Model selection for monetary policy analysis – How important is empirical validity?

Q. Farooq Akram*
Research Department, Norges Bank

Ragnar Nymoen
University of Oslo

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Abstract

We investigate the economic significance of trading off empirical validity of models against other desirable model properties, and the potential loss from assuming model uncertainty to be higher than justified and basing monetary policy on a relatively robust model, or on a suite of models. We find that differences in model specification and in estimates of key parameters across comparable models may entail widely different monetary policy and macroeconomic performance. Our results therefore caution against compromising the empirical validity of models when selecting a model for policy analysis. We also find that potential costs from basing monetary policies on the relatively robust model or on a suite of models, even when it contains the valid model by assumption, can be substantial. This suggests an important role for econometric modelling and evaluation in exploitation of available information to reduce model uncertainty as much as possible. Our investigation is based on three alternative econometric systems of wage and price inflation in Norway.

Keywords: *Model uncertainty; Economic significance; Econometric modelling.*

JEL Codes: *C52, E31, E52.*

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1 Introduction

Macroeconomic models generally influence monetary policy. Often, policy makers have to choose among several models with different properties and different policy implications. There are, however, several criteria for selecting among models, even when they conform with one's views on important characteristics of an economy. Common criteria for choosing among models include consistency with some preferred economic theory and data. In addition, one may evaluate models on the basis of how easily they would enable one to communicate their mechanisms and properties to a wider community, including users of models within the model-developing institution. Accordingly, one may use criteria such as the wider community's familiarity with a model, its similarity with models already in use, and its transparency. Parsimony of a model is another sought-after property, as it enhances a model's transparency and facilitates its updating.

However, model selection often involves a trade-off between desirable characteristics of a model since a model may perform well on one set of criteria and poorly on another set. For example, an economic theory may prefer a different model than available evidence suggests, while different statistical tests may favour different models; cf. Mankiw (1989) and Pagan (2003). One may also encounter cases where a model that is overwhelmingly supported by available evidence is more demanding to use and/or communicate than a model that is not fully data consistent.

Furthermore, several models may be available that appear almost equally attractive in light of economic theory and/or the empirical evidence considered. In the face of such model uncertainty, one may let policy be informed by the least fault-tolerant model, i.e. the model in which a deviation from the optimal policy has the most severe consequences, as suggested by the robust control approach towards designing monetary policy under model uncertainty; see e.g. Hansen and Sargent (2001). Alternatively, one may refrain from selecting one particular model and decide to base policy on a suite of models consistent with the Bayesian approach towards dealing with model uncertainty; see e.g. Levin *et al.* (1999). Such approaches may also be adopted, despite clear empirical evidence in favour of one particular model, if the policy maker has strong preferences for implementing monetary policy that is robust to model uncertainty; see Cogley and Sargent (2005).[1]

In this paper we investigate the economic significance of trading off empirical validity of models with other desirable model properties, and the potential loss from assuming the model uncertainty to be higher than justified and basing monetary policy on a relatively robust model, or on a suite of models. Model uncertainty may seem to be high if available information is underutilized and/or

[1] Cogley and Sargent (2005) have investigated the consequences of basing monetary policy on a suite of models for the inflation experience of the US since the 1960s. They ascribed the Fed's failure to curb the inflation of the 1970s to its preferences for implementing robust monetary policy in the face of perceived model uncertainty. Accordingly, the Fed placed too little weight on the natural rate model, even after accumulation of strong statistical evidence in its favour. The suite of models was assumed to contain three competing models of the Phillips curve, including a downward-sloping one, a vertical one in the long run, and one that did not allow any exploitable trade-off between inflation and unemployment at any horizon.

some information sources are left unexplored. Potential loss from basing policy on a model that has been selected on the basis of other criteria than empirical validity and/or robustness properties can be higher than in the case of a relatively robust model or a suite of models. Therefore, we do not explicitly consider model selection on the basis of such criteria. The focus of our investigation is consistent with Granger (1992, 2001), who emphasizes that consequences of one's decision regarding model choice for policy analysis should be evaluated in terms of their economic significance. Accordingly, possible loss from choosing one model specification ahead of another should not be assessed only in terms of predictive power forsaken or in terms of other desirable statistical properties, but also in terms of economic/welfare loss that may arise from the model's influence on policy decisions.

We find that differences in model specification and even differences in estimates of key parameters across similar models may entail widely different interest rate setting to counteract effects of a shock. Hence, we find that economic performance can suffer substantially if policy is based on a model other than the one that represents the closest approximation to the economy. We also find substantial costs of perceiving model uncertainty to be higher than justified and responding to such uncertainty by adopting robust monetary policies, either based on the least-fault tolerant model, or on a suite of models, even when it contains the valid model by assumption. Thus, adoption of a robust model or a wide suite of models does not seem to be preferable to using available information and methods to derive an empirically valid model or a narrow set of empirically comparable models for policy making.

Our investigation is based on three alternative econometric systems of wage and price inflation in Norway. They represent three competing perspectives on modelling wage and price inflation in the Norwegian debate since the early 1980s; see Bårdsen *et al.* (2005). One of the systems is derived in the light of open economy models of imperfect competition in product markets and a wage-bargaining framework, consistent with the institutional features of the Norwegian economy. The other two systems consist of open economy Phillips curves for prices and wages, where one of them specifies vertical Phillips curves for wage and price inflation through parameter restrictions.

The three systems may be considered alternative blocks of the supply side in a macroeconometric model for medium-term analysis. To close each of the three systems, we conduct our policy analysis by embedding, in turn, the three wage and price systems in a well specified macroeconometric model of Norway; see Bårdsen *et al.* (2003, 2005) and Akram *et al.* (2005). This model is part of the suite of models maintained by Norges Bank.

We proceed as follows. Section 2 develops the three systems and examines their econometric properties. The three systems seem to explain wage and price inflation almost equally well, despite significantly different econometric properties. A broad set of tests favours the system based on open economy models of imperfect competition in product markets and a wage-bargaining framework.

Section 3 derives monetary policy response to demand and supply shocks based on each of the three systems. We assume that the central bank is a flexible inflation targeter, such as Norges Bank; see Norges Bank (2007). Specifically, it is assumed that the central bank decides on an interest rate path that minimizes variability in deviations from the inflation target and variability in the output gap, while ensuring that inflation will reach its target in the short or medium run. The central bank is assumed to accommodate concern for output stabilisation by choosing an appropriate horizon for achieving the inflation target. This characterization of monetary policy seems consistent with the actual practice of inflation-targeting central banks; see e.g. Meyer (2004), Giavazzi and Mishkin (2006) and Blinder (2006).

Section 4 investigates the potential loss in terms of macroeconomic performance, viewed in the light of the central bank's objectives, from basing monetary policy on a different model than the one representing (the closest approximation to) the actual wage and price inflation process. This analysis helps to bring forward the potential macroeconomic loss when the econometric properties of a model are traded off against other desirable properties. It also sheds light on the potential loss from basing monetary policy on the least-fault tolerant model owing to concern for robustness in the face of perceived model uncertainty. To shed light on the potential loss that can result from basing monetary policy on a suite of models, we let monetary policy be informed by the three alternative econometric systems of wage and price inflation and compare the performance of such a policy with that based on the model representing the actual wage and price inflation process.

Finally, Section 5 presents our main conclusions. Appendix A contains precise definitions of the time series of the variables, while Appendix B sketches the derivation of interest rate rules and their implementation.

2 Alternative dynamic models of wages and prices

This section develops and evaluates three alternative empirical systems of Norwegian wage and price inflation. The three systems are derived from a common VAR model by imposing restrictions in the light of relevant economic theories and following a "general to specific"modelling strategy.[2]

2.1 The VAR for wages and prices

The three alternative economic models of wage and price inflation are estimated on quarterly time series for the mainland economy of Norway, i.e. exclusive of its offshore sector. The sample period is 1972q4–2001q4 and the time series are seasonally non-adjusted; see Appendix A for precise definitions. The three models are encompassed by a statistical model in the form of an open VAR. The endogenous variables are the (natural logarithm) of average hourly wages (w_t) and the

[2]A set of PcGive batch and data files that replicate our results in Section 2 can be downloaded as SelectionForPolicy.zip from http://folk.uio.no/rnymoen/.

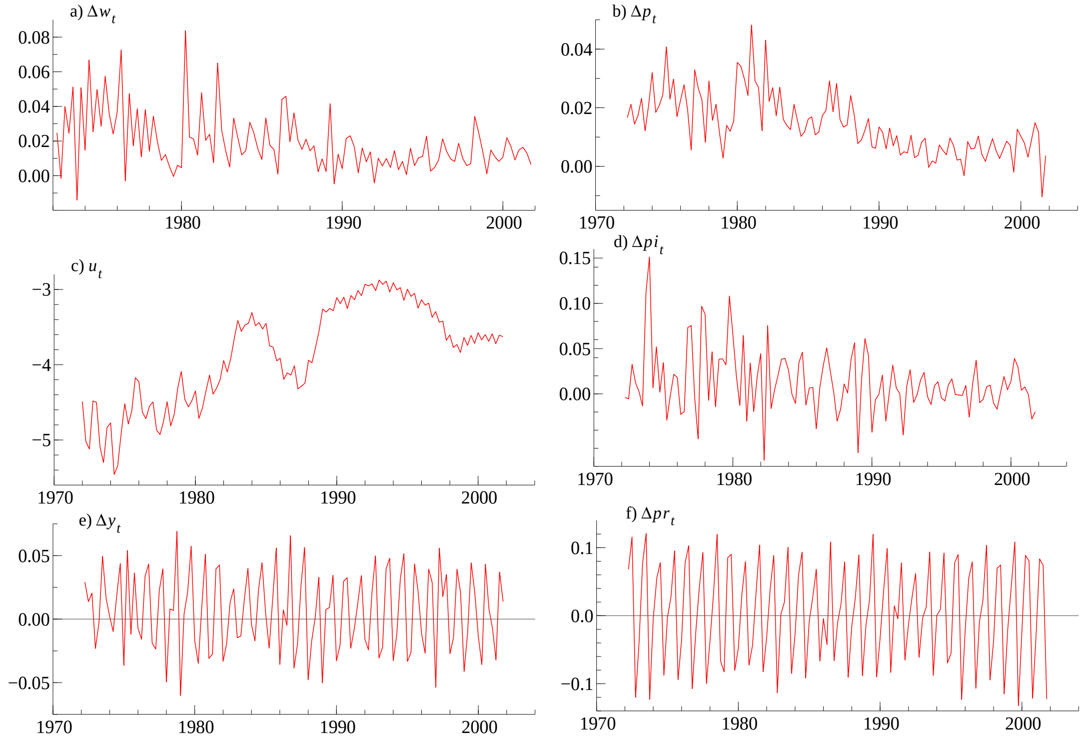

Figure 1: *Quarterly time series of key variables over the period: 1972q1–2001q4. a) Wage growth Δw_t, b) consumer price inflation Δp_t, c) the log of unemployment rate u_t, d) import price inflation Δpi_t, e) mainland-Norway GDP growth Δy_t, and f) productivity growth Δpr_t.*

consumer price index (p_t). Long run trends in the wages and consumer prices are explained by labour productivity (pr_t), import prices (pi_t) and the unemployment rate (u_t). The VAR also includes growth in the mainland GDP (Δy_t), as the unemployment rate may not be a sufficient indicator of "demand pressure"in consumer products markets. We condition on the variables pr_t, pi_t, u_t and Δy_t since these variables have been found to be weakly exogenous with respect to the parameters defining cointegration vectors and the associated adjustment coefficients. In particular, the cointegrating relationships for wages and prices are statistically insignificant in the dynamic models for these variables; see Akram *et al.* (2005). The weak exogeneity assumption has also been tested, and not rejected, by Bårdsen *et al.* (2003) and Bårdsen *et al.* (2005, Ch. 9.3) on earlier vintages of the data set employed in this paper.

Figure 1 shows the time series of the first differences of wages and consumer prices, Δw_t and Δp_t, in panels a) and b). Seasonality is evident, but also a tendency towards reduced variance for particularly Δw_t and a reduction in their means over time. Panel c) indicates that this development may be partly explained by the markedly higher and more stable unemployment rate in the 1990s. Panel d) shows that imported inflation (Δpi_t) has also declined and become less volatile over time. Panels e) and f) show the growth rates of output (Δy_t) and labour productivity (Δpr_t), which largely reflects the output growth.

