Electron phase coherence

Due to quantum interference

The positions of the oscillation minima are plotted vs. 1/ B as full circles in the inset. Here, the line is a least squares fit to theory, which gives an electronic wire width of 158 nm.

\[ \omega(B) = \sqrt{\omega_c^2 + \omega_0^2}, \]

\[ \omega_c = \frac{eB}{m_e^*} \]

The magnetoresistivity of the wire shown earlier.

The most prominent feature is the oscillation as a function of B, which detects the magnetic depopulation of the wire modes.

\[ j(n_{QWR}, \omega_0, 1/B) = \left( \frac{3\pi}{4} n_{QWR} \omega_0 \right)^{2/3} \left( \frac{\hbar}{2m^*} \right)^{1/3} \frac{1}{\omega(B)} \]
**Interference** is the addition (superposition) of two or more waves that result in a new wave pattern.

Simple example: Two slit experiment

\[
\cos(kL_1 + \chi_1) + \cos(kL_2 + \chi_2) = 2 \cos \frac{k(L_1 - L_2) + \chi_-}{2} \cos \frac{k(L_1 + L_2) + \chi_+}{2}
\]

\[
\approx 2 \cos \left( \frac{kd \cos \varphi + \chi_-}{2} \right) \cos \left( kL + \frac{\chi_1}{2} \right)
\]

\[
\chi_\pm = \chi_1 \pm \chi_2
\]
Important questions

Q1: Why do we use a two-slit set?
Q2: What do we need to observe interference from two different sources?

A: We can observe interference of two sources if they are coherent.

Two signals are coherent if the phase difference, $\chi$, is stable.
According to quantum mechanics, electrons can behave as waves.

What is the role of phase coherence?

To answer this question let us discuss the effect which would not exist in the absence of interference - the Aharonov-Bohm effect.

An important difference between electrons and electromagnetic waves is that electrons have a finite charge.
In classical mechanics the motion of a charged particle is not affected by the presence of magnetic fields in regions from which the particle is excluded. For a quantum charged particle there can be an observable phase shift in the interference pattern recorded at the detector. The Aharonov-Bohm effect demonstrates that the electromagnetic potentials, rather than the electric and magnetic fields, are the fundamental quantities in quantum mechanics.
Let us make a confined tube of magnetic field. Will the interference pattern feel this magnetic field?

For a plane wave, the wave function

$$\psi \propto e^{-i\mathbf{p}\cdot\mathbf{r}/\hbar}$$

The phase gain along some way is then

$$\phi = \frac{1}{\hbar} \int_{1}^{2} \mathbf{p}(\mathbf{r}) \, d\mathbf{r}$$

As we know, in a magnetic field

$$\mathbf{p} \rightarrow \mathbf{p} + e\mathbf{A}$$

Additional phase difference

$$\frac{e}{\hbar} \int \mathbf{A} \, d\mathbf{r} = \frac{e}{\hbar} \Phi$$

Electron phase coherence
Aharonov-Bohm Effect for Nanowires

\[ t_j = t_0 \cdot e^{i\phi_j}, \quad \delta \phi_l = -\delta \phi_u = \pi \frac{\Phi}{\Phi_0} \Rightarrow \]

\[ T = (t_u + t_l)^*(t_u + t_l) \]
\[ = |t_0|^2 \left[ 2 + e^{i(\phi_u - \phi_l)} + e^{i(\phi_l - \phi_u)} \right] \]
\[ = 2T_0 \left[ 1 + \cos \left( \phi_0 + 2\pi \frac{\Phi}{\Phi_0} \right) \right] \]

\[ \Phi_0 \equiv \hbar/e, \Phi - \text{magnetic flux} \]

Magnetic flux quantum

\[ t \text{ is transmission amplitude} \]
Aharonov-Bohm effect: experiment

Aharonov-Bohm oscillations, Webb 1985, Au

Fourier analysis shows that there are also weak oscillations with half period

1. Describe Aharonov–Bohm effect. How is it possible that a charged particle is affected by magnetic field in a region where magnetic field is absent? What characteristic length is important for this effect? Does it take place in diffusive regime? What value does oscillate in this effect and with what period?
Electron phase coherence

High order interferences

Here the clock-wise and counter-clockwise paths are exactly the same:

The waves “strengthen” each other, constructive interference

The backscattering probability is enhanced.

Finite magnetic field destroys the interference, the period being a half of the period in the ring.

