Dynamic models

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This note is written for the course ECON 3410 /4410 International macroeconomics and finance. The note reviews the key concepts and models needed to start addressing economic dynamics in a systematic way. The level of mathematics used does not go beyond simple algebra. References to the textbooks by Burda and Wyplosz (B&W hereafter) and Rødseth’s Open Economy Macroeconomics (OEM hereafter) are integrated several places in the text.

Section 8 applies the models and concepts to wage and price dynamics. After you have worked through the note, you should have a good understanding of concepts and models with a wide range of applications in macroeconomics, as well as an understanding of wage and price dynamics of small open economies that goes beyond Ch. 12 in B&W.

1 Introduction

In many areas of economics, time plays an important role: firms and households do not react instantly to changes in for example taxes, wages and business prospects but take time to adjust their decisions and habits. Moreover, because of information and processing lags, time goes by before changes in circumstances are even recognized. There are also institutional arrangements, social and legal agreements and norms that hinder continuous adjustments of economic variables. Annual (or even biannual) wage bargaining rounds is one important example. The manufacturing of goods is usually not instantaneous but takes time, often several years in the case of projects with huge capital investments. Dynamic behaviour is also induced by the fact that many economic decisions are heavily influenced by what firms, households and the government anticipate. Often expectation formation will attribute a large weight to past developments, since anticipations usually have to build on past experience.

Because dynamics is a fundamental feature of the macroeconomy, all serious policy analysis is based on a dynamic approach. Hence those responsible for fiscal and monetary policy use dynamic models as an aid in their decision process. In recent years, monetary policy had taken a more prominent and important role in activity regulation, and as we will explain later in the course, central banks in many countries have defined the rate of inflation as the target variable of economic policy. The instrument of monetary policy nowadays is the central banks sight deposit rate, i.e., the interest rate on banks’ deposits in the central bank. However, no central bank hopes for an immediate and strong effect on the rate of inflation after a change in the interest rate. Rather, because of the many dynamic effects triggered by a change in the interest rate, central bank governors prepare themselves to wait a
substantial amount of time before the full effect of the interest rate change hits the target variable. The following statement from the web pages of Norges Bank [The Norwegian Central Bank] is typical of many central banks’ view:

A substantial share of the effects on inflation of an interest rate change will occur within two years. Two years is therefore a reasonable time horizon for achieving the inflation target of $2\frac{1}{2}$% per cent.\(^1\)

One important aim of this course is to learn enough about dynamic modeling to be able to understand the economic meaning of a statement like this, and to start forming an opinion about its realism (or lack thereof).

As students of economics you are well acquainted to model based analysis, graphical or algebraic. Presumably, most of the models you have used have been static, since time has played no essential part in the model formulation or in the analysis. This note therefore starts, in section 2, by contrasting static models with models which have a dynamic formulation. Typically, dynamic models give a better description of macroeconomic time series data than static models. A variable \(y_t\) is called a time series if we observe it over a sequence of time periods represented by the subscript \(t\), i.e., \(\{y_T, y_{T-1}, \ldots, y_1\}\) if we have observations from period 1 to \(T\). Usually, we use the simpler notation \(y_t, t = 1, \ldots, T\), and if the observation period is of no substantive interest, that too is omitted. The interpretation of the time subscript varies from case to case, it can represent a year, a quarter or a month. In macroeconomics other periods are also considered, such as 5-year or 10 year averages of historical data, and daily or even hourly data at the other extreme (e.g., exchange rates, stock prices, money market interest rates). In section 2 we discuss in some detail an example where \(y_t\) is (the logarithm) of private consumption, and we consider both static and dynamic models of consumption (consumption functions).

The Norges Bank quotation above is interesting because it is a clear statement about the time lag between a policy change and the effect on the target variable. Formally, response lags correspond to the concept of the dynamic multiplier which is introduced in section 3. The dynamic multiplier is a key concept in this course, and once you get a good grip on it, you also have a powerful tool which allows you to calculate the dynamic effects of policy changes (and of other exogenous shocks for that matter) on important variables like consumption, unemployment, inflation or other variables of your interest.

After having emphasized the difference between static and dynamic models, in section 2 and 3, the next two sections (4 and 5) shows that there is a way of reconciling the two approaches (section 4), and that for some purposes we can be comfortable with using a static model formulation as long as we are aware of its limitations (section 5).

Section 6 shows briefly that underlying both dynamic policy analysis and the correspondence between dynamic and static formulations, is the nature of the solution of dynamic models. Section 7 sketches how the analysis can be extended to systems of equations with a dynamic specification. Finally, section 8 applies the analysis to wage and price dynamics.

\(^1\)http://www.norges-bank.no/english/monetary_policy/in_norway.html.

Similar statements can be found on the web pages of the central banks in e.g., Australia, New Zealand, The United Kingdom and Sweden.
2 Static and dynamic models

When we consider economic models to be used in an analysis of real world macro data, care must be taken to distinguish between static and dynamic models. The well known textbook consumption function, i.e., the relationship between private consumption expenditure \( C \) and households’ disposable income \( Y \) is an example of a static equation

\[
C_t = f(INC_t), \quad f' > 0.
\]

Consumption in any period \( t \) is strictly increasing in income, hence the positive signed first order derivative \( f' \)—the marginal propensity to consume. To be able to apply the theory to observations of the real economy we have to specify the function \( f(INC_t) \). Two of the most popular functional forms are the so called linear and log-linear specifications:

\[
\begin{align*}
C_t &= \beta_0 + \beta_1 INC_t + \epsilon_t, \quad \text{(linear)} \\
\ln C_t &= \beta_0 + \beta_1 \ln INC_t + \epsilon_t, \quad \text{(log-linear)}
\end{align*}
\]

For simplicity, we use the same symbols for the coefficients in the two equations but it is important to note that the slope coefficient \( \beta_1 \) has a different economic interpretation in the two cases. In (2.2), \( \beta_1 \) it is the marginal propensity to consume (MPC for short), and is assumed to be a constant parameter. In the log linear model (2.2) \( \beta_1 \) is the elasticity of consumption in period \( t \) with respect to income, thus \( \beta_1 \) measures the percentage increase in \( C_t \) following a 1% increase in \( INC \). Hence the log-linear specification in (2.2) implies that the marginal propensity to consume is itself a function of income. In that sense, the log-linear model is the least restrictive of the two, and in the rest of this example we use that specification.

**Exercise 1** Show that, after setting \( \epsilon_t = 0 \) (for convenience), \( MPC \equiv \partial C_t / \partial INC_t = k \cdot \beta_1 INC_t^{\beta_1-1} \), where \( k = \exp(\beta_0) \).

Macroeconomic textbooks usually omit the term \( \epsilon_t \) in equation (2.2), but for applications of the theory to actual data it is a necessary to get an intuitive grip on this disturbance term in the static consumption function. So: let us consider real data corresponding to \( C_t \) and \( Y_t \), and assume that we have really good way of quantifying the intercept \( \beta_0 \) and the marginal propensity to consume \( \beta_1 \). You will learn about so called least-squares estimation in courses in econometrics, but intuitively, least-squares estimation is a way of finding the numbers for \( \beta_0 \) and \( \beta_1 \) that give the on average best prediction of \( C_t \) for a given value of \( Y_t \). Using quarterly data for Norway, for the period 1967(1)-2002(4)—the number in brackets denotes the quarter—we obtain by using the least squares method in PcGive:

\[
\ln \hat{C}_t = 0.02 + 0.99 \ln INC_t
\]

where the “hat” in \( \hat{C}_t \) is used to symbolize the fitted value of consumption given the income level \( INC_t \). Next, use (2.2) and (2.3) to define the residual \( \epsilon_t \):

\[
\hat{\epsilon}_t = \ln C_t - \ln \hat{C}_t,
\]
which is the empirical counterpart to $e_t$.

In figure 1 we show a cross-plot of the 140 observations of consumption and income (in logarithmic scale), each observation is marked by a +. The straight line represents the linear function in equation (2.3), and for each observation we have also drawn the distance up (or down) to the line. These “projections” are the graphical representation of the residuals $\hat{e}_t$.

Clearly, if we are right in our arguments about how pervasive dynamic behaviour is in economics, (2.2) is a very restrictive formulation. All adjustments to a change in income are assumed to be completed in a single period, and if income suddenly changes next period, consumer’s expenditure changes suddenly too. A dynamic model is obtained if we allow for the possibility that also period $t - 1$ income affects consumption, and that e.g., habit formation induces a positive relationship between period $t - 1$ and period $t$ consumption:

$$\ln C_t = \beta_0 + \beta_1 \ln I N C_t + \beta_2 \ln I N C_{t-1} + \alpha \ln C_{t-1} + \varepsilon_t$$

The literature refers to this type of model as an autoregressive distributed lag model, ADL model for short. “Autoregressive” is due to the presence of $\ln C_{t-1}$ on the right hand side of the equation, so that consumption today depends on its own past. “Distributed lag” refers to the presence of lagged as well as current income in the model.

How can we investigate whether equation (2.5) is indeed a better description of the data than the static model? The answer to that question brings us in the direction of econometrics, but intuitively, one indication would be if the empirical
counterpart to the disturbance of (2.5) are smaller and less systematic than the errors of equation (2.2). To test this, we obtain the residual \( \hat{\varepsilon}_t \), again using the method of least squares to find the best fit of \( \ln C_t \) according to the dynamic model:

\[
\ln \hat{C}_t = 0.04 + 0.13 \ln INC_t + 0.08 \ln INC_{t-1} + 0.79 \ln C_{t-1}
\]

Figure 2 shows the two residual series \( \hat{\varepsilon}_t \) and \( \hat{\varepsilon}_t \), and it is immediately clear that the dynamic model in (2.6) is a much better description of the behaviour of private consumption than the static model (2.3). As already stated, this is a typical finding with macroeconomic data.

![Figure 2: Residuals of the two estimated consumptions functions (2.3), and (2.6).](image)

Judging from the estimated coefficients in (2.6), one main reason for the improved fit of the dynamic model is the lag of consumption itself. That the lagged value of the endogenous variable is an important explanatory variable is also a typical finding, and just goes to show that dynamic models represent essential tools for empirical macroeconomics. The rather low values of the income elasticities (0.130 and 0.08) may reflect that households find that a single quarterly change in income is “too little to build on” in their expenditure decisions. As we will see below, the results in (2.6) imply a much higher impact of a permanent change in income.

