Macroeconomic stability or cycles? The role of the wage-price spiral

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Abstract

We formulate a small and stylized dynamic macroeconomic model, and study how different specifications of the supply side affect the model's dynamic properties. The wage-price equilibrium-correction model (ECM) and the Phillips curve model (PCM), that both can be used to represent the supply side of a New Keynesian macro model, are synthesised in a generalised model of the wage-price spiral. We show that the choice of ECM or PCM has implications for the long-run stability of the macro model, without need of a NAIRU. We also find that the range of theoretically admissible dynamics is wide. For example, both the ECM and PCM may display endogenous cyclical fluctuations in inflation and unemployment, showing that even simple structures can give rise to complex dynamics. In practice that may entail that forecasting the effects of shocks and policy changes is difficult even in the best of circumstances.

Keywords Equilibrium-correction, macrodynamics, Phillips curve, unemployment, wage-price spiral.

JEL classification E24, E30, J50.

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1 Introduction

Medium-term macroeconomic models typically assume a natural rate of unemployment (NAIRU) mechanism that brings the activity level in line with the capacity of the economy. Shocks may drive wages and prices off their equilibrium paths, but the dynamic process is stable and reinstalls the structural equilibrium of the economy. In this paper we do not assume from the outset that the model economy obeys the natural rate principle. Instead we specify a dynamic model that has long-run theoretical relationships as "mere" attractors. We present two different and well known theories of wage and price dynamics — collective bargaining and the Phillips curve — in one coherent framework, and find that both stability and dynamics are genuine *system properties.* Real stability depends on information feedback (and not a certain equilibrating rate of unemployment). Cyclical fluctuations depend on the values of *all* parameters in the model. The implication is that stability is a fragile property of theoretical wage-price systems.

Hence, the paper also has a methodical motivation which is interconnected with the economic motivation above. The methodical purpose is to analyse how interdependence and inertia (rigidities) in a model with feedback-dynamics can secure stability and/or generate cyclical fluctuations. The novel contribution of the paper is to integrate and execute the economics and methodics in a small theoretical macroeconomic model. Qualitative properties of the model (real stability or trends) are analysed algebraically. Quantitative properties (cyclical fluctuations) are investigated by numerical eigenvalues and simulations. The lesson is that one should *not* start the modelling by asserting stability from the outset. Rather, as is common in the natural sciences, one should model (observable) change instead of (logical) equilibrium, and then derive the dynamic properties of the system. This paper is an example of that.

We formulate a linear model for a small *open economy* that can secure stability or generate a real trend, and exhibit cyclical fluctuations in inflation, unemployment, the real exchange rate and the wage share. The theoretical model exhibits a full range of integrated short-term and long-run dynamic properties on a domain of plausible parameter values. Feedback between a *wage-price spiral* on the supply side and unemployment dynamics on the demand side may prevent nominal trends from causing real trends. At the same time simultaneous and mutual wage and price changes and feedback-inertia may cause cyclical fluctuations. We explain and demonstrate *how* stability and dynamics are (endogenous) system properties. No exogenous impulses (shocks), extreme parameter values, nonlinearities or explosive expectations paths are needed to generate real trends or persistent cyclical fluctuations. Such system dynamics are more general than conceived long-run stability properties of partial models and models that are less dynamic and less interconnected.

We study how dynamics arise among aggregate variables that are *interdependent* and *inertial.* Since the concern of this paper is macro interaction dynamics, and not agents' motivation and optimising behavior, we do not derive our model from microfoundations. However, we do relate the macroeconomic equations to microeconomic theory. Our work is of the same spirit and to some extent methodologically related to the ongoing work of Chiarella, Flaschel and several collaborators on mathematical macrodynamics, cf. Asada et al. (2006). Our work is more limited in content and scope, focusing on a single economic mechanism (the wage-price spiral). Our model is (much) smaller, (much) simpler and in discrete-time. But, it still brings out the same basic lesson, that interactions and inertia may create different dynamics than individual optimisation and foresight.

While analyses of macromodels tend to focus on several transmission channels, the supply side is often held constant as a Phillips curve model (PCM) for (price) adjustments. Recently, we have seen new interest in the specification of the supply side and in the consequences of different specifications for the system properties of the medium-term macro model. Blanchard and Galí (2007) have developed a New Keynesian macro model with real-wage rigidities that extends the New Keynesian PCM of Gali and Gertler (1999). Forslund et al. (2008) show formally that the wage dynamics implied by a theoretical model with collective (rather than individualistic) bargaining is distinct from the dynamics of a wage-PCM. Hence, there is a case for investigating the system implications of different models of the wage-price spiral. In the present paper we do exactly that.

Our main interest is the *dynamic functioning* of a wage-price spiral, and how it secures real stability or generates cycles. To keep the model small and analytically tractable we supplement the wage and price equations with only a single additional relationship. The rate of unemployment represents the demand side. It can be interpreted as an inverse proxy for the activity level. Theory suggests that the wage-price spiral may affect the activity level through more than one channel. We only make use of the real exchange rate channel to get a feedback-loop between the supply and demand side. The supply-side specification encompasses the Phillips curve model, but is more general. When coupled to a simple demand-side equation, the wage-price spiral on the supply side is able to generate a range of dynamics. Our work shows that macroeconomic real (in)stability or fluctuations can be caused by interaction and inertia on the supply side as well as on the demand side.

We set up a dynamic open-economy model for *simultaneous* determination of nominal wage, price and unemployment conditional on exogenous processes for productivity and import price. The model determines the productivity-corrected real-wage (wage share) and the price-competitiveness (real exchange rate). We investigate dynamic implications of two hypotheses of wage and price setting. Theoretical propositions of wage bargaining and monopolistic pricing imply a model with adjustments of nominal wage and producer price toward real-wage goals. The Phillips curve model has *no* such *equilibrium-correction.* On the supply side, *nominal rigidity* is synonymous with partial and delayed responses of wage and price to changes in each other and to changes in other variables in the model. *Real rigidity* relates to the demand side and how sensitive unemployment (or the activity level) is to changes in price-competitiveness of the supply side facing exogenous import prices. These rigidities determine the *dynamic properties* of the model.

The model is small and stylized. That allows us to analyse its long-run stability properties algebraically. Despite trends in nominal wage, productivity, and prices, the domestic wage and price growth are *coordinated* in the wage-price spiral in such a way that the wage share and the real exchange rate become stationary. The Phillips curve model lacks the required coordination of wage growth and domestic inflation with the exogenous productivity growth and foreign inflation, and the wage share has a trend. For the investigation of medium-term dynamic properties we use a mix of algebraic analysis, numerical investigations and simulations. The purpose of the simulations is not to compare simulated data with real-world data nor to evaluate different model versions, but to explore the range of possible system dynamics for realistic parameter values. Due to interactions of inertial endogenous variables, the range is wider than one might expect in light of the relatively simple and well-known building blocks that constitute the model. Endogenous cyclical fluctuations appear for plausible parameterisations in all model versions. Even when the cycles are damped, in the medium-term the cycles may *dominate* long-run stability properties. Long-run stability has been the main concern of the literature, but in the more policy-relevant time-perspective cyclical fluctuations might be more important.

The algebraic and numerical investigations show that (i) long-run stability of real variables and nominal growth rates requires the existence of certain transmission or feedback channels (non-zero parameters), while (ii) the relative strengths of these channels compared to instantaneous wage and price changes determine the dynamics. Hence, we might say that *stability requires the presence of information*, while *dosing and timing of information determine dynamics.* Inefficiency and sluggishness affect dynamics rather than stability (the exception is persistent cyclical fluctuations). The long-run stability properties (trend or not) depends on certain parameters being non-zero, while the medium-term dynamics (cyclical fluctuations) depend on the values of all parameters in the model. The long-run and medium-term dynamic properties are *integrated*. We find that dynamics is more than fluctuations around a predefined long-run equilibrium — which is a common DSGE modelling assumption. The dynamics are system properties. Our integrated study, which links the dynamics of the reduced form of the model to specific economic properties of the structural form of the model, yields results that are neither common nor well known to economists and econometricians. Hence, it deserves attention.

