{ \sum_{i=1}^{n} \int_{0}^{T} v^i(s) dL^i_s - \sum_{i=1}^{n} \int_{0}^{T} \Theta^i(v^i(s)) ds \right\} dQ^M \quad (23)\]

is a martingale measure for \( F(t, \tau), (t, \tau) \in \mathcal{D} \).

In order to study the electricity market (22) under the martingale measure \( Q^F \) defined by (23) we need the distribution of \( L \) under \( Q^F \). By Girsanov’s Theorem for semimartingales (cf. Theorem III.3.24 in [13]), \( L \) is a semimartingale under \( Q^F \). In particular, if the process \( v(t) \) appearing in (21) and (23) is deterministic, we get from Girsanov’s Theorem the following Proposition:

**Proposition 3.1.** \( L = (L^1, \ldots, L^n) \) is a (non-homogeneous) Lévy process with independent components under the measure \( Q^F \), where for every \( j = 1, \ldots, n \), the characteristic triplet of \( L^j \) w.r.t. \( Q^F \) is given by

\[
b_j^{Q^F}(t) := b_j + c_j v^j(t) + \int_{\mathbb{R}} (e^{v^j(t)x} - 1)x I_{|x| \leq 1}(x) \nu_j(dx), \quad (24)\]

\[
c_j^{Q^F}(t) := c_j, \quad (25)\]

\[
\nu_j^{Q^F}(dt, dx) := e^{v^j(t)x} \nu_j(dx)dt. \quad (26)\]

For the definition of a non-homogeneous Lévy process we refer to [11].

**Remark 3.2.** Note that if \( v(t) \) is a constant function, then by Proposition 3.1 \( L \) is a time-homogeneous Lévy process under \( Q^F \).

For the reminder of the paper we assume that \( v(t) \) is a deterministic, continuous function.
4 Markov property

In this section we examine the Markov property of the spot price process $S$ given by (22). To begin with, applying Proposition 3.1, we compute the dynamics of $S$ under $Q^F$ as follows.

**Lemma 4.1.** The dynamics of $S$ under $Q^F$ is given by

$$dS(t) = S(t)[-r(t) + \frac{1}{2} \sum_{i=1}^{n} c_i(v^i(t))^2 + \sum_{i=1}^{n} \Theta^i(v^i(t))]dt - S(t-) \sum_{i=1}^{n} v^i(t)dL^i_t + \int_{\mathbb{R}^n} S(t-)(e^{(v(t-),x)} - 1 + (v(t-),x))J^Q_{L}(dx \times dt),$$

where $J^Q_{L}$ is the jump measure of $L$ under $Q^F$.

**Proof.** First, by (21)–(22)

$$S(t) = \exp\{-\int_{0}^{t} r(u)du + \sum_{i=1}^{n} \int_{0}^{t} \Theta^i(v^i(s))ds - \sum_{i=1}^{n} \int_{0}^{t} v^i(s)dL^i_s\},$$

where $\Theta(t)$ and $v(t)$ are deterministic, continuous functions and $L$ is a non-homogeneous Lévy process satisfying Propostion 3.1 The rest follows from the Itô formula. \hfill \Box

Hence, since $v$ is deterministic, we get the following result:

**Proposition 4.2.** Suppose the short rate process $r$ is a Markov process. Then the vector process $(S,r)$ is a Markov process.

**Proof.** Since $r$ is a Markov process and $v$ is deterministic, $(S,r)$ is a Markov process by (27). \hfill \Box

**Remark 4.3.** Note that if the volatility $\eta$ is deterministic, the short rate process $r$ is a Markov process by (9).