### 3 Dynamic multipliers

The quotation from Norges Bank’s web pages on monetary policy shows that the Central Bank has formulated a view about the dynamic effects of a change in the interest rate on inflation. In the quotation, the Central Bank states that the effect
will take place within two years, i.e., 8 quarters in a quarterly model of the relationship between the rate of inflation and the rate of interest. That statement may be taken to mean that the effect is building up gradually over 8 quarters and then dies away quite quickly, but other interpretations are also possible. In order to inform the public more fully about its view on the monetary policy transmission mechanism (see topic 5 in our course), the Bank would have to report a set of dynamic multipliers. Similar issues arise in almost all areas of applied macroeconomic, it is of vital interest to form an opinion on how fast an exogenous shock or policy change affects a variable of interest. The key concept needed to make progress on this is the dynamic multiplier.

In order to explain the derivation and interpretation of dynamic multipliers, we first show what our estimated consumption function implies about the dynamic effects of a change in income, see section 3.1. We then derive dynamic multipliers using a general notation for autoregressive distributed lag models, see 3.2.

3.1 Dynamic effects of increased income on consumption

We want to consider what the estimated model in (2.6) implies about the dynamic relationship between income and consumption. For this purpose there is no point to distinguish between fitted and actual values of consumption, so we drop the \(^\hat{C}_t\) above.

Assume that income rises by 1% in period \(t\), so instead of \(INC_t\) we have \(INC'_t = INC_t(1 + 0.01)\). Since income increases, consumption also has to rise. Using (2.6) we have

\[
\ln(C_t(1 + \delta_{c,0})) = 0.04 + 0.13 \ln(INC_t(1 + 0.01)) + 0.08 \ln(INC_{t-1}) + 0.79 \ln(C_{t-1})
\]

where \(\delta_{c,0}\) denotes the relative increase in consumption in period \(t\), the first period of the income increase. Using the approximation \(\ln(1 + \delta_{c,0}) = \delta_{c,0}\) when \(-1 < \delta_{c,0} < 1\), and noting that

\[
\ln C_t - 0.04 - 0.13 \ln INC_t - 0.08 \ln INC_{t-1} - 0.79 \ln C_{t-1} = 0,
\]

we obtain \(\delta_{c,0} = 0.0013\) as the relative increase in \(C_t\). In other words, the immediate effect of a one percent increase in \(INC\) is a 0.13% rise in consumption.

The effect on consumption in the second period depends on whether the rise in income is permanent or only temporary. It is convenient to first consider the dynamic effects of a permanent shock to income. Note first that equation (2.6) holds also for period \(t + 1\), i.e.,

\[
\ln C_{t+1} = 0.04 + 0.13 \ln INC_{t+1} + 0.08 \ln INC_t + 0.79 \ln C_t
\]

before the shock, and

\[
\ln(C_{t+1}(1 + \delta_{c,1})) = 0.04 + 0.13 \ln(INC_{t+1}(1 + 0.01)) + 0.08 \ln(INC_t(1 + 0.01)) + 0.79 \ln(C_t(1 + \delta_{c,0}))
\]

after the shock. Remember that in period \(t + 1\) not only \(INC_{t+1}\) have changed, but also \(INC_t\) and period \(t\) consumption (by \(\delta_{c,0}\)). From this, the relative increase in \(C_t\) in period \(t + 1\) is

\[
\delta_{c,1} = 0.0013 + 0.0008 + 0.79 \times 0.0013 = 0.003125,
\]
or 0.3%. By following the same way of reasoning, we find that the percentage increase in consumption in period \( t + 2 \) is 0.46\% (formally \( \delta_{c,2} \times 100 \)).

Since \( \delta_{c,0} \) measures the direct effect of a change in \( INC \), it is usually called the *impact multiplier*, and can be defined directly by taking the partial derivative \( \partial \ln C_t/\partial \ln INC_t \) in equation (2.6) (more on the relationship between derivative and multipliers in section 3.2 below). The *dynamic multipliers* \( \delta_{c,1}, \delta_{c,2}, ... \delta_{c,\infty} \) are in their turn linked by exactly the same dynamics as in equation (2.6), namely

\[
\delta_{c,j} = 0.13\delta_{inc,j} + 0.08\delta_{inc,j-1} + 0.79\delta_{c,j-1}, \text{ for } j = 1, 2, ..., \infty.
\]

For example, for \( j = 3 \), and setting \( \delta_{inc,3} = \delta_{inc,2} = 0.01 \) since we consider a permanent rise in income, we obtain

\[
\delta_{c,3} = 0.0013 + 0.0008 + 0.79 \times 0.0046 = 0.005734
\]
or 0.57\% in percentage terms. Clearly, the multipliers increase from period to period, but the increase is slowing down since in (3.7) the last multiplier is always multiplied by the coefficient of the autoregressive term, which is less than 1. Eventually, the sequence of multipliers are converge to what we refer to as the *long-run multiplier*. Hence, in (3.7) if we set \( \delta_{c,j} = \delta_{c,j-1} = \delta_{c,\text{long-run}} \) we obtain

\[
\delta_{c,\text{long-run}} = \frac{0.0013 + 0.0008}{1 - 0.79} = 0.01,
\]

meaning that according to the estimated model in (2.6), a 1\% permanent increase in income has a 1\% long-run effect on consumption.

Remember that the set of multipliers we have considered so far represent the dynamic effects of a permanent rise in income, and they are shown for convenience in the first column of table 2. In contrast, a temporary rise in income (by 0.01) in equation (2.6) gives rise to another sequence of multipliers: The impact multiplier is again 0.0013, but the second multiplier becomes \( 0.13 \times 0 + 0.08 \times 0.01 + 0.79 \times 0.0013 = 0.0018 \), and the third is found to be \( 0.79 \times 0.0018 = 0.0014 \), so these multipliers are rapidly approaching zero, which is also the long-run multiplier in this case.

Table 1: Dynamic multipliers of the estimated consumption function in (2.6), percentage change in consumption after a 1 percent rise in income.

<table>
<thead>
<tr>
<th>Impact period</th>
<th>Permanent 1% change</th>
<th>Temporary 1% change</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. period after shock</td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td>2. period after shock</td>
<td>0.31</td>
<td>0.18</td>
</tr>
<tr>
<td>...</td>
<td>0.46</td>
<td>0.14</td>
</tr>
<tr>
<td>long-run multiplier</td>
<td>1.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 2: Dynamic multipliers of the estimated consumption function in (2.6), percentage change in consumption after a 1 percent rise in income.

If you supplement the multipliers in the column to the right with a few more periods and then sum the whole sequence you find that the sum is close to 1, which is the long-run multiplier of the permanent change. A relationship like this always holds,
no matter what the long-run effect of the permanent change is estimated to be. Heuristically, another way to think about the effect of a permanent change in an explanatory variable, is as the sum of the changes triggered by a temporary change. In this sense, the dynamic multipliers of a temporary change is the more fundamental of the two, since the dynamic effects of permanent shock can be calculated in a second step. Also, perhaps for this reason, many authors reserve the term dynamic multiplier for the effects of a temporary change and use a different term—cumulated multipliers—for the dynamic effects of a permanent change. However, as long as one is clear about which kind of shock we have in mind, no misunderstandings should occur by the term dynamic multipliers in both cases.

Figure 3 shows graphically the two classes of dynamic multipliers, again for our consumption function example. First, we have the temporary change in income, and below that the consumption multipliers. Correspondingly, to the right we show the graphs with permanent shift in income and the (cumulated) dynamic multipliers.²

### 3.2 General notation

As noted in the consumption function example, the impact multiplier is (after convenient scaling by 100) identical to the (partial) derivative of $C_t$ with respect to

²These graphs were constructed using PcGive and GiveWin, but it is of course possible to use Excel or other programs.
We now establish more formally that also the second, third and higher order multipliers can be interpreted as derivatives. At this stage it is also convenient to introduce the general notation for the autoregressive distributed lag model. In (3.8), \( y_t \) is the endogenous variable while the \( x_t \) and \( x_{t-1} \) make up the distributed lag part of the model:

\[
y_t = \beta_0 + \beta_1 x_t + \beta_2 x_{t-1} + \alpha y_{t-1} + \varepsilon_t.
\]

In the same way as before, \( \varepsilon_t \) symbolizes a small and random part of \( y_t \) which is unexplained by \( x_t \) and \( x_{t-1} \) and the lagged endogenous variable \( y_{t-1} \).

In many applications, as in the consumption function example, \( y \) and \( x \) are in logarithmic scale, due to the specification of a log-linear functional form. However, in other applications, different units of measurement are the natural ones to use. Thus, frequently, \( y \) and \( x \) are measured in million kroner, in thousand persons or in percentage points. Mixtures of measurement are also frequently used in practice: for example in studies of labour demand, \( y_t \) may denote the number of hours worked in the economy (or by an individual) while \( x_t \) denotes real wage costs per hour. The measurement scale does not affect the derivation of the multipliers, but care must be taken when interpreting and presenting the results. Specifically, only when both \( y \) and \( x \) are in logs, are the multipliers directly interpretable as percentage changes in \( y \) following a 1% increase in \( x \), i.e., they are (dynamic) elasticities.

To establish the connection between dynamic multipliers and the derivatives of \( y_t, y_{t+1}, y_{t+2}, \ldots \), it is convenient to define \( x_t, x_{t+1}, x_{t+2}, \ldots \) as functions of a continuous variable \( h \). When \( h \) changes permanently, starting in period \( t \), we have \( \partial x_t / \partial h > 0 \), but no change in \( x_{t-1} \) or in \( y_{t-1} \) since those variables are predetermined, in the period of the shock. Since \( x_t \) is a function of \( h \), so is \( y_t \), and the effect of \( y_t \) of the change in \( h \) is founds as

\[
\frac{\partial y_t}{\partial h} = \beta_1 \frac{\partial x_t}{\partial h}.
\]

It is customary to consider “unit changes” in the explanatory variable (corresponding to the 1% change in income in the consumption function example), which means that we let \( \partial x_t / \partial h = 1 \). Hence the first multiplier is

\[
\frac{\partial y_t}{\partial h} = \beta_1.
\]

The second multiplier is found by considering the equation for period \( t + 1 \), i.e.,

\[
y_{t+1} = \beta_0 + \beta_1 x_{t+1} + \beta_2 x_t + \alpha y_t + \varepsilon_{t+1}.
\]

and calculating the derivative \( \partial y_{t+1} / \partial h \). Note that, due to the change in \( h \) occurring already in period \( t \), both \( x_{t+1} \) and \( x_t \) have changed, i.e., \( \partial x_{t+1} / \partial h > 0 \) and \( \partial x_t / \partial h > 0 \). Finally, we need to keep in mind that also \( y_t \) is a function of \( h \), hence:

\[
\frac{\partial y_{t+1}}{\partial h} = \beta_1 \frac{\partial x_{t+1}}{\partial h} + \beta_2 \frac{\partial x_t}{\partial h} + \alpha \frac{\partial y_t}{\partial h}
\]

Again, considering a unit change, and using (3.9), the second multiplier can be written as

\[
\frac{\partial y_{t+1}}{\partial h} = \beta_1 + \beta_2 + \alpha \beta_1 = \beta_1 (1 + \alpha) + \beta_2
\]
To find the third derivative, consider

\[ y_{t+2} = \beta_0 + \beta_1 x_{t+2} + \beta_2 x_{t+2} + \alpha y_{t+1} + \varepsilon_{t+2}. \]

Using the same logic as above, we obtain

\[
\frac{\partial y_{t+2}}{\partial h} = \beta_1 \frac{\partial x_{t+2}}{\partial h} + \beta_2 \frac{\partial x_{t+1}}{\partial h} + \alpha \frac{\partial y_{t+1}}{\partial h}
\]

\[ = \beta_1 + \beta_2 + \alpha \frac{\partial y_{t+1}}{\partial h} \]

\[ = \beta_1 (1 + \alpha + \alpha^2) + \beta_2 (1 + \alpha) \]

where the unit-change, \( \partial x_t/\partial h = \partial x_{t+1}/\partial h = 1 \), is used in the second line, and the third line is the result of substituting \( \partial y_{t+1}/\partial h \) out with the right hand side of (3.11).