The VAR also condition on payroll and indirect tax rates ($\tau 1_t$ and $\tau 3_t$), changes in standard working hours (Δh_t) and the growth in electricity prices (Δpe_t). The tax-rates are needed to link wage-earnings of employees to wage-costs of employers, which may be partly or completely passed over to consumer prices. Changes in standard working hours are required to account for wage growth owing to wage compensations for reductions in the length of the working week over the sample period; see Nymoen (1989). Electricity prices have become an important explanatory variable for CPI-inflation since the deregulation of the electricity market in the year 1991. The price of electricity may vary substantially with weather conditions because of fluctuations in demand and Norway's hydroelectric-based energy system.

The VAR also includes the following deterministic terms: Two intercepts, three (centered) seasonal dummies and two incomes policy dummies ($P_{d,t}$ and $W_{d,t}$) to account for price controls and occasions of highly centralized wage settlements. Inclusion of deterministic terms is important for the statistical adequacy of the VAR, and therefore for the validity of the cointegration analysis; see Andreou and Spanos (2003). The empirical relevance of the income policy dummies have been documented in several earlier studies including Nymoen (1991) and Bowitz and Cappelen (2001).

The VAR was eventually specified with three lags for w_t and p_t, two lags for the non-modelled levels variables pr_t, pi_t and u_t, and just one lag for the tax rates. Changes in standard working hours and growth in electricity prices were included in the VAR contemporaneously while growth in mainland GDP (over two quarters) was included with a lag. In sum, the preferred VAR includes 54 parameters that are estimated on 117 observations. An econometric evaluation of the VAR suggests that it is well specified. None of the standard tests were rejected at the 10% level of significance.[3] The next two subsections present and evaluate the three wage and price systems that are derived from the VAR.

2.2 Wage-price equilibrium correction models (ECMs)

The following long-run relationships for wages and prices in Norway are consistent with open economy models of imperfect competition in product markets and a wage-bargaining framework:

$$w_t = p_t + \gamma_{w1} pr_t - \gamma_{w2} u_t + \gamma_{w0} + \varepsilon_{w,t}, \tag{1}$$

$$p_t = \gamma_{p1}(w_t + \tau 1_t - pr_t) + (1 - \gamma_{p1}) pi_t + \gamma_{p2} \tau 3_t + \gamma_{p0} + \varepsilon_{p,t}, \tag{2}$$

where the slope coefficients are non-negative and γ_{w0} and γ_{p0} are intercepts. A detailed derivation is given in Bårdsen *et al.* (2005, Ch 5) based on the assumption that after removal of deterministic location shifts by dummies, w_t, p_t, pr_t and pi_t are integrated of degree one, but cointegrated. The unemployment and tax rates are assumed to be without unit-roots, though non stationarity due to location shifts in these variables such as shifts in the mean of u_t is likely. Constant parameters

[3] The diagnostics for this (unrestricted) VAR are $F = 0.68[0.85]$ (autocorrelation), $F = 0.98[0.58]$ (heteroscedasticity), and $\chi^2 = 4.09[0.40]$ (normality); see Table 2 for more details.

in (1) and (2) then require co-breaking, such that $\varepsilon_{w,t}$ and $\varepsilon_{p,t}$ become stationary deviations from the two long-run relationships.

Bårdsen *et al.* (2003) show that (1) and (2) represent identified cointegrating relationships. Equation (1) is interpreted as a steady-state wage equation, which is implied by a bargaining framework where wages are determined by domestic prices and productivity, while the rate of unemployment affects the mean level of the implied wage share: $w_t - p_t - pr_t$. Equation (2) is interpreted as a steady-state price equation which incorporates both the effects of mark-up pricing behaviour on unit labour costs and import prices, represented by the elasticities γ_{p1} and $(1 - \gamma_{p1})$, respectively. Since the price variable p_t is (log of) the consumer price index, it is also affected by a measure of indirect taxes, $\tau 3$.

We find support for full-rank consistent with two cointegrating relationships. The estimation of (1) and (2) then gives the following estimates: $\gamma_{w1} = 1$, $\gamma_{w2} = 0.15$, $\gamma_{p1} = 0.6$ and $\gamma_{p2} = 0.5$.[4] The estimates of the intercept terms, γ_{w0} and γ_{p0}, are close to the sample means of the cointegrating relationships defined by the elasticity estimates. These estimates are consistent with those found in a number of previous studies using data samples of different lengths, periods and level of aggregation; see e.g. Nymoen (1991), Johansen (1995), Bårdsen *et al.* (1998), Bårdsen and Nymoen (2003), Bårdsen *et al.* (2003).

We derive a system of structural equilibrium correction models (ECMs) of wages and prices, (3), to represent wage-price dynamics consistent with (1) and (2) being long-run cointegrating relationships. The ECMs were estimated by the FIML method.[5]

$$
\begin{aligned}
\Delta w_t = {} & -\underset{(0.01)}{0.11}[w_{t-3} - p_{t-1} - pr_{t-1} + 0.15u_{t-2}] + \underset{(0.07)}{0.16}\Delta w_{t-1} \\
& + \underset{(0.02)}{0.06}\Delta pr_t - \underset{(0.12)}{0.54}\Delta h_t - \underset{(0.002)}{0.02} W_{d,t} \\
\\
\Delta p_t = {} & \underset{(0.01)}{-0.06}[p_{t-3} - 0.6(w_{t-3} - pr_{t-1} + \tau 1_{t-1}) - 0.4pi_{t-1} + 0.5\tau 3_{t-1}] \\
& + \underset{(0.05)}{0.16}\Delta p_{t-2} + \underset{(0.03)}{0.21}\Delta w_t + \underset{(0.03)}{0.13}\Delta w_{t-1} + \underset{(0.02)}{0.04}\Delta_2 y_{t-1} \\
& - \underset{(0.01)}{0.01}\Delta pr_t + \underset{(0.01)}{0.03}\Delta pi_t + \underset{(0.01)}{0.06}\Delta pe_t - \underset{(0.001)}{0.01} P_{d,t}
\end{aligned}
\tag{3}
$$

The equation for wages in system (3) shows that nominal quarterly wage growth, Δw_t, adjusts to correct deviations from its long-run relationship (1).[6] The 't-value' associated with the equilibrium correction coefficient is -11, suggesting that one of the main implications of the underlying

[4] See the section on cointegration analysis in the available batch file.

[5] The seasonal dummies and intercepts are suppressed from the equation. Coefficient standard errors are given below their respective coefficients.

[6] The 3-quarter lag in wages and the 2-quarter lag in unemployment only affect how the dynamics is parameterized, not the interpretation of $[w_{t-3} - p_{t-1} - pr_{t-1} + 0.15u_{t-2}]$, which is consistent with (1).

theoretical framework is strongly supported by the evidence. The remainder of the equation first contains a tendency of positive autocorrelation in the quarterly wage growth rate which arises mainly because most wage adjustments usually take place in the second and the third quarters. The last part of the wage equation contains short-run effects of productivity growth (Δpr_t) and wage compensation for changes in standard working hours (Δh_t).

The price equation in system (3) shows that also the hypothesized long-run relationship for prices, (2), is supported by the data; the estimate of the equilibrium-correction coefficient has a 't-value' of -6. The other variables also have straightforward interpretations: There is a statistically significant positive autoregressive coefficient consistent with commonly observed inflation persistence. Actually, the autoregressive coefficient would have been larger without the inclusion of the other explanatory variables in the model. Wage growth has a relatively strong effect with an elasticity of 0.34 over two quarters. There is also a small positive effect of product demand growth if sustained over two quarters. The short-run effects of productivity and import prices are quite small when compared with the corresponding long-run elasticities in the equilibrium-correction term. In contrast, the estimated elasticity of growth in electricity prices (Δpe_t) and their effect are numerically significant since they fluctuate widely; Δpe_t varies in the range of $\pm 25\%$.

2.3 Phillips curve models

The Phillips curve models originate from the same unrestricted VAR as the ECM. The system of Phillips curves favoured by data is reported in (4):

$$
\begin{aligned}
\Delta w_t = & -\underset{(0.07)}{0.20}\,\Delta w_{t-1} + \underset{(0.13)}{0.27}\,\Delta p_{t-1} + \underset{(0.13)}{0.28}\,\Delta p_{t-2} \\
& - \underset{(0.004)}{0.01}\,\Delta u_t - \underset{(0.001)}{0.01}\,u_{t-1} - \underset{(0.002)}{0.016}\,W_{d,t} \\
\Delta p_t = & \ \underset{(0.07)}{0.10}\,\Delta p_{t-1} + \underset{(0.06)}{0.20}\,\Delta p_{t-2} + \underset{(0.05)}{0.31}\,\Delta w_t + \underset{(0.04)}{0.16}\,\Delta w_{t-1} + \underset{(0.01)}{0.05}\,\Delta_2 y_{t-1} \\
& + \underset{(0.01)}{0.03}\,\Delta pi_t + \underset{(0.01)}{0.07}\,\Delta pe_t - \underset{(0.001)}{0.01}\,P_{d,t}
\end{aligned}
\tag{4}
$$

This system is consistent with an open-economy triangular model of inflation, whereby inflation is determined by demand-pull, cost-push and expectations inherent in the wage-price spiral; see e.g. Gordon (1997), Stock and Watson (1999) and Bårdsen *et al.* (2003). The long-run Phillips curve implied by this system is downward sloping since the maximum likelihood estimates of the coefficients of the price and wage inflation terms do not add up to homogeneity.[7]

[7] However, since we have an open economy model, the Phillips curve does not rationalize that there is an exploitable trade-off between steady-state inflation and unemployment. This is because, in the steady state of the complete macroeconometric model used inflation would be determined by foreign inflation and nominal exchange rate depreciation consistent with PPP, while the wage growth would be equal to foreign inflation (in domestic currency) plus productivity growth.

In the short run, effects of lagged inflation appear in both the wage equation and the price equation. This is in contrast to the system of wage-price ECMs, (3), but not unexpected since lagged inflation is correlated with both of the two equilibrium correction terms in (3), which by the definition of Phillips curves are omitted from the current system.

The Phillips curve model with a vertical long-run Phillips curve is obtained by imposing homogeneity restrictions on the combined effects of wages and prices in both of the equations in system (4):

$$
\begin{aligned}
\Delta w_t &= -\underset{(--)}{0.18}\Delta w_{t-1} + \underset{(0.12)}{0.58}\Delta p_{t-1} + \underset{(0.12)}{0.60}\Delta p_{t-2} \\
&\quad - \underset{(0.006)}{0.01}\Delta u_t - \underset{(0.0013)}{0.003}u_{t-1} - \underset{(0.003)}{0.017}W_{d,t} \\
\Delta p_t &= \underset{(0.07)}{0.21}\Delta p_{t-1} + \underset{(0.06)}{0.26}\Delta p_{t-2} + \underset{(--)}{0.26}\Delta w_t + \underset{(0.03)}{0.16}\Delta w_{t-1} + \underset{(0.01)}{0.07}\Delta_2 y_{t-1} \\
&\quad + \underset{(0.01)}{0.04}\Delta pi_t + \underset{(0.01)}{0.07}\Delta pe_t - \underset{(0.001)}{0.01}P_{d,t}
\end{aligned}
\tag{5}
$$

A notable difference between system (4) and system (5) is that the coefficient estimate of u_{t-1} is substantially lower in the latter: -0.003 vs -0.01 in (4).[8] The price equation in (5), however, does not differ much from that in (4). This suggests that the homogeneity restrictions of the long-run Phillips curve model are largely data-consistent in the price equation.

2.4 Model evaluation

In the following, we examine the econometric properties of the three systems. It appears that the choice between them may not seem obvious if one only considers their in-sample explanatory power, or focuses on only some of their econometric properties. However, it becomes easier to choose between them if one uses information from a broad set of standard diagnostic tests.

The overall explanatory power of all of the models is fairly high and does not seem to differ much across models. In particular, both the systems of Phillips curves, (4) and (5), provide nearly the same level of fit to actual wage and price growth over the sample period; see Figure 2.

More precisely, Table 1 shows that the overall explanatory power, as measured by the standard deviations of the equation residuals, which are denoted $\hat{\sigma}_{\Delta w}$ and $\hat{\sigma}_{\Delta p}$, is less than 1% for (growth in) wages and less than 0.5% for prices. This may be reckoned as quite satisfactory since the data are seasonally unadjusted. The explanatory power of the three models, especially that of the Phillips curves, is lower than that of the unrestricted as well as the cointegrated VAR models.[9]

[8]One can set this estimate at some preferred value and re-estimate the model. However, the econometric performance of the resulting model would become inferior to that of system (5).

