Single electron tunneling

https://www.youtube.com/watch?v=UguFUpcV5Ew
Building blocks for nanodevices

- Two-dimensional electron gas (2DEG) structures
- Quantum wires and quantum point contacts
- Phase coherent structures
- **Single-electron** tunneling devices based on coulomb blockade
- Quantum dots
By carefully measuring how the cathode rays were deflected by electric and magnetic fields, Thomson was able to determine the ratio between the electric charge \( e \) and the mass \( m \) of the rays. Thomson's result was

\[
e/m = 1.8 \times 10^{11} \text{ coulombs/kg}
\]

The particle that J.J. Thomson discovered in 1897, the electron, is a constituent of all the matter we are surrounded by. All atoms are made of a nucleus and electrons. He received the Nobel Prize in 1906 for the discovery of the electron, the first elementary particle.
The Nobel Prize in Physics 1906

**Joseph John Thomson**

"in recognition of the great merits of his theoretical and experimental investigations on the conduction of electricity by gases"

The Nobel Prize in Physics 1923

**Robert Andrews Millikan**

"for his work on the elementary charge of electricity and on the photoelectric effect"

The Nobel Prize in Physics 1921

**Albert Einstein**

"for his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect"

The Nobel Prize in Physics 1937

**Clinton Joseph Davisson and George Paget Thomson**

"for their experimental discovery of the diffraction of electrons by crystals"
Established by the Duke of Devonshire and extended by Lord Paneth (1908) and Lord Austin (1940), the Cavendish Laboratory housed the Department of Physics from the time of the first Cavendish Professor James Clerk Maxwell until its move to new laboratories in West Cambridge.

In 1897 at the old Cavendish Laboratory, JJ Thomson discovered the electron. Subsequently recognised as the first fundamental particle of physics and the basis of chemical bonding, electronics, and computing.
Robert A. Millikan’s Nobel Prize lecture
(May 23, 1924)

• ‘The terms of this year’s award state that it is given for work on the fundamental electrical unit and on photoelectricity. In both fields my own work has been that of the mere experimentalist whose main motive has been to devise, if possible, certain crucial experiments for testing the validity or invalidity of conceptions advanced by others.’

• ‘The conception of electrical particles or atoms goes back a hundred and seventy years to Benjamin Franklin who wrote about 1750: "The electrical matter consists of particles extremely subtle since it can permeate common matter, even the densest, with such freedom and ease as not to receive any appreciable resistance."

• ‘This theoretical conception was developed in no little detail by Wilhelm Weber in papers written in 1871. The numerical value of the ultimate electrical unit was first definitely estimated by G. Johnstone Stoney in 1881, and in 1891 this same physicist gave to it the name the electron.’
Robert A. Millikan’s Nobel Prize lecture
(May 23, 1924)

• ‘In 1897 the experimental foot came forward with J. J. Thomson’s and Zeeman’s determinations of e/m by two wholly distinct methods. It was these experiments and others like them which in a few years gained nearly universal acceptance among physicists for the electron theory. There remained, however, some doubters, even among those of scientific credentials, for at least two decades - men who adopted the view that the apparent unitary character of electricity was but a statistical phenomenon…’

• ‘The measurement of the electron, not as above in terms of the speed that it imparts to a given oil-drop, but in absolute electrostatic units, involved observations of the foregoing sort upon thousands of drops of various sizes, made from a number of different substances, surrounded by a large number of different gases at widely differing pressures, varying from atmospheric down to a millimeter and a half of mercury. It involved also years of work in finding accurate values of gaseous viscosities…’
Single Electron Tunneling

Electron charge is $1.60217657 \times 10^{-19}$ coulombs

1 A is 1 coulomb/s or $6.24 \times 10^{18}$ el/s

1 $\mu$A is $6.24 \times 10^{12}$ el/s

1 mm$^2$ $\rightarrow$ 1 nm$^2$ $\rightarrow$ 6.24 el/s

When a bias voltage is applied, there is a tunnelling current proportional to the bias voltage. In electrical terms, the tunnel junction behaves as a resistor with a constant resistance, also known as an Ohmic resistor. The resistance depends exponentially on the barrier thickness. Typical barrier thicknesses are on the order of one to several nanometers.