Let us consider some examples. In particular, we refer to [4], Examples 3.4.4-3.4.5 where we show that our model for the electricity market contains the case, where the spot price process is an exponential of an Ornstein-Uhlenbeck process. Now we introduce an example, that also includes the seasonality effect.
Example 4.4. Let $W_t^Q$ be a standard Brownian motion under $Q$ and a Lévy process $L$ is given by

$$L_t = W_t^Q + \int_0^t \int_{\mathbb{R}} x J_L^Q (dx \times ds)$$

for some Poisson random measure $J_L^Q$ on $\mathbb{R} \times (0, \infty)$. Let $X_t$ be an Ornstein-Uhlenbeck process driven by $L$, i.e.

$$dX_t = -X_t dt + dL_t, \quad X_0 = x_0 \in \mathbb{R}, \quad t \leq T.$$ 

Then

$$X_t e^{-(T-t)} = x_0 e^{-T} + \int_0^t e^{-(T-s)} dL_s. \quad (29)$$

Further, we assume that the spot price process $S$ is given by

$$S_t = e^{\theta(t) + X_t}, \quad t \leq T, \quad (30)$$

where $\theta(t) : [0, T] \rightarrow \mathbb{R}$ is a deterministic, differentiable, periodic function characterising seasonality. We show in this example that we can find a forward rate structure giving a bond price process $P(t, T)$, $t \in [0, T]$, such that the futures price process $F(t, T) := S(t) P(t, T)$, $t \leq T$, is a $Q^F$-martingale. Note that this futures price model coincides with the model (22), where the spot price process is given by (30).

Assume that

$$\int_{\mathbb{R}} e^x (1 + |x|) \nu_Q^F (dx) < \infty, \quad (31)$$

where $\nu_Q^F$ is the Lévy measure of $L$ under $Q$, and define

$$P(t, T) := \exp \left\{ \theta(T) - \theta(t) + (e^{-(T-t)} - 1) X_t + \frac{1}{2} \int_t^T e^{-2(T-s)} ds \right\}$$

$$+ \int_t^T \int_{\mathbb{R}} \left\{ e^{-(T-s)} x - 1 \right\} \nu_Q^F (dx) ds. \quad (32)$$
Since \( P(T, T) = 1 \), we can consider \( P(t, T) \), for \( t \leq T \), as a bond price process. Furthermore, by (29) and (30)

\[
F(t, T) = S(t)P(t, T) = F(0, T) \exp \left\{ \int_0^t e^{-(T-s)}dL_s - \frac{1}{2} \int_0^t e^{-2(T-s)}ds 
\right. \\
- \int_0^t \int \left( e^{-(T-s)x} - 1 \right) \nu^F(dx)ds \\
= F(0, T) \exp \left\{ \int_0^t e^{-(T-s)}dW^Q_{s} - \frac{1}{2} \int_0^t e^{-2(T-s)}ds 
\right. \\
+ \int_0^t \int e^{-(T-s)x}dJ^Q_{s}(dx \times ds) \\
\left. - \int_0^t \int \left( e^{-(T-s)x} - 1 \right) \nu^F(dx)ds \right\}, \quad t \leq T, \quad (33)
\]

where

\[
F(0, T) = \exp \left\{ k + \theta(T) + \frac{1}{2} \int_0^T e^{-2(T-s)}ds 
\right. \\
+ \int_0^T \int \left( \exp\{e^{-(T-s)x}\} - 1 \right) \nu^Q(dx)ds \right\},
\]

and \( k \in \mathbb{R} \) is defined in (29). By the exponential formula for Poisson random measures (see e.g. [5], Proposition 3.6) the process \( F(t, T) \), given in (33), is a \( Q^F \)-martingale. Note also that the bond price process \( P(t, T) \) defined in (32) satisfies obviously conditions M1-M4 given in introductory Section 2.

We now derive the forward rate that gives us the bond \( P(t, T) \) as in (32):

\[
f(t, T) = -\frac{\partial}{\partial T} \ln P(t, T) \\
= e^{-(T-t)}X_t - \theta'(T) - \frac{1}{2} + \int_t^T \int \exp\{e^{-(T-s)x}\}e^{-(T-s)}xe^{Q^F(dx)ds} \\
- \int \nu^Q(dx) + \int_t^T e^{-2(T-s)}ds. \quad (34)
\]
In particular, the corresponding short rate process is then given by

\[ r(t) = f(t, t) = X_t - \theta'(t) - \frac{1}{2} \int_R (e^x - 1) \nu^Q(dx). \]

Note that condition (31) guarantees that \( f(t, T) \) in (34) and \( P(t, T) \) in (32) are well-defined.