The joint determination of unemployment and the real exchange rate is common of macroeconomic theories as different as the model of Layard, Nickel and Jackman (1991), and the 'new open macroeconomics', e.g. Lane and Milesi-Feretti (2004). In our model we get a dynamic solution for the real exchange rate from the same set of assumptions that determine the solutions for unemployment, inflation and the real wage. Contrary to the Layard, Nickel and Jackman model, no separate assumption about the current account is required for the determination of the real exchange rate and the equilibrium rate of unemployment. The solution, when it is stable, is generic in our model. However, since our model can be extended to include a fuller representation of the current account, it complements the static Layard, Nickel and Jackman model. The new open-macroeconomics theory is also complementary to our model. The theory addresses the long-term determinants of the equilibrium real exchange rate, but does not integrate the dynamic evolutions of the real exchange rate and the rate of inflation, which is a main feature of our model.

The rest of this paper is organised as follows. In section 2 we lay out the model and its two versions, an equilibrium-correction model (ECM) consistent with wage-bargaining and monopolistic mark-up pricing, see Bårdsen et al. (2005), and a Phillips curve model (PCM), see Fuhrer (1995), Gordon (1997). In section 3.1 and 3.2 we analyse theoretically the long-run stability properties of the ECM and the PCM. The medium-term dynamic properties of the model is investigated in section 3.3. In section 4 we summarise and discuss our findings. To improve the readability of the paper we have moved all the mathematics and all the numerical and simulation details to the appendices.

2 The model

The basic nominal variables in the model we formulate are: hourly wage w, domestic producer price q, domestic consumer price p, and import price pi in domestic currency. The average labour productivity a and the unemployment rate u are real variables. All variables are in logarithmic scale to facilitate relationships that are linear in the parameters. Appendix A lists all variables and parameters.

2.1 Nominal and real trends

We begin by defining the exogenous trends in the model. There are two: one nominal trend and one real trend. The nominal trend is the price of imports pi in domestic currency. We write the equation as a random-walk with a positive drift:

$$\Delta pi_t = g_{pf} + \varepsilon_{pi,t}, \quad \text{with} \quad g_{pf} > 0 \quad \text{and} \quad \varepsilon_{pi,t} \sim \mathsf{IN}(0, \sigma_{pi}^2), \tag{1}$$

where $\Delta pi_t \equiv pi_t - pi_{t-1}$ and subscript t denotes the time period. The positive trend impulse g_{pf} represents underlying foreign or 'world price' inflation. The residual term $\varepsilon_{pi,t}$ may include international price shocks, a stationary nominal foreign currency exchange rate (normalised to zero mean), and domestic shocks to import prices. It is well known that a null hypothesis of random walk behavior is rarely rejected for nominal price indices. Equation (1) is thus intended as a simple, but realistic assumption.

In the following we will define the consumer price as a weighted sum of the domestic producer price and the import price[1]:

$$p = \phi\, q_t + (1-\phi)\, pi_t \quad \text{with } 0 < \phi < 1 \text{ reflecting the openness of the economy.} \tag{2}$$

Exogenous productivity a_t is an important conditioning variable of the price and wage system. For simplicity, we assume a random walk with a positive drift:

$$\Delta a_t = g_a + \varepsilon_{a,t}, \quad \text{with } \; g_a > 0 \; \text{ and } \; \varepsilon_{w,t} \sim \mathsf{IN}(0, \sigma_w^2). \tag{3}$$

The equation reflects a trend-like growth that we typically observe for average labour productivity[2]. The approximation residual $\varepsilon_{a,t}$ may also represent productivity shocks. The specific processes for pi and a are not important. What matters in this context is that both exogenous variables have a trend that affects the model economy.

The import price is going to represent foreign inflation and foreign price level in the wage-price spiral. The import price is all we need to get an open economy model, where the purpose is to illustrate the effect on the domestic economy of a foreign 'nominal anchor'. Using (2), we define price competitiveness or the *real exchange rate* $re \equiv pi - q = (p-q)/(1-\phi)$.

2.2 Optimal price and wage levels

Following custom, we refer to the price and wage levels that firms and workers (or their union representatives) would decide if there were no costs or constraints on adjustment, as the 'optimal' or 'target' values of prices and wages. Another interpretation, following from the essentially static nature of these models, says that optimal prices are those that would prevail in a hypothetical and completely deterministic steady-state situation.

The firms' equilibrium price is defined in accordance with the theory of monopolistic price competition:

$$q^f = m_q + w_t - a_t - \vartheta\, u, \tag{4}$$

with $m_q, \vartheta \geq 0$, see e.g. Benassy (2011, Ch 5.3.4), Rødseth (2000, Ch 8.2). The parameter ϑ represents the joint effect of changes in the elasticity of demand and in marginal costs of production, see Layard et al. (2005, Ch 7). Normal cost pricing requires $\vartheta = 0$. For nominal wage we have

$$w^b = m_w + q + a - \varpi\, u + \omega\,(p-q)\,, \tag{5}$$

with $m_w > 0$, $0 \leq \omega \leq 1$, $\varpi \geq 0$, see e.g. Nymoen and Rødseth (2003). The variable w^b represents the theoretical concept of a *bargained wage*. The right hand side contains variables that might systematically influence the bargained wage. The producer price q and productivity a are the central variables in wage setting according to the theory of collective bargaining, see e.g. Forslund et al. (2008). They also give a rationale for setting the elasticity with respect to productivity equal to one, as we have done in (5). The impact of unemployment on the bargained wage is given by the elasticity $-\varpi \leq 0$, which is the slope of the wage curve, see Blanchflower and Oswald (1994). Equation (5) is seen to include the variable $p - q$, called the wedge (between the producer and the consumer real wage), with elasticity ω. If wage bargaining is only about the sharing of the value-added created by capital and labour then $\omega = 0$ is an implication from the theory of Forslund et al. (2008). However, this is a strong assumption when we have the total economy in mind. In the service sectors, where unions have less bargaining power, wage setting might be dominated by efficiency wage considerations. Equation (5) is formulated to be consistent with both theories.

Even though they are static relationships, equation (4) and (5) will play an important role in the dynamic model of the wage-price spiral, as attractors for wage adjustments (Δw_t) and domestic inflation (Δq_t, where the subscript t, denote time period).

[1] Note that, due to the log-form, $\phi = im/(1-im)$ where im the import share in private consumption.

[2] Cyclical behaviour of a_t is a natural extension that we put down for future work.

2.3 The wage-price spiral

We first use (4) and (5) to define the optimal *real* wage for firms: $rw_t^f \equiv w_t - q_t^f = -m_q + a_t + \vartheta\, u_t$, and for workers: $rw_t^b \equiv w_t^b - q_t = m_w + \omega\,(p_t - q_t) + a_t - \varpi\, u_t$. Productivity a_t is a common driving factor in both variables, and both real wage targets inherit the trending property of productivity. With trending rw_t^f and rw_t^b, logical consistency requires that also the actual real wage $rw_t \equiv w_t - q_t$ is a random walk with a positive drift. Next, define the firms' and the workers' real wage "gap":

$$ecm_t^f \equiv rw_t - rw_t^f = q_t^f - q_t = ws_t - \vartheta\, u_t + m_q, \tag{6}$$

$$ecm_t^b \equiv rw_t - rw_t^b = w_t - w_t^b = ws_t - \omega\,(1-\phi) re_t + \varpi\, u_t - m_w. \tag{7}$$

where we have substituted the productivity corrected real wage $ws \equiv w - q - a$ as the *wage share*, and $(1-\phi)re_t$ for the wedge $p - q$. If the economic theory is empirically relevant then both ecm_t^b and ecm_t^f are stationary variables, i.e. they have finite variability around constant levels. This is tantamount to assuming two cointegrating relationships between the three random walk variables rw_t^b, rw_t^f, and rw_t, cf. Engle and Granger (1987).