Sharvin, 1981

2. What is the difference between Aharonov–Bohm and Altshuler–Aronov–Spivak (AAS) oscillations? Can you explain both in ring geometry? What oscillations supposed to survive ensemble averaging and why?
Altshuler, Aronov, Spivak (AAS) oscillations

The period is $\Phi_0/2$

Experiment by Sharvin, 1981, Mg-coated human hair (different samples, 1.12 K)

$\Phi_0/2 = h/2e$

AB oscillations vanish in an ensemble of small rings since the phases $\varphi_0$ are random.

In contrast, AAS oscillations survive ensemble averaging.

3. Describe experimental realisations of Aharonov–Bohm and Altshuler–Aronov–Spivak (AAS) oscillation effects. What could be concluded from the experiments? Why are small sizes and low temperatures needed in these experiments?
Electron phase coherence

Test of the ensemble averaging, Umbach 1986

Ag loops, 940x940 nm², width of the wires 80 nm

N-dependence of the AB oscillations amplitude

Fourier series

Altshuler, Aronov, Spivak oscillations in set of rings
Let us calculate the probability for an electron to move from point 1 to point 2 during time $t$ in terms of the transition amplitude, $A_i$, along different paths.

$$P(r_1, r_2, t) = \left| \sum_i A_i \right|^2 = \sum_i |A_i|^2 + \sum_{i \neq j} A_i A_j$$

**Classical probability**

**Interference contribution**

4. What is weak localization? Is the resistance enhanced or reduced in coherent, as compared to the incoherent case? Can you explain why? At what magnetic fields takes this effect place?

Vanishes for most paths since phases are almost random
Consider now a close loop with two identical paths traversed in opposite directions.
Then the amplitude $A_j$ is just a time reversal of $A_i$. Hence

$$|A_i + A_j|^2 = |A_i + A_i^*|^2 = 4|A_i|^2$$

The backscattering probability is enhanced by factor 2!

This is a predecessor of localization.

This effect is called the weak localization since the relative number of closed loops is small. However, the effect is very important since it is sensitive to very weak magnetic fields.
Weak localization: magnetic field influence

As the magnetic field is increased, the contribution of the largest rings in the ensemble to the resistivity will oscillate rapidly, while the phase difference in the smallest rings will remain essentially unchanged. Hence, the larger the magnetic field, the fewer rings will contribute to the constructive interference, and the resistance should drop to its classical value.

5. What role plays time reversal trajectories in weak localization? How magnetic time and phase coherence time are mixed in weak localization? What weak localization experiment do you know? What temperature dependence of the phase coherence time is found in the experiment?
Weak localization is due to coherent backscattering. It allows to find the phase coherence time and its temperature dependence.
Altshuler et al. have derived the magnetic field dependence of the WL correction to the classical conductivity:

\[ \Delta \sigma_{xx}^{WL}(B) = \frac{e^2}{2\pi^2\hbar} \left[ \Psi \left( \frac{1}{2} + \frac{\tau_B}{2\tau_\phi} \right) - \Psi \left( \frac{1}{2} + \frac{\tau_B}{2\tau} \right) + \ln \left( \frac{\tau_\phi}{\tau} \right) \right] \]

\( \ell_B = (\hbar/eB)^{1/2} \)

\( \ell_e = v_F \tau \)

\( \Psi(x) \) is the digamma function. For large arguments, it can be approximated by

\[ \Psi(x) \approx \ln(x) - \frac{1}{x} \]

At \( B = 0 \) it reduces to

\[ \delta \sigma_{xx}^{WL} = -\delta g_s \frac{e^2}{4\pi^2\hbar} \ln \left( 1 + \frac{\tau_\phi}{\tau} \right) \]
It is widely accepted that the dephasing occurs via quasi-elastic electron-electron collisions. For such a type of dephasing, one expects a characteristic $\tau_\phi \propto 1/T$ dependence, which is usually found in experimental data, except at very low temperatures. Here, $\tau_\phi$ saturates. A plausible explanation is that the electron temperature deviates from the bath temperature at very low temperatures.
Let us look at the resistance of the wire we have already seen, in a weak magnetic field, $\omega_c < 0.45\omega_0$.

The patterns are not random, they are fingerprints of the system.
Specific features of UCF

- Fluctuations are not noise: the features are fairly symmetric with respect to magnetic field inversion.
- Fluctuations smear out rapidly as temperature is increased to about 1K.
- The typical amplitude is $\sim 0.2e^2/h$. Characteristic fluctuation periods are $\approx 20\text{mT}$.
- Fluctuations are parametric, i.e. they are perfectly reproducible as a function of the parameters, but there is no way to control their individual appearance.
- Look different in samples with identical macroscopic properties, and change in an individual sample when warmed to room temperature and cooled down again.