In the next section we consider the Markov property of the spot price under \( Q^F \) in the special case, where \( \sigma = (\sigma^1, \ldots, \sigma^n) \) appearing in (18) is deterministic.

4.1 Deterministic coefficients

First, we note that by (22), (21), and (20) we can factorize electricity price processes as follows

\[ F(t, \tau) = F(0, \tau) \exp\left\{ \sum_{i=1}^n \int_0^t \delta^i(s, \tau) dL^i_s - \sum_{i=1}^n \int_0^t \psi^i(s, \tau) ds \right\}, \quad (35) \]

where

\[ \delta^i(s, \tau) := \sigma^i(s, \tau) - v^i(s), \quad \text{and} \]
\[ \psi^i(s, \tau) := \Theta^i(\sigma^i(s, \tau)) - \Theta^i(v^i(s)). \quad (37) \]

Setting \( \tau = t \) in (35) we obtain the electricity spot price process

\[ S(t) = F(t, t) = F(0, t) \exp\left\{ \sum_{i=1}^n \int_0^t \delta^i(s, t) dL^i_s - \sum_{i=1}^n \int_0^t \psi^i(s, t) ds \right\}. \quad (38) \]

Note that by assumption the coefficients \( \delta = (\delta^1, \ldots, \delta^n) \) and \( \psi = (\psi^1, \ldots, \psi^n) \) are deterministic, since \( \sigma = (\sigma^1, \ldots, \sigma^n) \) is deterministic. For the sake of simplicity we will only consider the one-dimensional case, i.e. we assume \( n = 1 \). However, all results of this subsection still hold in the case of multi-dimensional non-homogeneous Lévy process with independent components.

In other words, we examine the Markov property of the spot price process \( S \) given by

\[ S(t) = F(0, t) \exp\left\{ \int_0^t \delta(s, t) dL_s - \int_0^t \psi(s, t) ds \right\}, \quad t \in [0, T], \quad (39) \]
under the futures martingale measure $Q^F$ when $\delta(s, t)$ and $\psi(s, t)$ are deterministic and continuous. Because $F(0, t)$ is also deterministic by assumptions, $S$ is a Markov process iff the process

$$Z_t = \int_0^t \delta(s, t) dL_s, \quad t \in [0, T],$$

is Markovian. Recall that $L$ is a non-homogeneous Lévy process under $Q^F$ by Proposition 3.1.

**Proposition 4.5.** We assume that there are constants $\epsilon, \eta > 0$ and functions $c(t), \gamma(t) : [0, T] \to \mathbb{R}^+$, such that for all $t \in [0, T]$

1. $\int_0^t c(s)ds < \infty,$

2. $\gamma(t) \geq \epsilon,$

3. $\Re \Phi_t(u) \leq c(t) - \gamma(t)|u|^\eta$, for every $u \in \mathbb{R}$, where $\Phi_t(\cdot)$ is the characteristic exponent of $L_t$ under $Q^F$ defined by

$$E^{Q^F}[e^{iuL_t}] = e^{\Phi_t(u)}, \quad u \in \mathbb{R}.$$

Then the spot price process $S$ is Markovian iff for all fixed $w$ and $u$ with $0 < w < u \leq T$ there exists a real constant $\xi = \xi_w$ (which may depend on $w$ and $u$) such that

$$\delta(t, u) = \xi_w \delta(t, w), \quad \forall t \in [0, T],$$

where $\delta$ is the volatility structure of $S$ in (39).

**Corollary 4.6.** Under the hypotheses of Proposition 4.5 the spot price process $S$ is Markovian iff its volatility structure $\delta$ admits the representation

$$\delta(t, \tau) = \zeta(t)\rho(\tau), \quad \forall(t, \tau) \in D,$$

where $\zeta, \rho : [0, T] \to \mathbb{R}$ are continuously differentiable functions.

The proofs of Proposition 4.5 and Corollary 4.6 are omitted, since they can be recovered from the ones given in Section 4 in [7] under minimal technical changements.