Comparing, equation (3.10) and the first line of (3.12) there is a clear pattern: The third and second multipliers are linked by exactly the same form of dynamics that the govern the \( y \) variable itself. This also holds for higher order multipliers, and means that the multipliers can be computed recursively: Once we have found the second multiplier, the third can be found easily using the second line of (3.12).

Table 3 shows summary of the results. In the table, we use the notation \( \delta_j \) (\( j = 0, 1, 2, ... \)) for the multipliers. For, example \( \delta_0 \) is identical to \( \partial y_t/\partial h \), and \( \delta_2 \) is identical to the third multiplier, \( \partial y_{t+2}/\partial h \) above. In general, because the multipliers are linked recursively, multiplier number \( j + 1 \) is given as

\[
\delta_j = \beta_1 + \beta_2 + \alpha \delta_{j-1}, \text{ for } j = 1, 2, 3, \ldots
\]

In the consumption function example, we saw that as long as the autoregressive parameter is less than one, the sequence of multipliers is converging towards a long run multiplier. In this more general case, the condition needed for the existence of a long run multiplier is that \( \alpha \) is less than one in absolute value, formally \(-1 < \alpha < 1\). In the next section, this condition is explained in more detailed. section. For the present purpose we simply assume that the condition holds, and define the long run multiplier as \( \delta_j = \delta_{j-1} = \delta_{\text{long-run}} \). Using (3.13), the expression for \( \delta_{\text{long-run}} \) is found to be

\[
\delta_{\text{long-run}} = \frac{\beta_1 + \beta_2}{1 - \alpha}, \text{ if } -1 < \alpha < 1.
\]

Clearly, if \( \alpha = 1 \), the expression does not make sense mathematically, since the denominator is zero. Economically, it doesn’t make sense either since the long run effect of a permanent unit change in \( x \) is an infinitely large increase in \( y \) (if \( \beta_1 + \beta_2 > 0 \)). The case of \( \alpha = -1 \), may at first sight seem to be acceptable since the denominator is 2, not zero. However, as explained below, the dynamics is essentially unstable also in this case meaning that the long run multiplier is not well defined for the case of \( \alpha = -1 \).
Table 3: Dynamic multipliers of the general autoregressive distributed lag model.

<table>
<thead>
<tr>
<th>ADL model:</th>
<th>$y_t = \beta_0 + \beta_1 x_t + \beta_2 x_{t-1} + \alpha y_{t-1} + \varepsilon_t$.</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Permanent unit change in $x^{(1)}$</th>
<th>Temporary unit change in $x^{(2)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. multiplier:</td>
<td></td>
</tr>
<tr>
<td>$\delta_0 = \beta_1$</td>
<td>$\delta_0 = \beta_1$</td>
</tr>
<tr>
<td>2. multiplier:</td>
<td></td>
</tr>
<tr>
<td>$\delta_1 = \beta_1 + \beta_2 + \alpha \delta_0$</td>
<td>$\delta_1 = \beta_2 + \alpha \delta_0$</td>
</tr>
<tr>
<td>3. multiplier:</td>
<td></td>
</tr>
<tr>
<td>$\delta_2 = \beta_1 + \beta_2 + \alpha \delta_1$</td>
<td>$\delta_2 = \alpha \delta_1$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>j+1 multiplier</td>
<td></td>
</tr>
<tr>
<td>$\delta_j = \beta_1 + \beta_2 + \alpha \delta_{j-1}$</td>
<td>$\delta_j = \alpha \delta_{j-1}$</td>
</tr>
<tr>
<td>long-run</td>
<td></td>
</tr>
<tr>
<td>$\delta_{\text{long-run}} = \frac{\beta_1 + \beta_2}{1-\alpha}$</td>
<td>0</td>
</tr>
</tbody>
</table>

**notes:**
1. As explained in the text, $\partial x_{t+j} / \partial h = 1$, $j = 0, 1, 2, \ldots$
2. $\partial x_t / \partial h = 1$, $\partial x_{t+j} / \partial h = 0$, $j = 1, 2, 3, \ldots$
If $y$ and $x$ are in logs, the multipliers are in percent.

**Exercise 2** Use the numbers from the estimated consumption function and check that by using the formulae of Table 3, you obtain the same numerical results as in section 3.1.

**Exercise 3** Check that you understand, and are able to derive the results in the column for a temporary change in $x_t$ in Table 3.

### 3.3 Multipliers in the text books to this course

As already noted in the introduction, the distinction between short and long-run multipliers permeates modern macroeconomics, and so is not special to the consumption function example above! The reader is invited to be on the look-out for expressions like short and long-run effects/multipliers/elasticities in the textbook by Burda and Wyplosz (2001). One early example in the book is found in Chapter 8, on money demand, Table 8.4. Note the striking difference between the short-run and long-run multipliers for all countries, a direct parallel to the consumption function example we have worked through in this section. Hence, the precise interpretation of (log) linearized money demand function in B&W’s Box 8.4 is as a so called steady state relationship, thus the parameter $\mu$ is a long-run elasticity with respect to income. In the next section of this note, the relationship between long-run multipliers and steady state relationships in explained. The money demand function also plays an important role in Chapter 9 and 10, and in later chapters in B&W.

In the book by B&W, the distinction between short and long-run is a main issue in Chapter 12, where short and long-run supply curves are derived. For example, the slope of the short-run curves in figure 12.6 correspond to the impact multipliers of the respective models, while the vertical long-run curve suggest that the long-run multipliers are infinite. In section 8 below, we go even deeper into wage-price dynamics, using the concepts that we have introduced here.
Later on in the course we will encounter models of the joint determination of inflation, output and the exchange rates which are also dynamic in nature, so a good understanding the logic of dynamic multipliers will prove very useful. Interest rate setting with the aim of controlling inflation is one specific example. However, we will not always need (or be able to) give a full account of the multipliers of these more complex model. Often we will concentrate on the impact and long-run multipliers.

4 Reconciling dynamic and static models

If we compare the long-run multiplier of a permanent shock to the estimated regression coefficient (or elasticity) of a static model, there is often a close correspondence. This is indeed the case in our consumption function example where the multiplier is 1.00 and the estimated coefficient in equation (2.3) is 0.99. This is not a coincidence, since the dynamic formulation in fact accommodates a so called steady-state relationship which is similar to the static model.

To show this, consider again the ADL model:

\[ y_t = \beta_0 + \beta_1 x_t + \beta_2 x_{t-1} + \alpha y_{t-1} + \varepsilon_t. \]

In the last section we introduced two quite general properties of this model. First it usually explains the behaviour of the dependent variable much better than a simple static relationship, which imposes on the data that all adjustments of \( y \) to changes in \( x \) takes place without delay. Second, it allows us to calculate the very useful dynamic multipliers. But what does (4.15) imply about the long-run relationship between \( y \) and \( x \), the sort of relationship that we expect to hold when both \( y_t \) and \( x_t \) are growing with constant rates, a so called steady-state situation? To answer this question it is useful to re-write equation (4.15), so that the relationship between levels and growth becomes clear.

As a first step, we subtract \( y_{t-1} \) from either side of the equation, and then subtract and add \( \beta_1 x_{t-1} \) on the right hand side, we obtain

\[ \Delta y_t = \beta_0 + \beta_1 x_t + \beta_2 x_{t-1} + (\alpha - 1)y_{t-1} + \varepsilon_t \]

\[ = \beta_0 + \beta_1 \Delta x_t + (\beta_1 + \beta_2)x_{t-1} + (\alpha - 1)y_{t-1} + \varepsilon_t \]

where \( \Delta \) is known as the difference operator, defined as \( \Delta z_t = z_t - z_{t-1} \) for a time series variable \( z_t \). If \( y_t \) and \( x_t \) are measured in logarithms (like consumption and income in our consumption function example) \( \Delta y_t \) and \( \Delta x_t \) are their respective growth rates. Hence, for example, in the previous section

\[ \Delta \ln C_t = \ln(C_t/C_{t-1}) = \ln(1 + \frac{C_t - C_{t-1}}{C_{t-1}}) \approx \frac{C_t - C_{t-1}}{C_{t-1}}. \]

The model in (4.16) is therefore explaining the growth rate of consumption by, first, the income growth rate and, second, the past levels of income and consumption. Since the disturbance term is the same in (4.15) and in (4.16), the transformation is often referred to as an I-1 transformation. In practice this means that if we instead of (2.5) above, estimated a relationship with \( \Delta \ln C_t \) on the left hand side, and
\[ \Delta \ln INC_t, \ln INC_{t-1} \text{ and } \ln C_{t-1} \] on the right hand side, we would obtain exactly the same residuals \( \hat{\varepsilon}_t \) as before.

For economic interpretation, is useful to collect the level terms \( y_{t-1} \) and \( x_{t-1} \) inside a bracket:

\[
\begin{equation}
\Delta y_t = \beta_0 + \beta_1 \Delta x_t - (1 - \alpha) \left\{ y - \frac{\beta_1 + \beta_2}{1 - \alpha} x \right\}_{t-1} + \varepsilon_t,
\end{equation}
\]
and let us assume that we have the following economic theory about the long-run average relationship between \( y \) and \( x \)

\[
\begin{equation}
y^* = k + \gamma x
\end{equation}
\]
Comparison with (4.17) shows that we can identify the parameter \( \gamma \) in the following way

\[
\begin{equation}
\gamma \equiv \frac{\beta_1 + \beta_2}{1 - \alpha}, \quad -1 < \alpha < 1
\end{equation}
\]
meaning that the theoretical slope coefficient \( \gamma \) is identical to the long-run multiplier of the dynamic model.