Cointegration between real wages is the same as cointegration between q_t and q_t^f, and between w_t and w_t^b. Cointegration implies equilibrium-correction dynamics, and we specify the following equilibrium-correction model for wages and prices:

$$\Delta q_t = c_q + \psi_{qw}\,\Delta w_t + \psi_{qpi}\,\Delta pi_t - \varsigma\, u_{t-1} + \theta_q\, ecm_{t-1}^f + \varepsilon_{q,t}, \tag{8}$$

$$\Delta w_t = c_w + \psi_{wq}\,\Delta q_t + \psi_{wp}\,\Delta p_t - \varphi\, u_{t-1} - \theta_w\, ecm_{t-1}^b + \varepsilon_{w,t}, \tag{9}$$

where $\psi_{qw}, \psi_{qpi}, \psi_{wq}, \psi_{wp}, \varsigma, \varphi, \theta_q, \theta_w \geq 0$, $\varepsilon_{q,t} \sim \mathsf{IN}(0, \sigma_q^2)$ and $\varepsilon_{w,t} \sim \mathsf{IN}(0, \sigma_w^2)$. A change in the nominal wage (Δw_t) or the import price (Δpi_t) induces a simultaneous change in the producer price (Δq_t), while a change in the producer price or the consumer price (Δp_t) induces a simultaneous change in the nominal wage. At the same time the price changes toward the real wage targets of the firms (ecm_{t-1}^f) while the wage changes toward the real wage targets of the workers (ecm_{t-1}^b).

Dynamic homogeneity is often regarded as a necessary feature of a model that is to be used for policy advise in order to avoid a false impression of a trade-off between unemployment and inflation. *Dynamic price and wage homogeneity* of order 1 entails the following restrictions $\psi_{wq} + \psi_{wp} = \psi_{qw} + \psi_{qpi} = 1$. The dynamic consequence of the restrictions is that an exogenous change in Δpi_t (i.e. $\varepsilon_{pi,t}$) is transfered in its entirety to *both* Δw_t and Δq. Then — per definition — re_t and ws_t are both unaffected by foreign inflation pi_t. We shall return to this issue later.

Substituting the right hand sides of (6) and (7) for the *ecm*s in (8) and (9), we obtain a dynamic system that corresponds to the supply side of standard macroeconomic models for medium-term analysis:

$$\Delta q_t = (c_q + \theta_q\, m_q) - \mu_q\, u_{t-1} + \theta_q\, ws_{t-1} + \psi_{qw}\,\Delta w_t + \psi_{qpi}\,\Delta pi_t + \varepsilon_{q,t}, \tag{10}$$

$$\begin{aligned}\Delta w_t = {} & (c_w + \theta_w\, m_w) - \mu_w\, u_{t-1} - \theta_w\, ws_{t-1} + \theta_w\,\omega\,(1-\phi) re_{t-1} \\ & + \psi_{wq}\,\Delta q_t + \psi_{wp}\,\Delta p_t + \varepsilon_{w,t},\end{aligned} \tag{11}$$

$$\Delta p_t = \phi\,\Delta q_t + (1-\phi)\,\Delta pi_t, \tag{12}$$

We have introduced $\mu_q = \theta_q\,\vartheta + \varsigma$ and $\mu_w = \theta_w\,\varpi + \varphi$. They will be discussed in section 2.4. Equation (12) is equation (2) in differenced form[3].

The coefficient θ_w in (11) is a key parameter. It determines the degree or speed of equilibrium-correction in the wage setting. In the case of $\theta_w > 0$, the wage increase

[3] For the coefficients ψ_{wq}, ψ_{qw} and ψ_{wp}, ψ_{qpi}, the non-negative signs are standard in economic models. Negative values of θ_w and θ_q imply explosive evolution in wages and prices (hyperinflation), which is different from the low to moderately high inflation scenario that we have in mind for this paper.

in the current period is negatively affected by last period's real wage and the rate of unemployment, and positively affected by productivity and the wedge.[4] As noted above, this case captures the main implication of both wage bargaining models and efficiency wage models. A strictly positive θ_w also implies that when we consider (11) as a single equation model for wages, that model is asymptotically stable and the long-run steady-state solution takes the form given in (5), so the dynamic relationship and the long-run wage equation are internally consistent.

Finally, we note that replacing Δw_t and Δq_t that appear on the right hand sides by their expectations will not change the economic interpretation fundamentally, unless those expectations are fully model consistent and rational. It is well known that rational expectations models can have multiple solutions, some with explosive dynamics. In this paper we show that "imperfect" coordination in a system may lead to its instability. The source of instability does *not* lie in expectations formation or in exogenous nominal rigidity of the Calvo (1983) type.

2.4 Two models: ECM and PCM

As already mentioned, there are two main types of models nested within in the framework. The model with wage bargaining and price mark-up implies wage and price equilibrium-correction ($\theta_w, \theta_q > 0$), and is denoted ECM. Equations (4)-(5) and (8)-(9) show that the rate of unemployment affects wage and price growth via the terms $\theta_w \varpi u_{t-1}$ and $\theta_q \vartheta u_{t-1}$. Then the only logically consistent value of φ and ς is zero. In the following we use

$$\text{ECM: } \theta_w, \theta_q, \varpi, \vartheta > 0 \text{ and } \varphi, \varsigma = 0 \;\Rightarrow\; \mu_w = \theta_w \varpi \text{ and } \mu_q = \theta_q \vartheta. \quad (13)$$

The Phillips curve model (PCM), without wage bargaining and mark-up pricing, implies no wage and price equilibrium-correction ($\theta_w, \theta_q = 0$). We define

$$\text{PCM: } \theta_w = \theta_q = 0 \text{ and } \varphi, \varsigma > 0 \;\Rightarrow\; \mu_w = \varphi \text{ and } \mu_q = \varsigma. \quad (14)$$

The specifications of the supply side do not exclude the case in which expectations errors can be added in a more elaborated version of the model. In its present form the model conforms to perfect expectations about current period wage and price increases.

2.5 Unemployment

To close the model we need to take account of how the rate of unemployment is related to the supply side. Aggregate demand, or unemployment, is influenced by one or more of the variables that appear in the supply-side equations above. Because focus is on the role of equilibrium-correction and nominal rigidity in the supply side, we keep the model of the demand side down to a minimum. The real exchange rate $re \equiv pi - q$ reflects the price competitiveness of the domestic production relative to the imports. According to standard macroeconomic theory, aggregate demand increases if there is a real depreciation (re increases), and, with reference to *Okun's law,* the rate of unemployment is reduced. Aggregate demand is represented by the log of the unemployment rate:

$$u_t = c_u + \alpha\, u_{t-1} - \rho\, re_{t-1} + \epsilon_{u,t}, \quad \text{with } \epsilon_{u,t} \sim \mathsf{IN}(0, \sigma_w^2). \quad (15)$$

Except for c_u the coefficients are logically non-negative: $\alpha, \rho \geq 0$. We presume that $\alpha < 1$, but Appendix B shows that this limitation is generally *not* necessary for stationarity.

[4] Although equilibrium correction in wage setting ($\theta_w > 0$) and price setting ($\theta_q > 0$) stabilize the dynamics of the system, "too much" equilibrium correction, for example $\theta_w \geq 2$, can endanger stability. However, values of θ_w in the region $1 < \theta_w < 2$ are usually not regarded as economically meaningful, because the implied negative autocorrelation ("volatility") in the nominal wage level is unrealistic.

An increase in price competitiveness (re) reduces unemployment (or increases capacity utilisation). The error term $\epsilon_{u,t}$ might represent a temporary shock to the aggregated demand or to labour supply. The lagged the real exchange rate, re_{t-1}, simply reflects that it takes time to gather information and act on it. Using a predetermined explanatory variable in (15) simplifies the algebra without loss of generality.

The most obvious omission from (15) is perhaps the real interest rate, which will have to be included in more realistic versions of the model. A possible interpretation of the present formulation of the model is that the real interest rate is kept constant at a long-run equilibrium level, by nominal interest rate adjustments, perhaps motivated by a wish to keep an 'even flow' of real investments. Logically, the monetary policy will then have to be accommodative in order to equilibrate the domestic money market (through quantitative easing and tightening). Including the lagged real exchange rate as the only explanatory variable in (15) provides a minimal representation of feedback from the supply side to the demand side. Together with the feedback from unemployment (or inverse activity level) in the wage-price spiral (10)-(11) closes a feedback-loop and creates bidirectional causation between the supply and the demand side. That is crucial for the possibility of cyclical fluctuations in all nominal and real endogenous variables.

