Static and dynamic analysis: basic concepts and examples
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The lecture plan is at

http://folk.uio.no/rnymoen/ECON3410_h08_index.html,

which is the workpage of the course. The **workpage** is for practical posting of slides, exercises sets etc. But refer to the Department’s webpage

http://www.uio.no/studier/emner/sv/oekonomi/ECON4410/h09/

for all official information: credits, overlap, exam dates and so on.
3 main topics

   - *Introductory Dynamic Macroeconomics* (IDM), posted on the workpage.

   - *Introducing Advanced Macroeconomics* (IAM) by Birch-Sørensen and Whitta Jacobsen

3. Critical assumptions of the standard model and alternative models of the supply-side.
The main focus: medium-run macro dynamics

- Review of the "building blocks" of the dynamic AD-AS model
- Closed economy AD-AS model:
  - The short run-and the long-run version of the model.
- Full dynamic analysis
- Application: Stabilization policy, rules versus discretion
- A different perspective: The RBC model.
- Open economy AD-AS model
- Short-and long run (again)
- Monetary policy regimes.
Dynamics is a typical feature of the real world (1)
If it was not, what would the world look like?

- Economic variables would jump whenever incentives changed.
- Time graphs would show:
  - a step-wise evolution, or
  - very erratic (volatile) behaviour, or
  - a combination if some incentives are huge, and some are small.
Dynamics is a typical feature of the real world (2)

- For some real world variables graphs look a little like the blue graph in picture.
- Daily data of stock prices, and exchange rates (under some monetary poly regimes) are examples.
- But for most macroeconomic variables, persistence is a dominant feature: It takes times before a change in incentives, or in legislation, or in policy, obtain full effect on macroeconomic variables.
- Main sources of persistence (and therefore of dynamics) are:
  - Information and recognition lags,
  - Adjustments cost,
  - Uncertainty and expectations,
  - Aggregation of individual decisions to the macro level.
Persistence in the response to a shock is typical of dynamics.

- The graph shows static (red) and dynamic (blue) responses to the same sequence of shocks.
- Note how dynamics add persistence to the series, because shocks are propagated through time.
GDP per capita can fluctuate in the medium-run time perspective.

But in longer perspective the dominating trait is growth!
The unemployment rate can fluctuate in the medium-run time perspective. But in longer time perspective, the dominating trait is no-growth!
The “end of inflation” is typical for many countries

Note the *financial crisis* at the end of the sample?
Norway and Sweden: Inflation and unemployment

- Note again the typical propagation of shocks.
- In Swedish unemployment in particular
Norges Bank [The Norwegian Central Bank] is typical of many central banks’ view:

“Monetary policy influences the economy with long and variable lags. Norges Bank sets the interest rate with a view to stabilizing inflation at the target within a reasonable time horizon, normally 1-3 years”

Policy decisions are based on Norges Banks beliefs about the dynamic nature of the monetary transmission mechanism.

In economics, beliefs means models, implicit or explicit.

Therefore the citation illustrates two theses: policy is model based, and policy models are dynamic.
Formal dynamic analysis in economics is a relatively new invention.

Ragnar Frisch worked intensively with the foundations of the discipline he dubbed *macrodynamics* in the early 1930s. His definition of dynamics was:

*A dynamic theory or model is made up of relationships between variables that refer to different time periods. Conversely, when all the variables included in the theory refer to the same time period (or, more generally, the model is conceptualized without time as an entity), the system of relationships is static.*

In a dynamic model: *time plays an essential role.*
A static model demand schedule

A linear demand function is

\[ X_t = aP_t + b + \varepsilon_{d,t}, \]

with \( a < 0 \) and \( b > 0 \) as parameters.

The three variables: \( X \) and \( P \), and \( \varepsilon_d \) (denoting a random demand shock) are all provided with time subscript \( t \).

\( t \) might represent for example a year (the time period is annual); or a quarter (the period is quarterly); or month (the period is monthly).