Consider now a theoretical steady state situation in which growth rates are constant, \( \Delta x_t = g_x, \Delta y_t = g_y \), and the disturbance term is equal to its mean, \( \varepsilon_t = 0 \). Imposing this in (4.17), and using (4.19) gives

\[
\begin{equation}
g_y = \beta_0 + \beta_1 g_x - (1 - \alpha) \left\{ y^* - \gamma x \right\}_{t-1}.
\end{equation}
\]
The term in curly brackets is constant since we consider a steady state, so the time subscript can be dropped. Re-arranging with \( y^* \) on the left hand side gives

\[
\begin{equation}
y^* = \frac{-g_y + \beta_0 + \beta_1 g_x}{1 - \alpha} + \gamma x,
\end{equation}
\]
which is valid if \(-1 < \alpha < 1\). Comparing the two expressions for \( y^* \) in (4.18) and (4.20) shows that for consistency, the parameter \( k \) in (4.18) must be taken to depend on the steady state growth rates which are however parameters just as \( \alpha, \beta_0 \) and \( \beta_1 \). Often we only consider a static steady states, with no growth, in that case \( k \) is simply \( \beta_0/(1 - \alpha) \).

In sum, there is an important correspondence between the dynamic model and a static relationship like (4.18) motivated by economic theory:

1. A theoretical linear relationship \( y^* = k + \gamma x \) can be retrieved as the steady state solution of the dynamic model (4.15). This generalizes to theory models with more than one explanatory variable (e.g., \( y^* = k + \gamma_1 x_1 + \gamma_2 x_2 \)) as long as both \( x_{1t} \) and \( x_{2t} \) (and/or their lags) are included in the dynamic model. In section 8 we will discuss some details of this extension in the context of models of wage and price setting (inflation).

2. The theoretical slope coefficient \( \gamma \) are identical to the corresponding long-run multiplier (of a permanent increase in the respective explanatory variables).
3. Conversely, if we are only interested in quantifying a long-run multiplier (rather than the whole sequence of dynamic multipliers), it can be found by using the identity in (4.19).

A reasonable objection to # 3 is that, if we are only interested in the theoretical long-run slope coefficient, why don’t we simply estimate $\gamma$ from a static model, rather than bother with a dynamic model? After all, in the consumption function example, the direct estimate (2.3) is practically identical to the long-run multiplier which we derived from the estimated dynamic model? The short answer is that, as a rule, static models yields poor estimates of long-run multipliers. To understand why takes us into time series econometrics, but intuitively the direct estimate (from the static model) is only reliable when the theoretical relationship has a really strong presence in the data. This seems to be the case in our consumption function example, but in a majority of applications, the theoretical relationship, though valid, is obscured by slow adjustment and influences from other factors. Therefore, it generally pays off to follow the procedure in of first formulating (and estimating) the dynamic model.

Returning to the beginning of this section, we note that the transformation of the ADL model into level and differences is known as the error correction transformation. The name reflects that according to the model, $\Delta y_t$ corrects past deviation from the long run equilibrium relationships. The error correction model, ECM, not only helps clarify the link between dynamics and the theoretical steady state, it also plays an essential role in econometric modelling of non-stationary time series. In 2003, when Clive Granger and Rob Engle were awarded the Noble Price in economics, part of the motivation was their finding that so called cointegration between two or more non-stationary variables implies error correction, and vice versa.

In common usage, the term error correction model is not only used about equation (4.17), where the long run relationship is explicit, but also about (the second line) of (4.16). One reason is that the long run multipliers (the coefficients of the long run relationship) can be easily established by estimating the linear relationship in (4.17) by OLS, and then calculating the ratio $\gamma$ in (4.19). Direct estimation of $\gamma$ requires a non-linear estimation method.

5 Role of static models

After all we have said in favour of dynamic models, why don’t we give up static equations altogether in macroeconomics? The reason, for not taking this extreme view is that after all static models remain the main workhorses of economic analysis, but we need to be clear about their interpretation and limitation. As we have argued, static models may have two (valid) interpretations in macroeconomics:

1. As (approximate) descriptions of dynamics

2. As corresponding to long-run, steady state, relationships.

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3 Error correction models became popular in econometrics in the early 1980s. Since $\Delta y_t$ is actually bringing the level of $y$ towards the long run relationship, a better name may be equilibrium correction model. However the term error correction model has stuck.
The first rationale may sound like a contradiction of terms, but what we mean is that it sometimes realistic to assume that the dynamic adjustment process is so fast that the adjustment to a change in an exogenous variable is completed within the period that we have in mind for our analysis. Hence, as a first approximation, we can do without formulating the dynamic adjustment process in full. An example of this rationale “in operation” is the usual IS-LM model (with sticky prices). When we use that model we typically focus on the short-run effects of for example a rise in government expenditure. In the longer term, prices will be affected by a change in activity, but that dynamic process is not specified in the IS-LM model. Another, very carefully argued case for a static model, is the model of the market for foreign exchange in the textbook *Open Economy Macroeconomics* (OEM for short) by Asbjørn Rødseth (2000), see chapter 1 and chapter 3.1. In those models the idea that the agents in the market adjust so fast that the full effect of a change in a policy instrument emerges within short time period.

The second rationale for (interpretation of) static models is, of course, the interpretation that we have highlighted most in this note. Ideally, the two interpretations should not be mixed! Nevertheless authors do exactly that, perhaps because they need to make short-cuts in order to complete their models and to be able to “tell a story”. Consider for example Ch 12.3 in B&W where equations which reasonably can be interpreted as long-run tendencies (interpretation 2 above), namely equation (12.6) and (12.7) are turned into a seemingly dynamic model by using an algebraic trick (see section 12.3.4 in B&W) and an ad hoc theory of dynamics in the mark-up. In section 8 we present two other models of wage-and-price formation where the short-run is reconciled with the long-run in a consistent manner.

## 6 Solution and stability

The reader will probably have noted that the existence of a finite long-run multiplier, and thereby the validity of the correspondence between the ADL model and long-run relationships, depends on the autoregressive parameter $\alpha$ in (4.15) being different from unity. Not surprisingly, $\alpha$ is also all important for the nature and type of solution of (4.15).

Mathematically speaking, as long as we have a known initial condition which is given from history, $y_0$, a solution exists to (4.15).4 The solution can however be stable, unstable or explosive, and in this section discuss the nature of the different solutions.

The condition

\[
-1 < \alpha < 1
\]

is the necessary and sufficient condition for the existence of a (globally asymptotically) *stable* solution. The stable solution has the characteristic that asymptotically there is no trace left of the initial condition $y_0$. This is easy to see by solving (4.15) forward, starting with period 1 (treating $y_0$ as known). For simplicity, and without

---

4If we open up for the possibility that the initial condition is not determined by history, but that it may “jump” due to market forces, expectations etc, there are other solutions to consider. Such solutions play a large role in macroeconomics, but they belong to more advanced courses.
loss of generality set $\beta_2 = 0$ and assume that the exogenous variable $x_t$ (sometimes called the “forcing variable”) takes a constant value $m_x$ for the whole length of the solution period. We also set $\varepsilon_t = 0$. For the three first periods we obtain

$$
\begin{align*}
y_1 &= \beta_0 + \beta_1 m_x + \alpha y_0 \\
y_2 &= \beta_0 + \beta_1 m_x + \alpha y_1 \\
&= \beta_0 (1 + \alpha) + \beta_1 m_x (1 + \alpha) + \alpha^2 y_0 \\
y_3 &= \beta_0 + \beta_1 m_x + \alpha y_2 \\
&= (\beta_0 + \beta_1 m_x) (1 + \alpha) + \alpha^2 y_0
\end{align*}
$$

and thus, by induction, for period $t$

$$
(6.22) \quad y_t = (\beta_0 + \beta_1 m_x) \sum_{s=0}^{t-1} \alpha^s + \alpha^t y_0, \quad t = 1, 2, ...
$$

which is the general solution of (4.15) (given the simplifying assumptions just stated $\beta_2 = 0$, $x_t = m_x$, $\varepsilon_t = 0$). Next, we consider the stable solution and two unstable solutions:

**Stable solution** In this case, condition (6.21) holds. Clearly, as the distance in time between $y_t$ and the initial condition increases, $y_0$ exerts less and less influence on the solution. When $t$ becomes large (approaches infinity), the influence of the initial condition becomes negligible. Since $\sum_{s=0}^{t-1} \alpha^s \to \frac{1}{1-\alpha}$ as $t \to \infty$, we have asymptotically:

$$
(6.23) \quad y^* = \frac{(\beta_0 + \beta_1 m_x)}{1 - \alpha}
$$

where $y^*$ denotes the equilibrium of $y_t$. As stated, $y^*$ is independent of $y_0$. Note that $\frac{\partial y}{\partial m_x} = \beta_1/(1 - \alpha)$, the long-run multiplier with respect to $x$. Finally, note also that (4.20) above, although derived under different assumption about the exogenous variable (namely a constant growth rate), is perfectly consistent with (6.23).

The stable solution can be written in an alternative and very instructive way. Note first that by using the formula for the sum of the $t-1$ first terms in a geometric progression, $\sum_{s=0}^{t-1} \alpha^s$ can be written as

$$
\sum_{s=0}^{t-1} \alpha^s = \frac{1 - \alpha^t}{1 - \alpha}.
$$

Using this result in (6.22), and next adding and subtracting $(\beta_0 + \beta_1 m_x) \alpha^t/(1 - \alpha)$ on the right hand side of (6.22), we obtain

$$
(6.24) \quad y_t = \frac{(\beta_0 + \beta_1 m_x)}{1 - \alpha} + \alpha^t (y_0 - \frac{\beta_0 + \beta_1 m_x}{1 - \alpha})
$$

$$
= y^* + \alpha^t (y_0 - y^*), \text{ when } -1 < \alpha < 1.
$$
Thus, in the stable case, the dynamic process is essentially correcting the initial discrepancy (disequilibrium) between the initial level of \( y \) and its long-run level.

**Unstable solution (hysteresis)** When \( \alpha = 1 \), we obtain from equation (6.22):

\[
y_t = (\beta_0 + \beta_1 m_x) t + y_0, \quad t = 1, 2, ...
\]

showing that the solution contains a linear trend and that the initial condition exerts full influence over \( y_t \) even over infinitely long distances. There is of course no well defined equilibrium of \( y_t \), and neither is there a finite long-run multiplier. Nevertheless, the solution is perfectly valid mathematically speaking: given an initial condition, there is one and only one sequence of numbers \( y_1, y_2, ... y_T \) which satisfy the model.