[9]The standard errors of the unrestricted VAR are 0.82% for wage growth and 0.32% for inflation.

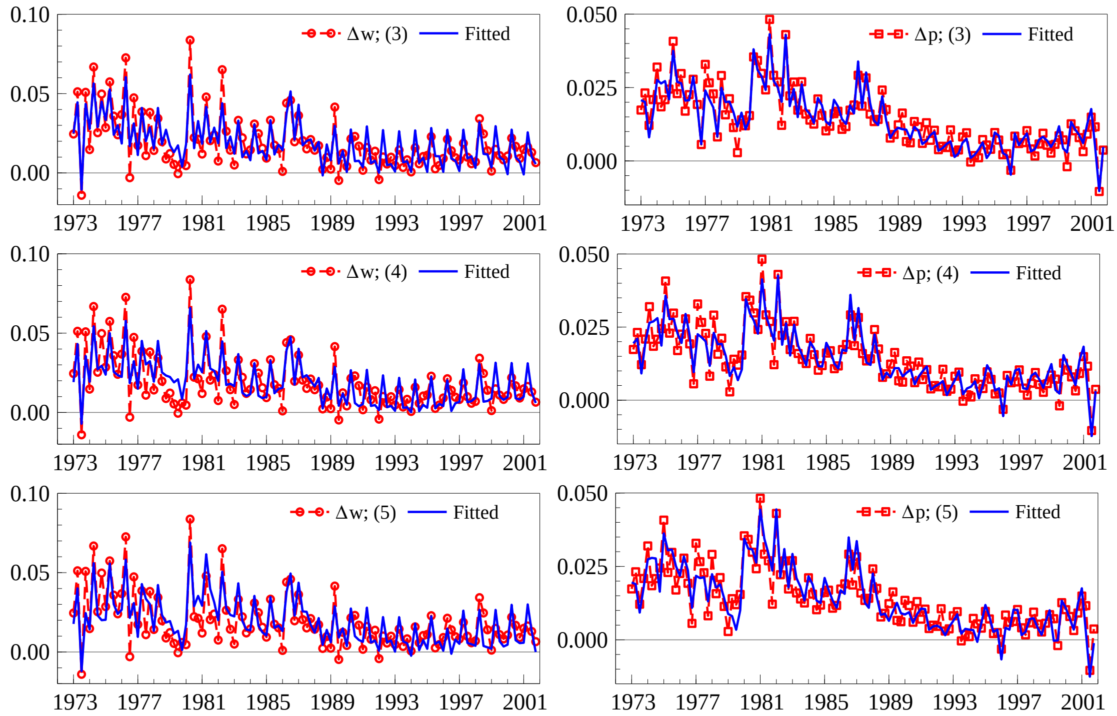

Figure 2: *Explanatory power of the three models (3)–(5) for (quarterly) growth in wages and prices over the sample period: 1972q4–2001q4. The left-hand column presents the explanatory power of the three models for growth in wages (Δw), while the right-hand column presents that for growth in prices. Dashed lines with circles represent actual values of growth in wages, while dashed lines with boxes represent the actual growth in prices. Solid lines represent the corresponding fitted values.*

For wages, the model with the vertical Phillips curve, (5), has lower explanatory power than the model with the downward-sloping Phillips curve, (4), and the model of wage-price ECMs, (3). For prices, however, both Phillips curve models provide the same explanatory power, although lower than the ECM of prices.

Table 1: Explanatory power of the cointegrated VAR and the three systems

System	*VAR*	**(3)**	**(4)**	**(5)**
$\hat{\sigma}_{\Delta w}$	0.85%	0.88%	0.93%	0.98%
$\hat{\sigma}_{\Delta p}$	0.33%	0.36%	0.48%	0.46%

Notes: The three systems have been estimated by FIML on a sample for the period 1973q1–2001q4. The columns show diagnostics for the (cointegrated) VAR and the three systems of wages and prices obtained as special cases of the VAR.

Yet, given the relatively high explanatory power of all three models, there may not seem to be any harm in selecting one of them to e.g. facilitate communication with the wider community, including financial markets, politicians, academics and the general public. On such grounds, one could select e.g. the system of the vertical Phillips curve for policy analysis instead of the other two systems: (3) and (4).

However, choosing the system with the vertical Phillips curve, or that of the downward-sloping one, may seem less obvious in the light of a further examination of their econometric properties.

One may start by examining the validity of employing the FIML method for estimation since it rests on specific assumptions regarding residuals; see e.g. Andreou and Spanos (2003). Any violation of these assumptions on available data may signal model misspecification, such as omitted variables and/or wrong functional form. It is also of interest to test formally whether the explanatory power of the three models is comparable to that of the VAR model from which they originate.

Table 2 shows that the evidence is not favourable to the systems of Phillips curves, especially to that of the vertical Phillips curve, while it does not reject the validity of the system of wage-price ECMs. For all three systems and the VAR model, the null hypotheses of normally distributed errors are not rejected by the chi-square distributed test. The corresponding p-values are well above 50%. However, the hypotheses of no-autocorrelation and heteroscedasticity are strongly rejected for the system with the vertical Phillips curve (5). The F-distributed tests of autocorrelation (up to order 5) and heteroscedasticity in the column for system (5) suggest that the two null hypotheses are rejected at even the 1% level of significance.

Table 2: Diagnostics for the cointegrated VAR and the three systems

System		*VAR*	(**3**)	(**4**)	(**5**)
Autocorrelation	F	0.71[0.80]	1.12[0.33]	1.41[0.12]	2.17[0.00]**
Heteroscedasticity	F	1.08[0.32]	1.09[0.29]	1.32[0.05]	1.82[0.00]**
Normality	χ^2	2.98[0.56]	2.34[0.67]	0.95[0.92]	3.02[0.55]
Overidentification	χ^2		34.3[0.10]	69.1[0.00]**	118.8[0.00]**

Notes: The number in square brackets are p-values for the respective statistics, which are adjusted for degrees of freedom. References:
Autorcorrelation test: Godfrey (1978), Doornik (1996); Heteroscedasticity test: White (1980), Doornik (1996); Normality test: Doornik and Hansen (1994); and Overidentification test: Anderson and Rubin (1950) and Sargan (1988, p. 125).

In contrast to the results for system (5), the F-tests do not reject the null hypotheses of no-autocorrelation and heteroscedasticity at the 5% level of significance for systems (4) and (3). The corresponding tests for the VAR model confirm that it constitutes a valid foundation for deriving the three systems.

We now apply an encompassing test to examine whether the three systems originating from the VAR model explain the data nearly as well as the VAR model itself; cf. Hendry and Mizon (1993). This test supplements the impression gained by comparing the standard deviations of the residuals from the different systems in Table 1. Specifically, we test whether or not the sets of over-identifying restrictions distinguishing each of the three models from the VAR model are accepted by a chi-square distributed test. Table 2 shows that the set of (24) over-identifying restrictions specifying system (4) is strongly rejected, the insignificance of the misspecification tests notwithstanding. As one may expect, system (5) is also strongly rejected. In contrast, the set of (25) over-identifying restrictions specifying system (3) is not rejected at the 5% significance level; the p-value is 10%.

To further investigate which of the restrictions that particularly harm the empirical validity of model (4), we have employed a likelihood-ratio test for the null hypothesis that the equilibrium correction terms, which appear in system (3), are insignificant in system (4). This null hypothesis was strongly rejected by the test on the full data sample and different subsamples. On the full sample the test-statistic was 36.2 (=$\chi^2(2)$), with a p-value of 0.00%. On a subsample starting in 1981q1, the test-statistic was 12.5 (=$\chi^2(2)$), with a p-value of 0.2%. For the vertical Phillips-curve model, (5), the statistical rejection of the dynamic homogeneity restrictions, in addition to the invalid exclusion of the equilibrium correction terms, explains the relatively higher value of the associated *Overidentification* statistic; see Table 2.

To summarize this section, one may say that competing dynamics wage and price equations can be formulated and estimated in such a way that requirements like a reasonable fit and correctly signed coefficients are met. If a model evaluation ends with checking such properties, one might argue that one faces model 'uncertainty' since available data are not sufficiently informative and do not strongly favour one model ahead of another, i.e. show little recalcitrance. However, if one extends model assessment and employs a number of standard misspecification tests, data may be quite recalcitrant and helpful in model selection; cf. Hoover (2001). Section 3 suggests that econometric differences between models may signal substantial differences in their policy implications.

2.5 Macroeconometric model

To investigate the policy implications, we embed the three alternative systems for wage and price inflation in a well specified and well documented macroeconometric model of Norway. This way we are able to close the wage and price system while conditioning on foreign prices.

The macroeconometric model is a version of the model developed in Bårdsen *et al.* (2003, 2005) which has been documented and employed in several studies, including Akram *et al.* (2005).[10] The model is (log) linear and estimated on quarterly aggregate data for the period 1972–2001. In addition to a system of wages and prices, the model contains equations for aggregate demand, unemployment, import prices, labour productivity, credit demand, and three asset prices: house prices, domestic equity prices and the nominal exchange rate. Foreign variables and domestic government expenditures and electricity prices are treated as exogenous variables.

In particular, short-run fluctuations in aggregate demand are determined by the real exchange rate, real interest rate and wealth effects from house prices and equity prices. Thus, a change in consumer prices affects aggregate demand through its effect on the real exchange rate and the real interest rate. However, an increase in domestic consumer prices mainly depresses aggregate demand because of subsequent appreciation of the real exchange rate dominates effects of lower real interest rates. The unemployment rate follows growth in output in the short run, as in an Okun's law relationship. In addition, it exhibits reversion towards its equilibrium rate over time.

[10] Available from http://www.norges-bank.no/upload/import/publikasjoner/arbeidsnotater/pdf/arb-2005-09.pdf.

Monetary policy, represented by short-term interest rates, has direct effects on the asset prices, credit and aggregate demand, but is neutral in the long run. The model may be considered a backward-looking model in the sense that it does not make the expectation-formation processes explicit. The whole model may be considered econometrically well-specified, when system (3) is included, with apparently invariant parameters with respect to changes in monetary policy over the sample; see Bårdsen *et al.* (2003, 2005) for documentation. The lack of evidence for significant parameter instability in the face of shifts in monetary policy is in line with Ericsson and Irons (1995) and Rudebusch (1995). In the following, we assume that the model will remain invariant to the monetary policy decisions we consider as they are undertaken within the monetary policy regime. That is, we consider the monetary policy decisions to be too modest to induce noticeable changes in the model; cf. Leeper and Zha (2003).

3 Monetary policy implications of the models

We start by presenting the monetary policy objectives, which are pursued by following horizon-dependent interest rate rules in real time; see Appendix B for a detailed derivation. In Section 3.2, we derive interest rate rules based on the three versions of the macroeconometric model and analyse the economic and policy implications of the three systems.

3.1 Monetary policy objectives and the interest rate rule

To devise an optimal response to an observable shock that occurs at time τ, we assume that a forward-looking central bank minimises the following loss function with respect to an interest rate path i_τ, $i_{\tau+1}$, $i_{\tau+2}$,..$i_{\tau+H-1}$, $i_{\tau+H}$, $i_{\tau+H+1}$,...:

$$L_\tau = V(\pi_t - \pi^*) + \lambda V(y_t), \tag{6}$$

subject to the constraint that the conditional mean of inflation in period $\tau + H$ is close to the constant inflation target, π^*:

$$\mathsf{E}_\tau \pi_{\tau+H} \approx \pi^*. \tag{7}$$

The term $V(\cdot)$ is a variance function while the variables $\pi_t - \pi^*$ and y_t denote the inflation gap and the output gap, respectively. We assume that the inflation target is constant in line with the Norwegian monetary policy. The inflation gap as well as the output gap are assumed to be stationary variables, i.e. integrated of order zero. The constant parameter λ indicates the degree of concern for fluctuations in the output gap relative to that for fluctuations in inflation; t is a period indicator. The loss function is a reformulation of a quadratic loss function assuming that the discount factor is close to one. E_τ is an expectation operator conditional on available information at time τ e.g. regarding observed and predicted shocks to the economy and their properties.

This constrained optimization approach implies that the central bank does not compromise its inflation-targeting objective; cf. Smets (2003). Ensuring stability of inflation around its target is given less importance as the central bank is willing to trade off the variance of inflation against that of the output gap in accordance with its value of λ. This approach is consistent with what Faust and Henderson (2004) regard as best-practice monetary policy.[11]

We use H to represent the policy horizon, which we define as the number of periods of appropriate length, here quarters, during which the policy *interest rate* will deviate from its neutral value and stimulate or cool off the economy. The policy horizon H can take on any discrete value from zero onwards. Thus, the precise policy horizon, when measured as the number of periods, would be $H + 1$, because $H \geq 0$.