The heuristic of the reduced form equation (15) can be rationalised by job search theory and the concepts of matching and separation. The change in unemployment is the difference between job destruction and job creation. Following Pissarides (2000), the change in unemployment can be written as

$$\Delta u_t = s\,(1 - u_{t-1}) - f(u_{t-1}, v_t) \tag{16}$$

where u_{t-1} is the unemployment rate, $1 - u_{t-1}$ is the employment rate and v_t is the job vacancy rate, both measured as a fraction of the labour force. The constant rate (or exogenous probability) of separation of workers from their jobs is denoted by s. The rate f at which vacant jobs are filled is a function of the unemployment rate and the vacancy rate, with $\partial f/\partial u \geq 0$ and $\partial f/\partial v \geq 0$. The simplest log-linear formulation of f is

$$f(u_{t-1}, v_t) = c_{f0}\, u_{t-1} + c_{f1}\, v_t, \quad \text{with coefficients } c_{f0}, c_{f1} \geq 0. \tag{17}$$

This function can be written as $f(u_{t-1}, v_t) = [c_{f0}\,(u_{t-1}/v_t) + c_{f1}]v_t$. The substitutions $\theta_t \equiv v_t/u_{t-1}$ and $\theta_t\, u_{t-1} = v_t$ make the function equivalent to $f_1(\theta_t, u_t) = [c_{f0}\,\theta_t^{-1} + c_{f1}]\,\theta_t\, u_{t-1} \equiv m(\theta_t, 1)\,\theta_t\, u_{t-1} \equiv \theta_t\, q(\theta_t)\, u_{t-1}$, which is our dynamic version of the form used by Pissarides (2000, note that his variable $\theta_t \neq$ our parameters θ_q, θ_w).

The vacancy rate v_t is probably a complex function that depends, among other variables, on price competitiveness re_{t-1}. For simplicity, it is convenient to assume that

$$v_t = v_0 + v_1\, re_{t-1} + \varepsilon_{v,t}, \quad \text{with coefficient } v_1 > 0, \tag{18}$$

so that more vacancies open when price competitiveness improves. Note that the vacancy rate depends indirectly and negatively on the wage level w through the price competitiveness variable re since $\partial re/\partial w = -\partial q/\partial w \leq 0$.

Inserting (18) in (17) and (17) in (16) yields a relationship like (15), with coefficients $c_u = s - c_{f1}\, v_0$, $\alpha = (1 - s - c_{f0})$, $\rho = c_{f1}\, v_1$ and error term $\varepsilon_{u,t} = c_{f1}\, \varepsilon_{v,t}$. Within the scope of the present paper, the important property of equation (15) is the negative feedback from price competitiveness ($-\rho$) on unemployment. That closes a feedback-loop between the supply and demand side of the model economy. To facilitate analytic tractability and ease the exposition, we avoid further (realistic) complexity.

2.6 Stability, trend and cycles

The nominal wage-price spiral (10)-(12) is characterised by nominal rigidity. The responses of the nominal variables to each other are partial (parameters < 1) and delayed (explanatory variables are lagged). Shocks and inertia cause the variables to develop differently over time. At the same time, the equilibrium-correcting terms serve as 'attractors' that coordinate the long-run development of the nominal variables. Coordination of growth rates of constituent variables is necessary to secure that certain composite or real variables, the real exchange rate $re \equiv pi - q$ and the wage share $ws \equiv w - q - a$, are free of trends. Stability of the model requires that the endogenous real variables re, ws and u all converge to constant steady-state levels in the absence of shocks. It follows that the endogenous nominal variables q, w and p must converge to constant steady-state growth-rates determined by constant exogenous productivity growth g_a and constant exogenous foreign inflation g_{pf}. While 'first order stability', i.e. absence of a real trend (caused by a real unit root) is a necessary condition for overall stability, it is not sufficient.

Cyclical fluctuations are well known features of economic variables. Persistent or slowly damped cycles around stable levels and trends constitute 'second order instability'. The dynamic interaction of the three endogenous real variables in the model might possibly create endogenous cycles (due to complex conjugate roots of certain magnitudes ≤ 1). Such cycles do *not* have exogenous causes. If the model is cyclical, all endogenous variables, nominal and real, move in cycles because they are interdependent. If there are cyclical fluctuations around constant levels, asymptotic stability requires that the cycles are quickly damped. If not, they might dominate in the short and medium run, and be revitalised by temporary shocks. In our model cyclical fluctuations occur if inflation and wage growth simultaneously influence each other with different strengths while both are relatively weakly equilibrium-corrected by lagged real variables. We shall see that equilibrium-correction is sufficient for nominal trends to cancel out, such that the real variables re and ws are stationary. We use the term first order stability if there is no real trend. We shall also see that the strength of the equilibrium-corrections relative to the instantaneous and simultaneous effects of wage growth and inflation on each other determines whether all endogenous variables display cyclical fluctuations. We use the term second order stability if there is no persistent endogenous cyclical fluctuations. While first order stability (no real trends) depends on the presence of equilibrium-corrections, but not their strengths, second order stability (no lasting cyclical fluctuations) depends on the strength of the equilibrium-corrections relative to the impact effects of wage growth and inflation.

In the ECM, we shall see that the coordination of inflation and wage growth by the equilibrium-correction terms (6) and (7) is able to neutralise nominal trends. Real stability thus requires that ws and re (directly or through u) influence Δw and Δq. The actual levels of the real variables do not matter for stability, only that they influence and thus coordinate the nominal developments. Hence, a certain rate of unemployment (natural or NAIRU) is *not* necessary for stability of ws and re. The PCM has no equilibrium-correction, and we shall thus see that the wage share becomes trending. Because re still influences q through u, it remains stationary. Both models display cyclical fluctuations, persistent or damped, for plausible parameter values.

3 Dynamic properties of the model

The model consists of 6 equations: (1), (3), (10)-(12) and (15). They determine time series for q_t, w_t, p_t, pi_t, a_t and u_t as functions of initial values and error/shock $\varepsilon_{i,t} \sim \mathsf{IN}(0, \sigma_i^2)$, $i = q, w, pi, a, u$. Nominal wage w, producer price q and unemployment u are simultaneously determined, the consumer price p is an identity, while the import price pi and productivity a are autonomous and effectively exogenous.

In this section we derive analytic expressions for the long-run or steady-state levels of re, ws and u in the ECM For the PCM, we find the expressions for the level of a trending ws. We also report how short-term dynamics, including damped or persistent cyclical fluctuations, depend on certain parameters determining the interplay between temporary impulses and equilibrium-correction.

3.1 Reduced-form model

The structural-form model (10)-(12) for the two interacting *nominal* variables q and w can be transformed into a reduced-form model for two interacting *real* variables $re \equiv pi - q$ and $ws \equiv w - q - a$, conditional on *lagged* u and exogenous Δpi and Δa. The unemployment rate (15) is a real variable which is already on a reduced form. The dynamic system of three reduced-form equations can be expressed as a single vector equation $\mathbf{y}_t = \mathbf{R}\,\mathbf{y}_{t-1} + \mathbf{P}\,\mathbf{\Delta x}_t + \boldsymbol{\epsilon}_t$, where the vector $\mathbf{y} = (re, ws, u)'$ contains the endogenous variables, the vector $\Delta\mathbf{x} = (\Delta pi, \Delta a, 1)'$ contains the exogenous variables and 1 for the constant terms, and the vector $\boldsymbol{\epsilon}$ contains the reduced-form errors/shocks. The reduced-form coefficients are the elements of the 3×3 matrices $\mathbf{R}$ and $\mathbf{P}$. The vector equation for the reduced form of the model is

$$\underbrace{\begin{pmatrix} re_t \\ ws_t \\ u_t \end{pmatrix}}_{\mathbf{y}_t} = \underbrace{\begin{pmatrix} l & -k & n \\ \lambda & \kappa & -\eta \\ -\rho & 0 & \alpha \end{pmatrix}}_{\mathbf{R}} \underbrace{\begin{pmatrix} re_{t-1} \\ ws_{t-1} \\ u_{t-1} \end{pmatrix}}_{\mathbf{y}_{t-1}} + \underbrace{\begin{pmatrix} e & 0 & -d \\ \xi & -1 & \delta \\ 0 & 0 & c_u \end{pmatrix}}_{\mathbf{P}} \underbrace{\begin{pmatrix} \Delta pi_t \\ \Delta a_t \\ 1 \end{pmatrix}}_{\Delta\mathbf{x}_t} + \underbrace{\begin{pmatrix} \epsilon_{re,t} \\ \epsilon_{prw,t} \\ \epsilon_{u,t} \end{pmatrix}}_{\boldsymbol{\epsilon}_t}. \tag{19}$$

Appendix B contains the derivation of (19) and explicit expressions for the reduced-form coefficients and errors/shocks. The coefficients are functions of all structural parameters in the wage and price formation and the unemployment equation. The domains of the structural parameters in equation (10)-(12) and (15) imply that all reduced-form coefficients in (19), except d and δ, lie in the closed interval $[0, 1]$.