The instability is however apparent when we consider a sequence of solutions. Assume that we first find a solution conditional on \( y_0 \), and denote the solution \( \{y^0_1, y^0_2, ... y^0_T\} \). After one period, we usually want to recalculated the solution because something unexpected has happened in period 1. The updated solution is \( \{y^1_2, y^1_3, ... y^1_{T+1}\} \) since we now condition on \( y_1 \). From (6.25) we see that as long as \( y^0_1 \neq y_1 \) (the same as saying that \( \varepsilon_1 \neq 0 \)) we will have \( y^0_2 - y^1_2 \neq 0 \), \( y^0_3 - y^1_3 \neq 0, ..., y^0_T - y^1_T \neq 0 \). Moreover, when the time arrives to condition on \( y_3 \), the same phenomenon is going to be observed again. The solution is indeed unstable in the sense that any (small) change in initial conditions have a permanent effect on the solution. Economists like to refer to this phenomenon as *hysteresis*, see Burda and Wyplosz (2001, p 538). In the literature on wage-price setting, the point has been made that failure of wages to respond properly to shocks to unemployment (in fact the long-run multiplier of wages with respect to is zero) may lead to hysteresis in the rate of unemployment.

The case of \( \alpha = -1 \) is more curious, but it is nevertheless useful to check the solution and dynamics implied by (6.22) also in this case.

**Explosive solution** When \( \alpha \) is greater than unity in absolute value the solution is called explosive, for reasons that should be obvious when you consult (6.22).
Figure 4: Two stable solutions of 4.15 (corresponding to two values of $\alpha$, and two unstable solutions (corresponding to two different initial conditions, see text)

**Exercise 4** Explain why the two stable solutions in 4 behave so differently!

**Exercise 5** Indicate, in the graph for the unstable case, the predicted value for $y_1$ given the initial condition $y_0$. What about $\varepsilon_1$?

**Exercise 6** Sketch a solution path for the explosive solution in a diagram like 4.

## 7 Dynamic systems

In macroeconomics, the effect of a shock or policy change is usually dependent on system properties. As a rule it is not enough to consider only one (structural/behavioural equation) in order to obtain the correct dynamic multipliers. Consider for example the consumption function of section 2 and 3, where we (perhaps implicitly) assumed that income ($INC$) was an exogenous variable. This exogeneity assumption is only tenable given some further assumptions about the rest of the economy: for example if there is a general equilibrium with flexible prices (see Burda and Wyplosz (2001, ch 10.5)) and the supply of labour is fixed, then output and income may be regarded as independent of $C_t$. However, with sticky prices and
idle resources, i.e., the Keynesian case, $INC$ must be treated as endogenous, and to use the multipliers that we derived in section 3 are in fact misleading.

Does this mean that all that we have said so far about multipliers and stability of a dynamic equation is useless (apart from a few special cases)? Fortunately things are not that bad. First, it is often quite easy to bring the system on a form with two reduced form dynamic equations that are of the same form that we have considered above. After this step, we can derive the full solution of each endogenous variable of the system (if we so want). Second, there are ways of discussing stability and the dynamic properties of systems, without first deriving the full solution. One such procedure is the so called phase-diagram which however goes beyond the scope of this course. Third, in many cases the dynamic system is after all rather intuitive and transparent, so it is possible to give a good account of the dynamic behaviour, simply based on our understanding of the economics of the problem under consideration.

In this section, we give a simple example of the first approach (finding the solution) based on the consumption function again. However, it is convenient to use a linear specification:

\[ C_t = \beta_0 + \beta_1 INC_t + \alpha C_{t-1} + \varepsilon_t \]

together with the stylized product market equilibrium condition

\[ INC_t = C_t + J_t \]

where $J_t$ denotes autonomous expenditure, and $INC$ is now interpreted as the gross domestic product, GDP. We assume that there are idle resources (unemployment) and that prices are sticky. The 2-equation dynamic system has two endogenous variables $C_t$ and $INC_t$, while $J_t$ and $\varepsilon_t$ are exogenous.

To find the solution for consumption, simply substitute $INC$ from (7.27), and obtain

\[ C_t = \tilde{\beta}_0 + \tilde{\alpha} C_{t-1} + \tilde{\beta}_2 J_t + \tilde{\varepsilon}_t \]

where $\tilde{\beta}_0$ and $\tilde{\alpha}$ are the original coefficients divided by $(1 - \beta_1)$, and $\tilde{\beta}_2 = \beta_1 / (1 - \beta_1)$, (what about $\tilde{\varepsilon}_t$?).

Equation (7.28) is yet another example of a ADL model, so the theory of the previous sections applies. For a given initial condition $C_0$ and known values for the two exogenous variables (e.g. $\{J_1, J_2, ..., J_T\}$) there is a unique solution. If $-1 < \tilde{\alpha} < 1$ the solution is asymptotically stable. Moreover, if there is a stable solution for $C_t$, there is also a stable solution for $INC_t$, so we don’t have to derive a separate equation for $INC_t$ in order to check stability of income.

The impact multiplier of consumption with respect to autonomous expenditure is $\tilde{\beta}_2$, while in the stable case, the long-run multiplier is $\tilde{\beta}_2 / (1 - \tilde{\alpha})$.

Equation (7.28) is called the final equation for $C_t$. The defining characteristic of a final equation is that (apart from exogenous variables) the right hand side only contains lagged values of the left hand side variable. It is often feasible to derive a final equation for more complex system than the one we have studied here. The conditions for stability is then expressed in terms of the so called characteristic roots of the final equation. The relationship between $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ and the characteristic roots goes beyond the scope of this course, but a sufficient condition for stability of a second order difference equation is that both $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ are less than one.
**Exercise 7** In a dynamic system with two endogenous variables, explain why we only need to derive one final equation in order to check the stability of the system.

**Exercise 8** In our example, derive the final equation for $INC_t$. What is the relationship between the demand multipliers that you are familiar with from simple Keynesian models, and the impact and long-run multipliers of INC with respect to a one unit change in autonomous expenditure?
8 Wage-price dynamics

Inflation denotes the rates of change of the general price level, and is thus a dynamic phenomenon almost by definition. From an empirical point of view we seldom see that prices and wages make big jumps form one period to the next. Thus inflation rates are typically finite, and in economies with public trust in the monetary and fiscal system, it is even quite moderate for long periods of time. Nevertheless, inflation can be highly persistent, and towards the end of last century, the governments of the “Western world” invested heavily in curbing inflation. Institutional changes took place (crushing of labour unions in the UK, revitalization of incomes policies in Norway, a new orientation of monetary policy, the EU's stability pact, etc.), and governments even allowed unemployment to rise to level not seen since the 2WW.

Models of wage-price dynamics is therefore of the greatest interest in a course like ours, as they also represent substantive usage of several of the concepts reviewed above. In this section we present two very important models of inflation in small open economies: the Norwegian model and the Phillips curve. The Norwegian model is an extremely interesting example of macroeconomic theory which is based on realistic assumptions of behavioural regularities. In Norway, it has been a very important framework for analysis and policy. The Phillips curve is of course covered in every textbook in macroeconomics, and in B&W it is derived in Ch. 12. Below, we attempt to give a fresh lick to the Phillips curve by contrasting it with the Norwegian model of inflation. In fact we show that the Phillips curve can be seen as a special case of the richer dynamics implied by the Norwegian model. We end the section by a brief look at the evidence, for the Norwegian economy.

8.1 The Norwegian main-course model

The Scandinavian model of inflation was formulated in the 1960s. It became the framework for both medium term forecasting and normative judgements about “sustainable” centrally negotiated wage growth in Norway. In this section we show that Aukrust’s (1977) version of the model can be reconstructed as a set of propositions about long-run relationships and causal mechanisms. The reconstructed Norwegian model serves as a reference point for, and in some respects also as a corrective to, the modern models of wage formation and inflation in open economies.

---

5In fact there were two models, a short-term multisector model, and the long-term two sector model that we re-construct using modern terminology in this chapter. The models were formulated in 1966 in two reports by a group of economists who were called upon by the Norwegian government to provide background material for that year’s round of negotiations on wages and agricultural prices. The group (Aukrust, Holte and Stolzt) produced two reports. The second (dated October 20 1966, see Aukrust (1977)) contained the long-term model that we refer to as the main-course model. Later, there was similar development in e.g., Sweden, see Edgren et al. (1969) and the Netherlands, see Driehuis and de Wolf (1976).

6In later usage the distinction between the short and long-term models seems to have become blurred, in what is often referred to as the Scandinavian model of inflation. We acknowledge Aukrust’s clear exposition and distinction in his 1977 paper, and use the name Norwegian main-course model for the long-term version of his theoretical framework.

6On the role of the main-course model in Norwegian economic planning, see Bjerkholt (1998).

7For an exposition and appraisal of the Scandinavian model in terms of current macroeconomic theory, see Rodseth (2000, Chapter 7.6).
8.1.1 A model of long-run wage and price setting

Central to the model is the distinction between a tradables sector where firms act as price takers, either because they sell most of their produce on the world market, or because they encounter strong foreign competition on their domestic sales markets, and a non-tradables sector where firms set prices as mark-ups on wage costs.\(^8\)

Let \(Y_e\) denote output (measured as value added) in the tradeable or exposed (e), and let \(Q_e\) represent the corresponding price (index). Moreover, let \(L_e\) denote labour input (total number of hours worked), and use \(W_e\) to represent the hourly wage. The wage share is then \(W_eL_e/Q_eY_e\) or equivalently \(W_e/Q_e\). Thus, using lower case letters to denote natural logarithms (i.e., \(w = \log(W)\) for the wage rate), the first equation in the system (8.29)-(8.32) below says that the log of the wage share \(w_e - a_e - q_e\) is a constant denoted \(m_e\). \(w_s\), \(q_s\) and \(a_s\) are the corresponding variables of the non-tradables or sheltered (s) sector, so the second and third equations imply a constant relative wage \((m_{es})\) between the two sectors, and a constant wage share \((m_s)\) in the sheltered sector also. Finally, \(p\) is the overall domestic price level and (8.32) gives \(p\) as a weighted sum of \(q_e\) and \(q_s\). \(\phi\) is a coefficient that reflects the weight of non-traded goods in private consumption.\(^9\)

\[
\begin{align*}
(8.29) & \quad w_e - q_e - a_e = m_e \\
(8.30) & \quad w_e - w_s = m_{es} \\
(8.31) & \quad w_s - q_s - a_s = m_s \\
(8.32) & \quad p = \phi q_s + (1 - \phi) q_e, \quad 0 < \phi < 1.
\end{align*}
\]

The exogenous variables in the model are \(q_e, a_e, \text{ and } a_s\), while \(w_e, w_s, q_s\) and \(p\) are endogenous.