This model of demand is static.

Note that the “appearance of time” (in the form of the time subscript) is not enough to make the model dynamic, because time does not play an essential role!
A static equilibrium model

If we supplement the demand equation with a static supply equation, we obtain the *static market equilibrium model*

\[ X_t = a P_t + b + \varepsilon_{d,t}, \quad X_t = c P_t + d + \varepsilon_{s,t}, \]

determining the endogenous variables \( X_t \) and \( P_t \) for known values of the exogenous variables \( \varepsilon_{d,t} \) and \( \varepsilon_{s,t} \) (and fixed and known values of the 4 parameters \( a, b, c, d \)).

We assume that \( \varepsilon_{d,t} \) and \( \varepsilon_{s,t} \) are completely random variables. Their role is to represent shocks, or in Frischean terminology, *impulses* to the system. A variable that represents random technology shocks is important in the Real Business Model (RBC) that we will discuss later in the course.
The model written in structural form

\[ 1 \cdot X_t - aP_t = b + \varepsilon_{d,t} \]
\[ 1 \cdot X_t - cP_t = d + \varepsilon_{s,t} \]

Cramer’s rule gives:

\[
    X_t = \frac{1}{a-c} \begin{vmatrix} b + \varepsilon_{d,t} & -a \\ d + \varepsilon_{s,t} & -c \end{vmatrix} = \frac{ad - bc + a\varepsilon_{s,t} - c\varepsilon_{d,t}}{a-c}
\]
\[
P_t = \frac{1}{a-c} \begin{vmatrix} 1 & b + \varepsilon_{d,t} \\ 1 & d + \varepsilon_{s,t} \end{vmatrix} = \frac{d - b + \varepsilon_{s,t} - \varepsilon_{d,t}}{a-c}
\]
The initial (before the shock) equilibrium is at A.

B, C and D are new equilibria, corresponding to different types of shocks.
The graph shows 50 simulated equilibrium values for $P_t$.

$P_t$ is direct reflection of “excess demand”.
The effect of a single shock
(A graph of a dynamic multiplier from a static model)

In the static model, the full effect of a temporary shock occurs in the first period.

In the periods after the shock, there are no responses in $P_t$.

The impact multiplier is non-zero, all other dynamic multipliers are zero.
Summary of properties of the static model

- The whole effect of a shock is contained in the equilibrium values of $P$ and $X$ in the period of the shock.
- There are no spill-over effects of a shock in period $t = 1$ to period $2, 3,$ and later periods.
- We say that impulses in period 1 are not propagated to later periods.
  - The time series of $P_t$ (and $X_t$) are perfect mirror images of the shocks $\varepsilon_{d,t}$ and $\varepsilon_{st}$.
  - The sequence of dynamic multipliers, for example $\frac{\partial P_t}{\partial \varepsilon_{s,1}}$ ($t = 1, 2, 3...$) are zero, except for $\frac{\partial P_1}{\partial \varepsilon_{s,1}}$. 
A dynamic equilibrium model

\[ X_t = a P_t + b + \varepsilon_{d,t}, \text{ demand, and} \]
\[ X_t = c P_{t-1} + d + \varepsilon_{s,t}, \text{ supply.} \]

- The only change is in the supply equation, where \( P_{t-1} \) replaces \( P_t \).
- Interpretation: In some markets supply is fixed in the short-run. No matter how high or low the price is in the current period, the supply of the good is ‘frozen’ by decisions of the past.
- Classic example: agricultural products such as pork and wheat. Relevance today: “Salmon farming”, and China food price inflation; but also the market for oil and for raw materials.
The long-run supply function is \( X = cP + d \)

After a temporary demand shock, the sequence of equilibria is \( A \) \((t = 0)\), \( B \) \((t = 1)\), \( C \) \((t = 2)\), \( D \) \((t = 3)\) and so on in a cobweb pattern.