https://www.youtube.com/watch?v=cV2fkDscwvY

https://www.youtube.com/watch?v=EwU9Yin_2mM
Coherence and tunneling transport

Classical motion

Quantum tunneling

Equations:

1. \( \psi = e^{ikx} + re^{-ikx}, \)
2. \( \psi = \alpha e^{\kappa x} + \beta e^{-\kappa x}, \)
3. \( \psi = te^{ikx}, \)

Where:

\( k \equiv \hbar^{-1} \sqrt{2m^*E}, \)
\( \kappa \equiv \hbar^{-1} \sqrt{2m^*(V_0 - E)} \)

Ivar Giaever

https://www.youtube.com/watch?v=AJY8farPqdI
https://www.youtube.com/watch?v=cV2fkDscwvy
https://www.youtube.com/watch?v=EuU9Yin_2mM

Electron phase coherence
Double barrier tunneling

Single barrier properties:

For a symmetric system:

\[ r_{11} = r_{22} = r \]
\[ t_{12} = t_{21} = t \]

What will happen with transmission probability if we place two barriers in series?

Owing to conservation of probability current density, \( S \) has to be unitary:

\[ S^\dagger S = 1 \quad \Rightarrow \]
\[ |r_{11}| = |r_{22}|, \quad |t_{12}| = |t_{21}| \]
\[ r_{11}^* r_{11} + r_{22}^* r_{22} = 1 \]
\[ r_{11} t_{12}^* - t_{21} r_{22}^* = 0 \]

17. Introduce the double barrier tunnelling problem. What is the transmission probability for the double barrier structure as function of transmission and reflection in individual barriers? Give the condition of resonant tunnelling.
Transmission probability

\[ t_{sd} = t_1 e^{i\theta} t_2 + t_1 e^{i\theta} r_2 e^{i\theta} r_1 e^{i\theta} t_2 + \ldots \]

\[ = t_1 t_2 e^{i\theta} \sum_{j=0}^{\infty} (r_1 r_2 e^{i\theta})^j \]

Geometric series

Transmission probability:

\[ T = |t_{sd}|^2 = \frac{T_1 T_2}{1 + R_1 R_2 - 2 \cos \theta \sqrt{R_1 R_2}} \]

Electron phase coherence
Resonant tunneling

At some values of $\theta$, $\theta_s = 2\pi s$, there are resonances:

Coherent transmission of a double barrier as a function of the phase collected during one round trip between the barriers, shown for equal individual barrier transmissions $T_b = 0.9$, 0.5, and 0.1, respectively. Right: Lorentzian fit (bold lines) for $T_b = 0.9$ and 0.1 (thin lines).

18. Describe resonant tunnelling of the double barrier structure. Is it possible that transmission of double barrier is higher than transmission of individual barriers in series? Would it be possible to fit resonance curves with Lorentzian? Do you know resonant tunnelling analogue in electro-magnetism?
Coulomb blockade in a tunnel barrier

At $|q| < e/2$ the electron tunneling will increase the energy stored in the barrier - one has to pay for the tunneling by the bias voltage.

Energy stored is $q^2/2C$

\[ \text{Why } R \text{ matters?} \]
\[ \text{time delay } \delta t = eR/V \]
\[ \text{duration } \tau \sim \hbar/eV \]
\[ \delta t \gg \tau \rightarrow R \gg \hbar/e^2 \]

Because of environment resistances and capacitances it is difficult to observe CB in single junctions.
Single-electron tunneling and Coulomb blockade

- Size reduction goes along with a reduction of capacitances. For a plate capacitor of area $L^2$ at a separation $L$ its capacitance $C$ scales with $L$. At small sizes, the energy required to store an additional electron on it, $E = \frac{e^2}{2C}$, may become larger than the thermal energy. As a consequence, the quantization of charge can dominate the behaviour of circuits, in which tunnelling of single electrons carries the current.
- The tunnel junction capacitor is charged with one elementary charge by the tunnelling electron, causing a voltage buildup $U = \frac{e}{C}$, where $e$ is the elementary charge of $1.6\times10^{-19}$ coulomb and $C$ the capacitance of the junction. If the capacitance is very small, the voltage buildup can be large enough to prevent another electron from tunnelling.