Now we consider two examples of the volatility function $\delta$ that satisfies (41).
Example 4.7 (Vasicek volatility structure). Recall that
\[
\delta(t, \tau) = \sigma(t, \tau) - v(t),
\]
where \(\sigma\) is the volatility of the corresponding bond and \(v\) is a deterministic function. Let
\[
\sigma(t, \tau) = \frac{\hat{\sigma}}{a} (1 - e^{-a(\tau-t)}) \quad (\text{Vasicek volatility}),
\]
where \(\hat{\sigma} > 0\) and \(a \neq 0\). Then by Corollary 4.6 the spot price process \(S\) is Markovian iff there exist continuously differentiable functions \(\zeta, \rho : [0, T] \to \mathbb{R}\), such that
\[
v(t) = \frac{\hat{\sigma}}{a} (1 - e^{-a(\tau-t)}) - \zeta(t) \rho(\tau).
\]
Since \(v\) is constant in \(\tau\), by deriving we obtain
\[
\zeta(t) \rho'(\tau) = \hat{\sigma} e^{at} e^{-a\tau},
\]
and consequently
\[
\zeta(t) = \lambda \hat{\sigma} e^{at},
\rho'(\tau) = \frac{1}{\lambda} e^{-a\tau}
\]
for \((t, \tau) \in \mathcal{D}\) and some \(\lambda \neq 0\). Then \(\rho(\tau) = -\frac{1}{\lambda a} e^{-a\tau} + c\) for some \(c \in \mathbb{R}\), \(\lambda \neq 0\). Hence, in this example the spot price process \(S\) is Markovian iff \(v(t)\) is of the form
\[
v(t) = \frac{\hat{\sigma}}{a} - \hat{\sigma} c e^{at}
\]
for some \(c \in \mathbb{R}\).

Example 4.8 (Ho-Lee volatility structure). In case the bond volatility structure \(\sigma\) satisfies
\[
\sigma(t, \tau) = \hat{\sigma}(\tau - t) \quad \text{with} \quad \hat{\sigma} > 0 \quad (\text{Ho-Lee volatility}),
\]
Corollary 4.6 yields that the spot price \(S\) is a Markov process iff \(v(t)\) is of the form \(v(t) = \hat{\sigma}(c - t)\) for some \(c \in \mathbb{R}\).
We follow the approach of \cite{12} and, applying Corollary \ref{corollary:stationary}, characterize the class of stationary volatility structures $\delta$ that lead to Markovian spot price process $S$.

**Proposition 4.9.** Suppose the volatility structure $\delta$ is stationary, that means, there exists a twice continuously differentiable function $\tilde{\delta} : [0, T] \to \mathbb{R}^+$ such that $\delta(t, \tau) = \tilde{\delta}(\tau - t)$ for all $(t, \tau) \in D$. Then, under the hypotheses of Proposition \ref{proposition:stationary}, $S$ is a Markov process iff $\delta$ is of the form

$$\delta(t, \tau) = \hat{\delta} e^{a(\tau-t)} \quad (42)$$

with $a \in \mathbb{R}$ and $\hat{\delta} > 0$.

**Proof.** If $\delta$ is of the form \eqref{eq:stationary}, then $S$ is a Markov process by Corollary \ref{corollary:stationary}. Assume now that $S$ is Markovian. As $\delta(t, \tau)$ is stationary by assumption, the partial derivatives satisfy

$$\frac{\partial}{\partial \tau} \delta(t, \tau) = \tilde{\delta}'(\tau - t) = -\frac{\partial}{\partial t} \delta(t, \tau).$$

Corollary \ref{corollary:stationary} yields then

$$\zeta'(t)\rho(\tau) = -\zeta(t)\rho'(\tau),$$

i.e.

$$(\log \rho)'(\tau) = -(\log \zeta')(t)$$

for all $(t, \tau) \in D$. Since $t$ and $\tau$ are independent variables, neither of the last equality sides can actually depend on $t$ or $\tau$. Hence both sides are constant. Denoting their common value by $a$, we obtain

$$\rho(\tau) = e^{a\tau + K_1} \quad \text{and} \quad \zeta(t) = e^{-at + K_2}$$

with two real constants $K_1$ and $K_2$, and hence

$$\delta(t, \tau) = e^{K_1 + K_2 e^{a(\tau-t)}}.$$ 

Defining $\hat{\delta} := e^{K_1 + K_2}$, we get \eqref{eq:stationary}. \hfill $\Box$

