Estimating runoff by analytical modeling of groundwater levels

data by courtesy of NVE and met.no

Application of the Dupuit-Forchheimer assumption for calculating groundwater heads
http://folk.uio.no/nilsotto/

Nils-Otto Kitterød
The Norwegian Centre for Soil and Environmental Research
and University of Oslo, Dept. of Geoscience

nilsotto@geo.uio.no
Why runoff?? in groundwater and climatic change context??

Water balance: \( R = P - E + \Delta S \)
\[ \Delta S \rightarrow 0 \text{ if time} \rightarrow \infty \]
\( P \sim f(\text{atmc}, T) \)
\( E \sim f(T, \theta) \)
\( R \)

Changes in atmosphere circulation pattern → Runoff
\( P, E, T, \theta, \text{grw.levels} \)

Changes in atmosphere circulation pattern ← Runoff

Challenge: **Coupling** Atmosphere Circulation Modeling to Runoff

- weakly to get ‘correct runoff’ or
- strongly which include the land – atmosphere coupling
Idea:
1) Given present observations of runoff (R) and groundwater levels (G)
2) Calibrate hydraulic conductivity (k) or transmissivity (kH) in a groundwater model (f) that reproduce present observations of G
3) Validate f on independent R or G
4) Use paleo-ecological data to reconstruct groundwater levels (Gp)
5) Estimate paleo-runoff (Rp) by applying f on Gp

Problem: significant uncertainties of Gp
   → develop simple but robust groundwater model (f)

Assumption: f is ~ constant ??
   erosion rate will affect boundary conditions
   → include boundary location as a stochastic function
The inland glacier ~10500 B.P. (Andersen, B.G., 2000)

Sea level about 200 m higher than present sea level.
Average paleo-discharge: ~3000 m$^2$/s (Tuttle, K., 1998)
**Kettle lakes** may be used as 'paleo-sensor’s for changing groundwater levels (Stabell, B.)

~30 kettle lakes

Risa runoff station

Trandum Delta Paleo-portal

Helgebostad Delta Paleo-portal

Bonntjern & Svenskestutjern
Groundwater levels, Hauerseter

The Bonntjern kettle lake

-17
-18
-19
-20
-21
-22

1966
1970
1974
1978
1982
1986
1990
1994
1998
2002

Grw. level, m

1974
low groundwater levels

1991
high

2000
medium high
IAHS Hydrological decade (1965-75?), observations from NVE

problem: only 503 L. Olimb and 411 Gardermoen within the Trandum delta
Idea:
estimate runoff (or net precipitation) from ~ 6000 B.P. – present given a record of groundwater levels based on paleo-studies of kettle lake sediments

Runoff

Risa

catchment area: 54.4 km²
Note that variation of runoff is more related to temperature than precipitation due to the importance of snow at Gardermoen.
runoff

temp.

precipitation

runoff
Observations from the Gardermoen aquifer, Oslo.
paleo-distribution channels shows radial flow
Tuttle, K. (1997)
"The Gardermoen doughnut"
Yallahs River, Jamaica
(b) **Wave-dominated transgressive phase**

(c) **Fluvial-dominated/wave modified regressive phase**

*Figure 6.29* The Eocene La Trona (Montserrat) fan delta in northern Spain (from Marzo & Anadón, 1988) showing alternate phases of wave-dominated transgressions and fluvial-dominated/wave-modified regressions. (a) Cross-section of four transgressive–regressive phases

Darcy’s law + conservation of mass

→ Poisson’s equation: \( \nabla^2 \Phi = -N(t) \)

Precipitation \( N \), over an island/delta with radius \( R \):

The Dupuit-Forchheimer assumption: \( d\Phi/dz = 0 \)

Because \( R >> z \)

\[
\Phi = -\frac{N}{4} \left[ r^2 + R^2 \right] + \Phi_0
\]

phreatic aquifer: \( \Phi = \frac{1}{2} k h^2 \)

confined aquifer: \( \Phi = k \Delta \phi \)
groundwater levels (1992-94) at the Trandum delta
some grw.levels from 1967-today Helgebostad delta
and meteorological data
Forward problem: find $h$ given $k$
Inverse problem: determine $k$ given $h$

For the Gardermoen case, we suggest Poisson's equation as flow model:

\[ \nabla^2 \Phi = -N(t) \]
geology: $3d \rightarrow 2d$
because of symmetry
around paleo-portals

and

because $L \gg H$, $\frac{\partial \phi}{\partial z} \approx 0$
(but without assuming $q_z = 0$)

$\Rightarrow$ Dupuit-Forcheimer assumption

$2d \rightarrow 1d$

i.e. $\phi$ or $h$ is a function of $r$

hydrology: transient $\rightarrow$ steady state
Model:

purpose: $E\{k|\text{obs}\}$

from geology: 3d is overkill

from hydrology: transient flow is overkill
A) Steady state model for \( l \leq r < R_f \):

Balance of mass:

i) \( Q_A = N\pi l^2 - N\pi r^2 \)

ii) \( Q_r = -\frac{Q_A}{2\pi r} \)