1. Explain why just single electron can influence conducting properties of material on nanometre scale. Is charging effect of an island, through which electron is passing, important?
Coulomb blockade

Coulomb blockade is effect of trapping electron on an island due to its charging effect. To achieve Coulomb blockade, three criteria have to be met:

1. The bias voltage moving electron from the island must be lower than the elementary charge divided by the self-capacitance of the island: $V_{\text{bias}} < e/C$.

2. The thermal energy in the source contact plus the thermal energy in the island, i.e. $k_B T$, must be below the charging energy: $k_B T < e^2/2C$.

3. The tunnelling resistance, $R_\uparrow$, should be greater than $h/e^2 (~26 \, \text{KOhm})$. 

2. What is Coulomb blockade? What conditions need to be satisfied to observe Coulomb blockade?
Single-electron tunneling: history

• In 1951, Gorter suggested that experiments by van Itterbeek and coworkers, who measured the current through metal grains embedded in an isolated matrix, could be explained by single-electron charging.

• The first transistor that exploited this effect was built by Fulton and Dolan in 1987.

• The single-electron tunnelling can be used to design new types of devices.
Coulomb blockade in tunnel barrier

Single-electron transistor (SET)

\[ Q = Ne \]

\[ E(\text{Ne}) = E((\text{N+1}e)) \]

\[ Q_0 = V_g C \]

\[ E = QV_g + \frac{Q^2}{2C} \]

At the energy cost vanishes!

At

\[ V_g = - \left( N + \frac{1}{2} \right) \frac{e}{C} \]

3. What is a single-electron transistor? Please compare energy of transistor with N and N+1 electrons on the island. What is the energy condition for changing the number of electrons in the transistor? At what gate voltage can it take place?
5.4. Supersensitive Electrometer
The high sensitivity of single-electron transistors has enabled them as electrometers in unique physical experiments. For example, they have made possible unambiguous observations of the parity effects in superconductors. Absolute measurements of extremely low dc currents (~10⁻²⁰ A) have been demonstrated. The transistors have also been used in the first measurements of single-electron effects in single-electron boxes and traps. A modified version of the transistor has been used for the first proof of the existence of fractional-charge excitations in the fractional quantum hall effect.

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<th>SET Circuit</th>
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<tr>
<td>Maximum operation temperature</td>
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</table>

8. Advantages of SET
Following are the advantages of Single electron transistors (SETs):
- Low energy consumption
- High sensitivity
- Compact size
- High operating speed
- Simplified circuit
- Feature of reproducibility
- Simple principle of operation
- Straight forward co-integration with traditional CMOS circuits. [6]
Single electron tunneling

Isolated island and background charge

\[ \Delta E = \frac{\left[(n + 1)e + q_0\right]^2}{2C} - \frac{\left[ne + q_0\right]^2}{2C} \]

\[ = \frac{e}{C} \left[\left(n + \frac{1}{2}\right) + q_0\right] \]

At \( q_0 = -(n + 1/2)e \) the energy cost vanishes!

4. What is the influence of environment on the state of an island with electrons? Is environment important? Introduce the background (or induced) charge of the device. At what background charge does the difference in the energy of an island with \( n \) and \( n+1 \) electrons vanish?
Single-electron transistor

Evolution of the I-V characteristic of a single tunnel junction as the resistance of the environment $R_e$ is increased. The traces are shown for $R_e/R = 0, 0.1, 1, 10,$ and $\infty$

The I-V characteristic of Al-Al$_2$O$_3$-Al tunnel barrier fabricated by angle evaporation. The cross section of the Al wires is only $10\text{nm} \times 10\text{nm}$. The superconducting state has been suppressed by applying a magnetic field.