The volatility structure \eqref{eq:stationary} picks up the maturity effect for $a < 0$: the volatility increases when a future contract comes to delivery, since temperature forecasts, outages and other specifics about the delivery period become more and more precise. However, the model \eqref{eq:stationary} does not include seasonality: futures during winter months show higher prices than comparable
contracts during the summer. See [1], [18], and [16] for a description of electricity futures and options markets. In order to include the seasonality we can use, for example, the volatility model suggested in [8]:

\[ \delta(t, \tau) = a(t)e^{-b(\tau - t)}, \quad b \geq 0. \]

The seasonal part \(a(t)\) can be modeled, for example, as a truncated Fourier series

\[ a(t) = a + \sum_{j=1}^{J}(d_j \sin(2\pi jt) - f_j \cos(2\pi jt)), \]

where \(a \geq 0, d_j, f_j \in \mathbb{R}\), and \(t\) is measured in years. See [8] and [1] for more details on the modeling of volatility.

In the reminder of the paper we consider deterministic coefficients \(\delta, \psi, \sigma, \) and \(\eta\).

5 Valuation of options

5.1 Pricing of European options

For the valuation of the European options on the spot price we use Fourier transform method applied to the dampened payoff. For an overview of this method see [22]. We consider the pricing of the options only on the example of an electricity floor contract. Electricity calls, puts and caps can be priced similarly. See also [12] for the pricing of European options on the electricity spot price under the assumption of continuous futures and spot price processes.

A floor is a European type contract that protects against the low commodity prices within \([\tau_1, \tau_2]\). It ensures a cash flow at intensity \(((K - S(s))^+)_{s \in [\tau_1, \tau_2]}\) with strike price \(K > 0\) at any time \(s \in [\tau_1, \tau_2]\) of the contract.

In the remainder of this paper we suppose that the riskless interest rate \(r\) is constant. The fair price at time \(t\) of the floor option with strike price \(K > 0\) is equal to

\[ \text{Floor}(t, K) = E^Q \left[ \int_{t \vee \tau_1}^{\tau_2} e^{-r(\tau - t)} (K - S(\tau))^+ d\tau \right]_{\mathcal{F}_t}. \]

By Fubini’s Theorem we get

\[ \text{Floor}(t, K) = \int_{t \vee \tau_1}^{\tau_2} e^{-r(\tau - t)} E^Q \left[ (K - S(\tau))^+ \right]_{\mathcal{F}_t} d\tau. \]  

(43)
To simplify the notation we only consider the one-dimensional case under assumption of the deterministic coefficients, i.e. we assume the spot price process \( S(t) \) to be given by (39), where \( \delta \) and \( \psi \) are deterministic.

Recall that by (35)-(38) we can also factorize the spot price process \( S(t) \) in the one-dimensional case as follows

\[
S(\tau) = F(t, \tau) \exp\{\int_t^\tau \delta(s, \tau) dL_s - \int_t^\tau \psi(s, \tau) ds\} =: F(t, \tau) U_{t\tau},
\]

where \( F(t, \tau) \), for \( 0 \leq t \leq \tau \), is a \( Q^F \)-martingale, and \( L \) is a non-homogeneous Lévy process. Since \( F(t, \tau) \) is \( F_t \)-measurable and \( U_{t\tau} \) is independent of \( F_t \), by substituting (44) into (43) we obtain

\[
\text{Floor}(t, K) = \int_{t\tau}^\tau e^{-r(\tau-t)} E^{Q^F}\left[(K - F(t, \tau) U_{t\tau})^+ d\tau \right|F_t] d\tau
\]

where \( K(f) := K \exp\{\int_t^\tau \psi(s, \tau) ds\}, f > 0 \). In order to compute the expectation in (45), consider the integrable dampened pay-off function

\[
g(x) := e^x (K(f) - e^{i\pi})^+ \in L^1(\mathbb{R}).
\]

Denote by \( \hat{g} \) its Fourier transform:

\[
\hat{g}(u) := \int_{\mathbb{R}} e^{iux} g(x) dx = K(f)^{2+iu} \frac{1}{(1+iu)(2+iu)} \in L^1(\mathbb{R}).
\]