The target horizon, i.e. the number of periods *inflation* (forecast) will deviate from target, will generally be linked and be close to the policy horizon, but the exact relationship will depend on the shock and the model used. Inflation will typically converge asymptotically to its target rate in the wake of a shock in a dynamic model. Hence, imposing an exact target horizon is generally not meaningful. We assume that the inflation target will be largely achieved when the policy interest rate has almost converged with its neutral rate in period H. This assumption seems to be consistent with published future paths of interest rates and inflation; see e.g. Norges Bank (2007). Therefore, we will focus on the policy horizon in the following.

We envision that in the face of a shock, the central bank derives a set of interest rate paths, each of them satisfying the constraint (7) for different policy horizons, i.e. H values. Then, from this set of interest rate paths, it selects and implements the interest rate path, and the corresponding policy horizon, that would minimise the loss function (6).

However, there can be numerous interest rate paths that satisfy the constraint (7) for every possible value of H. By only considering interest rate paths that adhere to some reasonable pattern, however, the set of relevant interest rate paths can be limited to the number of policy horizons (H values) considered by the central bank.

We assume that the central bank initiates changes in the interest rate when the shock occurs at time τ and thereafter allows the interest rate to return gradually towards its neutral rate, (i_0), as commonly observed; see e.g. Sack and Wieland (2000).[12] Then, if the model is stable and linear, an interest rate path corresponding to a specific policy horizon H can be obtained from the

[11] Accordingly, "...best-practice policy can be summarized in terms of two goals: First, get mean inflation right; second, get the variance of inflation right.", but "...getting the mean right may be the goal of greatest importance"; see Faust and Henderson (2004, pp.117–118).

[12] It is quite common in the (relevant) literature to rule out interest rate paths that seem unreasonable. In contrast to our approach, this is typically obtained by including a measure of volatility in interest rates in the objective function of the central bank; see e.g. Smets (2003), Taylor (1999) and the references therein.

following interest rate rule:

$$i_{\tau+m} = i_0 + (1-\varrho_H)\frac{\beta_\varepsilon}{(1-\phi)}\varepsilon_\tau + \varrho_H(i_{\tau+m-1} - i_0) \quad ; \, m = \, 0, \, 1, \, 2, ..., H, \, H+1, ... \tag{8}$$

The response coefficient $\beta_{\varepsilon,H}$, which is defined as $(1-\varrho_H)\beta_\varepsilon/(1-\phi)$, determines how much the interest rate must deviate initially from the neutral rate to offset inflationary effects of the shock at time τ, ε_τ. This initial deviation is thereafter eliminated gradually, depending on the value of an interest rate smoothing parameter ϱ_H.[13] Both the response coefficient and the degree of smoothing depend on the policy horizon, as indicated by the subscript H.[14] The parameter ϕ denotes the degree of persistence in the shock and is assumed to be positive and less than one: $0 \leq \phi < 1$. It follows that a persistent shock requires a stronger initial response ($\beta_{\varepsilon,H}$) than a transitory shock (for which $\phi = 0$) for a given degree of interest rate smoothing (ϱ_H) and value of β_ε.

The value of β_ε depends on the shock and the model. It is a derived parameter whose value increases with the inflationary effects of the shock over the simulation horizon, but declines with the effectiveness of interest rates in checking inflation; see Appendix B. The derived parameter β_ε can be considered a constant (shock- and model-dependent) parameter, if the transmission mechanism of the shock and interest rate is super exogenous with respect to the policy changes considered; see Engle *et al.* (1983). We have assumed this to be the case since the policy decisions, which amounts to choosing H as explained below, may be considered modest; cf. Leeper and Zha (2003).

The policy horizon enters the interest rate rule through the interest rate smoothing parameter, ϱ_H. It is defined as $\delta^{1/(H+1)}$ and takes on a value in the range of $(0, 1)$ depending on H (for a chosen fraction δ). Interest rates are considered converged with the neutral rate when just a fraction δ of the initial interest rate deviation from the neutral rate remains. The choice parameter δ also determines how close inflation is to its target when monetary policy becomes neutral; cf. constraint (7).

The degree of smoothing increases with the policy horizon in a concave fashion since $\varrho_H = \delta^{1/(H+1)}$. In particular, $H = 0$ will lead to (almost) no interest rate smoothing ($\varrho_H = \delta$), while large values of H will imply a high degree of interest rate smoothing since $\varrho_H = \delta^{1/(H+1)} \longrightarrow 1$ when $H \longrightarrow \infty$. The case of $H = 0$ refers to the case when the policy maker allows interest rates to deviate from their reference rate in just a single period.

However, the value of the response coefficient $\beta_{\varepsilon,H}$ declines (in a geometric fashion) with the policy horizon or degree of interest rate smoothing. In particular, $\beta_{\varepsilon,H} \approx \beta_\varepsilon/(1-\phi)$ when $H = 0$, while $\beta_{\varepsilon,H} \longrightarrow 0$ when $H \longrightarrow \infty$ since $\varrho_H \longrightarrow 1$. This suggests that if a very long policy horizon is

[13]In practice, interest rates are changed in steps. By employing rule (8), we approximate interest rate moves as continuous. This simplification is consistent with the literature and with interest rate projections of central banks who publish such projections; see e.g. Norges Bank (2007).

[14]This rule resembles a Taylor-type rule with interest rate smoothing except that it is the determinant of the inflation gap, i.e. ε_τ, that enters the rule rather than the inflation gap itself; see Taylor (1999) and the references therein.

chosen, the interest rate needs to deviate only marginally from its neutral rate, but this deviation has to be quite persistent.

A long horizon would help subdue the required initial response to a relatively persistent shock. In particular, if persistence in a shock is matched by persistence in interest rates, i.e. $\varrho_H = \phi$, the response coefficient $\beta_{\varepsilon,H}$ becomes equal to β_ε. In contrast, a short horizon may imply a particularly large deviation from the neutral interest rate in the face of a persistent shock.

Clearly, the parameters characterising the interest rate rule depend on the policy horizon (H), ceteris paribus. By varying H, one can vary the interest rate rule and thus the complete interest rate path as well as the level of the loss, L_τ.

It follows that once the rule (8) is implemented in the model, the optimal policy response to a shock can be found by minimising the loss function (6) with respect to H. The optimal value of H, H^*, will then define the optimal interest rate change, β_{ε,H^*}, the optimal degree of interest rate smoothing, ϱ_{H^*}, as well as the optimal level of loss conditional on a given (version of the) macroeconometric model, $\mathcal{M}$. The optimal policy horizon H^* will depend on the degree of concern for fluctuations in the real economy, λ. Thus, β_{ε,H^*} and ϱ_{H^*}, will also depend on λ.

In the following we are particularly interested in analyzing the effect of model choice on the loss and consequently the policy, represented by the policy horizon. We therefore express the loss function (6) as an explicit function of H and $\mathcal{M}$:

$$L_\tau \equiv L(H, \mathcal{M}). \tag{9}$$

3.2 Monetary policy response to shocks

For the sake of brevity, we refer to the three versions of the macroeconometric model, which would differ from each other only by the wage and price system included, as 'ECM', 'PCM' and 'PCMr', respectively. Specifically, ECM includes the system of wage-price equilibrium correction models, (3); PCM includes the system of Phillips curves (4) while PCMr includes the system with the vertical Phillips curve (5), which is a restricted version of (4).

The difference between the three versions of the macroeconometric model essentially consists of difference in restrictions on the overall equilibrium correction behaviour. The version with the wage-price error correction model has more equilibrium correction mechanisms than the version with the downward-sloping Phillips curve; which in turn is more equilibrium correcting than the version with a vertical Phillips curve system.

3.2.1 Interest rate rules in response to demand and supply shocks

The monetary policy response to a shock is characterised by rule (8), where the response coefficient (β_ε) is entirely shock- and model-dependent. In the following, β_d and β_s, refer to the response

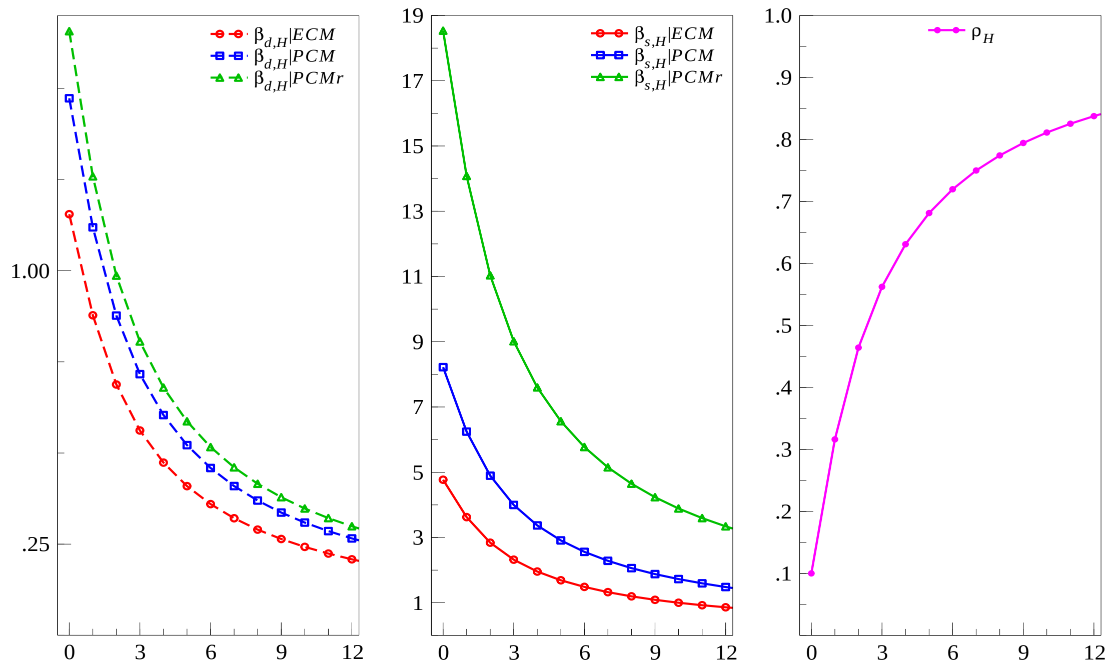

Figure 3: *Left: Initial interest rate responses to the demand shock (in percentage points) implied by different policy horizons (horizontal axes), $\beta_{d,H}$. They are suggested by the three versions of the macroeconometric model:* ECM, PCM *and* PCMr. *Middle: Initial interest rate responses to the supply shock implied by different policy horizons, $\beta_{s,H}$, suggested by the three model versions. Right: Interest rate smoothing, ϱ_H, associated with different policy horizons.*

coefficients in the case of a demand shock and a supply shock, respectively. Their estimates are derived for each of these model versions. A demand shock contributes to an initial growth of aggregate demand by one percentage point over four quarters, while a supply shock initially increases price inflation by one percentage point over four quarters; see Appendix B.3 for details.

Values of ϱ_H for different policy horizons are obtained from $\varrho_H = \delta^{1/(H+1)}$, where we set δ to, say, 0.1 to define convergence. That is, we would consider an interest rate deviation (of e.g. one percentage point) from the neutral rate/reference rate converged with the reference rate when it deviates not more than 1/10 of the initial deviation from the reference rate. Alternative values of δ do not bring about qualitatively different results.

Estimates of the horizon-specific response coefficients $\beta_{\varepsilon,H}$ for a given shock and model can be obtained from its defining formula, i.e. $(1 - \varrho_H)\beta_\varepsilon/(1 - \phi)$, for different degrees of persistence in the shock and interest rates, ϕ and ϱ_H, respectively.

The left and the middle frames of Figure 3 display values of the response coefficient for the (transitory, $\phi = 0$) demand shock and the supply shock, respectively, implied by the three model versions. The values of the response coefficients are presented for different policy horizons in the range 0–12 quarters. The right frame of Figure 3 depicts the degree of interest rate smoothing ϱ_H implied by the different policy horizons. Before analyzing the results for each of the two shocks, we note the following general features.