Dynamic homogeneity is often regarded as a necessary feature of a model that is to be used for policy advise in order to avoid 'monetary illusion' or false impression of a trade-off between unemployment and inflation. *Dynamic price and wage homogeneity* of order 1 entails the following restrictions on the structural parameters: $\psi_{wq} + \psi_{wp} = \psi_{qw} + \psi_{qpi} = 1$. The expressions in the appendices show that the homogeneity restrictions make $e = \xi = 0$, in which case there is *no* effect of foreign inflation on the the reduced form real exchange rate and wage share (19)[5]. It follows directly that this also holds for unemployment (15) as a function of the real exchange rate.

The stability properties of the system (19) depends on the recursion matrix $\mathbf{R}$ and its eigenvalues. The general analytic expressions for the eigenvalues of $\mathbf{R}$ are too large and complex to be of much help. Instead, we resort to a combined analytic and numeric approach, which is summarised in Appendix B and C. The appendices show that the model with 'realistic' parameter values is stable in the sense that with constant exogenous input $\Delta pi_t = \mathsf{E}\Delta pi_t = g_{pf}$ and $\Delta a_t = \mathsf{E}\Delta a_t = g_a$ all three endogenous variables re, ws and u converge to constant levels in the absence of stochastic shocks ($\boldsymbol{\epsilon}_t = 0$) The appendices also show that within a domain of plausible parameter values the model may exhibit endogenous cyclical fluctuations *without* exogenous impulses.

While the ECM is free of trends, the PCM is not. The parameter restrictions $\theta_w = \theta_q = 0$ remove all information about the wage share from the wage-price spiral (10)-(12). That causes a trend in the wage share ws. Since the real exchange rate indirectly affects the wage and price growth through the unemployment rate, the real exchange rate is not trending (unless unemployment is autonomous and insensitive to the real exchange rate).

3.2 Steady states

We derive the constant steady-state solution of the ECM by solving the reduced form $\mathsf{E}\mathbf{y}_t = \mathbf{R}\,\mathsf{E}\mathbf{y}_{t-1} + \mathbf{P}\,\mathsf{E}\mathbf{\Delta x}_t + \mathsf{E}\boldsymbol{\epsilon}_t$ under the constancy conditions: $\mathsf{E}\mathbf{y}_t = \mathsf{E}\mathbf{y}_{t-1} \equiv \mathbf{y} =$

[5] To be consistent with economic theory that implies no effect of foreign nominal inflation on the wage share (nominal neutrality), we need to impose dynamic price and wage homogeneity.

$(re, ws, u)'$, $\mathsf{E}\boldsymbol{\Delta}\mathbf{x}_t \equiv \mathbf{g} = (g_{pf}, g_a, 1)$ and $\mathsf{E}\boldsymbol{\epsilon}_t = 0$. Then $\mathbf{y} = \mathbf{R}\,\mathbf{y} + \mathbf{P}\,\mathbf{g} = (\mathbf{I} - \mathbf{R}L)^{-1}\,\mathbf{P}\,\mathbf{g}$, and the solution is

$$re = e_{ss}\, g_{pf} + \underset{\pm}{b_{ss}}\, g_a + \underset{\pm}{d_{ss}}, \tag{20}$$

$$ws = \underset{\pm}{\xi_{ss}}\, g_{pf} - \beta_{ss}\, g_a - \underset{\pm}{\delta_{ss}}, \tag{21}$$

$$u = -\,\mathfrak{e}_{ss}\, g_{pf} - \underset{\pm}{\mathfrak{b}_{ss}}\, g_a + \mathfrak{d}_{ss}. \tag{22}$$

Appendix B contains the derivation of (20)-(22) and explicit expressions for the steady-state coefficients. The expressions show that the stable level of each variable depends on *all* parameters in the model. The explicit sign of each steady-state coefficient follows analytically from the structural form. Depending on the parameterisation five of the twelve coefficients might be positive or negative, indicated by the $\pm$ labels. The other coefficients are positive. We discuss the sign of the coefficients in the section on comparative statics below.

In the constant steady state the real growth rates are $\Delta re \equiv \Delta pi - \Delta q \equiv 0$ and $\Delta ws \equiv \Delta w - \Delta q - \Delta a \equiv 0$. It follows from these definitions and equation (2) that in steady-state domestic nominal growth rates are determined by foreign inflation and productivity growth: $\Delta q = \Delta p = g_{pf}$ and $\Delta w = \Delta q + \Delta a = g_{pf} + g_a$.

The PCM has no equilibrium-correction. Inflation and wage growth are still influenced by the real exchange rate through unemployment, and that prevents the real exchange rate from trending. The wage share, on the other hand, exercise no influence on the wage and price growth. Nominal wage growth is unable to align with inflation and productivity growth, and the wage share is trending. Its long-run solution is

$$ws_t = ws_0 + t \times (\xi'_{ss}\, g_{pf} - g_a + \delta'_{ss}). \tag{23}$$

The steady-state expressions for the real exchange rate (20) and the unemployment rate (22) still hold, but with different expressions for the coefficients, cf. Appendix B. Without equilibrium-correction of domestic wage and price, productivity growth g_a affects *no* nominal variable in the model. Hence, productivity affects only the trending wage share ($b_{ss} = \mathfrak{b}_{ss} = 0$), and, by definition, productivity growth affects wage share growth in full, as expressed by (23).

A stable real exchange rate re implies $\Delta q = g_{pf}$ also in the PCM. From the long-run growth rate in the trending wage share (23) it follows that $\Delta ws = \xi'_{ss}\, g_{pf} - g_a + \delta'_{ss}$, and hence $\Delta w = (1 + \xi'_{ss})\, g_{pf} + \delta'_{ss}$. The trend in ws is caused by $\Delta w \neq g_{pf} + g_a$. Dynamic wage and price homogeneity eliminates any long-run effect of foreign nominal inflation on the stationary re and u, and on Δws.

3.2.1 Comparative statics

In the steady-state of the ECM (20)-(22), a permanent increase in exogenous productivity growth g_a implies a higher level of price competitiveness re, a lower wage share ws and, consequently, a lower unemployment rate u. Higher productivity growth g_a increases the productivity level a, which implies lower producer price growth Δq (10) and level q, and thus improved price competitiveness $re \equiv pi - q$. A permanent increase in exogenous productivity level a stimulates wage growth Δw (11) and inhibits price growth Δq. Despite this, the increase in the producer real wage $rw \equiv w - q$ is less than the increase in the productivity level a. The stable wage share $ws \equiv rw - a$ is therefore lower the higher the productivity growth g_a. Higher g_a implies higher capacity utilisation and lower unemployment u. *If* $\theta_w\, \psi_{qw} > \theta_q$ then both $b_{ss}, \mathfrak{b}_{ss} < 0$, and productivity growth Δa stimulates wage growth Δw so much that wage growth also stimulates domestic inflation Δq. Higher producer price q reduces the real exchange rate $re \equiv pi - q$, and through re also employment u.

A permanent increase in exogenous foreign inflation g_{pf} implies a higher level of price competitiveness $re \equiv pi - q$ and lower unemployment u. Higher g_{pf} increases domestic inflation Δq, but to a lesser degree, so that the real exchange rate re increases. The wedge $p - q$ (proportional to re) helps the wage level w increase more than the price level q, and consequently the wage share $ws \equiv w - q - a$ increases. With no wedge it may decrease. Increased price competitiveness re lowers the unemployment rate u directly. Since dynamic wage and price homogeneity holds for the short-term dynamics ($e = \xi = 0$ in (19)) it must also hold in the long run. Appendix B shows that the homogeneity restrictions imply $e_{ss} = \mathfrak{e}_{ss} = \xi_{ss} = 0$, hence foreign inflation has *no* effect on re, ws or u.