5. Is environment resistance strongly influencing a single tunnel junction device? What does it look like in experiment? At what environment resistance is the Coulomb blockade well-pronounced in a single-electron device? How to relax the influence of environment?
SET: Basic circuit and devices

Equivalent circuit of a single tunnel junction. The resistor $R_e$ represents the low-frequency impedance of the environment.
The limitations imposed by the need to decouple the environment from the tunnel junction can be relaxed by using two tunnel junctions in series.
Electrostatic Energy and Capacitance Matrix

The charges and potentials of the islands can be written in terms of an island charge vector $q_I$ and potential vector $V_I$, respectively. Similarly, charge and potential vectors can be written down for the electrodes, $q_E$ and $V_E$. The state of the system can be specified by the total charge vector $q = (q_I, q_E)$. The total potential vector: $V = (V_I, V_E)$. Charge and potential vectors are related via the capacitance matrix $C$.

A system of islands (floating) and electrodes (connected to voltage sources) and the corresponding equivalent circuit composed of potential nodes and mutual capacitances.

What is the capacitance matrix formalism for a basic single-electron tunnelling circuit? Please introduce island charge and potential vectors. How these scalar parameters can be vectors? How to treat single-electron tunnelling circuits? What is condition of the transition of the electron system to a new ground state? Can you describe terms of the basic equation that is used to analyse the motion of electron between island, source and drain?
Charge and potential vectors are related via the capacitance matrix $\mathbf{C}$. The capacitance submatrices between type A and type B conductors (A, B can be electrodes or islands) are denoted by $\mathbf{C}_{AB}$.

The matrix elements of $\mathbf{C}$ are given by:

\[
(C)_{ij} = \begin{cases} 
-C_{ij} & j = 1, \ldots, n + m; j \neq i \\
C_{iD} + \sum_{k=1; k \neq i}^{n+m} C_{ik} & j = i 
\end{cases}
\]

The electrostatic energy $E$ is given by the energy stored at the islands, minus the work done by the voltage sources. Minimizing this energy gives us the ground state. At the transition between states:

\[
\Delta E[\vec{V}_E, \vec{q}_I, \Delta \vec{q}] = \Delta \vec{q}_I C_{II}^{-1} [\vec{q}_I + \frac{1}{2} \Delta \vec{q}_I - C_{IE} \vec{V}_E] + \Delta \vec{q}_E \vec{V}_E
\]
Double barrier electron transfer

Electrostatics: 4 different charge transfer events are relevant

Condition of transfer

\[ \Delta E = E_{\text{new}} - E \leq 0 \]

\[ \Delta E = \frac{e}{C_{1S} + C_{1D}} \left[ \pm \left( ne - q_0 + C_{1D} V \right) \right] \]

The simplest situation is \( n = 0 \), no background charges \( (q_0 = 0) \), and identical junction capacitances \( C_{1S} = C_{1D} = C_{11}/2 \).

7. Describe the double-barrier single-electron device. What is its equivalent circuit? Formulate its capacitance matrix. What is the condition of the transfer between states of this device? Analyse simplest situation of zero voltage, zero background charges and zero number of electrons on island. Is Coulomb blockade possible in this case?
Voltage induced double barrier electron transfer

For $n=0$ and $q_0=0$, all transfers are suppressed until

$$-e/C_{11} \leq V \leq e/C_{11}$$

$$V = e/C_{11}$$

Electrons can tunnel both from drain onto island and from island to source.

Coulomb blockade of transport

$V=0$
$q_0=0$
During each cycle a single electron is transferred!