First, an increase in the policy horizon reduces the required initial interest rate response to a shock, but raises the degree of interest rate smoothing, ceteris paribus; see Figure 3. For example, the required initial interest rate response declines substantially if the policy horizon is increased from 0 to 8 quarters. This must, however, be accompanied by an increase in the degree of interest rate smoothing, ϱ_H, from 0.1 to 0.77 (right frame). And second, an increase in the policy horizon from a low level leads to a larger reduction in the response coefficient than an increase in the policy horizon from a relatively high level. This is due to the concave relationship between the degree of interest rate smoothing and the policy horizon, since $\varrho_H = \delta^{1/(H+1)}$, which in turn leads to a convex relationship of geometric form between the response coefficient and the policy horizon. A linear relationship between the degree of interest rate smoothing and the policy horizon would have implied a linear relationship between the response coefficient and the policy horizon. However, the results to be presented would not have changed qualitatively.

3.2.2 Economic and monetary policy implications

Figure 3 shows that both PCM and PCMr suggest a stronger response to both shocks than ECM, for all policy horizons. In particular, PCMr suggests a stronger response to both shocks than PCM and ECM. Figure 3 also reveals substantial differences between the monetary policy response to the two shocks across the three model versions.

In the case of the demand shock, the interest rate response is relatively low varying in the range of 0.25–1.75 percentage points across the three models. The differences across the three models are relatively small. This reflects that the interplay of the wage and price system with the rest of the model, particularly with aggregate demand and unemployment, is not that different across the three wage and price systems. A demand shock has relatively larger inflationary effects, while a monetary policy shock has relatively stronger deflationary effects in PCM and especially in PCMr, relative to ECM. This is also reflected in the response coefficients, but to a smaller extent since the two effects partly outweigh each other.

Figure 4 sets out the economic performance of the policies suggested by the three models in the face of the demand shock. The economic performance associated with every policy horizon is measured by the standard deviations of the inflation and the output gaps; see the left column. A policy horizon fully describes the interest rate rule for given values of the response coefficient, β_d; see Section 3.1. Hence, the optimal policy is found by minimizing the loss function (6) with respect to the policy horizon H. We assume that the parameter reflecting concern for output gap fluctuations, λ, is 0.5. The right column presents values of the loss functions under different policy horizons relative to their value under the optimal policy horizon (H^*) for a given model version.

We define the relative loss, $\Delta L(H; \mathcal{M})$, as:

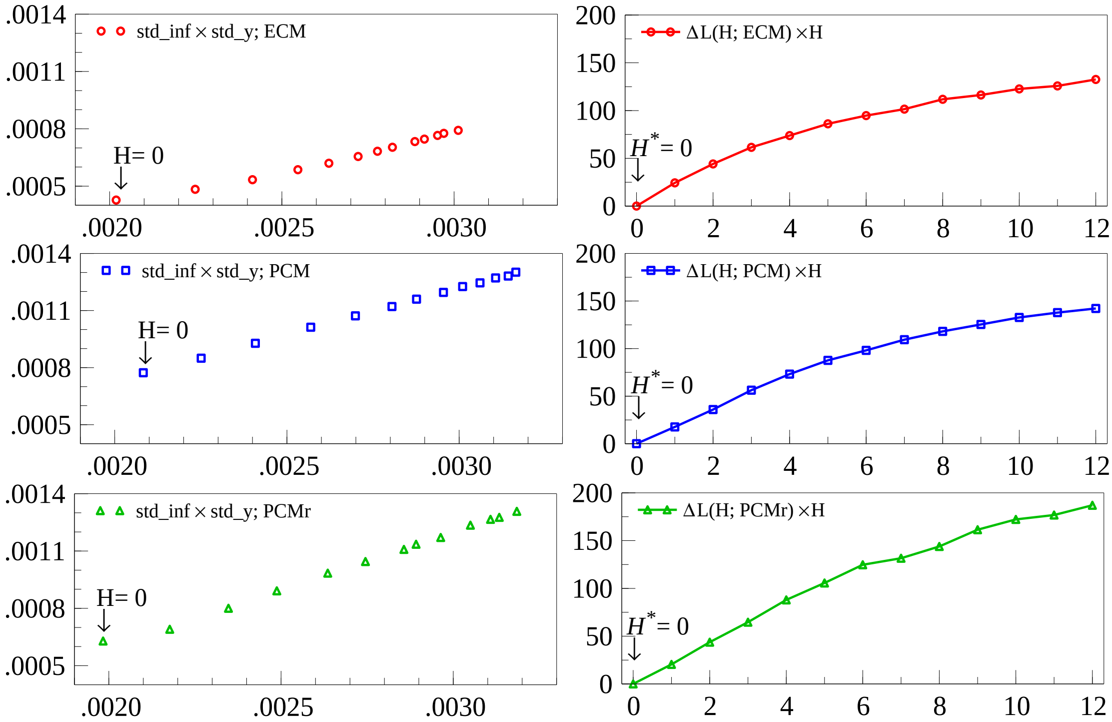

Figure 4: *Economic performance and optimal policy suggested by the three models in the face of the demand shock. Left column: Trade-offs between standard deviations of the inflation gap and the output gap (horizontal axis) associated with different (policy) horizon-specific rules in response to the demand shock. The trade-offs are plotted for rules associated with policy horizons (H) in the range of 0–12 quarters. Here and elsewhere, the trade-offs associated with different horizons follow each other, where that one for $H=0$ is indicated. Right column: Values of the relative loss function (in %), defined by equation (10), at the different policy horizons (horizontal axis).*

$$\Delta L(H;\mathcal{M}) \equiv \frac{L(H;\mathcal{M}) - L(H^*;\mathcal{M})}{L(H^*;\mathcal{M})}. \tag{10}$$

Here, $L(H;\mathcal{M})$ denotes the level of loss by choosing H conditional on a specific model (version) $\mathcal{M}$, while $L(H^*;\mathcal{M})$ expresses the loss under optimal policy horizon H^* conditional on model $\mathcal{M}$. It follows that $\Delta L(H;\mathcal{M}) > 0$ for $H \neq H^*$ while $\Delta L(H;\mathcal{M}) = 0$ when $H = H^*$, assuming the loss function is continuous in H and there is a unique optimum.

As expected, there is no conflict between the objectives of price stabilization and output stabilization in the case of the demand shock; see Figure 4, left column. Moreover, it appears that both objectives can be promoted by reducing the policy horizon as much as possible. Hence, a policy horizon of zero appears as the most efficient one. The values of the relative loss functions are zero, i.e. at their optimal level, for $H = 0$; see right column. Hence, the optimal policy horizon would be zero irrespective of which wage and price system we implement in the model. This finding is consistent with the bulk of studies suggesting that demand shocks should be counteracted as aggressively as possible, since inflation can be stabilized jointly with output. However, even though the three systems imply the same optimal policy horizon, they imply different interest rate

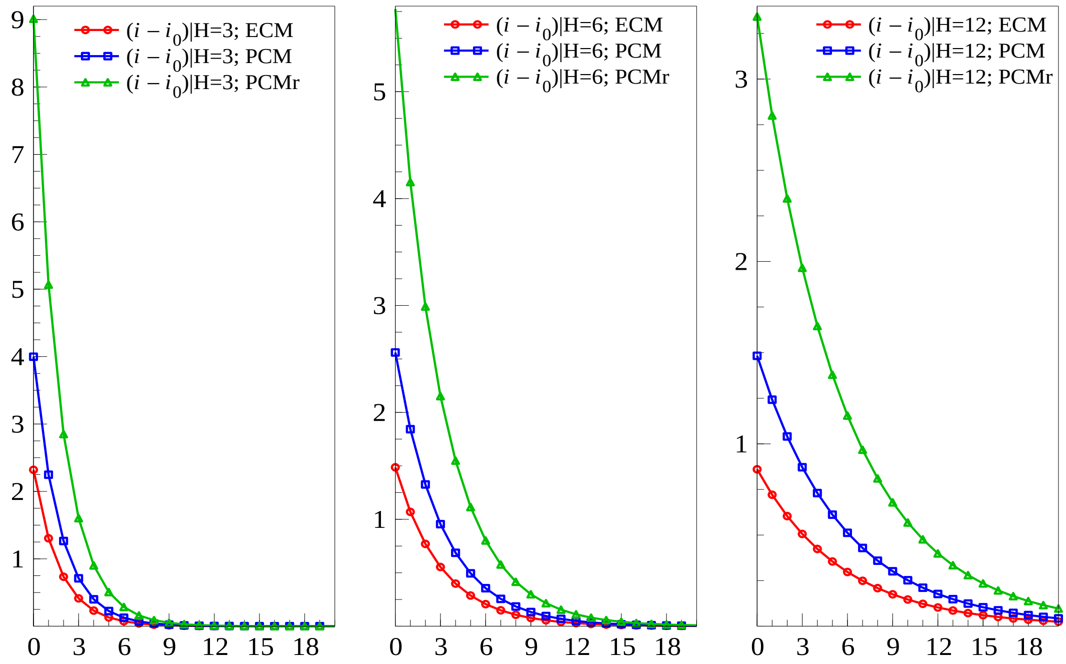

Figure 5: *Interest rate paths over time suggested by the three models in the face of the supply shock. The three frames show interest rate paths associated with the policy horizons of 3, 6 and 12 quarters, respectively. The interest rates are measured as deviation from the reference interest rate paths in percentage points, while the horizontal axes depict periods in quarters.*

response to the demand shock and macroeconomic performance; see Figure 3. It may therefore not be immaterial which system is adopted to derive and implement the optimal policy response to the demand shock, as suggested by the different levels of standard deviations of the inflation gap and the output gap implied by the different model versions; see the left column for e.g. $H = 0$.

In the case of the supply shock, the implied monetary policy response is much stronger and differs widely across the three models; see Figure 3, middle frame. Figure 5 depicts the interest rate paths implied by the three models for three different policy horizons: 3, 6 and 12 quarters. These paths exhibit clearly the differences in monetary policy response implied by the three models. Moreover, the gap in policy implications of PCM and PCMr is wider than the gap between those of PCM and ECM. Notably, if there was an exogenously provided fixed policy horizon, the three wage and price systems, particularly the two systems of Phillips curves, would have suggested substantially different monetary policy responses to the supply shock. One may therefore say that apparently minor parameter restrictions can alter the policy implications of a model substantially.

The large differences in the response coefficients across the three models can be ascribed to the associated wage and price systems, specifically to differences in the autoregressive coefficients across the three systems and to the effect of the unemployment term. The autoregressive coefficients largely determine the degree of persistence in the inflationary effects of the supply shock, i.e. how fast the inflationary effects of the transitory supply shock diminish. The larger the persistence, the more lasting the inflationary effects and the stronger the required interest rate response will be. Due to the relatively weak effect of unemployment on the wage growth in system (5), relative to those in systems (3) and (4), the monetary policy becomes less effective in PCMr, than in ECM and PCM. Hence, a relatively larger change in the interest rate is required in the case of PCMr than in the cases of ECM and PCM.

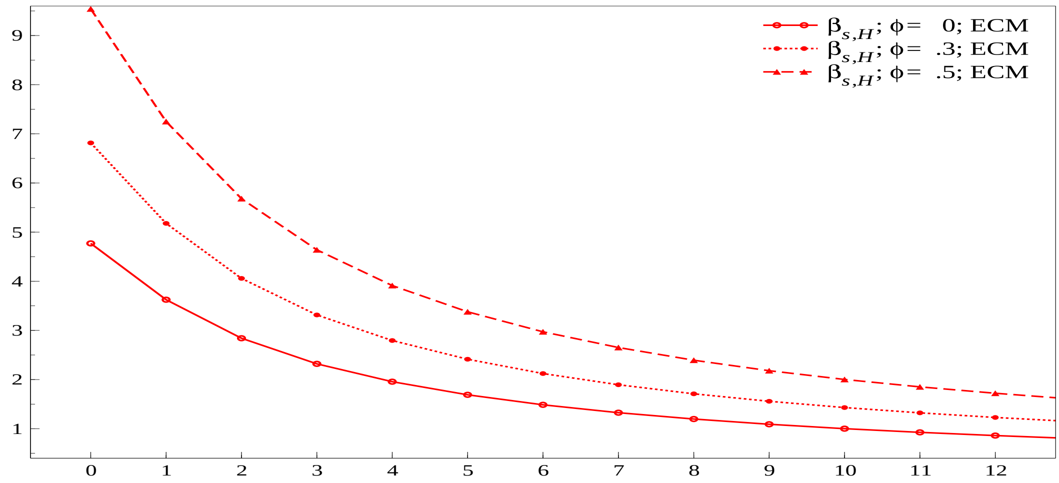

Figure 6: *Initial interest rate response suggested by ECM to supply shocks with different degrees of persistence, ϕ. The initial interest rate response is implied by policy horizons in the range 0–12 quarters (horizontal axis).*

For example, the degree of persistence implied by the lagged and contemporaneous terms of wages and prices in system (5) is higher than that implied by system (4), which itself implies higher persistence than system (3). Consequently, the inflationary effects of the transitory supply shock are more lasting in the case of PCMr than in the case of PCM, which itself implies more lasting effects than ECM. Accordingly, the required interest rate response is higher in the case of PCMr than in the case of PCM and relatively low in the case of ECM.