The expressions in Appendix B for the steady-state coefficients in (20)-(22) show that a permanent increase in u (by an increase in c_u) raises re and ws by lowering q more than w. A *temporary* increase in Δw, by a permanent increase in c_w or mark-up m_w, lowers re and raises ws and u. The permanent changes to the steady-state real levels counter the increase in c_w or m_w such that Δw returns to its steady-state value $g_{pf} + g_a$. In parallel, a temporary increase in Δq, by a permanent increase in c_q or mark-up m_q, lowers re and ws and raises u. Again, the permanent changes to the steady-state real levels counter the increase in c_q or m_q such that Δq returns to its steady-state value g_{pf}.

It is more difficult to decide the steady-state real effects in the ECM of a change in any structural parameter in the wage-price spiral (10)-(12). In addition to the complicated expressions in Appendix B, the effects depend on the other parameters as well. In the following we thus assume a basis parameterisation (E_b), which is explained in section 3.3 and Appendix C, and note that the directions of the changes may turn with a different parameterisation.

In the real-wage targets (4)-(5) a higher mark-up m_q or m_w lowers re, ws and u in steady state. A stronger moderating effect ϑ of unemployment on producers' price goal q^f increases re and ws, and lowers u. A more moderating effect ϖ of unemployment on workers' nominal wage goal w^b, or a stronger influence ω of price competitiveness $p - q$, raises re and reduces ws and u.

Stronger wage bargaining power, i.e. higher θ_w, decreases re and increases ws and u. More mark-up based pricing, i.e. higher θ_q, increases re and ws, and lowers u. If wage growth gets more sensitive to domestic inflation, i.e. ψ_{wq} or ψ_{wp} increases, then re decreases and ws and u increase. If domestic inflation gets more sensitive to wage growth or foreign inflation, i.e. ψ_{qw} or ψ_{qpi} increases, then re and ws decrease and u increases.

If u (15) becomes more responsive to the price competitiveness re due to an increase in ρ, or if u becomes more sluggish and persistent due to an increase in α, then the steady-state levels of u, re and ws all increase.

3.3 Dynamics

The long-run analysis above provides no information about short- to medium-term movements of the variables around their steady-state levels or trend. From an economic point of view shorter term dynamic properties can be equally important and more relevant for economic analyses with a limited horizon. If the economy is in equilibrium and experiences a permanent shock, say an exogenous shift in the unemployment level, how fast, how much and for how long do the variables react to the shift? To answer this question, we supplement the theoretical steady-state analysis with numerical analysis and simulations.

First we parameterise the model. The dynamic model is stylized to facilitate algebraic analysis. Its dynamics is therefore too simple to be suitable for estimation. Instead, we select parameter values guided by estimation results in a quarterly econometric model which encompases our model, see Bårdsen et al. (2005). We choose a wide range of numerical values for each parameter in the wage-price spiral and the unemployment equation. From the wide domain we select a large number of sets of parameter values in a fairly space-filling way. By a combination of numerical eigenvalue analysis and simulations we shrink the selection down to a handful. The remaining sets allow the ECM and the PCM to display a full range of short- to medium-term dynamics on top of the long-run (in)stability properties: smooth convergence toward steady-state levels or trend, and damped or persistent oscillations around the levels or trend.

The selected parameterisations are denoted b for basis, h for dynamic wage and price homogeneity in addition to the basis values, and 3 and 4 for other different values. parameterisation b is intended as typical, while 3 and 4 are realistic alternatives since the differences from b are not large in light of estimation uncertainty. parameterisation h is relevant in its own right, since homogeneity is a common assumption in theoretical models of the wage-price spiral. In all simulations the unemployment rate is subject to a permanent exogenous shock of a fixed size. This shift propagates through the model economy and influences all endogenous variables. Appendix C contains more information and details about the parameterisations, simulations, and dynamic properties of the model.

The model equations (10)-(12) and (15) do not tell how the interactions of level variables and growth variables might cause cyclical fluctuations (or stability for that matter). From the parameterisations, numerical eigenvector analysis and simulations of the model it appears that a balance between forces — how strongly the nominal changes and real levels influence on inflation and wage growth — is able to prevent endogenous cyclical fluctuations. It sounds reasonable that cyclical fluctuations depend on the relative strength of long-run stabilising forces and short-term (destabilising) changes. This tentatively suggests that rigidities and frictions *may* cause imbalances that lead to business cycles in a real-world economy. This issue deserves future studies.

Below we present simulations of the ECM in which the nominal wage and price growth adjust toward the real wage goals of workers and firms, and of the PCM which has no such corrections of either wage or price growth. In all simulations, the time period can be thought of as a quarter. Every model/parameterisation is simulated once with temporary shocks and once without. After 50 periods the unemployment rate u_t (15) experiences a permanent exogenous change (to c_u). To illustrate the stabilising function of unemployment, we have also simulated the model with autonomous unemployment ($\rho = 0$).

3.3.1 ECM simulations

Figure 1 shows simulations of the equilibrium-correction model (ECM) with three different parameterisations. Each column of panels displays simulations of the model with a specific parameterisation, denoted E_b (left), E_3 (centre) and E_4 (right). The basis parameterisation has parameters close to estimated values, and are shown in row E_b in Table 1 in Appendix C. The left panels in Figure 1 show the time paths of the unemployment rate u, the real exchange rate re, the wage share ws, the annual producer price inflation $\Delta_4 q$, the annual consumer price inflation $\Delta_4 p$ and the annual nominal wage growth $\Delta_4 w$. Bold solid graphs are simulations with endogenous u ($\rho > 0$) and no temporary shocks (all $\varepsilon = 0$). Ragged dotted graphs are a single stochastic simulation (with shocks: all $\varepsilon \neq 0$). Dashed straight lines are the analytical steady-state solutions (20)-(22). Thin solid graphs are simulations with autonomous u ($\rho = 0$). Dotted straight lines are analytical steady-state solutions for autonomous u. The differences in dynamics and levels between the bold and thin graphs in each panel reflect the interacting role of endogenous unemployment. Cyclical fluctuations are not possible in the model if u is autonomous.

parameterisations E_3 and E_4 imply dynamic wage and price homogeneity, slightly weaker equilibrium-correction, and a more responsive u. Rows E_3 and E_4 in Table 1 in Appendix C show the parameter values. Table 2 in Appendix C shows the complex conjugate eigenvalues of the recursion matrix $\mathbf{R}$ that cause the cyclical fluctuations which are clearly seen in the panels in the right two columns of Figure 1.