Darcy’s law:

iii) \( \frac{Q_r}{H} = q = -k\frac{dh}{dr} \)

\( Q_r = -\frac{d\Phi_A}{dr} \)

Trick:

\( \Phi = \frac{1}{2} kh^2 \), open
\( \Phi = kH\phi \), confined
i, ii and iii) \[ d\Phi_A = \frac{N}{2} \left( \frac{l^2}{r} \, dr - r \, dr \right) \]

solution: \[ \Phi_A = - \frac{N}{4} \left[ r^2 - R_1^2 \right] + \frac{N l^2}{2} \ln \left( \frac{r}{R_1} \right) + \Phi_I \] (iv)

B) Same procedure for \( R_2 \leq r < l \):

solution: \[ \Phi_B = - \frac{N}{4} \left[ r^2 - R_2^2 \right] + \frac{N l^2}{2} \ln \left( \frac{r}{R_2} \right) + \Phi_2 \] (v)

substitute \( l \) in iv and v, then \( \Phi_A = \Phi_B = \Phi \)

\[ \Phi = \left[ \frac{N}{4} \left( r^2 - R_1^2 \right) - \Phi_I \right] \left( \frac{\ln r - \ln R_2}{\ln R_2 - \ln R_1} \right) \]

\[ - \left[ \frac{N}{4} \left( r^2 - R_2^2 \right) - \Phi_I \right] \left( \frac{\ln r - \ln R_1}{\ln R_2 - \ln R_1} \right) \] (vi)
Problem: does not fit observed $l$ very well

blue is 'observations'
purple is the simple doughnut equation
Solution: let $H$ (or $k$) = $f(r)$

$$d\varphi_A = \frac{N}{2k} \left[ \frac{l^2}{rH} \, dr - \frac{r}{H} \, dr \right]$$

$$H = H_1 - a(r-R_1)$$

Two (simple) integrals to solve:

$$\int \frac{1}{r(H_0-ar)} \, dr \quad (1)$$

$$\int \frac{r}{(H_0-ar)} \, dr \quad (2)$$
Limitations of Dupuit-Forcheimer assumptions

For isotropic aquifer and constant $H$, the 3d effect is neglect able at distance $1-2H$ from the boundaries, $2H(k_h/k_v)^{1/2}$ if anisotropic.

What if $H \neq \text{constant}$?

The vertical gradient cannot be ignored if $H_1 \gg H_2$, but for $H_1/H_2 < 10$, max. relative error is $< 1\%$. 
\[ k = 1.73 \times 10^{-5} \text{ m/s} \quad N = 1.267 \times 10^{-8} \text{ m/s} \approx 1 \text{ mm/d} \quad L = 5100 - 1000 \text{ m} \]

\[ \frac{k}{N} \frac{L}{H} \approx \frac{100}{3} \]

\[ H = H_1 = H_2 = 100 \text{ m} \]

\[ H = 1000 \text{ m}, \quad H_2 = 100 \text{ m}, \quad H' = 0.5(H_1 + H_2) \]

\[ \frac{k}{N} \frac{L}{H'} \approx \frac{550}{3} \]
$H_2$ is constant

$H_1$ gradually opens up
$H_1$ is constant

$H_2$ gradually opens up
Results:

\[ h = f(r, R_1, R_2, h_1, h_2, N, k_1, k_2) \]

\[ k_1, k_2 \text{ unknown} \]

\[ \phi = f(r, R_1, R_2, \phi_1, \phi_2, N, k', H_1, H_2) \]

\[ H_1, H_2 \text{ unknown} \]

grw. obs from 1992 - 1994
“Validation” exercise:

R1 = 336 m, h1 = 171.5 m
R2 = 5100 m, h2 = 185 m

1) calculate optimal k1, k2 (or k’, H1, H2)
given R (or N_{eff}) and
grw. obs (1992/94)
- k1 = 2.89e-5 m/s
- k2 = 5.69e-6 m/s
- k’ = 1.73e-5 m/s
- H1 = 302 m
- H2 = 66 m

2) calculate R (or N_{eff})
given optimal k1, k2 and
a timeseries of groundwater levels

3) compare calculated R (or N_{eff}) with independent observations of R (or N_{eff})
from the Risa runoff station
The longest timeseries within the Trandum delta: well 503 L. Olimb
3) compare **observed** and **calculated** R

overall pattern OK  
⇒ steady state is sufficient

under estimate max values  
over estimate min values
In this case all data is available except evapotranspiration, which can be calculated:

\[ E = P - R \]

\( E \) is expected to correspond with temperature.
Conclusion:
why analytical modeling?

analytical solutions are *at hand*
focus *the essence*
easy *sensitivity analysis*
*continuous* in time and space

numerical modeling require *details*
*discretization* in time and space
*time consuming*
Climatic change:

The present is the key to the past

The past is the key to the future
Next step I:

Stochastic R1 and R2
→ sensitivity analysis
Next step II:

Levelling of kettle lakes → paleo runoff (proxy data to paleo-climatic modeling)
Thank you!