Static oligopoly models revisited

*Example*: Homogeneous duopoly

Linear demand and cost functions: \( p = 1 - Q \), \( C_i(q_i) = 0, i = 1, 2 \).

Equilibrium:
- Cournot: \( \Pi^i = \frac{1}{9}, i = 1, 2 \).
- Bertrand: \( \Pi^i = 0, i = 1, 2 \).

The Profit Possibility Frontier: \( \max \Pi^1 \text{ s.t. } \Pi^2 = \Pi \)

In the linear example: \( \Pi^2 = \Pi^m - \Pi^1, \Pi^m = \frac{1}{4} \).

*Figure* \((\Pi^1, \Pi^2)\)-space, Profit Possibility Frontier, equilibrium points).

Firms have a joint interest in moving towards the Profit Possibility Frontier.

Co-operation

Modelling co-operation:
- co-operative games – contracts
- non-co-operative games – tacit collusion

Tacit collusion: long-term and frequent interaction
- aggressive competition may be meet with aggressive response at a later date;
- such ‘threats’ may discipline firms, soften competition and lead to higher prices

Dynamic Bertrand duopoly game

Repeated games - supergames
the same game is played repeatedly

Technology assumptions
- homogenous products
- constant variable unit costs
- profits  for , \(i, j = 1, 2, i \neq j, t = 0, 1, 2, ..., T\) (\(T\) may be infinite)

Firms maximise

\[
\sum_{t=0}^{T} \delta^t \Pi(p_i, p_j)
\]

where \(\delta = e^{-rt} \in [0,1]\), \(r\) = interest rate and \(\tau\) = length of periods.

At the start of period \(t\), firms choose prices \((p_i, p_j)\) simultaneously.

Strategies: \(p_{it} = p_{jt}(H_t), H_t \equiv (p_{10}, p_{20}, ..., p_{it-1}, p_{jt-1}), \forall i, t\)

We are looking for Subgame Perfect Equilibria; that is, in any period, after any history, strategies maximise the discounted stream of profits for the rest of the game.

**Finite time horizon** \((T < \infty)\)

Backwards induction:

\[
p_{it} = p_{jt} \Rightarrow p_{it-1} = p_{jt-1} = c \Rightarrow ... \Rightarrow p_{10} = p_{20} = c
\]

That is; the perfectly competitive, or Bertrand, outcome.

**Infinite time horizon** \((T' = \infty)\)

Bertrand equilibrium still an equilibrium

- strategies: \(\{\forall t : p_{it} = c\}\)

Monopoly price as equilibrium outcome

- strategies: \(\{\text{if } \forall t < \tau : p_{1t} = p_{2t} = p^m, \text{choose } p_{it} = p^m; \text{otherwise } p_{it} = c\}\)
- ‘trigger strategies’

Profits obtained when adhering to strategy: \[
\left[1 + \delta + \delta^2 + ...\right] \frac{\Pi^m}{2} = \frac{1}{1-\delta} \frac{\Pi^m}{2}
\]
Upper limit on profits from deviating: $\Pi^m + 0 + 0 + ... = \Pi^m$

Consequently, if $\delta \geq \frac{1}{2}$, the strategies constitute an equilibrium.

There exist other equilibria also (if $\delta \geq \frac{1}{2}$).

Folk Theorem: Any pair $(\Pi^1, \Pi^2)$, where $\Pi^1 > 0, \Pi^2 > 0$, and $\Pi^1 + \Pi^2 \leq \Pi^m$ are the per period profits, may be constituted as an equilibrium outcome if $\delta$ is sufficiently close to 1.  

*Figure* $(\Pi^1, \Pi^2)$-space)

Co-ordination (focal points):

- symmetry (natural if firms are identical): $\Pi^1 = \Pi^2$.
- Pareto optimality (from point of view of firms): $\Pi^1 + \Pi^2 = \Pi^m$.

Renegotiation

- weakens punishment possibilities

Interpretation of infinite time horizon

- in any period, there is a positive probability that ‘the game goes on’
- $\delta' = \delta x$, where $x =$ probability that ‘the game goes on’

**Market concentration**

Extend the above model to the case of oligopoly.

Profits obtained when adhering to strategy: $\frac{1}{1-\delta} \frac{\Pi^m}{n}$

Upper limit on profits from deviating: $\Pi^m$

Monopoly price can be sustained as an equilibrium if $\delta \geq 1 - 1/n$

**Information lags and response times**

Aggressive responses will delayed the more difficult it is to

- detect price cuts
- adjust prices
Assume firms can change prices every second period only.