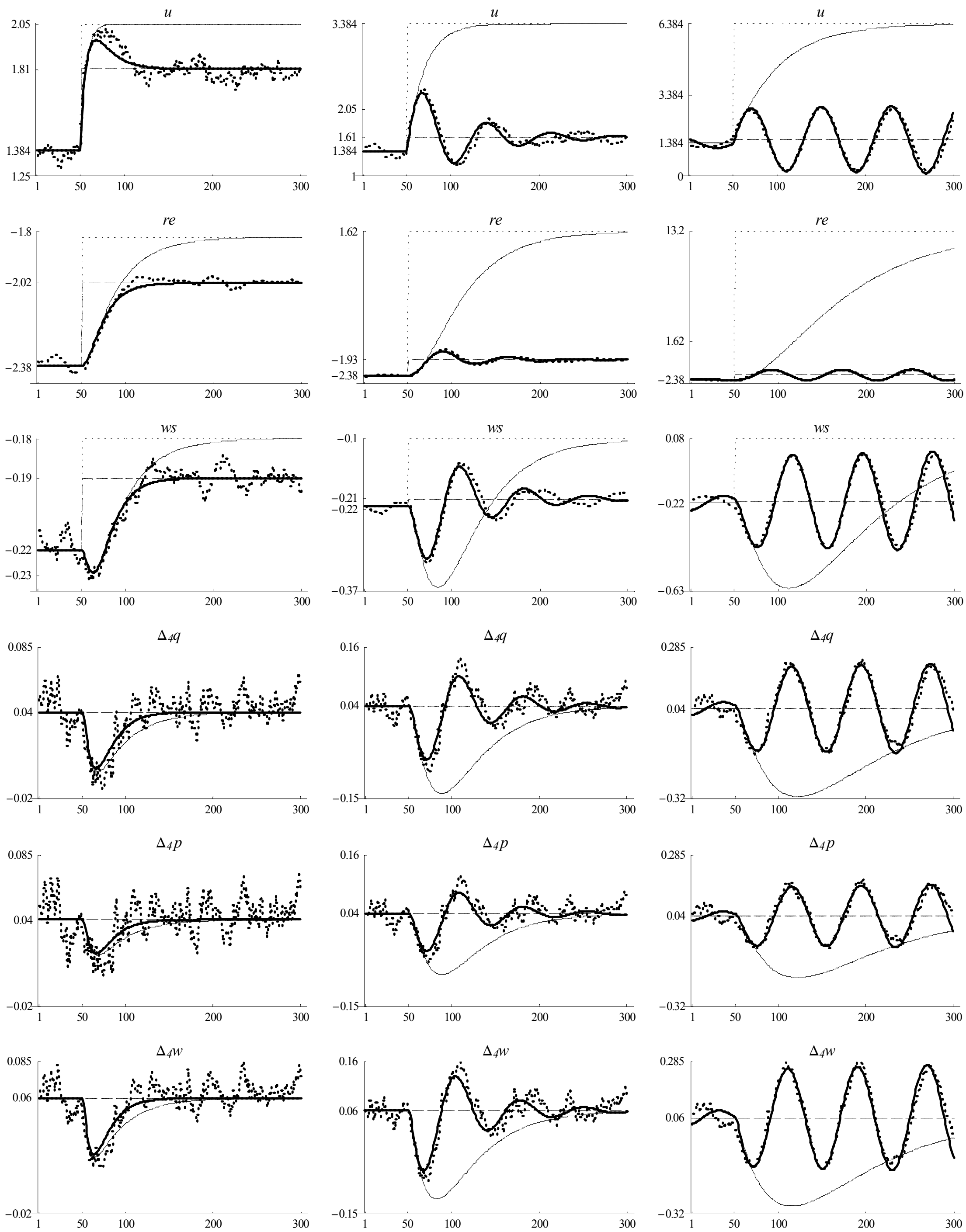

Figure 1: Simulations of the ECM with parameterization E_b (left panels), E_3 (centre panels) and E_4 (right panels). The upper 9 panels show levels of real variables, while the lower 9 panels show changes (growth rates) in nominal variables. Bold or thin graphs are simulations with endogenous or autonomous unemployment. Smooth graphs are steady state simulations without temporary shocks. Dotted ragged graphs are simulations with temporary shocks. Dashed and dotted straight lines are analytical steady states. The simulations start in period $t = 1$. In period $t = 51$ there is an upward exogenous shift in unemployment ($\Delta c_u = 0.1$). From then on the graphs show the dynamic responses to the shift. The main text explains the parameterizations and the simulations, while details are found in Appendix C.

The left panels show that all variables are stable in the model with the basis parameterisation (E_b), irrespective of whether u is endogenous or autonomous (effectively exogenous). The exogenous positive permanent shock to u gets multiplied almost seven times by its autoregression. The permanent upward shift in u causes a permanent depreciation of re. The rise in u makes domestic price inflation drop below the foreign inflation rate for a period of time, as seen in the fourth panel from the top. When u is endogenous, the depreciation of re partly counters the autoregressive multiplier. The new steady-state u is therefore below the autonomous level by about a third. The same holds for re. The wage share ws is also permanently affected by the increase in u. The immediate response to the increase is a reduction in ws, as predicted by bargaining theory and the reduced form equation in (19). But then the wage-price spiral kicks in, and Δq is reduced more than Δw. That increases ws, and makes its post-break level higher than the pre-break level. This is a general equilibrium result, and opposite of the partial equilibrium result from the single equation in (19).

The central panels show simulations of a model with slightly weaker equilibrium-correction, dynamic homogeneity in the wage-price spiral and u more responsive to the real exchange rate, cf. parameterisation E_3 in Table 1 in Appendix C. This explains the more lasting and larger responses to the exogenous shift if u is autonomous (thin graphs). Stronger interaction of endogenous u with the wage-price spiral via re causes damped cycles in all variables (bold graphs). These mechanisms are even stronger and the effects more pronounced in the simulations shown in the right panels. Due to a complex root of unit magnitudes the cycles do not cease, cf. E_4 in Table 2 in Appendix C.

The three parameterisations displayed in Figure 1 illustrate the three types of dynamics possible in a model with equilibrium-correction in the wage and price formation: stability, damped cycles, and constant or increasing cycles. Even though a trend is not possible in the ECM, pronounced and lasting cycles superimposed on the stable long-run levels constitute a significant second order instability (cf. Section 2.6).

The existence of a steady-state is independent of the level of unemployment, and whether it is endogenous or autonomous .The upper nine panels of Figure 1 show that. The model has *non-accelerationist* properties: the wage-price spiral stabilises wage and price inflation independent of the unemployment rate. There is no need for a unique natural rate of unemployment (NAIRU) to stabilise the variables' levels or growth rates. Inflation is stable at *any* constant rate of unemployment. The expected stable rate of inflation is given by the trend in foreign price growth ($\Delta_4 p = \Delta_4 q = 4 g_{pf} = 0.04$). This is contrary to conventional macro models that can be described as *accelerationist*: «there is a degree of supply-demand balance of the economy as a whole, measured by the unemployment rate although capacity utilisation or output-gap can also be used, with the property that inflation speeds up if the economy is tighter and decelerates if the economy is slacker. That special state of the real economy is usually called the 'natural rate' of unemployment, or the NAIRU» (Solow (1999)).

3.3.2 PCM simulations

Figure 2 shows simulations of the Phillips curve model (PCM), which differs from the ECM by lack of nominal wage and price adjustments toward real-wage targets. The PCM has many traits in common with the standard aggregate demand and supply (AD-AS) model found in modern textbooks in macroeconomics, see Sørensen and Whitta-Jacobsen (2010). The main difference is that we have an explicit model of the wage-price spiral, while the textbook model only includes a price Phillips curve, but that is due to simplification in the textbooks. The intended interpretation is always that the underlying process of nominal adjustments is of a wage-price spiral type. Another difference is that in textbooks the Phillips curve, and therefore also the AS schedule, are in terms of an