Using the Inversion Theorem for Fourier transform (cf. [19], Section 8.2) we get

\[
E^{Q^F}\left[(K(f) - e^{i\pi} \int_t^\tau \delta(s, \tau) dL_s)^+ \right] = E^{Q^F}\left[e^{-\int_t^\tau \delta(s, \tau) dL_s} g\left(\int_t^\tau \delta(s, \tau) dL_s\right)\right]
\]

\[
= E^{Q^F}\left[e^{-\int_t^\tau \delta(s, \tau) dL_s} \frac{1}{2\pi} \int_{\mathbb{R}} e^{-iu \int_t^\tau \delta(s, \tau) dL_s} \hat{g}(u) du\right]
\]

\[
= \frac{1}{2\pi} E^{Q^F}\left[\int_{\mathbb{R}} e^{-(1+iu) \int_t^\tau \delta(s, \tau) dL_s} \hat{g}(u) du\right]
\]

\[
= \frac{1}{2\pi} \int_{\mathbb{R}} E^{Q^F}\left[e^{-(1+iu) \int_t^\tau \delta(s, \tau) dL_s}\right] \hat{g}(u) du.
\]
where (46) allows to apply Fubini’s Theorem in the last equality. By Proposition 3.1 and Proposition 1.9 in [17]

\[ E_{Q}^{F} [e^{-\int_{t}^{\tau}(1+iu)\delta(s,\tau) dL_s}] = \exp\left\{ \int_{t}^{\tau} \Theta_{s}^{Q} (-(1+iu)\delta(s,\tau)) ds \right\}, \quad (48) \]

where \( \Theta_{s}^{Q} \) is given by

\[ \Theta_{s}^{Q}(z) = z\theta_{s}^{Q} + \frac{z^2}{2} c_{s}^{Q} + \int_{R} (e^{zx} - 1 - zx)e^{x(s)x} \nu(dx), \quad s \leq T. \]

Substituting (47), (46), and (48) into (45), we obtain

\[
\text{Floor}(t, K) = \int_{t \vee \tau_1}^{\tau_2} e^{-r(t-\tau)} F(t, \tau) e^{-\int_{t}^{\tau} \psi(s,\tau) ds}
\times \int_{R} \exp\left\{ \int_{t}^{\tau} \Theta_{s}^{Q} (-(1+ix)\delta(s,\tau)) ds \right\}
\times \left( \frac{K}{F(t, \tau)} e^{\int_{t}^{\tau} \psi(s,\tau) ds} \right)^{2+ix} \frac{1}{(1+ix)(2+ix)} dxd\tau
\]

\[
= K^2 e^{rt} \int_{t \vee \tau_1}^{\tau_2} e^{-r(\tau-t)} \int_{R} \exp\left\{ \int_{t}^{\tau} \Theta_{s}^{Q} (-(1+ix)\delta(s,\tau)) ds \right\}
\times \left( \frac{e^{\int_{t}^{\tau} \psi(s,\tau) ds}}{F(t, \tau)} \right)^{1+ix} \frac{K^{ix}}{(1+ix)(2+ix)} dxd\tau.
\]

5.2 Pricing of swing options

In this section we illustrate how the spot price model (27) can be used to valuate path dependent derivatives on an example of electricity swing options. For the sake of simplicity we consider a special case, where the process \( L \) is one-dimensional Lévy process under \( Q^M \) and \( v \in \mathbb{R} \) is a constant.

Then by Remark 3.2 \( L \) is also a Lévy process under \( Q^F \). Moreover, by Assumption (14) and Proposition 3.1 \( L \) admits the canonical representation:

\[
L_t = b^{Q^F} t + \sqrt{c^{Q^F}} W^Q_t + \int_{0}^{t} \int_{R} x J_{L}^{Q^F} (dx \times ds), \quad (49)
\]

where \( W^{Q^F} \) is a standard Brownian motion and \( J_{L}^{Q^F} \) is the compensated random measure of jumps under \( Q^F \). Recall that by (9) and (49) the short
rate process \( r \) follows the dynamics

\[
    dr(t) = \alpha(t,t)dt + \eta(t,t)dL_t
\]

\[
    = (bQF \eta(t,t) + \alpha(t,t))dt + \sqrt{c\eta(t,t)dW_t^{QF}}
\]

\[
    + \int_{\mathbb{R}} x\eta(t,t)J_L^{QF}(dx \times dt), \quad t \in [0, T].
\]  

(50)

Now we assume that the volatility \( \eta \) of the short rate process \( r \) is deterministic, and hence \( r \) is a Markov process.