8. Consider double-barrier single-electron device with an applied voltage in a simple situation of zero background charges and zero number of electrons on island. What is the condition of Coulomb blockade in this case? Describe the process of cycled transfer of electrons.
Influence of background charge

Coulomb blockade is established only if all energies of four differences are positive. This defines a voltage interval of vanishing current:

$$\text{Max}\left\{ \frac{2}{C_{11}}(-q_0 - \frac{e}{2}), \frac{2}{C_{11}}(q_0 - \frac{e}{2}) \right\}$$

$$< V < \text{Min}\left\{ \frac{2}{C_{11}}(-q_0 + \frac{e}{2}), \frac{2}{C_{11}}(q_0 + \frac{e}{2}) \right\}$$

By a non-zero $q_0$, the Coulomb gap can be reduced, but never be increased. In fact, for $q_0 = (j + 1/2)e$ with $j$ being an integer, the Coulomb gap vanishes completely.

Background charges can seriously hamper the observation of the Coulomb blockade.

9. What is the influence of background charge on the electron transfer in double-barrier single-electron device? Can background charge increase or reduce Coulomb gap? At what condition does Coulomb gap vanish completely?
First observation:

Giæver and Zeller, 1968 - granular Sn film, superconductivity was suppressed by magnetic filed

\[ \frac{dV}{dl} (\Omega) \]

Differential resistance

Coulomb gap manifests itself as increased low-bias differential resistance

The background charges, \( q_0 \), influence Coulomb blockade and can even lift it.

14. Give example of a typical mesoscopic structure in which Coulomb blockade oscillations take place. Does current there represent a single energy-level state? Is the peak in conductance temperature-dependent? Why is it said that Coulomb oscillations measure not the density of states on the island, but the addition spectrum? What is added?
I-V curves

How to calculate I-V curve of a tunneling device if the energy of electron changes during tunnelling?

It can be done for a stationary case through the transition rate, \( \Gamma \). The probability of finding \( n \) electrons on the island \( p(n) \) should also be introduced. This function is expected to be peaked around one number, which is given by the sample parameters and by applied voltage. The steady state condition requires that the probability for making a transition between two charge states (characterized by \( n \)) is zero. This means that the rate of electrons entering the island occupied by \( n \) electrons equals the rate of electrons leaving the island when occupied by \((n + 1)\) electrons:

\[
p(n)[\Gamma_{1 \rightarrow S}(\Delta E_{1 \rightarrow S}(n)) + \Gamma_{1 \rightarrow D}(\Delta E_{1 \rightarrow D}(n))] = p(n + 1)[\Gamma_{S \rightarrow 1}(\Delta E_{S \rightarrow 1}(n + 1)) + \Gamma_{D \rightarrow 1}(\Delta E_{D \rightarrow 1}(n + 1))]
\]
Fermi Golden Rule

The transition rate, $\Gamma$, can be calculated from the Fermi Golden Rule:

$$\Gamma_{i \rightarrow f} = \frac{2\pi}{\hbar} |\langle i | \hat{H} | f \rangle|^2 \delta(E_f - E_i - \Delta E)$$

To get the complete tunnel rate we have to multiply the probability by:

$$G_i G_f F(i) [1 - F(f)]$$

and then sum over all initial and final states.

Since only the vicinity of the Fermi level matters we can take the densities of states and matrix elements at the Fermi level.

10. How to take into account the change in energy $\Delta E$ during a tunnel event in a mesoscopic structure that has a transition rate $\Gamma(\Delta E)$? What is the Fermi Golden rule?
I-V curves: Coulomb staircase

For a steady state, the average charge at the island is constant, and the current from source to the island is given by:

\[ I(V) = e \sum_{n=-\infty}^{\infty} p(n) [\Gamma_{1\to S}(\Delta E_{1\to S}(n)) - \Gamma_{S\to 1}(\Delta E_{S\to 1}(n))] \]

Equivalently, \( I(V) \) can be expressed in terms of the drain tunnelling rates.