In the long-run, the equilibrium is back at \( A \).
Solution of the dynamic model (numerical)

Graph a) shows solution of $P_t$ from the dynamic model,

b) shows the sequence of dynamic responses in $P_t$ ($\partial P_t/\partial \varepsilon_{d,1}$, $t = 1, 2, ...$)

c) and d) show the corresponding for the static model
In the dynamic model, the whole effect of a shock is not contained in the equilibrium values of $P$ and $X$ in the period of the shock.

The sequence of the dynamic multipliers, for example $\frac{\partial P_t}{\partial \varepsilon_{d,1}}$ ($t = 1, 2, 3...$) are generally non-zero, but may approach zero for large values of $t$, if the dynamics is stable.

- There are spill-over effects of a shock in period $t = 1$ to period 2, 3, ....
- Impulses in period 1 are propagated to later periods
- The solution does $P_t$ (and $X_t$) are not perfect mirror images of the shocks $\varepsilon_{d,t}$ and $\varepsilon_{st}$ in each period.
- The cobweb pattern is however not general, as a second example will show.
A second dynamic model of market equilibrium

\[ X_t = aP_t + b_1 X_{t-1} + b_0 + \varepsilon_{d,t}, \text{ demand} \]
\[ X_t = c_0 P_t + c_1 P_{t-1} + d + \varepsilon_{s,t}, \text{ supply} \]

- The demand function is now a dynamic equation. The parameter \( b_1 \) measures by how much an increase in \( X_{t-1} \) shifts the short-run demand curve. This can be rationalized by consumer habits for example.
  
- \( 0 < b_1 < 1 \).

- In any given period, \( X_{t-1} \) is determined from history and cannot be changed. Hence in this model there are two pre-determined variables: \( P_{t-1} \) and \( X_{t-1} \).

- The supply equation is a generalization of the cobweb model: If we set \( c_0 > 0 \), short run supply is no longer completely inelastic as in the cobweb model.
Numerical solution of the model with habit formation

- Compare panel a) with c), and panel b) with d).
- It is typical that small changes in the model specification can significantly affect the solution of the dynamic model.
When is a static model relevant for the real world?

Frisch:

“Hence it is clear that the static model world is best suited to the type of phenomena whose mobility (speed of reaction) is in fact so great that the fact that the transition from one situation to another takes a certain amount of time can be discarded. If mobility is for some reason diminished, making it necessary to take into account the speed of reaction, one has crossed into the realm of dynamic theory.”

- We would add: Static models are also relevant when we only claim to analyze the very short-run effects (what we will call the impact multiplier) of a shocks, i.e. we know that the dynamic effects of a shock “are there”, but we do not (know how to) analyze them.
- In this way we can interpret the Keynesian IS-LM model as a short-run model.
The three steps in a dynamic analysis

1. The question we typically want to answer is: “What are the dynamic effects of a shock (of a certain type) on the endogenous variables of the model?”

2. It is often practical to break this question down to 3 “smaller” questions:
   1. What are the short-run effects of the shock?
   2. What are the long-run effects of the shock, given that the dynamic adjustment process is stable?
   3. What are the properties of the dynamic adjustment process (regarding stability in particular)?

3. To answer Q1 and Q2 we use two separate models!
Market equilibrium: the short-run model

\[ X_t = aP_t + b_1 X_{t-1} + b_0 + \varepsilon_{d,t}, \quad (1) \]
\[ X_t = c_0 P_t + c_1 P_{t-1} + d + \varepsilon_{s,t}. \quad (2) \]

Since \( P_{t-1} \) and \( X_{t-1} \) are pre-determined from history in each period \( t \), they are exogenous in this short-run model. The analytical solution:

\[ X_t = \frac{ad - b_0 c_0 - c_0 b_1 X_{t-1} + ac_1 P_{t-1} + a\varepsilon_{s,t} - c_0 \varepsilon_{d,t}}{a - c_0} \]
\[ P_t = \frac{d - b_0 - b_1 X_{t-1} + c_1 P_{t-1} + \varepsilon_{s,t} - \varepsilon_{d,t}}{a - c_0} \]

gives the short-run effects of the shocks \( \varepsilon_{d,t} \) and \( \varepsilon_{s,t} \) as derivatives, see Table 1.1 in IDM.
Market equilibrium: The long-run model