The patterns are not random, they are fingerprints of the system.

6. What are specific features of universal conductance fluctuations? Do they need phase coherence of electrons? Why are they frequently referred to as parametric fluctuations?
Diffusive transport

Between scattering events electrons move like free particles with a given effective mass.

In 1D case the relation between the final velocity and the effective free path, $l$, is then

$$v^2 - v_0^2 = -2 \frac{eE}{m^*} l$$

Assuming $v = v_0 + v_d$ where $v_d$ is the drift velocity while $v_0$ is the typical velocity and introducing the collision time as $\tau = l / v_0$ we obtain in the linear approximation:

$$v_d = \frac{e\tau}{m^*} E$$

Mobility
How universal conductance fluctuations can be explained?

For a rectangular sample \( L \times W \) the Drude formula can be written as

\[
G = \frac{\sigma W}{L} = e^2 \frac{n \pi}{m} \frac{W}{L} = e^2 \frac{l}{\hbar k_F} \frac{2\pi k_F^2}{4\pi^2} \frac{W}{L} = e^2 \frac{l \pi}{\hbar/2L} \frac{k_F W}{\pi^2}
\]

7. What is the variance of conductivity for universal conductance fluctuations? Do fluctuations characterize the specific impurity configuration? Are they linked with weak localization and do localization loops play role in UCF?
Universal conductance fluctuations: explanation with Landauer formula

\[ G = \frac{e^2}{h} \frac{\pi \ell}{2L} N, \quad N \equiv \frac{2k_F W}{\pi}. \]

**Conductance per mode**  **Number of modes**

From the Landauer formula,

\[ G = \frac{2e^2}{h} \sum_{\alpha, \beta=1}^{N} |t_{\alpha \beta}|^2 \]

\[ \sum_{\alpha, \beta=1}^{N} |t_{\alpha \beta}|^2 = N^2 \langle |t_{\alpha \beta}|^2 \rangle \]

\[ \langle |t_{\alpha \beta}|^2 \rangle = \frac{\pi \ell}{4NL} \]

Now we are interested in

\[ \text{Var} (G) \equiv \langle (G - \langle G \rangle)^2 \rangle \]

Electron phase coherence
Statistical approach

For the transition amplitude,

\[ \sum_{\alpha, \beta = 1}^{N} |t_{\alpha \beta}|^2 = N - \sum_{\alpha, \beta = 1}^{N} |r_{\alpha \beta}|^2 \]

\[
\text{Var} (G) = \left( \frac{e^2}{\hbar} \right)^2 \text{Var} \left( \sum_{\alpha, \beta = 1}^{N} |r_{\alpha \beta}|^2 \right) = \left( \frac{e^2}{\hbar} \right)^2 N^2 \text{Var} \left( |r_{\alpha \beta}|^2 \right)
\]

Since reflections are dominated by few events, they can be considered as uncorrelated. Then

\[
\text{Var} \left( |r_{\alpha \beta}|^2 \right) = \langle |r_{\alpha \beta}|^4 \rangle - \langle |r_{\alpha \beta}|^2 \rangle^2
\]

Now we have to calculate the variance of the reflections
\[
\langle |r_{\alpha\beta}|^4 \rangle = \sum_{i,j,k,l=1}^{M} \langle A^*(i)A(j)A^*(k)A(l) \rangle
\]
\[
= \sum_{i,j,k,l=1}^{M} \left[ \langle |A(i)|^2 \rangle \langle |A(k)|^2 \rangle \delta_{ij}\delta_{kl} + \langle |A(i)|^2 \rangle \langle |A(j)|^2 \rangle \delta_{il}\delta_{jk} \right]
\]
\[
= 2 \langle |r_{\alpha\beta}|^2 \rangle^2.
\]

**Result:**

\[
\text{Var} \left( |r_{\alpha\beta}|^2 \right) = \langle |r_{\alpha\beta}|^2 \rangle^2 \quad \Rightarrow \quad \text{Var} \left( |r_{\alpha\beta}|^2 \right) \approx \frac{1}{N^2}
\]

Averaging \[ \sum_{\alpha,\beta=1}^{N} |t_{\alpha\beta}|^2 = N - \sum_{\alpha,\beta=1}^{N} |r_{\alpha\beta}|^2 \] and substituting \( \langle |t_{\alpha\beta}|^2 \rangle = \pi\ell/4NL \) we get:

\[
\langle |r_{\alpha\beta}|^2 \rangle = \frac{1}{N} \left[ 1 - O \left( \frac{\ell}{L} \right) \right]
\]