The application of the expression above to double-junction single electron device produces I-V curve in the form of staircase.
I-V curves: Coulomb staircase

The steps become most pronounced if both the resistance and the capacitance of one junction are large compared to those of the second junction.

11. How to calculate current–voltage characteristic of the double-barrier structure in a steady state taking into account change of the energy of electron during the tunnelling? How is Coulomb blockade expressed in current–voltage characteristic? What is staircase current–voltage characteristic? What is the condition of its appearance in the double-barrier structure? What is the island-charge period in staircase? Suggest a qualitative explanation of the staircase effect. Can scanning tunnelling microscope be used to measure staircase effect? Please explain how.

Experiment: STM of a granule. The granule was a small indium droplet on top of an oxidized conducting substrate.
An extra electrode (gate) defined in a way to have very large resistance between it and the island.

That allows to tune induced charges by the gate voltage

Fulton & Dolan, 1987

The Coulomb gap is given by the onset of the same tunnelling events as for the single island studied above. Now, however, the Coulomb gap depends upon the gate voltage. The energy differences at electron tunnelling are:

\[
\Delta E[(V, V_G), -ne, \pm e(-1, 1)] = \frac{e}{C_{11}} \left[ \frac{1}{2} e \pm (C_{11} - C_{1S})V \pm ne \mp C_{1G}V_G \right]
\]

\[
\Delta E[(V, V_G), -ne, \pm e(-1, 0)] = \frac{e}{C_{11}} \left[ \frac{1}{2} e \mp C_{1S}V \pm ne \mp C_{1G}V_G \right]
\]
Stability diagram of a single-electron transistor. Within the diamonds, Coulomb blockade is established, while outside, a current flows between source and drain. The slopes of the boundaries are given by $C_{1G}/(C_{11} - C_{1S})$, and by $-C_{1G}/C_{1S}$.

12. Explain the principle of single electron tunnelling transistor (SET). What is its schematic diagram? How does gate voltage influence Coulomb gap? Draw the 'diamond' stability diagram of a single-electron transistor. What is on its axis? How does system behave inside and outside of diamonds?
The SET transistor: experiment

Experimental test: Al-Al$_2$O$_3$ SET, temperature 30 mK

13. Was current-voltage characteristic of single electron tunnelling transistor first measured experimentally? At what source-drain voltage do Coulomb blockade oscillations take place? What changes by one in each gate voltage period in Coulomb blockade oscillations? Was Coulomb staircase also seen in single electron tunnelling transistor? Why could it appear there?
In this device, the gate voltage keeps the island potential fixed at long time. There is only one Coulomb diamond, centred around \((V, V_G) = (0, 0)\). The transconductance is no longer oscillatory in \(V_G\), and the device is much less sensitive to fluctuating background charges. However, the heating and the stray capacitance is a problem. In addition, Coulomb diamond is sensitive to thermal smearing and noise.

16. What is the necessity for introducing a resistively coupled single-electron transistor? What is its schematic diagram? How does it work and what is its difference from SET? Does the gate voltage able to keep the island potential fixed for a long time? Does device have single or multiple ‘diamonds’? Do Coulomb blockade oscillations take place here? Does device very sensitive to fluctuating background charges? What are its main disadvantages comparable to SET?
Double island device

Two islands - each one can be tuned by a nearby gate electrode. The structure is symmetric.

Six electron transfers are important.

Equalities between direct and reverse processes define lines of the stability diagram.

It defines the regions of stable configurations characterized by specific charges of the island.

The inter-island capacitance $C_{12}$ is responsible for the interplay of the contributions from $V_A$ and $V_B$.

17. Describe a two-island, two-gate single electron transistor. What is its ground state as function of two gate voltages at zero and finite capacitances between islands? Are any points in the stability diagrams at which current could pass from source to drain? How can charge configuration of the double-island system be directly monitored and what is the result of this monitoring?
At $C_{12}=0$, the diagram is a set of squares, while at $C_{12}\neq 0$ it is a set of tilted hexagons.