Recall that, since \( r \) is Markovian, by Proposition 4.2 \((S(t), r(t))\) is also a Markov process. Furthermore, by (27) and (49) the dynamics of the electricity spot price \( S \) is given by

\[
    dS(t) = S(t)[-r(t) + \frac{1}{2}cv^2 + \Theta(v) - vbQF]dt - S(t)v\sqrt{cdW_t^{QF}}
\]

\[
    - S(t)v \int_{\mathbb{R}} xJ_L^{QF}(dx \times dt) + S(t)v \int_{\mathbb{R}} (e^{vx} - 1 + vx)J_L^{QF}(dx \times dt)
\]

\[
    = S(t)[-r(t) + \frac{1}{2}cv^2 + \Theta(v) - vbQF + \int_{\mathbb{R}} (e^{vx} - 1 + vx)vQF(dx)]dt
\]

\[
    - S(t)v\sqrt{cdW_t^{QF}} + S(t) \int_{\mathbb{R}} (e^{vx} - 1)\bar{J}_L^{QF}(dx \times dt)
\]

\[
    = -S(t)(r(t) - \beta)dt - S(t)v\sqrt{cdW_t^{QF}}
\]

\[
    + S(t) \int_{\mathbb{R}} (e^{vx} - 1)\bar{J}_L^{QF}(dx \times dt), \quad t \in [0, T],
\]  

(51)

where, by (16), (24), and (26) for \( \beta \) we get

\[
    \beta := \int_{\mathbb{R}} (e^{vx} - 1 - vx)\nu(dx) - v \int_{|x| \leq 1} (e^{vx} - 1)x\nu(dx)
\]

\[
    + \int_{\mathbb{R}} (e^{vx} - 1 + vx)e^{vx}\nu(dx)
\]

\[
    = \int_{\mathbb{R}} (e^{vx} - 1 - vxI_{|x|>1})\nu(dx) + \int_{\mathbb{R}} (e^{vx} - 1 + vxI_{|x|>1})e^{vx}\nu(dx).
\]

Moreover, we assume that there exists a unique solution \((S^u(t), r^u(t))\) of the system (51) – (50) satisfying the initial condition \((S^u(u), r^u(u)) = (s, r) \in \mathbb{R}^2\), and such that

\[
    \mathbb{E}^{QF}[(S^u(t))^2] < \infty \quad \text{for all } t \in [0, T].
\]

For instance, if \( u = 0 \) then \((s, r) = (1, r(0)) = (1, f(0, 0))\).
Let us consider a swing option on the spot price process (51). A swing option is an agreement to purchase energy at a certain fixed price over a specified time interval. Following [25] we define the payoff of a swing option settled at time $T$ as

$$
\int_u^T \nu(t)(S^u(t) - K)dt,
$$

(52)

where $\nu(t)$ is the production intensity, $S$ is the electricity spot price and $K > 0$ is the strike price of the contract. The holder of the contract has the right (within specified limits), to control the intensity of electricity production at any moment. The goal of the option holder is to maximize the value of the contract by selecting the optimal intensity process $\nu$ among the processes that are limited by contract specific lower and upper bounds:

$$
\nu_{\text{low}} \leq \nu(t) \leq \nu_{\text{up}} \text{ a.e. } t \in [u,T],
$$

under the constraint that the optimal intensity process $\nu$ is such that the total volume produced

$$
C^\nu(t) = c + \int_u^t \nu(x)dx, \quad u \leq t \leq T,
$$

(53)

does not exceed the maximum amount $\bar{C}$ that can be produced during the contract life time.