The long-run model applies to a hypothetical (or counterfactual) stationary situation where there are no new shocks, and all past shocks have worked their way through the system. The long-run model is therefore defined for the situation: 
\[ X_t = X_{t-1} = \bar{X}, \quad P_t = P_{t-1} = \bar{P} \] and \[ \varepsilon_{d,t} = \bar{\varepsilon}_d, \quad \varepsilon_{s,t} = \bar{\varepsilon}_s. \] The model is given by

\[ \bar{X} = a \bar{P} + b_1 \bar{X} + b_0 + \bar{\varepsilon}_d, \] long-run demand

\[ \bar{X} = c_0 \bar{P} + c_1 \bar{P} + d + \bar{\varepsilon}_s, \] long-run supply

or

\[ \bar{X} = \frac{a}{1 - b_1} \bar{P} + \frac{1}{1 - b_1} (b_0 + \bar{\varepsilon}_d), \]

\[ \bar{X} = (c_0 + c_1) \bar{P} + (d + \bar{\varepsilon}_s). \]

Solve to obtain analytical expressions for long-run effects, see Table 1.1 in IDM.
Graphically, we can represent the short-run and long-run models in one diagram,

Lines with different slopes define the short-run and the long-run.

We can then analyze the short-run effect of a shock, as well as the long-run effect.
The short-run model and the long-run model of the macroeconomy will be important tools in the following, in particular for the medium-term AS-AS model covered by the IAM book.

But we will also address systematically the third question:

What are the properties of the dynamic adjustment process (regarding stability in particular)?

To do this we need to develop several concepts more precisely than we have done in this introduction.

We do that within a class of dynamic equations which wide enough to cover many economic interpretations as special cases (Ch 2 of IDM).
The distinction between static models and dynamic modes is fundamental.

Whether dynamic models are expressed in terms of discrete time or continuous time is however *not* fundamental.

Often theories are expressed in continuous time, but since actual data series are recorded in discrete time, choosing discrete time keeps the theory closer to applications.

Refer to Box 1.1 in IDM for example. The point is that for dynamics to occur, time must play an essential role in model (discrete/continuous time is a secondary issue).
Dynamic models often include both flow and stock variables.

Flow: in units of (for example) million kroner per year

Stock: in units of (for example) million of kroner at a particular period in time (for example start or end of the year).

Population size, and capital stock are examples of stock variables. But so are also price indices: $P_t$ may represent the value of the Norwegian CPI in period $t$ (a month, a quarter or a year), and indicators of the wage level.

In practice: the values of $P$ will be index numbers. The number 100 (often 1 is used instead) refers to the base period of the index. If $P_t > 100$ it means that relative to the base period, prices are higher in period $t$. 
A flow variable is often a change in a stock variable

Starting from a stock variable like $P_t$, a flow variable results from obtaining the change of that variable, hence

$$x_t = P_t - P_{t-1},$$

the (absolute) change

$$y_t = \frac{P_t - P_{t-1}}{P_{t-1}},$$

the relative change, and

$$z_t = \ln P_t - \ln P_{t-1}$$

the approximate relative change

are examples of flow variables. Note that:

- $y_t \times 100$ is inflation in percentage points. In this course we often use to the rate formulation (hence, we omit the scaling by 100)

- $z_t \approx y_t$ by the properties of the (natural) logarithmic function, see for example the appendix of IDM, if in doubt.
A stock variable is the cumulated sum of a flow

\[
debt = -\text{current account} + \text{lagged debt.}
\]

If there is a primary account surplus for some time, this will lead to a gradual reduction of debt—or an increase in the nation’s net wealth. Conversely, a consistent current account deficit raises a nation’s debt.