There are “triple points”, where Coulomb blockade is lifted. This allows to “pump” electrons one by one.
Bias voltage moves lines at the stability diagram. It creates triangles where Coulomb blockade is lifted.

Let us adjust the gate voltages to start within a triangle, and then apply to the gates AC voltages shifted in phase. Then the path in the phase space is

\[(n_1, n_2) \rightarrow (n_1 + 1, n_2) \rightarrow (n_1, n_2 + 1) \rightarrow (n_1, n_2)\]

Exactly one electron has passed through the device!

18. How does the stability diagram of two-island, two-gate single electron transistor change in the presence of nonzero bias voltage? What is the principle of electron pump operation? What is the link between the current on the plateau of the diagram and the frequency of AC signal added to the gate voltage?
Charge configuration in the double island system

SETs 3 and 4 work as **electrometers** to measure charges at the islands 1 and 2.

**Conductance of the double island**

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15. What is single electron transistor with Coulomb blockade oscillations sensitive to? Can it be used as an electrometer? In what device? What is its expected sensitivity? What is the advantage of single electron transistor compared to other mesoscopic devices? Can it be used at room temperature? What are its limitations and how, in principle, could they be overcome?
Single electron pump as current standard

The current on plateau: \( I = -ef \)

Opposite phase shifts

This is an excellent current standard!

Comparison between the observed current plateau of the single-electron pump (circles) and the current \( I \) expected from \( I = e \cdot f \). Close to the center of the plateau, a relative error of \( 10^{-6} \) is found.

Six islands in series - the uncertainty is one electron for \( 10^8 \) pumped electrons.

19. Can double-island system be used as current standard? What is its current accuracy? How accurate is the number of electrons pumped? What is the co-tunnelling and how can it be prevented? How to make the capacitance standard? What could be its standard deviation?
Clocking single electrons through electrical circuits one-by-one:

**Single-Electron Tunneling (SET) Pump**

An animation made by

Hansjörg Scherer

Physikalisch-Technische Bundesanstalt Braunschweig
Germany
Another current standard: Quantized electron drag in quantum wires

Only integer number of electrons can be trapped in the potential wells – drag current is quantized in units of $e_f$, depending on the gate voltage.

Talyanskiy *et al.*, 1996
Single electron tunneling
Accuracy limitations of Coulomb-blockade devices

We discussed only sequential tunneling through a grain, which is \textit{exponentially} suppressed by the Coulomb blockade.

In addition, there is a \textit{coherent} transfer. Consider the initial and final states in different leads. Then the transition rate in the \textit{second order} of the perturbation theory is

\[
W_{i \rightarrow f} = \frac{2\pi}{\hbar} \left| \sum_{\psi} \frac{\langle i | H_{\text{int}} | \psi \rangle \langle \psi | H_{\text{int}} | i \rangle}{E_{\psi} - E_{i}} \right|^2 \delta(E_{i} - E_{f})
\]

These are \textit{virtual} transitions!
There is additional small tunneling transparency (one more factor containing conductance).

Here energy costs for tunneling enter as powers rather than exponents.

Due to its importance, the quantum co-tunneling has been thoroughly studied. It can lead to the contributions to the current proportional to $V^3$ and $V$. 

\[
\Gamma_{\text{cot}} = \frac{\hbar G_e G_c}{2\pi e^4} \int_e d\epsilon_k \int_g d\epsilon_q \int_g d\epsilon_q' \int_c d\epsilon_k' \delta(eV + \epsilon_k - \epsilon_q + \epsilon_q' - \epsilon_k')
\]

\[
\times \left[ \frac{1}{\Delta_{-e}(n+1)} + \frac{1}{\Delta_{+c}(n-1)} \right]^2.
\]
Graphene single electron transistors

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Single electron tunneling
Single electron tunneling

