Hall Effect in Semiconductors

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Hall Effect

When a magnet field is applied perpendicular to the direction in which a charged particle (electron or hole) is moving, the particle will be deflected as shown.

The force on the particle will be $F = q(\xi + vXB)$

In the x-direction, the force will be $F_x = q(\xi_x + v_yXB_z)$

To counter the flow of particles in the x-direction, a field $\xi_y = vxBz$ will be created so that the net force is zero. The applied field is called the Hall effect, and the resulting voltage, $V_H = \xi_y d$.
Drift velocity for an electron in the x-direction is:
\[ <v_x> = -\frac{J_x}{qn} \]

where \( J \) is the current density, \( n \) is the number of carriers and \( q \) is the charge.

Defining the Hall coefficient, \( R_H = \frac{1}{qn} \), then
\[ E_y = v_x B_z = -\frac{J_x B_z}{qn} = R_H J_x B_z \]

and
\[
\begin{align*}
n &= -\frac{1}{qR_H} = \frac{J_x B_z}{qE_y} = \frac{I_x / wt}{q(V_{AB} / w)} = \frac{I_x B_z}{qtV_{AB}}
\end{align*}
\]

Measuring the resistance gives the resistivity:
\[ \rho (\Omega \cdot m) = \frac{Rwt}{L} = \frac{V_{cd}}{I_x}(L/wt) \] where \( w \) is the width of the bar.

Since conductivity, \( \sigma = 1/\rho = q\mu_n n \), the mobility is:
\[ \mu_n = \frac{\sigma}{qn} = \frac{1}{\rho}/q(1/qR_H) = R_H/\rho \]
Moving electrons experience a force due to a perpendicular B field:

\[ F = q(\mathbf{\varepsilon} + \mathbf{\nu} \times \mathbf{B}) \]

An electric field develops in response to this force.
- The sign of this field perpendicular to the flow of current determines the carrier type.
- Carrier Density and mobility can also be calculated.

\[ \mu = \frac{1}{qn\rho} \]

\[ E_y = \frac{J x B_z}{qn} \]

\[ \rho = \text{resistivity} \]
Hall effect - principles

- Consider a block of conducting medium through which a current of electrons is flowing caused by an external field.
- A magnetic field $B$ is established across the conductor, perpendicular to the current $(\theta = 90^\circ)$.
- The electrons flow at a velocity $v$.
- A force perpendicular to both the current and field is established.

$$F = qvB\sin\theta_v b$$ [N]
Magnetic deflecting force

\[ F = q(v_d \times B) \]

Hall electric deflecting force

\[ F = qE_H \]

When an equilibrium is reached, the magnetic deflecting force on the charge carriers are balanced by the electric forces due to electric Field \( E_H \).

\[ q(v_d \times B) = qE_H \]

\[ E_H = (v_d \times B) \]

Where \( v_d \) is drift velocity.
The relation between current density and drift velocity is

\[ \nu_d = \frac{J}{ne} \]

Where \( n \) is the number of charge carriers per unit volume.

\[ E_H = (\nu_d \times B) \]
\[ E_H = \left( \frac{J}{ne} \right) \times B \]
\[ E_H = \left( \frac{1}{ne} \right) \times JB \]
\[ E_H = R_H \times JB \]

\[ R_H (\text{Hall coefficient}) = \frac{1}{ne} = \frac{E_H}{JB} \]
If $V_H$ be the Hall voltage in equilibrium , the Hall electric field.

$$E_H = \frac{V_H}{d}$$

Where $d$ is the width of the slab.

$$R_H = \frac{E_H}{J_B}$$

$$R_H = \frac{1}{J_B \times \frac{V_H}{d}}$$

If $t$ is the thickness of the sample,

Then its cross section is $dt$ and current density

$$J = \frac{I}{dt}$$

$$V_H = R_H J_B d$$

$$V_H = R_H \left( \frac{I}{t} \right) B$$

$$R_H = \frac{V_H t}{IB}$$
Since all the three quantities $E_H$, $J$ and $B$ are measurable, the Hall coefficient $R_H$ and hence the carrier density can be found out.

Generally for N-type material since the Hall field is developed in negative direction compared to the field developed for a P-type material, negative sign is used while denoting hall coefficient $R_H$. 
Hall voltage

- The electrons are pulled towards the front side surface of the conductor (holes in semiconductors move towards the back).
- A voltage develops between the back (positive) and front (negative) surface. This voltage is the Hall voltage and is given by:

\[ V_{out} = \frac{IB}{qnd} \]  

\( d \) is the thickness of the hall plate, 
\( n \) is the carrier density [charges/m\(^3\)] and 
\( q \) is the charge of the electron [C].
Hall voltage

- If the current changes direction or the magnetic field changes direction, the polarity of the Hall voltage flips.
- The Hall effect sensor is polarity dependent,
  - may be used to measure direction of a field
  - or direction of motion if the sensor is properly set up.
- The term $1/qn \ [m^3/C]$ is material dependent and is called the Hall coefficient $K_H$ (or $R_H$).
Hall coefficient

The hall voltage is usually represented as:

\[ V_{out} = K_H \frac{IB}{d} \] [V]

• Hall coefficients vary from material to material
• Are particularly large in semiconductors.
• Hall voltage is linear with respect to the field for given current and dimensions.
• Hall coefficient is temperature dependent and this must be compensated if accurate sensing is needed.
Hall coefficient - cont.

- Hall coefficient is rather small - of the order of 50 mV/T
- Most sensed fields are smaller than 1 T
- The Hall voltage can be as small as a few µV
- Must in almost all cases be amplified.

**Example**, the earth’s magnetic field is only about 50 µT so that the output is a mere 25 µV
Hall Effect and Mobility

The Hall effect is easier to measure in semiconductors than in metals, since the carrier concentration is smaller:

When one carrier dominates, we have a Hall coefficient:

$$R_H = \frac{E_H}{JB} = \frac{1}{nq}$$

where

$$V_H = wE_H$$

Hall measurements can tell us whether a semiconductor is n-type or p-type from the polarity of the Hall voltage:

When one carrier dominates, we can write the conductivity:

$$\sigma = ne\mu_e \quad (pe\mu_h)$$

So the mobility can be written:

$$\mu = |R_H| \sigma$$

Measuring $R_H$ and $\sigma$ will thus give: sign, concentration, and mobility of carrier,
For a semiconductor with significant concentrations of both types of carriers:

\[ R_H = \frac{E_H}{JB} = \frac{p\mu_h^2 - n\mu_e^2}{e(n\mu_e + p\mu_h)} \]

So if holes predominate \((p\mu_h > n\mu_e)\), \(R_H > 0\) and the material is said to be p-type, while if \(R_H < 0\) (as for simple metals), the material is said to be n-type.
Hall effect sensors - practical considerations

- Hall voltages are easily measurable quantities
- Hall sensors are among the most commonly used sensors for magnetic fields:
  - simple, linear, very inexpensive, available in arrays
  - can be integrated within devices.
- Errors involved in measurement are mostly due to temperature variations and the averaging effect of the Hall plate size
- These can be compensated by appropriate circuitry or compensating sensors.
Hall effect sensors - applications

- Example: measuring power
- The magnetic field through the hall element is proportional to the current being measured
- The current is proportional to voltage being measured
- The Hall voltage is proportional to product of current and voltage - power
Hall element power sensor

- LOAD
- IRON CONTAINING MAGNETIC FLUX
- HALL DEVICE
- POWER INDICATOR
- TRANSFORMER
- R_C
- LINE
Hall sensors used to control a CDROM motor