Profits obtained when adhering to strategy (duopoly): \[ \frac{1}{1 - \delta} \frac{\Pi^m}{2} \].

Upper limit on profits from deviating: \[ [1 + \delta] \Pi^m \].

Monopoly price can be sustained as an equilibrium if \[ \delta \geq \frac{1}{\sqrt{2}} > 1/2 \].

If prices can be observed only imperfectly (i.e. with an error), price wars may be result even if no-one ‘cheated’ (i.e. deviated from collusive play).

While under full information ‘punishment’ (i.e. price wars) never occurs at equilibrium, with uncertainty/asymmetric information price wars may be an equilibrium phenomenon (ref Green and Porter, 1984).

**Fluctuating demand**

Assume that, with equal probability, demand may be either low or high in any period \( (D_H(p) \geq D_L(p)) \).

The state of the world is observed before firms set prices.

Equilibrium profits, starting from state \( s = H, L \): \[ \frac{\Pi^m_s}{2} + \delta \left[ \frac{1}{2} \frac{\Pi^m_H}{2} + \frac{1}{2} \frac{\Pi^m_L}{2} \right] \].

Upper limit on profits from deviating: \( \Pi^H_s \).

More profitable to deviate in high-demand periods.

Conclusion: ‘price wars’ in good times.

Note: with uncertainty/asymmetric information price wars will typically start in low-demand periods, since then observing low prices is more likely.

**Multi-market contact**

Assume the duopolists meet in two identical markets.

In market 1, prices may be changed every period, whereas in market 2 prices may only be changed every second period.

Consequently, if \( \delta \) is the per period discount rate, the effective discount rate in market 2 equals \( \delta^2 \) (i.e. it is as if, in market 2, each period is twice as long).
We can infer that, when each market is considered in isolation, (tacit) collusion may be sustained

- in market 1, if $\delta \geq 1/2$; and
- in market 2, if $\delta^2 \geq 1/2$ (i.e. $\delta \geq 1/\sqrt{2}$).

Suppose $\delta^2 < 1/2 < \delta$. It can then be shown that collusion may be sustained in both markets.

Note: the incentive to deviate and undercut rivals will be greater when both prices can be changed:

- profits along equilibrium path: $2\frac{1}{1-\delta} \frac{\Pi_m}{2}$;
- upper limit on profits from deviating: $[2 + \delta \Pi_m]$. 

Collusion can be sustained if $\delta \geq \frac{1}{4} \left[\sqrt{33} - 1\right] \approx 0.59$.

Conclusion: multi-market contact enhances the possibility of sustaining collusion in any one market.

**Reputation effects**

We have seen that under full information and with a finite time horizon collusion cannot be sustained as an equilibrium phenomenon.

Suppose now that Firm 1 (for some reason) consider Firm 2 as being ‘co-operative’ with probability $\alpha$, in which case Firm 2 follows the strategy {play $p_{2t} = p^m$ so long as $\forall t < T: p_t = p^m$; and $p_{2t} = c$ otherwise}. With probability $1 - \alpha$ Firm 2 is ‘rational’ and plays $p_{2t} = c$ always.

Let $\delta = 1$.

Backwards induction implies that $p_2 = p_{2T} = c$, all $t$, is an equilibrium. Moreover, if Firm 1 plays $p_{1t} < p^m$ in any given period, profits are at most $\Pi_m$ (if Firm 2 is of the ‘co-operative’ type).

If, instead, Firm 1 plays $p_{1t} = p^m$ and then goes on to play $p_2 = p^m$ for $\bar{T}$ more periods if Firm 2 co-operates, but $p_2 = c$ if Firm 2 does not, Firm 2’s profits is $\alpha [1 + \bar{T}] \frac{\Pi_m}{2} + [1 - \alpha] 0$. This is optimal if $\bar{T}$ is sufficiently large.
Consequently, collusion may be sustained if the remaining time horizon is sufficiently long, but will break down when the game reaches its end (ref Kreps et al, 1982).

Note:

- if Firm 2’s ‘co-operative’ type were {play $p^m$ whatever} collusion would break down (Firm 1 would always undercut);

- it can be shown that with a suitable choice of Firm 2’s ‘co-operative’ type, any profit outcome can be sustained as an equilibrium outcome.

- exogenous strategies like ‘tit-for-tat’ lead to co-operation.

**Evolutionary theories**

Strategies are exogenously given (not result of optimising behaviour).

Mutations and the fight for survival.

Question: what strategies are robust against other strategies?