**Thus**

\[
\text{Var} \left( G \right) = \left( \frac{e^2}{2\hbar} \right)^2 N^2 \text{Var} \left( |r_{\alpha\beta}|^2 \right) = \left( \frac{e^2}{2\hbar} \right)^2
\]

Universal conductance fluctuations (UCF)
Electron phase coherence

Specifics of universal conductance fluctuations

Typical fluctuation is very large, and relative fluctuation does not decay \( \propto 1/\sqrt{LW} \) as it would follow from statistical physics. Quantum low-dimensional systems do not possess the property of self-averaging.

At relatively large temperatures this property is restored due to decoherence.

At \( W \ll L_\varphi \ll L \),

\[
\delta G = \frac{g_s g_v \sqrt{12}}{2\sqrt{\beta}} \frac{e^2}{h} \left( \frac{L_\varphi}{L} \right)^{3/2} \left[ 1 + \frac{9}{2\pi} \left( \frac{L_\varphi}{L_T} \right)^2 \right]^{-1/2}
\]

\[
L_T = v_F \sqrt{\tau \hbar / kT}; \quad L_T^2 = v_F^2 \tau_{unc}; \quad \tau_{unc} = \hbar / kT; \quad E\tau_{unc} = \hbar;
\]

8. Does average universal conductance fluctuation amplitude depend on sample size, strength, configuration and number of the elastic scatterers? Why are these fluctuations named universal? What do UCF depend on?
Usually it is assumed that in a magnetic field the sample properties change significantly. Thus one can think that at each magnetic field a sample represents a realization of an ensemble (so-called ergodic hypothesis). Consequently, one can average over magnetic fields rather than over different samples.

All theoretical models for parametric UCF are based on the ergodicity theorem. A system is called ergodic (with respect to $q$) if averaging ensemble and parameter procedures give the same mean value and the same variance of $q$. Non-ergodic samples are samples for which tuning the external parameter does not induce sufficient transitions between micro-states. This can be due to metastable states.

9. What theorem are theoretical models describing parametric UCF based on? Describe the principle of ergodicity.
Let us look at the resistance of the wire we have already seen, in a weak magnetic field, $\omega_c < 0.45\omega_0$.

QWR is tuned by varying the two in-plane gates. A constant electric field displaces a confining parabolic potential without changing its shape.
Reproducible conductance fluctuations are observed. The interference pattern is changed by shifting the wire through the crystal. The average period and amplitude are \( \delta y \approx 2 \text{ nm} \) and \( \delta G \approx 0.15e^2/h \).

One finds \( \delta y = 2.7 \text{ nm} \) for distance between defects \( d = l_q \), the quantum scattering length. The fluctuations are caused by all scatterers, and not just by the large-angle scatterers that determine \( l_e \).

The density of bumps in the scattering potential is \( 1/d^2 \). On average, the number of bumps inside the wire should change by one as the wire is displaced by \( \delta y = d^2/2l_e \).

10. What conclusions can be drawn from the observations of universal conductance fluctuations in quantum wires with shifted parabolic potential? Are UCF caused just by the large-angle scatterers that determine mean free path \( l_e \)?
Phase coherence in ballistic systems

\[ E = -eV \rightarrow \varphi = -\frac{Et}{\hbar} = \frac{eVt}{\hbar} \]

Electrostatic Aharonov-Bohm Effect

(a) Split gates
(b) Interference pattern

11. Is electrostatic Aharonov-Bohm Effect possible? What is the phase shift in this effect?
Assuming a linear relation between the gate voltage $V_g$ and the Fermi energy $E_F$ in the 2DEG, the phase shift that the electrons acquire underneath the gate is given by:

$$\delta \phi = W k_{F,0} \delta \left( \sqrt{1 - \frac{V_g}{V_T}} \right)$$

Resulting current should be periodic with a period of:

$$\delta \phi = 2\pi \quad \implies \quad \delta \left( \sqrt{1 - \frac{V_g}{V_T}} \right) = \frac{2\pi}{k_{F,0}} W$$

The oscillation amplitude $a$ is used to calculate $l_\phi$ considering that dephasing takes place via single electron–electron scattering:

$$l_\phi = -\frac{L}{\ln[a(L, V_{E,DC})]}$$

An application of a small, negative gate voltage $V_g$ to $A$ results in a partial depletion of the 2DEG underneath the gate and, hence, in a phase change $\delta \phi$ in all paths pertaining to $A$. The transmission coefficient is expected to oscillate as a function of $V_g$ with a period corresponding to the optical path of $A$-group by one wavelength. The oscillation should be periodic in $(1-\frac{V_g}{V_T})^{0.5}$ with a period given by $2\pi/k_{F,0} W$. Here, $W$ is the electrical gate width and $V_T$ is the gate voltage needed to deplete the 2DEG underneath the gate.