Hence the option holder tries to maximize the expected profit, i.e. to find

$$
V(u,s,r,c) := \sup_{\nu \in N} \mathbb{E}^Q\left[ \int_u^{T \wedge \tau_C} \nu(t)(S^u(t) - K)dt \right]
$$

(54)

$$
= \mathbb{E}^Q\left[ \int_u^{T \wedge \tau_C} \nu^*(t)(S^u(t) - K)dt \right],
$$

(55)

where

$$
N := \{ \nu \text{ progressively measurable: } \nu(t) \in [\nu_{\text{low}}, \nu_{\text{up}}] \text{ for a.e. } t \in [u,T] \}
$$

is the control set, and

$$
\tau_C := \inf\{ t > u | C^\nu(t) = \bar{C} \}
$$

is the first time when all of production rights are used up. Note that the value function $V$ satisfies the boundary conditions

$$
V(T,s,r,c) = 0 \quad \text{and} \quad V(u,s,r,\bar{C}) = 0.
$$

(56)
If we assume that the value function $V$ is sufficiently smooth, then by Itô formula and by (51), and (50) we get
\[
0 = V(T \land \tau_C, S^u(T \land \tau_C), r^u(T \land \tau_C), C^u(T \land \tau_C)) \\
= V(u, s, r, c) + \int_u^{T \land \tau_C} \mathcal{A}^\nu V(t, S^u(t), r^u(t), C^u(t))dt \\
- \sqrt{c} \int_u^{T \land \tau_C} (\partial_s V S^u(t)v(t) + \partial_r V \eta(t, t))dW^Q_t \\
+ \int_u^{T \land \tau_C} \int_\mathbb{R}(\partial_r V \eta(t, t)x + S^u(t)(e^{vx} - 1)\partial_s V)J_{Q^F_t}(dx \times dt), \tag{57}
\]
where
\[
\mathcal{A}^\nu V(t, S^u(t), r^u(t), C^u(t)) := \partial_t V + \partial_s V S^u(t) - \partial_s V S^u(t)(r^u(t) - \beta) \\
+ \partial_r V (\mathbb{Q}^{Q^F_t} \eta(t, t) + \alpha(t, t)) + \frac{C}{2} (S^u(t))^2 v^2(t)\partial^2_{ss} V - 2S^u(t)v(t)\eta(t, t)\partial^2_{sr} V \\
+ \eta^2(t, t)\partial^2_{rr} V + \int_\mathbb{R} (V(t, S^u(t)e^{vx}, r^u(t) + x\eta(t, t), C^u(t)) \\
- V(t, S^u(t), r^u(t), C^u(t)) - \partial_s V S^u(t)(e^{vx} - 1) - \partial_r V \eta(t, t))e^{vx} \nu(dx).
\tag{58}
\]
Applying Dynkin formula (Theorem 1.24 in [21]) we can now formulate a verification theorem for the optimal control problem (54) analogous to the classical result for the Hamilton-Jacobi-Bellman equation for jump diffusions (see Theorem 3.1 in [21]):

**Proposition 5.1.** Let $S = [u, T] \times \mathbb{R}_+^2 \times [0, \bar{C})$. Assume that there exist $\hat{V} \in C^2(S) \cap C(\bar{S})$ and $\hat{\nu} \in N$, such that $(\hat{\nu}, \hat{V})$ is a solution of the Hamilton-Jacobi-Bellman equation
\[
\mathcal{A}^\hat{\nu} V(t, s, r, c) + \hat{\nu}(s - K) = 0 \quad \text{for each } (t, s, r, c) \in S, \tag{59}
\]
satisfying
\[
\mathbb{E}^{Q^F}[\int_u^{T \land \tau_C} |\mathcal{A}^\hat{\nu} \hat{V}(t, S^u(t), r^u(t), C^\hat{\nu}(t))|dt] < \infty. \tag{60}
\]
Moreover, suppose that $\hat{V}$ fulfills the terminal and boundary conditions (56). Then $\hat{V}$ is the value function of the swing option defined in (54).

Note that the Markov property of the process $(S, r)$ is essential for the proof of Proposition 5.1. We refer to [21] for more details on stochastic optimal control problems.
References