The systems of Phillips curves, (4) and (5), which have relatively stronger autoregressive effects than the system of wage-price ECMs, (3), effectively make the transitory supply shock a more persistent one than the system of ECMs. In the system of ECMs, persistence is to a large extent modelled by lagged wage and price growth variables and the equilibrium correction terms. In terms of a VAR in levels, this entails that some of the characteristic roots are on the unit circle, while others are on the stable side of the unit circle. The system of the downward-sloping Phillips curve (4), however, implies a reduction in the number of stable roots since the direct equilibrium correction in wage and price setting is omitted, and as a consequence, an increase in the degree of persistence of any shock. The system of vertical Phillips curve system (5), has even more in-built persistence, because extra unit-roots are implied by the homogeneity restrictions; cf. Bårdsen and

Nymoen (2006).

The analytical expression for the required interest rate response suggests that an increase in the degree of persistence in a shock increases the required interest rate response; see rule (8). Figure 6 illustrates that the required interest rate responses in the case of ECM can become comparable to those implied by PCM and PCMr if we raise the persistence in the supply shock; cf. Figure 5.

Figure 7 presents the economic performance of (optimal and suboptimal) policies employed in response to the supply shock. The left column of the figure shows that there is a trade-off between price and output stabilization for different ranges of policy horizons. Specifically, in the case of ECM and PCM there is a trade-off in the range of 0 to 8 quarters. Policy horizons that are longer than 8 quarters appear inefficient as both price and output stabilization can be improved by shortening the policy horizon. The opposite is the case for PCMr. In this case, the trade-off curve is associated with policy horizons that are longer than 6 quarters, while policy horizons shorter than 6 seem inefficient.

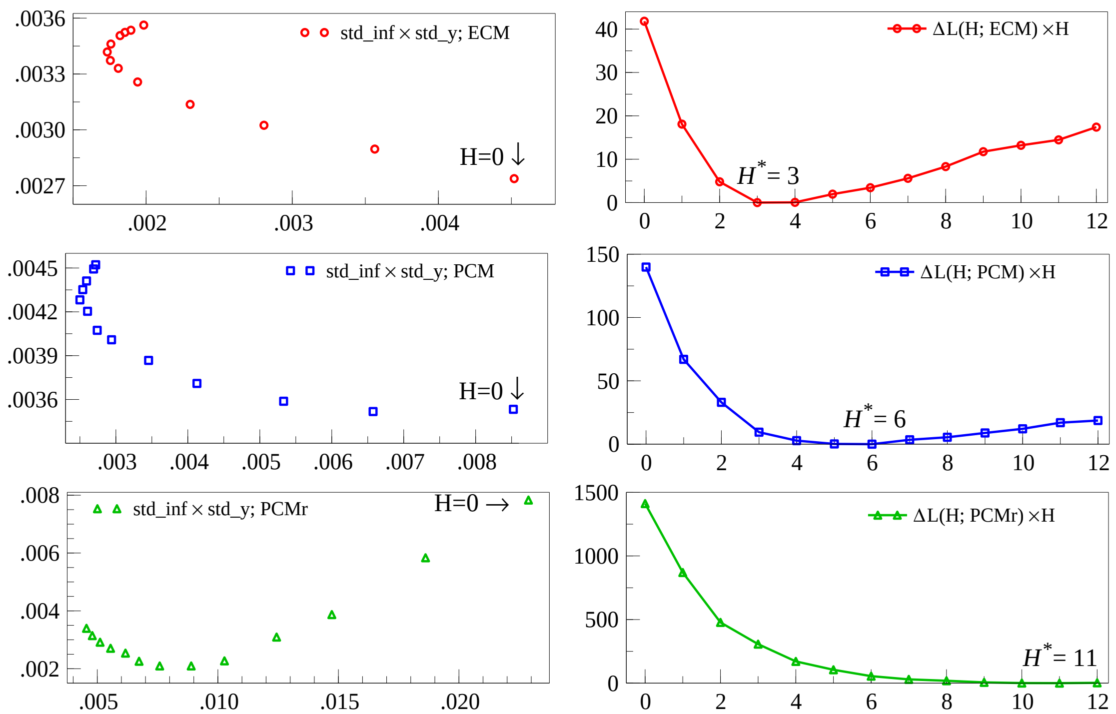

Figure 7: *Economic performance and optimal policy suggested by the three models in the face of the supply shock. Left column: Trade-offs between standard deviations of the inflation gap and the output gap (horizontal axis) associated with different horizon-specific rules in response to the supply shock. The trade-offs are plotted for rules associated with policy horizons (H) in the range of 0–12 quarters, where that for H = 0 is indicated. Right column: Values of the relative loss function (in %) for the different policy horizons (horizontal axis); see definition (10).*

Figure 7 shows, in right column, that the three models recommend quite different policy horizons. Even though the efficiency frontiers for ECM and PCM are defined by almost the same policy horizons, the optimal horizon is 3 quarters in the case of ECM, but 6 quarters in the case of PCM.

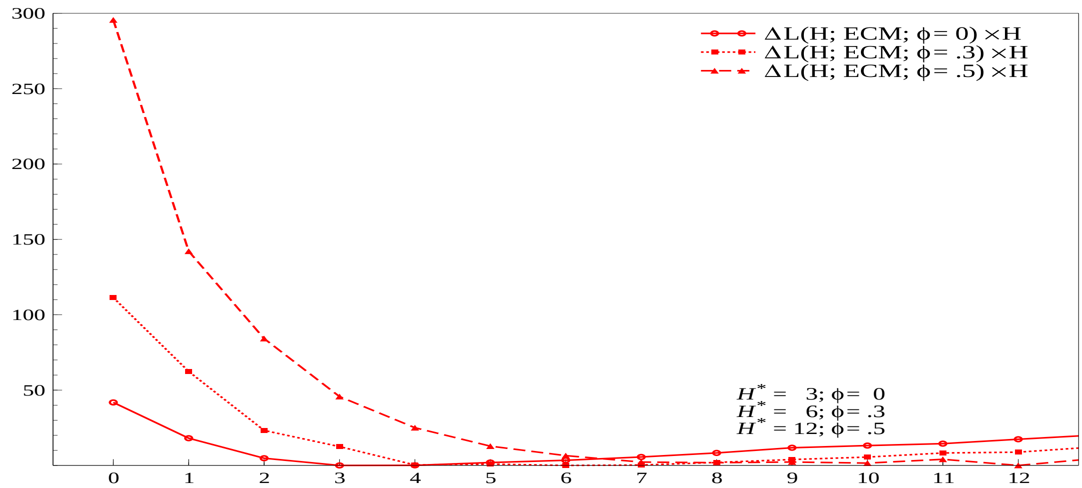

Figure 8: *Relative losses at different policy horizons, 0–12, when monetary policy responds to supply shocks with different degrees of persistence. The relative losses (in %), defined by (10), are based on* ECM.

In the case of PCMr the optimal policy horizon is 11 quarters. (An increase in the value of λ from 0.5 would have increased the optimal policy horizons in all three models.)

The quite different optimal policy horizons imply widely different interest rate paths. They can be seen from Figure 5, where the interest rate path favoured by ECM appears in the left frame, that by PCM in the middle frame, while that favoured by PCMr would be close to that for $H = 12$ in the right frame. Both ECM and PCM suggest about the same initial interest rate increase for $H = 3$ and $H = 6$, respectively; 2.25 and 2.5. However, the degree of interest rate smoothing associated with $H = 6$ is relatively higher, i.e. 0.72, which makes monetary policy contractionary over a relatively longer period than when $H = 3$. PCMr suggests an initial interest rate increase of about 4 for $H = 11$, while the implied interest rate smoothing is 0.83; see Figure 3. Hence, PCMr suggests a more aggressive as well as a more prolonged contractionary monetary policy stance than the other two models.

In sum, the more persistent the inflationary effects are in a model, the longer is the preferred policy horizon. Both a high degree of persistence in the inflationary effects of the shocks and a strong policy response, which also increases with the degree of persistence, contribute to relatively large economic fluctuations, i.e. high standard deviation of prices and the output gap. This is especially the case at especially short policy horizons. A relatively long horizon leads to a less aggressive policy response and a more prolonged contractionary policy. This helps to achieve a better synchronization between the destabilizing effect of the persistent inflationary effects with the stabilizing effect of monetary policy. Monetary policy thereby becomes more effective in stabilizing the economy.

The above results underscore the importance of imposing valid coefficient estimates when conducting monetary policy analysis. As demonstrated, restrictions on parameter values can have a more profound effect on monetary policy than alterations in model specification. The differences in suggested policy horizons can be mainly ascribed to alteration in the persistence of inflation by changes in the model specification. For example, Figure 8 shows that ECM would have produced similar results if the shock had been more persistent. If the persistence in the shock had been 0.3, ECM would have suggested an optimal policy horizon of 6 quarters, and of 12 quarters if the persistence had been 0.5. Imposing seemingly weak restrictions to make the model e.g. more presentable or transparent may therefore not be an innocuous act.

4 Costs of basing monetary policy on an invalid model

In the following, we investigate potential costs of basing monetary policy on a model that turns out to be invalid, or on a suite of models containing the valid model. For brevity, we only focus on monetary policy response to the supply shock and only consider monetary policy rules that are optimal in their respective models. Moreover, we only present results where ECM is assumed to be the valid model of the economy, as suggested by the empirical evidence.[15]

4.1 Potential costs when the model selected turns out to be invalid

The following analysis sheds light on the potential loss when the econometric properties of a model are traded-off against other desirable properties, and monetary policy is based on the least-fault tolerant model owing to concern for robustness in the face of perceived model uncertainty.

Figure 9 suggests the potential costs of choosing rules that are optimal in PCM and in PCMr in response to the supply shock when ECM is by assumption the valid model; the former rules are referred to as the PCM-rule and the PCMr-rule, respectively. The upper frame of Figure 9 depicts the outcomes in terms of standard deviations of inflation and the output gap when the PCM-rule and PCMr-rule are implemented in ECM. For comparison, the outcomes under (suboptimal and optimal) rules based on ECM itself, referred to as ECM-rules, are also plotted. The lower frame of the figure presents the relative losses under the ECM-rules as well as under the (optimal) PCM-rule and PCMr-rule.

The losses are measured relative to the level under the optimal ECM-rule: $L(\text{ECM-rule}; \mathsf{ECM})$, defined by $H = 3$. For example, the relative loss under the PCMr-rule is defined as:

$$\Delta L(\text{PCMr-rule}) \equiv \frac{L(\text{PCMr-rule}; \mathsf{ECM}) - L(\text{ECM-rule}; \mathsf{ECM})}{L(\text{ECM-rule}; \mathsf{ECM})}, \tag{11}$$

where $L(\text{PCMr-rule}; \mathsf{ECM})$ expresses that the value of the loss function (6) has been obtained by implementing the rule that is optimal in PCMr, in ECM. As shown above, $H^* = 11$ defines the

[15] The results for the cases where PCM and in PCMr are assumed to be valid models do not affect our conclusions.