Equation (8.29) has a very important implication for actual data of wages prices and productivity in the e-sector: Since both \(q_e\) and \(a_e\) are variables that show trendlike growth over time, the data for the nominal wage \(w_e\) should show a clear positive trend, more or less in line with price and productivity. Hence, the sum of the technology trend and the foreign price plays an important role in the theory since it traces out a central tendency or long-run sustainable scope for wage growth. Aukrust (1977) aptly refers to this as the main-course for wage determination in the exposed industries.\(^10\) Thus, for later use we define the main-course variable:

\[
(8.33) \quad mc = a_e + q_e
\]

Aukrust clearly meant equation (8.29) as a long-run relationship between the e-sector wage level and the main-course made up of product prices and productivity.

The relationship between the “profitability of E industries” and the “wage level of E industries” that the model postulates, therefore, is a

---

\(^8\)In France, the distinction between sheltered and exposed industries became a feature of models of economic planning in the 1960s, and quite independently of the development in Norway. In fact, in Courbis (1974), the main-course theory is formulated in detail and illustrated with data from French post-war experience (we are grateful to Odd Aukrust for pointing this out to us).

\(^9\)Due to the log-form, \(\phi = x_s/(1 - x_s)\) where \(x_s\) is the share of non-traded good in consumption.

\(^10\)The essence of the statistical interpretation of the theory is captured by the hypothesis of so-called cointegration between \(w_e\) and \(mc\) see Nymoen (1989) and Rodseth and Holden (1990)).
certainly not a relation that holds on a year-to-year basis”. At best it is valid as a long-term tendency and even so only with considerable slack. It is equally obvious, however, that the wage level in the E industries is not completely free to assume any value irrespective of what happens to profits in these industries. Indeed, if the actual profits in the E industries deviate much from normal profits, it must be expected that sooner or later forces will be set in motion that will close the gap. (Aukrust, 1977, p 114-115).

Aukrust goes on to specify “three corrective mechanisms”, namely wage negotiations, market forces (wage drift, demand pressure) and economic policy. For example

The profitability of the E industries is a key factor in determining the wage level of the E industries: mechanism are assumed to exist which ensure that the higher the profitability of the E industries, the higher their wage level; there will be a tendency of wages in the E industries to adjust so as to leave actual profits within the E industries close to a “normal” level (for which however, there is no formal definition). (Aukrust, 1977, p 113).

![Figure 5: The ‘Wage Corridor’ in the Norwegian model of inflation.](image)

Aukrust coined the term ‘wage corridor’ to represent the development of wages through time and used a graph similar to figure 5 to illustrate his ideas. The
main-course defined by equation (8.33) is drawn as a straight line since the wage is measured in logarithmic scale. The two dotted lines represent what Aukrust called the “elastic borders of the wage corridor”.

Equation (8.30) incorporates two other substantive hypotheses of the Norwegian model of inflation: A constant relative wage between the two sectors (normalized to unity), and wage leadership of the exposed sector. Thus, the sheltered sector is a wage follower and wage setting is recursive, with exposed sector wage determinants also causing sheltered sector wage formation. Equation (8.31) implies that sheltered sector price setters mark-up their prices on average variable costs. Thus sheltered sector price formation adheres to so called normal cost pricing.

Just as the main-course was meant to define a long-term tendency for e-sector wages, also the two other hypotheses apply to the long-run, or alternatively, to a hypothetical steady state. In order to make the long-term nature of the theory explicit we can summarize Aukrust’s model in three propositions about the long-run behaviour of wages and prices

\[
\begin{align*}
H_{1mc} & \quad w_e^* - q_e - a_e = m_e, \\
H_{2mc} & \quad w_e^* - w_s^* = m_{es}, \\
H_{3mc} & \quad w_s^* - q_s^* - a_s = m_s
\end{align*}
\]

where the * indicates a long-run or steady state level. Actual data may show considerable deviations from the long run tendencies, and, as we have seen, Aukrust expected that (e-sector) wages followed a corridor with the main course as the central tendency. We may translate this into a proposition saying that the observed wage share in the e-sector is not exactly constant, but the average wage share over a relatively long period of time should be constant and equal to \(m_e\). Hence, according to the theory, \(m_e\) in \(H_{1mc}\) is the long-run mean of the observed wage share \(w_{e,t} - q_{e,t} - a_{e,t}\).\(^{11}\)

The set of institutional arrangements surrounding wage and price setting change over time, and for that reason \(m_e\) may itself change—not in a trendlike manner like wages and prices, but in smaller “steps” up- or downwards. For example, bargaining power and unemployment insurance systems are not constant factors but evolve over time, sometimes abruptly too. Aukrust himself, in his summary of the evidence for the theory, noted that the assumption of a completely constant mean wage share over long time spans was probably not tenable. However, no internal inconsistency is caused by replacing the assumption of unconditionally stable wage shares with the weaker assumption of conditional stability. Thus, we consider in the following

\[\hat{m}_e = 1/T \sum_{t=1}^{T} (w_{e,t} - a_{e,t} - q_{e,t}),\]

\(\hat{m}_e\) should be an unbiased estimate of \(m_e\), and the standard deviation of \(\hat{m}_e - m_e\) should decline with the size of the sample (i.e., with how large a number \(T\) is).
an extended main-course model, namely the following generalization of H1

$$H1_{mc} \quad u_e^* = m_{e,0} + mc + \gamma_{e,t} u_t,$$

where $u_t$ is the log of the rate of unemployment, thus, in $H1_{mc}$, $m_{e,0}$ denotes the mean of the extended relationship, rather than of the wage share itself. Graphically, the main course in figure 5 is no longer necessarily a straight unbroken line, unless the rate of unemployment stays constant for the whole time period considered.

Other candidate variables for inclusion in an extended main-course hypothesis include the ratio between unemployment insurance payments and earnings (the so called replacement ratio, see for example Burda and Wyplosz (2001, p 89-90, and Table 4.8)), and variables that represent unemployment composition effects (unemployment duration, the share of labour market programmes in total unemployment), see Nickell (1987), Calmfors and Forslund (1991).

Following the influence of trade union and bargaining theory, it has also become popular to also include a so called wedge between real wages and the consumer real wage, i.e., $p - q_e$. However, inclusion of a wedge variable in the long-run wage equation of an exposed sector is inconsistent with the main-course hypothesis, and finding such an effect empirically may be regarded as evidence against the framework. On the other hand, there is nothing in the main-course theory that rules out substantive short-run influences of the consumer price index.

The other two long-run propositions (H2 and H3) in Aukrust’s model have not received nearly as much attention as H1 in empirical research, but exceptions include Rødseth and Holden (1990) and Nymoen (1991). In part, this is due to lack of high quality data wage and productivity data and for the private service and retail trade sectors. Another reason is that both economists and policy makers in the industrialized countries place most emphasis on understanding and evaluating wage setting in manufacturing, because of its continuing importance for the overall economic performance.

8.1.2 Causality

The main-course model specifies the following three hypothesis about causation:

$$H4_{mc} \quad mc \rightarrow w_e^*,$$

$$H5_{mc} \quad w_e^* \rightarrow w_s^*,$$

$$H6_{mc} \quad w_s^* \rightarrow q_e^* \rightarrow p^*,$$

where $\rightarrow$ denotes one-way causation. In his 1977 paper, Aukrust sees the causation part of the theory (H4-H6) as just as important as the long term relationships (H1-H3). If anything Aukrust seems to put extra emphasis on the causation part. For example, he argues that exchange rates must be controlled and not floating, otherwise $q_e$ (world price in USDs + the Kroner/USD exchange rate) is not a pure causal factor of the domestic wage level, but may itself reflect deviations from the main course, thus

In a way.....the basic idea of the Norwegian model is the “purchasing power doctrine” in reverse: whereas the purchasing power doctrine assumes floating exchange rates and explains exchange rates in terms of

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relative price trends at home and abroad, this model assumes controlled exchange rates and international prices to explain trends in the national price level. If exchange rates are floating, the Norwegian model does not apply (Aukrust 1977, p. 114).

From a modern viewpoint this seems to be something of a strait-jacket, in that the steady state part of the model can be valid even if Aukrust’s one-way causality is untenable. For example H1_{mc}, the main course proposition for the exposed sector, makes perfect sense also when the nominal exchange rate, together with wage adjustments are stabilizing the wage share around a long-run mean. In sum, it seems unduly restrictive to a priori restrict Aukrust’s model to a fixed exchange rate regime. But with a floating exchange rate regime, care must be taken to formulate a relevant equation of the nominal exchange rate.

8.1.3 The Norwegian model and the Battle of the Mark-ups

Chapter 12.3 in the book by B&W contains a general framework for thinking about inflation. The basic idea is that in modern economies firms typically attempt to mark-up up their prices on unit labour costs, while workers and unions on their part strive to make their real wage reflect the profitability of the firms, thus their real wage claim is a mark-up on productivity. Hence there is a conflict between workers and firms, both are interested in controlling the real wage, but they have imperfect control: Workers influence the nominal wage, while the nominal price is determined by firms.

The Norwegian model of inflation fits nicely into this (modern) framework. Hence, using H1_{mc}-H3_{mc} above we have that

$$w^* = m_e + q_e + a_e,$$
$$q_s^* = -m_s + w^* - a_s,$$

saying that the desired wage level is a mark up on prices and productivity, exactly as in equation (12.5') in B&W, albeit in the exposed sector of the economy, while the desired (s-sector) price level is a mark-up on unit-labour costs (as in B&W’s equation (12.4')). There are however two notable differences:

First: since causality is one-way in the Norwegian model, we still don’t have the full “circular process” emphasized by B&W (see p 287). However, if the Norwegian model is extended to incorporate also effects of consumer prices (which would be an average of \(q_e\) and \(q_s\), in e-sector wage setting, full circularity would result.

Second, B&W present the battle of mark-ups model in a static setting. The implication is that (if only workers’ price expectations are correct in each period) actual wages and prices are determined by the static model. This of course runs against our main message, namely that actual wage and prices are better described by a dynamic system. In the last subsection on the Norwegian model, we sketch how we can make a consistent story about the long-run and the dynamics of wage setting.

8.1.4 Dynamic adjustment

As we have seen, Aukrust was clear about two things. First, the three main-course relationships should be interpreted as long-run tendencies. For example, actual
observations of e-sector will fluctuate around the theoretical main-course. Second, if e-sector wages deviate too much from the long-run tendency, forces will begin to act on wage setting so that adjustments are made in the direction of the main-course. For example, profitability below the main-course level will tend to lower wage growth, either directly or after a period of higher unemployment.