12. Describe an experiment to test the phase coherence in ballistic 2DEGs. What is the phase shift that the electrons acquire underneath the control gate of a width $W$ assuming a linear relation between the gate voltage $V_G$ and the Fermi energy $E_F$ in the 2DEG? Define $V_T$ as the voltage that completely depletes electron density below the gate.
**Coherent ballistic system: an experiment**

It can be concluded that complete dephasing takes place via single electron-electron scattering events, which occur randomly.

This is different from the result in the diffusive regime ($1/\Theta$).

The $l_\phi \approx 100 \, \mu m$ was found for $V_{E,DC} = 0$, which corresponds in the ballistic regime to a dephasing time of $\tau_\phi = l_\phi/v_F \approx 37 \, ps$, which is the same order of magnitude as found in diffusive systems.

13. What conclusions can be drawn from the experiments on ballistic point contacts connected in series and controlled by the depleting gate? How could coherence length and time be found? What are their values in ballistic point contacts? What is the origin of electron dephasing?
Coherence and tunneling transport

Classical motion

Quantum tunneling

Ivar Giaever

https://www.youtube.com/watch?v=AJYBfarPqdI

https://www.youtube.com/watch?v=cV2fkDscwVY

https://www.youtube.com/watch?v=EuU9Yin_2mM

14. What is advantage of tunnel barriers comparable to direct resistive nanostructures? Is electron coherence possible in tunnel structures? What is the difference between classical motion and quantum tunnelling? Please describe transmission of an electron through the tunnelling barrier in terms of wave functions.

Electron phase coherence
Following on Esaki's discovery of electron tunnelling in semiconductors in 1958, Giaever showed that tunnelling also took place in superconductors (1960).

Giaever's demonstration of tunnelling in superconductors stimulated Brian Josephson to work on the phenomenon, leading to his prediction of the Josephson effect in 1962. Esaki and Giaever shared half of the 1973 Nobel Prize, and Josephson received the other half.
4 unknown coefficients, and 4 boundary conditions at $x = \pm a/2$:

continuity of wave functions and their derivatives (currents) gives:

one finds **transition** and **reflection** amplitudes, $t$ and $r$, respectively.

The expression for the transition **probability**:

$$ T(E) = |t|^2 $$

Quantum effects

$$ T(E) = \begin{cases} \frac{4E(V_0-E)}{4E(V_0-E)+V_0^2 \sinh^2 \kappa a}, & E \leq V_0 \\ \frac{4E(V_0-E)}{4E(V_0-E)+V_0^2 \sin^2 \kappa a}, & E \geq V_0 \end{cases} $$

Over-barrier transmission

15. What is the expression for the transition probability of a single rectangular tunnelling barrier? Please describe tunnelling regime and over-barrier transmission. What are their criteria?
Resonant tunneling and S-matrices

Transmission of a single rectangular barrier. The inset shows a logarithmic plot of the tunnelling regime, showing that, in the tunnelling regime, $T$ increases approximately exponentially with energy. The S-matrix of a tunnel barrier relates the outgoing wave functions to the incoming wave functions.

$$
\vec{b} = S \vec{a}, \quad \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} r_{11} & t_{12} \\ t_{21} & r_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}
$$

16. Introduce tunnelling S-matrix for a single rectangular tunnelling barrier. What are reflection and transmission terms? What are the unitarity conditions for the matrix?
**Double barrier tunneling**

**Single barrier properties:**

For a symmetric system:

\[
\begin{align*}
    r_{11} &= r_{22} = r \\
    t_{12} &= t_{21} = t
\end{align*}
\]

What will happen with transmission probability if we place two barriers in series?