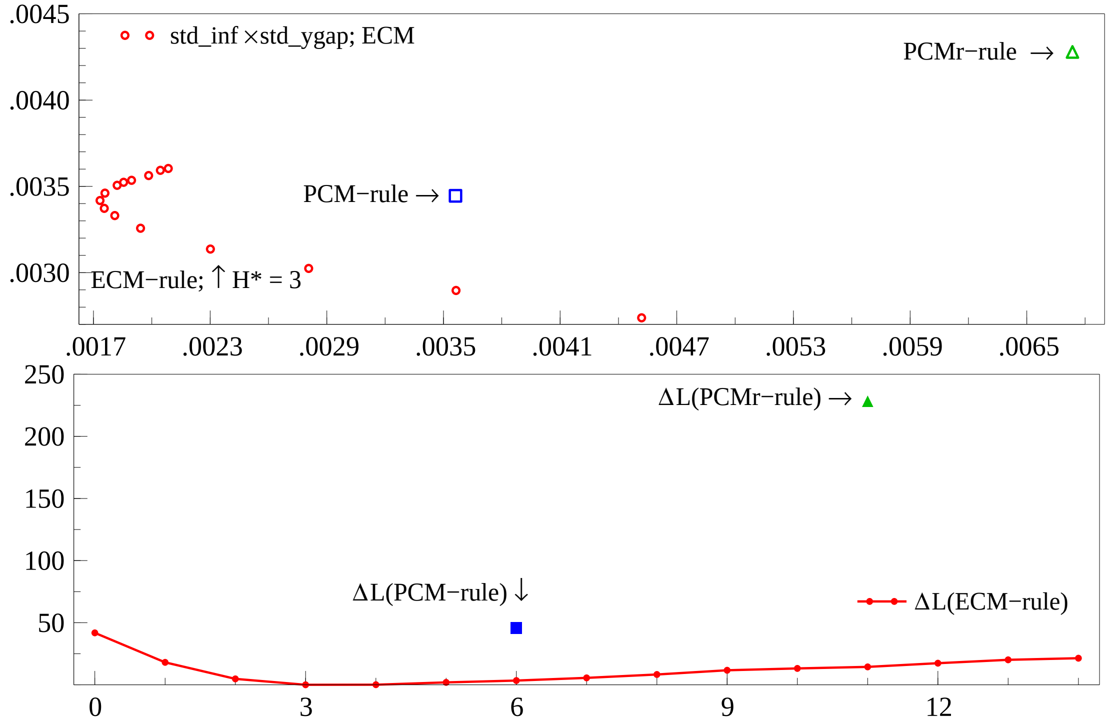

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Figure 9: *Economic performance and relative losses (in %) under different rules in response to the supply shock when* ECM *is the valid model. In the upper column, we plot outcomes in terms of standard deviations of the inflation gap (vertical axis) and output gap conditional on rules that would have been optimal if* PCM *or* PCMr *was the valid model. For comparison, we also reproduce the outcomes associated with (policy) horizon-specific rules based on* ECM *itself; cf. Figure 7. The policy horizon is varied in the range of 0-14; thus outcomes under 15 rules based on the valid model itself are reported. In the lower frame, we report values of the relative loss function (in %) under the optimal rules based on* PCM *and* PCMr*. The relative losses are calculated relative to the optimal loss if the optimal rule based on* ECM *itself was implemented; cf. (11) for a definition. The lower frame also reproduces values of the relative loss for different policy horizons under* ECM *itself; cf. Figure 7.*

optimal rule conditional on PCMr, while $H^* = 3$ defines the optimal rule conditional on ECM.

The relative losses summarizes the loss due to both bias in achieving the inflation target and excess variances of inflation and output gap. Clearly, when we implement the interest rate path minimizing the loss function (6) while satisfying the constraint (7) conditional on a different model than the valid model, the constraint (7) will not be satisfied while the linear combination of variances around the inflation target and that of output gap will not be minimized.

Figure 9 shows that if ECM is the valid model, while the policy rules are based on PCM and PCMr, both inflation and the output gap will become relatively more unstable, especially if the PCMr-rule is implemented. We note that under the PCM-rule, the loss would be 46% higher relative to that under the optimal ECM-rule. Under the vertical PCM-rule, however, the relative loss would be much higher: 228%.

It also seems that choosing the valid model and the corresponding rule is much more important than choosing the optimal policy horizon, and thereby the corresponding interest rate path. The bottom frame of the figure shows that the relative losses under both the PCM-rule and particularly under the PCMr-rule are higher than under rules that are based on ECM but are defined by policy horizons that differ from the optimal policy horizon, 3. It can be shown that, at least for policy horizons within the range of 0–20 quarters, the relative loss under such rules never exceeds 42%, which is for $H = 0$.

A decomposition of the relative losses to derive possible bias in achieving the inflation target over the simulation horizon suggests that both the PCM-rule and the PCMr-rule imply a downward bias in average inflation. Precisely, in the case of the transitory supply shock considered here, the average inflation is about 1/10 of a percentage point and 1/4 of a percentage point lower than the inflation target under the PCM-rule and the PCMr-rule, respectively. The bias is downward as both of these rules imply stronger monetary policy response to the supply shock than the ECM-rule. It can be shown that the downward biases in the case of a rather persistent supply shock would be much higher.

4.2 Potential costs when policy is based on a suite of models

Here, we assume that the economy is adequately characterised by one of the three models considered. However, we consider them equally probable, and hence also the associated monetary policy rules as equally relevant. Therefore, instead of implementing one specific monetary policy rule in the face of a given shock, we implement an 'average-rule'. Specifically, we define the required interest rate response, β_s, in the rule (8) as 1/3 of the sum of the β_ss implied by the three models. Thereafter, we determine response coefficients $\beta_{s,H}$ for a transitory shock for different policy horizons and implement the interest rate rules that follow as above. If we only take into account the fit of the models, equal weighting of the required interest rate responses could be justified in light of the comparable in-sample predictive power of the models. Measures of in-sample or out-of-sample predictive power of models are often used to weigh the results of different models. Below, we point out how alternative weighting schemes would affect our results.

Figure 10 presents the outcome of the average-rule if the economy is characterised by ECM. For comparison, we also present the performance under the "valid rule", i.e. the optimal rule implied by ECM. The upper frame depicts the curves presenting the trade-off between stability in inflation and output gap, while the lower frame depicts the loss under both rules relative to the loss under the optimal ECM-rule, as defined in (11).

It appears that the average-rule will lead to much higher variation in both inflation and the output gap than the ECM-rule, irrespective of the policy horizon; in the figure this is shown only for policy horizons in the range of 0–14 quarters. Under average-rules, which can be defined by different policy horizons, there is a trade-off between price and output stability for policy horizons

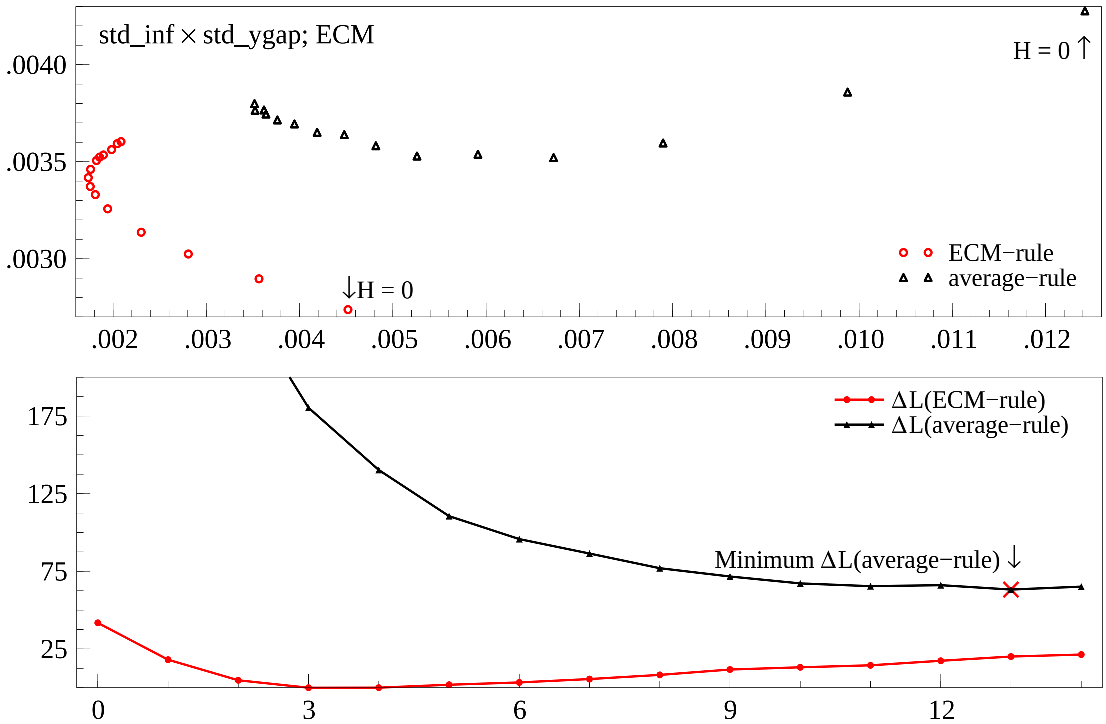

Figure 10: *Economic performance and relative losses (in %) under the 'average-rule' in response to the supply shock when* ECM *is the valid model. In the upper column, we plot outcomes in terms of standard deviations of the inflation gap (vertical axis) and the output gap conditional on the average-rule for different policy horizons (horizontal axis). For comparison, we also reproduce the outcomes associated with horizon-specific rules based on* ECM*; cf. Figure 7. The policy horizon is varied in the range of 0–14 quarters; thus outcomes under 15 rules based on the average-rule as well as* ECM *itself are reported. In the lower frame, we report values of the relative loss function under the average-rule as well as* ECM*. The relative losses are calculated relative to the loss if the optimal rule based on* ECM *was implemented; cf. equation (11) for a definition.*

above 4 quarters. The poor performance of average-rules is mainly because they suggest a much stronger policy response than favoured by the valid model, by assumption, ECM. Relatively short policy horizons under an average-rule are especially destabilizing because they suggest relatively strong immediate interest rate hikes; cf. Figure 3. A comparison of the trade off curves in Figure 10 suggests that the performance of an average-rule will be considered inferior to even that of an suboptimal ECM-rule, irrespective of policy horizon.

The lower frame of Figure 10 shows relative values of the loss function under both (suboptimal and optimal) ECM-rules and average-rules. The difference between relative losses indicates the costs of implementing the average-rule relative to that of implementing ECM-rules. It appears that the loss will be higher, the lower is the policy horizon under an average-rule. Notably, if we implement an average-rule, the loss when ECM is valid will tend to decrease with the policy horizon. Thus, even though we choose the policy horizon for which the loss under an average-rule will be at a minimum, which is at $H = 13$, the relative loss under the average-rule will be ca. 63% higher than that under the optimal ECM-rule, defined by $H = 3$. It should also be noted that possible costs of basing policy on a suite of models may become higher if the suite does not include the valid model.

In addition, if an average-rule based on any of the policy horizons considered is implemented, it will generally make inflation deviate from its target rate. In the case of transitory shock considered here, the average inflation over the simulation horizon is found to be below the inflation target by 1/10 of a percentage point. The bias is downward as the average rule is more contractionary relative to the optimal ECM-rule.

As observed above, it also seems more important to choose the rule consistent with the valid model than choosing the optimal policy horizon. We note that under an ECM-rule defined by $H \neq H^* = 3$, the relative loss does not exceed that under an average-rule even when the relative loss under an average-rule is at its minimum, i.e. when $H = 13$.

Figure 9 can be used to assess the range of potential loss if the average-rule had been determined by other weights than equal weights. Assuming that ECM is the valid model, Figure 9 suggests that the relative loss would be in the range 0–228%, which would also imply average deviation from the inflation target (bias) in the range 0 to -0.25 percentage point, depending on how much weight was assigned to the PCMr-rule at the expense of especially the optimal ECM-rule.

However, if the weights to the policy implications of the different models were assigned in light of indicators of their empirical validity in addition to in-sample predictive power, one would obtain weights favouring ECM and the optimal ECM-rule and thereby reduce the potential loss including the bias.

5 Conclusions

We have investigated the economic significance of trading off the empirical validity of models against other desirable model properties. The investigation has also shed light on the potential loss from assuming model uncertainty to be higher than justified and basing monetary policy on a relatively robust model, or on a suite of models. Our results also apply to model selection on the basis of other criteria than empirical validity and/or robustness properties since the potential loss in such cases would be even higher than in the case of employing a relatively robust model, or a suite of models.

We have based the analysis on three alternative econometric systems for wage and price inflation in Norway and embedded them as the supply side in a well specified macroeconometric model for medium-term analyses. The three econometric systems have been tested econometrically with the aid of a VAR model, which contains the three systems as special cases. The empirically preferable system is the ECM of wages and prices which is an adequate statistical representation of the VAR model. The two rejected systems may be considered versions of the Phillips curve, which often constitutes the supply side in models used for policy analysis. We have analysed optimal interest rate paths suggested by the three systems, to investigate whether the difference in empirical validity had any discernible consequences for monetary policy and macroeconomic performance.

Our results support the view that a model for policy analysis should be empirically valid and caution against compromising this property for other desirable model properties.[16] We find that the different models entail widely different monetary policy response to shocks, and macroeconomic outcomes. This in turn implies that robust monetary policies in response to perceived model uncertainty are quite costly, irrespective of whether they are based on the least-fault tolerant model, or on a suite of models, even when it contains the valid model by assumption.