FIG. 1. (a) Schematic of a SQUIDET. Left (1) and right (2) superconducting leads act as source and drain, respectively, whereas the central gate electrode is capacitively-coupled to the N metal island. The latter is connected to the lateral proximity N regions via tunnel junctions. The loops are threaded by $\Phi_1$ and $\Phi_2$ magnetic fluxes. (b-d) Sketches of the low-temperature energy band diagrams of the SQUIDET biased at $V = V_0$ under a sweep of magnetic flux with $\Phi_1 - \Phi_2 = \Phi_0/2$. $\Delta_0$ is the zero-temperature superconducting energy gap, and $\Phi_0$ is the flux quantum. $\Phi_1$ was set to 0.15$\Phi_0$ (b), 0.25$\Phi_0$ (c) and 0.35$\Phi_0$ (d). The island charging energy ($E_C$) leads to a discrete level spacing vertically shifted by the gate electrode ($\eta = 0.5$). For the above schemes we set $E_C = \Delta_0$.

FIG. 2. (a-d) DC current vs voltage ($V$) characteristics of a SQUIDET calculated at different magnetic fluxes and for different $\eta$ values. $\Phi_1$ and $\Phi_2$ are supposed to be related by $\Phi_1 - \Phi_2 = \Phi_0/2$. Aluminum (Al) is the superconductor chosen for the loops, and we set $\Delta_0(0) = \Delta_0(0) = \Delta_0 = \xi = 185\mu$V. The tunneling occurs at thermal equilibrium at 35 mK. The electric current is blocked by the two energy gaps in a phase shifted and modulated in amplitude by the magnetic flux.

FIG. 3. Stability diagrams at different $\phi_1/\phi_0$ values (0, 0.25 and 0.5 from left to right) in a SQUIDET with $\Phi_1 = \Phi_2 = \Phi_0/2$. The plots surfaces have been colored having the red, green and blue channels proportional to $p_{11}$, $p_{11}$ and $p_{11}$, respectively. Here we set $\Delta_0(0) = \Delta_0(0) = \Delta_0 = \xi = 500\mu$V. The surfaces are determined for the stability of the states $|11\rangle$, $|20\rangle$, $|01\rangle$ and $|1\rangle$.

FIG. 4. (a-c) SQUIDET stability diagram vs $\Phi = \Phi_1 = \Phi_2 = \Phi_0/2$ and $V$ at fixed gate voltage ($\eta = 0.5$). The surface has been colored having the green and blue channels proportional to $p_{12}$ and $p_{12}$, respectively, and shows the periodicity in the blockade regime states as a function of magnetic flux. The red arrow represents Eq. (6) with $N_p = 3$. (b) Basic pumping cycle of a SQUIDET with $N_p = 1$, a fully symmetrical configuration having $\Delta_0(0) = \Delta_0(0) = \Delta_0 = \xi = 185\mu$V. Here $I_{12}^{c}$ and $I_{12}^{a}$, normalized by $I_0 = \Delta_0/2\xi$, represent the dominant rates moving the system between $p_{10}$ and $p_{10}$ states. During this particular cycle from Eq. (6) the bias voltage in set to $V = \Delta_0/\xi$, and $\eta = 0.5$. (c) Frequency dependence of the normal source-drain current for different $N_p$ values.
InAs nanowire with epitaxial aluminium as a single-electron transistor with fixed tunnel barriers

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We report on fabrication of a single-electron transistor using InAs nanowires with epitaxial aluminium with fixed tunnel barriers made of aluminium oxide. The device exhibits a hard superconducting gap induced by the proximized aluminium cover shell and it behaves as its metallic counterpart. We confirm that unwanted extra quantum dots can appear at the surface of the nanowire, but can be prevented either by covering the nanowire with aluminium, or by inserting a layer of GaAs between the InAs and Al. Our work provides another approach to study proximized semiconducting wires.
Open questions

In the previous lecture we discussed electrons in terms of waves. However, in this lecture we spoke about particles, their charge, etc.

Are we running two horses at the same time?
How single-electron effects interplay with quantum interference?
These problems are solved to some extent and we will discuss them later.