Owing to conservation of probability current density, \( S \) has to be unitary:

\[
S^\dagger S = 1 \quad \Rightarrow
\]

\[
|r_{11}| = |r_{22}|, \quad |t_{12}| = |t_{21}|
\]

\[
r_{11}^* r_{11} + r_{22}^* r_{22} = 1
\]

\[
r_{11} t_{12}^* - t_{21} r_{22}^* = 0
\]

17. Introduce the double barrier tunnelling problem. What is the transmission probability for the double barrier structure as function of transmission and reflection in individual barriers? Give the condition of resonant tunnelling.
Transmission probability

\[
t_{sd} = t_1 e^{i\theta} t_2 + t_1 e^{i\theta} r_2 e^{i\theta} r_1 e^{i\theta} t_2 + \ldots
\]

\[
= t_1 t_2 e^{i\theta} \sum_{j=0}^{\infty} (r_1 r_2 e^{i\theta})^j = \frac{t_1 t_2 e^{i\theta}}{1 - r_1 r_2 e^{i\theta}}
\]

**Geometric series**

Transmission probability:

\[
T = \left| t_{sd} \right|^2 = \frac{T_1 T_2}{1 + R_1 R_2 - 2 \cos \theta \sqrt{R_1 R_2}}
\]

Electron phase coherence
Resonant tunneling

At some values of $\theta_s = 2\pi s$, there are resonances:

Coherent transmission of a double barrier as a function of the phase collected during one round trip between the barriers, shown for equal individual barrier transmissions $T_b = 0.9$, 0.5, and 0.1, respectively. Right: Lorentzian fit (bold lines) for $T_b = 0.9$ and 0.1 (thin lines).

18. Describe resonant tunnelling of the double barrier structure. Is it possible that transmission of double barrier is higher than transmission of individual barriers in series? Would it be possible to fit resonance curves with Lorentzian? Do you know resonant tunnelling analogue in electro-magnetism?
We observe that at $E=E_s$ ($\theta=\theta_s$) and $T_1=T_2$ the total transparency is one though partial transparency is very small. This phenomenon is called the resonant tunneling - it is fully due to quantum interference.

Often people introduce the attempt frequency as

$$\nu = \frac{\nu}{2L} = \frac{\hbar k}{2m^*L}$$

and partial escape rates, $\Gamma_i = \hbar \nu T_i$. Then near the s-resonance

$$T_s(E) = \frac{\Gamma_1 \Gamma_2}{(\Gamma_1 + \Gamma_2)^2/4 + (E - E_s)^2}$$

19. What is the attempt frequency and partial escape rate for the double tunnelling barrier structure? Can you write the transmission probability for the double barrier junction in terms of energy and partial escape rates of individual barriers?
Resonant tunneling: analogy

The double barrier can be thought of as an electron interferometer. A resonance occurs when the Fermi wavelength is commensurable with barriers distance $L$, i.e. $n \times \lambda_F = L$.

Will resonant tunneling survive if the barriers are far from each other?

Unfortunately, NO, since then the coherence would be lost. Decoherence can be modeled by introducing a reservoir between the barriers which can absorb and re-emit the electrons whose phase memory gets lost.
Electron phase coherence

Transmission through a ballistic quantum ring

Calculation can be done using S-matrix approach

Novel feature - triple junctions, described by the matrix

\[
S = \begin{pmatrix}
    c & \sqrt{1 - c^2}/2 & \sqrt{1 - c^2}/2 \\
    \sqrt{1 - c^2}/2 & -(1 + c)/2 & (1 - c)/2 \\
    \sqrt{1 - c^2}/2 & (1 - c)/2 & -(1 + c)/2
\end{pmatrix}
\]

Here \( c \) represents the reflection amplitude for electrons hitting the junction from lead 1.

20. Can you outline the description of the transmission through a ballistic quantum ring in terms of S-matrix and the transmission through a ballistic quantum ring locked between two tunnel barriers, using Shapiro matrix? What is the transmission of an ideal quantum ring as a function of dynamic \( \theta \) and magnetic \( \varphi \) phase for different reflection amplitudes at the ring entrances?
Transmission of an ideal quantum ring as a function of dynamic $\theta$ and magnetic $\phi$ phase for different reflection amplitudes at the ring entrances. Black corresponds to $T_{\text{ring}} = 0$, white to $T_{\text{ring}} = 1$.

$$\phi = \frac{2\pi \Phi}{\Phi_0}, \quad \theta \propto \sqrt{E} - \sqrt{E_S}$$

Electron phase coherence
In this lecture, we have discussed important electron interference effects:

- The Aharonov-Bohm effect in mesoscopic conductors
- Weak localization
- Universal conductance fluctuations
- Phase coherence in ballistic 2DEGs
- Resonant tunneling

Their understanding is crucial for nanotechnology applications.