This idea fits into the autoregressive distributed lag framework, ADL for short, in section 4 above. To illustrate, assume that $w_{e,t}$ is determined by the dynamic model

$$w_{e,t} = \beta_0 + \beta_{11} mc_t + \beta_{12} mc_{t-1} + \beta_{21} u_t + \beta_{22} u_{t-1} + \alpha w_{e,t-1} + \varepsilon_t. \tag{8.34}$$

Compared to the ADL model in equation (4.15) above, there are now two explanatory variables (“x-es”): the main-course variable $mc_t (= a_{e,t} + q_{e,t})$ and the (log of) the rate of unemployment. To distinguish the effects of the two variables, we have added a second subscript to the $\beta$ coefficients. Equation (4.15) is consistent with workers forming expectations about future values of $mc$ and $u$, based on current and historical information.

The endogenous variable in (8.34) is clearly $w_{e,t}$. As explained above, $mc_t$ is an exogenous variable which displays a dominant trend. The rate of unemployment is also exogenous. Assuming exogenous unemployment is a simplification of Aukrust’s model, we have already seen that he included unemployment rises/falls as potential corrective mechanisms that makes wages return towards the main-course. However, it is interesting to study the simple version with exogenous unemployment first, since we will then see that corrective forces are at work even at any constant rate of unemployment. This is a thought provoking contrast to “natural rate models” which dominates modern macroeconomic policy debate, and which takes it as a given thing that unemployment has to adjust in order to bring about wage and inflation stabilization, see 8.2 below.

Using the same error-correction transformation as in section 4 gives:

$$\Delta w_{e,t} = \beta_0 + \beta_{11} \Delta mc_t + \beta_{21} \Delta u_t$$

$$+ (\beta_{11} + \beta_{12}) mc_{t-1} + (\beta_{21} + \beta_{22}) u_{t-1} + (\alpha - 1) w_{e,t-1} + \varepsilon_t$$

and

$$\Delta w_{e,t} = \beta_0 + \beta_{11} \Delta mc_t + \beta_{21} \Delta u_t$$

$$- (1 - \alpha) \left\{ w_{e,t-1} - \frac{\beta_{11} + \beta_{12}}{1 - \alpha} mc_{t-1} - \frac{\beta_{21} + \beta_{22}}{1 - \alpha} u_{t-1} \right\} + \varepsilon_t$$

Next, invoking H1$_{gmc}$, this can be expressed as

$$\Delta w_{e,t} = \beta_0 + \beta_{11} \Delta mc_t + \beta_{21} \Delta u_t$$

$$- (1 - \alpha) \left\{ w_{e,t-1} - mc_{t-1} - \gamma_{e,1} u_{t-1} \right\} + \varepsilon_t$$

as long as also the following restriction is imposed in (8.34)

$$\beta_{11} + \beta_{12} = (1 - \alpha) \tag{8.38}$$
(8.38) embodies that the long-run multiplier implied by (8.34) is unity, meaning that a one percent increase in the main-course (either a permanent increase in output prices or in labour productivity) result in a long-run rise in hourly wages by one percent. The short-run multiplier with respect to the main-course is of course $\beta_{11}$, which can be considerably smaller than unity without violating the main-course hypothesis $H_{1.gmc}$.

The formulation in (8.37) is often called an equilibrium correction model, ECM for short, since the term in brackets captures that wage growth in period $t$ partly corrects last periods deviation from the long-run equilibrium wage level. In fact, since we have imposed the restriction in (8.38) on the dynamic model, we can write

$$\Delta w_{e,t} = \beta_0' + \beta_{11} \Delta mc_t + \beta_{21} \Delta u_t$$

$$- (1 - \alpha) \{w_e - w^*\}_{t-1} + \varepsilon_t$$

where $w^*_e$ is given by the left hand side of the extended main-course relationship $H_{1.gmc}$ (simplified by setting $\gamma_{e,2} = 0$).

Figure 6 illustrates the dynamics: We consider a hypothetical steady state with a constant rate of unemployment (upper panel) and wages growing along the main-course. In period $t_0$ the steady state level of unemployment increases permanently. Wages are now out of equilibrium, since the steady state path ($w^*$ is shifted down in period $t_0$) but because of the corrective dynamics wages adjusts gradually towards the new steady state growth path. Two possible paths are indicated by the two thinner line. In each case the wage is affected by $\beta_{21} < 0$ in period $t_0$. Line a corresponds to the case where the short-run multiplier is smaller in absolute value than the long-run multiplier, (i.e., $-\beta_{21} < -\gamma_{e,1}$). A different situation, is shown in adjustment path b, where the short-run effect of an increase in unemployment is larger than the long-run multiplier.

**Exercise 9** Is $\beta_{22} > 0$ a necessary and/or sufficient condition for path b to occur?

**Exercise 10** What might be the economic interpretation of having $\beta_{21} < 0$, but $\beta_{22} > 0$?

**Exercise 11** Assume that $\beta_{21} + \beta_{22} = 0$. Try to sketch the wage dynamics (in other words the dynamic multipliers) following a rise in unemployment in this case!

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12 The interested reader will have noted that, for consistency, the intercept $\beta_0'$ is given as $\beta_0' = \beta_0 + (1 - \alpha) m_{e,0}$. 

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There are at least two additional remarks worth making. First, looking back at the consumption function example, it is clear that the dynamics there also has an equilibrium correction interpretation. Hence, the ECM is a 1-1 transformation of the autoregressive model is generic and is not confined to the main-course model. Second, below we will show that also the Phillips curve has an ECM interpretation. The main difference is the nature of the corrective mechanism: In Aukrust’s model there is enough collective rationality in the system to secure dynamic stability of wages setting at any rate of unemployment (also very low rates). Wage growth and inflation never gets out of hand or out of control. In the Phillips curve model on the other hand, unemployment has to adjust to a special level called the “natural rate” and/or NAIRU for the rate of inflation to stabilize.

8.2 The Phillips curve version of the main-course model

The Norwegian model of inflation and the Phillips curve are rooted in the same epoch of macroeconomics. But while Aukrust’s model dwindled away from the academic
scene, the Phillips curve literature “took oﬀ” in the 1960s and achieved immense impact over the next four decades. In the 1970s, the Phillips curve and Aukrust’s model were seen as alternative, representing “demand” and “supply” model of inﬂation respectively. However, as pointed at by Aukrust, the diﬀerence between viewing the labour market as the important source of inﬂation, and the Phillips curve’s focus on product market, is more a matter of emphasis than of principle, since both mechanism may be operating together. In this section we show formally how the two approaches can be combined by letting the Phillips curve take the role of a short-run relationship of nominal wage growth, while the main-course thesis holds in the long-run.

Although the Phillips curve started out as an empirical regularity, it has subsequently been derived by the use of economic theory (see B&W Ch 12). In fact, the Phillips curve can be rationalized in a number of ways. In this section we build on Calmfors (1977), and reconcile the Phillips curve with the Norwegian model of inﬂation.

Without loss of generality we concentrate on the wage Phillips curve, and recall that according to Aukrust’s theory it is assumed that

1. \( w^*_e = m_e + mc \), i.e., \( H1_{mc} \) above.

2. \( u^*_t = m_u \), i.e., unemployment has a stable long-run mean \( m_u \).

3. the causal structure is “one way” as represented by \( H4_{mc} \) and \( H5_{mc} \) above.

A Phillips curve ECM system is deﬁned by the following two equations

\[
\begin{align*}
\Delta w_t &= \beta_{w0} + \beta_{w1}mc_t + \beta_{w2}u_t + \varepsilon_{w,t}, \\
\beta_{w2} &\leq 0, \\
\Delta u_t &= \beta_{u0} + \alpha_u u_{t-1} + \beta_{u1}(w - mc)_{t-1} + \varepsilon_{u,t}, \\
0 &< \alpha_u < 1,
\end{align*}
\]

where we have simpliﬁed the notation somewhat by dropping the “e” sector subscript. Compared to equation (8.35) above, we have simpliﬁed by assuming that only the current unemployment rate aﬀects wage growth. On the other hand, since we are considering a dynamic system, we have added a \( w \) in the subscript of the coeﬃcients. Note that compared to (8.35) the autoregressive coeﬃcient \( \alpha_w \) is set to unity in (8.39)—this is of course not a simpliﬁcation but a deﬁning characteristic of the Phillips curve.

Equation (8.40) represents the basic idea that low proﬁtability causes unem- ployment. Hence if the wage share is too high relative to the main-course unemployment will increase in most situations, i.e., \( \beta_{w1} \geq 0 \).

To establish the main-course rate of equilibrium unemployment, rewrite first (8.39) as

\[
\begin{align*}
\Delta w_t &= \beta_{w1}mc_t + \beta_{w2}(u_t - \frac{\beta_{w0}}{-\beta_{w2}}) + \varepsilon_{w,t},
\end{align*}
\]

13See Aukrust (1977, p. 130).

14Alternatively, given \( H2_{mc} \), \( \Delta w_t \) represents the average wage growth of the two sectors.
Next, assume a steady state situation where $\Delta mc_t = g_{mc}$. From assumption 1, we have that $\Delta w^* - g_{mc} = 0$ where $\Delta w^*$ is the steady state growth rate of wages. Then (8.41) defines the main-course equilibrium rate of unemployment which we denote $u^{phil}$:

$$0 = \beta_{w2}[u^{phil} - \frac{\beta_{w0}}{-\beta_{w2}}] + (\beta_{w1} - 1)g_{mc}.$$ 

or

(8.42)

$$u^{phil} = \left(\frac{\beta_{w0}}{-\beta_{w2}} + \frac{\beta_{w1} - 1}{-\beta_{w2}}\right)g_{mc},$$

In fact, $u^{phil}$ represents the unique and stable steady state of the two dynamic equations (8.39) and (8.40).\(^{15}\)

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Figure 7: Open economy Phillips curve dynamics and equilibrium.

One way of illustrating the dynamics of the Phillips curve is shown in Figure 7. Assume that the economy is initially running at a low level of unemployment, i.e., $u_0$ in the figure. The short-run Phillips curve (8.39) determines the rate of wage inflation $\Delta w_0$. The corresponding wage share consistent with equation (8.40) is above its long-run equilibrium, implying that unemployment starts to rise and wage

\(^{15}\text{again using the convention that all shocks are switched off, } \varepsilon_{wt} = \varepsilon_{ut} = 0 \text{ for all } t,$
growth is reduced. During this process, the slope of the Phillips curve becomes steeper, illustrated in the figure by the rightward rotation of the short-run Phillips curve. The steep Phillips curve in the figure has slope $\beta_w/\left(1 - \beta_{w1}\right)$ and is called the long-run Phillips curve.\(^{16}\) The stable equilibrium is attained when wage growth is equal to the steady state growth of the main-course, i.e., $g_{mc}$ and the corresponding level of unemployment is given by $u^{phil}$. The issue about the slope of the long-run Phillips curve is seen to hinge on the coefficient $\beta_{w1}$, the elasticity of wage growth with respect to the main-course. In the figure, the long-run curve is downward sloping, corresponding to $\beta_{w1} < 1$ which is conventionally referred to as dynamic inhomogeneity in wage setting. The converse, dynamic homogeneity, implies $\beta_{w1} = 1$ and a vertical Phillips curve. Subject to dynamic homogeneity, the equilibrium rate $u^{phil}$ is independent of world inflation $g_{mc}$.