We interpret our results as suggesting that there may be huge gains from employing econometric tools rigorously to select an empirically valid model for policy analysis. Some times, however, available data may not be sufficiently informative to discriminate between models, for instance when the economy is undergoing a transition between economic regimes. Also, the preferred model may not (yet) incorporate relevant aspects of economic behaviour. In such cases, there could be large gains from using econometric tools to narrow down the set of models and assign appropriate weights to their policy implications.

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[16] This is consistent with e.g. Granger (1992) who states that "...it should be generally agreed that a model that does not generate many properties of actual data cannot be claimed to have any 'policy implications'...".

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A Appendix: Data definitions

The time series have been extracted from databases maintained by Norges Bank (the central bank of Norway). The variables are precisely defined in Rikmodnotat 140, Norges Bank. The variables as named in the RIMINI database are noted in hard brackets [.] below. Where relevant, the base year is 1991 and the unit of measurement is mill. NOK. *Mainland economy* is defined as the total Norwegian economy excluding oil and gas production and international shipping. Impulse dummies are denoted as $i\boldsymbol{yy}q\mathbf{x}$. For example, *i80q2* is 1 in the second quarter of 1980 and 0 in all other quarters. The definitions are:

H Normal working hours per week; [NH].

P Consumer price index; [CPI].

PE Electricity component of consumer price index; [CPI-E].

PI Deflator of total imports; [PB].

Y Total value added at market prices in the mainland economy; [YF].

PR Mainland economy value added per man-hour at factor costs; [ZYF].

RS 3 month Euro-krone interest rate; [RS].

$\tau 1$ Employers' tax rate; [$T1$].

$\tau 3$ Indirect tax rate; [T3].

U Unemployment rate; [UTOT].

W Mainland economy hourly nominal wages; [WF].

W_d Composite dummy for wage freeze: 1 in 1979q1, 1979q2, 1988q2 and 1988q3.

P_d Composite dummy for introduction and removal of direct price regulations. 1 in 1971q1, 1971q2, 1976q4, 1979q1; -1 in 1975q1, 1980q1, 1981q1, 1982q1; and zero otherwise.

B Appendix: The interest rate rule

In the following, we draw on Akram (2007) and present the assumptions behind rule (8) and sketch how it can be derived using a simple model of the inflation gap $(\pi_t - \pi^*)$. This approach is particularly useful when employing large-scale macroeconometric models.

B.1 Assumptions

1.A Assume the following linear model of inflation π:

$$\pi_t - \pi^* = \sum_{l=0}^{\widetilde{T}_0} \alpha_l \Delta z_{t-l} - \sum_{m=0}^{T_0} \gamma_m \Delta i_{t-m}, \tag{12}$$

where π^* is the constant inflation target, z is an exogenous shock variable, while i is the nominal interest rate. Here, we use Δ to denote deviation from the neutral or steady state level. Thus, Δi_t denotes an interest rate deviation relative to the neutral interest rate, and hence the extent of a non-neutral monetary policy stance. The inflation gap at time t, $\pi_t - \pi^*$, depends on finite lagged and contemporaneous effects of shocks and nominal interest rates. Thus, effects of a transitory shock will eventually die out, even when conditioned on the neutral nominal interest rate path, i.e. $\Delta i_{t-m} \equiv i_{t-m} - i_0 = 0$ for all m. The partial effects, αs and γs, are assumed to be constant parameters and their sums are assumed to be strictly positive: $\sum_{l=0}^{\widetilde{T}_0} \alpha_l > 0$, while $\sum_{m=0}^{T_0} \gamma_m > 0$, where $\widetilde{T}_0$ and T_0 are finite values. α_l represents the effect of Δz_t in period $t + l$, while γ_m represents the effect of Δi_t in period $t + m$.

2.A A shock Δz_t with persistence $\phi \in [0, 1)$ is characterised as an AR(1) process:

$$\Delta z_t = \phi \Delta z_{t-1} + \varepsilon_t, \tag{13}$$

where ε_t is an exogenous shock term whose expected value is zero. Any change in Δz_t from zero will die out asymptotically, but we assume that it diminishes after $N + 1$ periods, i.e. $\Delta z_{t+N+1} = \phi^{N+1} \Delta z_t \approx 0$, and neglect it afterwards.

Monetary policy is operationalised as follows, for simplicity.

3.A The overriding objective of monetary policy is $\mathsf{E}\pi_t = \pi^*$, which is pursued by ensuring $\mathsf{E}_t\pi_{t+H} \approx \pi^*$ for an appropriate H; constraint (7). In response to shocks, the monetary policy authority only considers implementing interest rate paths that are consistent with the overriding objective as well as with the operational objective (7). $\mathsf{E}\pi_t = \pi^*$ is represented by

$$\overline{\pi} = \pi^*, \tag{14}$$

where $\overline{\pi}$ is the mean of inflation forecasts ($\widehat{\pi}$s) over a sufficiently large number of periods: t–T.

4.A In response to a shock, the interest rate is shifted abruptly to a non-neutral level but is thereafter brought more or less gradually to its neutral level over time. A monetary policy response to a shock is initiated when the shock occurs; assuming away possible observation and decision lags.[17]

Such an interest rate behaviour in response to a shock at time t can be characterised as:

$$\Delta i_{t+m} = \beta\Delta z_t + \varrho\Delta i_{t+m-1};\ m = 0, 1,\ 2, ..H, H+1, ... \tag{15}$$

where $0 < \varrho < 1$. The interest rate is brought gradually towards i_0, in principle over infinite periods.

However, interest rates are considered converged with the neutral rate after a finite number of periods. Specifically, in period $t + H + 1$ just a small fraction, say δ, of the initial interest rate deviation is assumed to remain:

$$\Delta i_{t+H+1} = \delta\Delta i_t. \tag{16}$$

where $0 < \delta < 1$ is a sufficiently small ratio (of choice). Equations (15) and (16) imply that $\varrho^{H+1} = \delta$ and hence, $\varrho = \delta^{1/(H+1)}$, where $H \geq 0$.

B.2 Derivation

We now derive the value of β and ϱ (or H) that will define an interest rate path consistent with the monetary policy objectives, as expressed by 3.A, for the given specification of the interest rate rule, the shock process and the inflation model.

[17]Under such simplifying assumptions, 3.A–4.A, inflation rate targeting becomes effectively equal to price path targeting, however. This is because a potential base drift in prices, which may arise under inflation targeting because of observation and decision lags in actual monetary policy making, do not arise here. Price path targeting with a positive drift in prices differs from inflation targeting by not letting 'bygones be bygones'; price level targeting is a special case of price path targeting.

Assume that a state of equilibrium is disturbed in period τ by a shock Δz_τ with persistence ϕ. That is, $\Delta z_\tau = \varepsilon_\tau$, while $\Delta z_t = 0$ for $t < \tau$ and $\varepsilon_t = 0$ for $t \neq \tau$.

3.A implies that the interest rate in response to the shock is set such that:

$$\sum_{l=0}^{T} (\widehat{\pi}_{\tau+l} - \pi^*) = 0. \tag{17}$$

This equation would be satisfied if the accumulated effects on inflation of current and future interest rate deviations (from i_0) outweigh the accumulated effects of the shock on inflation over time.[18]

The model given by 1.A implies that a transitory shock in period τ, i.e. $\Delta z_\tau \neq 0$, will affect inflation contemporaneously and in $\widetilde{T}_0$ future periods. Under 2.A, the accumulated effects of a persistent shock Δz_τ will become:

$$\left[\frac{1-\phi^{N+1}}{1-\phi}\right] \left[\sum_{l=0}^{\widetilde{T}_0} \alpha_l\right] \Delta z_\tau. \tag{18}$$

The model also implies that an interest rate deviation in period τ, $\Delta i_\tau \neq 0$, will affect inflation contemporaneously and in T_0 future periods. Under 4.A, the accumulated effects of an interest rate deviation Δi_τ, which is eliminated gradually over $H+1$ periods, will amount to:

$$\left[\frac{1-\varrho^{H+1}}{1-\varrho}\right] \left[\sum_{m=0}^{T_0} \gamma_m\right] \Delta i_\tau. \tag{19}$$

Consistency with 3.A (and (17)) would require that:

$$\left[\frac{1-\phi^{N+1}}{1-\phi}\right] \left[\sum_{l=0}^{\widetilde{T}_0} \alpha_l\right] \Delta z_\tau - \left[\frac{1-\varrho^{H+1}}{1-\varrho}\right] \left[\sum_{m=0}^{T_0} \gamma_m\right] \Delta i_\tau = 0. \tag{20}$$

4.A implies that $\Delta i_\tau = \beta \Delta z_\tau$ if the interest rate is initially at the neutral rate, i.e. $\Delta i_{\tau-1} = 0$. Thus, β will be:

$$\beta = \left[\frac{1-\varrho}{1-\varrho^{H+1}}\right] \left[\frac{1-\phi^{N+1}}{1-\phi}\right] \left[\frac{\sum\limits_{l=0}^{\widetilde{T}_0} \alpha_l}{\sum\limits_{m=0}^{T_0} \gamma_m}\right]. \tag{21}$$

Hence, (15) can be formulated as:

$$i_{\tau+m} = i_0 + \left[\frac{1-\varrho}{1-\varrho^{H+1}}\right] \left[\frac{1-\phi^{N+1}}{1-\phi}\right] \beta_\varepsilon \varepsilon_\tau + \varrho(i_{\tau+m-1} - i_0);\ m = 0, 1,\ 2, ..H, H+1, ... \tag{22}$$

where $\beta_\varepsilon \equiv \sum_{l=0}^{\widetilde{T}_0} \alpha_l / \sum_{m=0}^{T_0} \gamma_m$, $\Delta z_\tau = \varepsilon_\tau$ and e.g. $\Delta i_t \equiv i_t - i_0$. Different values of ϱ, or H, since

[18] $T = \sup\{\widetilde{T}_0 + N, T_0 + H\}$.

$\varrho = \delta^{1/(H+1)}$ where δ is a given fraction, will specify different interest rate paths satisfying 3.A, i.e. (14) and (17).

The interest rate rule (22) can be closely approximated by rule (8) as $\varrho^{H+1} = \delta$, which is close to zero by assumption (4.A) and $\phi^{N+1} \approx 0$ can be reasonable when N is sufficiently large. In rule (8), we use $\varrho_H \equiv \varrho$ to indicate explicitly that the degree of interest rate smoothing depends on the policy horizon.

B.3 Implementation

To implement the interest rate rule (8) in response to a specific shock ε with a given persistence $0 \leq \phi < 1$, we need to derive estimates of ϱ_H and β_ε.[19] Values of interest rate smoothing for different H, ϱ_H, can then be obtained from the relationship $\varrho_H = \delta^{1/(H+1)}$. We assume that $\delta = 0.1$, but our empirical results are quite invariant to the choice of δ-values if they are relatively small.

To estimate β_ε, we derive impulse responses of inflation to a transitory shock ε, conditional on a given nominal interest rate path, and impulse responses of inflation to a transitory increase in interest rates. Then, in line with its definition, $\left(\beta_\varepsilon \equiv \sum_{l=0}^{\widetilde{T}_0} \alpha_l / \sum_{m=0}^{T_0} \gamma_m\right)$, the estimate of β_ε is obtained as the ratio of accumulated impulse responses of inflation to shock ε to the accumulated impulse responses of inflation to the interest rate increase.

To provide a transitory demand shock (ε_d) to a given version of the model (ECM, PCM or PCMr), we raise the residual of the aggregate demand equation (y-equation) over the period 1995q1–1995q4 such that the growth in aggregate demand initially increases by one percentage point over the year 1995. Thereafter, we set the residual of the demand equation to zero and let the dynamic model adjust on its own over the simulation period: 1995q1–2000q4. This is a sufficiently long period to let the partial effects of the demand shock (as well as those of the supply shock and an interest rate change) work out. We let the policy interest rates remain invariant to the shock and the subsequent adjustment by letting them follow their reference path. We follow the same procedure in the case of the supply shock (ε_s), but raise the residual of the *cpi*-equation over the period 1995q1–1995q4 such that inflation increases by one percentage point over the year 1995.

To estimate the partial effects of a transitory increase in interest rates, we raise the interest rate by one percentage point relative to its reference path over the period 1995q1–1995q4, ceteris paribus, and thereafter set it back to its reference path.

The variances of the inflation gap and the output gap entering the loss function conditional on a given horizon-specific interest rate rule are estimated using the implied projections of the inflation gap and the output gap over the simulation period.

[19] The results are almost the same if we employ the exact rule (22) rather than its approximation (8).