The slope of the long-run Phillips curve represented one of the most debated issues in macroeconomics in the 1970 and 1980s (this is not reflected in B&W Ch 12, though). One arguments in favour of a vertical long-run Phillips curve is that workers are able to obtain full compensation for CPI inflation. Hence $\beta_{w1} = 1$ is a reasonable restriction on the Phillips curve, at least if $\Delta q_t$ is interpreted as an expectations variable. The downward sloping long-run Phillips curve has also been denounced on the grounds that it gives a too optimistic picture of the powers of economic policy: namely that the government can permanently reduce the level of unemployment below the natural rate by “fixing” a suitably high level of inflation, see e.g., Romer (1996, Ch 5.5). In the context of an open economy this discussion appears as somewhat exaggerated, since a long-run trade-off between inflation and unemployment in any case does not follow from the premise of a downward-sloping long-run curve. Instead, as shown in figure 7, the steady state level of unemployment is determined by the rate of imported inflation $m_{mc}$ and exogenous productivity growth. Neither of these are normally considered as instruments (or intermediate targets) of economic policy.\(^{17}\)

In the real economy, cost-of-living considerations play an significant role in wage setting, see e.g., Carruth and Oswald (1989, Ch. 3) for a review of industrial relations evidence. Thus, in applied econometric work, one usually includes current and lagged CPI-inflation, reflecting the weight put on cost-of-living considerations in actual wage bargaining situations. The above framework can easily be extended to accommodate this, but with the expense of additional notation and is omitted here.

In sum, the open economy wage Phillips curve represents one possible specification of the dynamics of the Norwegian model of inflation. Clearly, the dynamic features of our Phillips curve model apply to other versions of the Phillips curve, in that it implies the natural rate (or a NAIRU) rate of unemployment as a stable stationary solution. It is important to note that the Phillips curve needs to be supplemented by an equilibrating mechanism in the form of an equation for the rate of unemployment. Without such an equation in place, the system is incomplete and we have a missing equation. The question about the dynamic stability of the natural rate (or NAIRU) cannot be addressed in the incomplete Phillips curve system.

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\(^{16}\)By definition, $\Delta w_t = \Delta q_t + \Delta a_t$ along this curve.

\(^{17}\)To affect $u^{phil}$, policy needs to incur a higher or lower permanent rate of currency depreciation.
8.3 Estimated wage dynamics for Norway

In this section we first show an open economy Phillips curve for the Norwegian manufacturing sector. We then show an estimated extended main-course model.

8.3.1 A Phillips curve system

We use an annual data set for the period 1965-1998. In the choice of explanatory variables and of data transformations, we build on existing studies of the Phillips curve in Norway, cf. Stølen (1990, 1993). The variables are in log scale (unless otherwise stated) and are defined as follows:

\[ wc_t = \text{hourly wage cost in manufacturing}; \]
\[ q_t = \text{index of producer prices (value added deflator)}; \]
\[ p_t = \text{the official consumer price index (CPI index)}; \]
\[ a_t = \text{average labour productivity}; \]
\[ tu_t = \text{rate of total unemployment (i.e., unemployment includes participants in active labour market programmes)}; \]
\[ rpr_t = \text{the replacement ratio}; \]
\[ h = \text{the length of the "normal" working day in manufacturing}; \]
\[ t1 = \text{the manufacturing industry payroll tax-rate (not log)}. \]

The estimated Phillips curve is shown in (8.43). Beside effects of product price changes and the rate of unemployment, it contain a strong effect of last years rise in CPI and a catch-all variable for incomes policies \((IP_t)\). The number in brackets below the coefficients are the estimated standard errors, which is helpful in judging statistical significance, but we will leave that for econometrics courses.\(^{18}\)

\[
\Delta wc_t = -0.0683 + 0.26 \Delta p_{t-1} + 0.20 \Delta q_t + 0.29 \Delta q_{t-1} - 0.0316 tu_t - 0.07 IP_t
\]

(8.43)

As discussed above, a key parameter of interest in the Phillips curve model is the equilibrium rate of unemployment, i.e., \(u^{phil}\) in (8.42). Using the coefficient estimates in (8.43), and setting the growth rate of prices \((\delta_f)\) and productivity growth equal to their sample means of 0.06 and 0.027, we obtain \(\hat{u}^{phil} = 0.0305\), which is as nearly identical to the sample mean of the rate of unemployment (0.0313).

Figure 8 shows the sequence of \(u^{phil}\) estimates for over the last part of the sample— together with ±2 estimated standard errors and with the actual unemployment rate for comparison. The figure shows that the estimated equilibrium

\(^{18}\)Batch file: aar_NmcPhil_gets.f1
rate of unemployment is relatively stable, and that it is appears to be quite well
determined. 1990 and 1991 are exceptions, where \( \hat{u}_{phil} \) increases from to 0.033 and
0.040 from a level 0.028 in 1989. However, compared to confidence interval for 1989,
the estimated NAIRU has increased significantly in 1991, which represents an internal
inconsistency since one of the assumptions of this model is that \( u_{phil} \) is a time
invariant parameter.

Another point of interest in figure 8 is how few times the actual rate of un-
employment crosses the line for the estimated equilibrium rate. This suggest very
sluggish adjustment of actual unemployment to the purportedly constant equilib-
rium rate. In order to investigate the dynamics formally, we graft the Phillips curve
equation (8.43) into a system that also contains the rate of unemployment as an en-
dogenous variable, i.e., an empirical counterpart to equation (8.40) in the the-
ory of the main-course Phillips curve. As noted, the endogeneity of the rate of
unemployment is just an integral part of the Phillips curve framework as the wage
Phillips curve itself, since without the “unemployment equation” in place one cannot
show that the equilibrium rate of unemployment obtained from the Phillips curve
corresponds to a steady state of the system.

Next we therefore turn to the properties of an estimated version of the dynamic
Phillips curve system (8.39)-(8.40) above. Figure 9 offers visual inspection of some
of the empirical properties of such a model. The first four graphs shows the actual
values of \( \Delta p_t, tu_t, \Delta wc_t \) and the wage share \( wc_t - q_t - a_t \) together with the results
from dynamic simulation. As could be expected, the fits for the two growth rates are quite acceptable. However, the “near instability” property of the system manifests itself in the graphs for the level of the unemployment rate and for the wage share. In both cases there are several consecutive years of under- or overprediction. The last two displays contain the cumulated dynamic multipliers (of $u$ and and to the wage share) resulting from a 0.01 point increase in the unemployment rate. As one might expect from the characteristic roots, the stability property is hard to gauge from the two responses. For practical purposes, it is as if the level of unemployment and the wage share are “never” return to their initial values. Thus, in the Phillips curve system equilibrium correction is found to be extremely weak.

![Figure 9: Dynamic simulation of the Phillips curve model. Panel a)-d) Actual and simulated values (dotted line). Panel e)-f): multipliers of a one point increase in the rate of unemployment.](image)

As already mentioned, the belief in the empirical basis a the Phillips curve natural rate of unemployment was damaged by the remorseless rise in European unemployment in the 1980s, and the ensuing discovery of great instability of the estimated natural rates. In that perspective, the variations in the Norwegian natural rate estimates in figure 8 are quite modest, and may pass as relatively acceptable as a first order approximation of the attainable level of unemployment. However, the estimated model showed that equilibrium correction is very weak. After a shock to the system, the rate of unemployment is predicted to drift away from the natural rate for a very long period of time. Hence, the natural rate thesis of asymptotically stability is not validated.

There are several responses to this result. First, one might try to patch-up the estimated unemployment equation, and try to find ways to recover a stronger
relationship between the real wage and the unemployment rate. In the following we focus instead on the other end of the problem, namely the Phillips curve itself. In fact, it emerges that when the Phillips curve framework is replaced with a wage model that allows equilibrium correction to any given rate of unemployment rather than to only the “natural rate”, all the inconsistencies are resolved.

8.3.2 A main-course system

Equation (8.44) shows an empirical version of an equilibrium correction model for wages, similar to equation (8.37) above

\[
\Delta w_t = -0.197 - 0.478 ecm_{w,t-1} + 0.413 \Delta p_{t-1} + 0.333 \Delta q_t - 0.835 \Delta h_t - 0.0582 IP_t
\]

\[
\begin{align*}
(0.0143) & \quad (0.0293) & \quad (0.0535) & \quad (0.0449) & \quad (0.129) & \quad (0.00561)
\end{align*}
\]

\(ecm_{w,t-1}\) corresponds to \(w_e,t - w_e^*\) in the notation of section 8.1.4. The estimated \(w_e^*\) is a function of \(mc\), with restriction (8.38) imposed. The estimated value of \(\gamma_{e,1}\) is \(-0.01\). The variable \(\Delta h_t\) represent institutional changes in the length of the working week, and the estimated coefficient captures that the pay-losses that would have followed the reductions in working time have been largely compensated in the short-run.

It is interesting to compare this model to the Phillips curve system. First note that the coefficient of \(ecm_{w,t-1}\) is relatively large, a result which is in direct support of Aukrust’s view that there are wage-stabilizing forces at work even at a constant rate of unemployment. To make further comparisons with the Phillips curve, we have also grafted (8.44) into dynamic system that contains the rate of unemployment as an endogenous variable (in fact, the estimated equation for \(tu_t\) is almost identical to its counterpart in the Phillips curve system). Some properties of that system is shown in figure 10.
For each of the four endogenous variables shown in the figure, the model solution ("simulated") is closer to the actual values than in the case in figure for the Phillips curve system. The two last panels in the figure show the cumulated dynamic multiplier of an exogenous shock to the rate of unemployment. The difference from figure 9, where the steady state was not even "in sight" within the 35 years simulation period, are striking. In figure 10, 80% of the long-run effect is reached within 4 years, and the system is clearly stabilizing in the course of a 10 year simulation period. It is clear that this system is more convincingly stable than the Phillips curve version of the main-course model.

Sometimes macroeconomists give the impression that a Phillips curve with a natural rate is necessary for dynamic stability. However, the evidence here shows not only that the wage price system is stable when the Phillips curve is substituted by a wage equation which incorporates direct adjustment also with respect to profitability (consistent with the Norwegian model and with modern bargaining theory). In fact, the system with the alternative wage equation has much more convincing stability properties than the Phillips curve system.

**References**