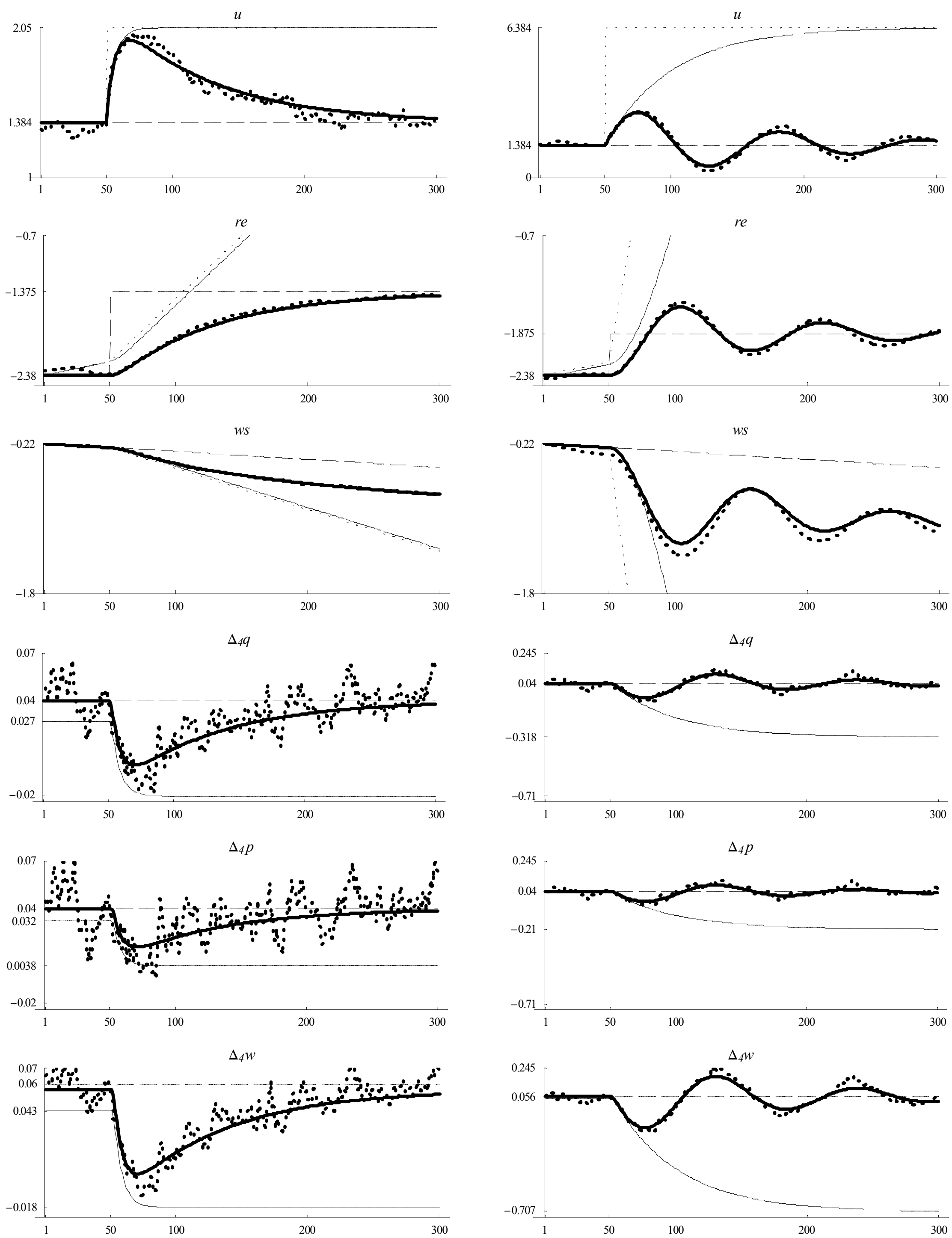

Figure 2: Simulations of the PCM with parameterization P_b (left panels), and P_4 (right panels). See Figure 1 for explanations of the panels and graphs. The real exchange rate re is trending in the regime with autonomous unemployment (thin solid graph in the second row of panels). The trend is caused by domestic inflation $\Delta_4 q$ being less than foreign inflation $4\,g_{pf} = 0.04$. The negative trend in the wage share ws — with endogenous as well as autonomous (exogenous) unemployment — is caused by the wage growth $\Delta_4 w$ being less than the sum of domestic inflation $\Delta_4 q$ and productivity growth $4\,g_a = 0.02$, both before and after the shock to unemployment. The trends in the real exchange rate and in the wage share can be both positive and negative, as long as they are of opposite sign. For a full explanation of the parameterizations and the simulations, confer the main text and Appendix C.

output-gap variable. We use unemployment as a proxy for capacity utilisation, but due to Okun's law this difference does not affect the interpretation. Finally, the textbook version has more variables that represent determinants of aggregate demand, while our model only includes the real exchange rate. We focus on the stability properties of the model when there is a single endogenous variable in the AD schedule (15): the real exchange rate.

The PCM is simulated with parameterisation P_b (left panels) and P_4 (right panels). The wage share ws is trending since it has no coordinating influence on the wage and price growth. The real exchange re is trending only if unemployment u is autonomous. The real exchange re does not influence the wage and price growth directly (like in the ECM). It can only do so indirectly, through an endogenous u. The rest of the variables are non-trending, and return gradually to their pre-break level when u is endogenous. Contrary to the ECM, the steady-state u is independent of the exogenous permanent shock. The rise in the steady-state re exactly counters it, hence *there is a "natural rate" of unemployment in the PCM. This is a general algebraic and not numerical result.* Equation (27) in Appendix B shows that the steady-state unemployment rate depends only on the parameters in the wage-price spiral, and *not* on *any* coefficient in the unemployment equation (15). Like in the ECM, cyclical fluctuations might dominate the steady states and the trend for a long time after the shock.

4 Summary and further work

We have formulated a model for the *simultaneous* determination of nominal wage, prices and unemployment, and explored the model's dynamic properties by a combination of theoretical analysis, numerical investigations and computer simulations. The results show that the dynamic properties of the endogenous variables are *system properties*. The choice of model for the supply side conditions many important system properties. For instance, the equilibrium-correction model (ECM) has *no* "natural" rate of unemployment (NAIRU). The ECM is dynamically stable for any stable rate of unemployment. The Phillips curve model (PCM), on the other hand, has a NAIRU that depends only on the parameters of the wage-price spiral. But the PCM is not stable. Its wage share trends.

The ECM's supply side is a wage-price spiral, with wage bargaining and mark-up pricing characterised by nominal rigidity and adjustments towards real-wage goals. The real-wage goals bring real attractors into the wage-price spiral. The attractors are able to coordinate the nominal wage and price growth, and thereby eliminate a trend in the real exchange rate and the wage share *independent of the unemployment rate*. The steady-state levels as well as the nominal wage growth and domestic inflation are determined by exogenous productivity growth and foreign inflation. In the PCM there is no information about the wage share in the wage-price spiral, and that causes the wage share to trend. This result does *not* depend on the assumption that unemployment reacts to a real depreciation with a lag[6]. In both the ECM and the PCM, stationarity or trend is independent of the actual unemployment rate.

The quantitative dynamic properties of the ECM and the PCM depend on all parameters in the model. The qualitative dynamic properties of the ECM and the PCM are independent of the actual processes for the exogenous variables[7]. The dynamic properties

[6] Without a lag, (15) would not be the reduced form equation for unemployment u. Substituting the reduced form for re in (19) into (15) would replace the 0 in the $\mathbf{R}$ matrix in (19) with ρk. According to Appendix B, in the PCM: $\theta_q = \theta_w = 0 \Rightarrow k = 0$. Hence, the 0 would reappear in $\mathbf{R}$ and there would be a trend in the wage share.

[7] There is a chain of implications from the exogenous trends to endogenous trends to the equilibrium correction formulation of a wage-price spiral (10)-(11). But, endogenous trends in the wage-price spiral do not require exogenous trends. The wage-price spiral passes a trend in the exogenous import price pi and/or a trend in productivity a onto domestic wage w and prices q and p. But, in the absence of exogenous trends, the wage-price spiral is still able to keep domestic wage and prices growing. The reason

are fully endogenous, and are determined by the existence and strength of transmission and feedback channels in the model. Explorations of short/medium term dynamics by numerical analysis and simulations reveal that the interplay of stabilising attractors (level variables in the wage-price spiral (10)-(11)) and destabilising impact forces (shocks to nominal wage growth ($\varepsilon_{w,t}$), domestic and imported inflation ($\varepsilon_{q,t}, \varepsilon_{pi,t}$) that all affect the Δ-variables in the wage-price spiral) may give rise to cyclical fluctuations that are entirely endogenous. It appears that some degree of balance of strength between transmission/feedback channels and impact effects is necessary to avoid cycles. In other words, rigidities (persistence) and frictions (unequal responses) may cause cyclical fluctuations. Our modelling and discussion of a mechanism for economic fluctuations links back to the 1930s and the different views of Frisch and Kalecki on whether the economy propagates (exogenous/non-economic) shocks into cyclical fluctuations or generates its own cycles. Frisch believed cyclical fluctuations to be *highly damped*, but revitalised by shocks[8] while Kalecki saw *persistent* cycles as an intrinsic feature of a capitalist economy. Our model accommodates both views on economic fluctuations (simply by different relative parameter values, cf. Appendix C and Figure 1 and 2). However, we do not allude to any similarity in economic contents and forms of models by Frisch, Kalecki and us.

We have demonstrated a possible coordinating role that a dynamic wage-price spiral on the supply side and unemployment on the demand side might play in an economy. In the ECM's wage-price spiral the real exchange rate and the wage share coordinates nominal wage and price growth so that real variables become stationary. The interaction between the supply and demand side by the real exchange rate and unemployment might cause damped or persistent cyclical fluctuations that can dominate in the medium or long run. The PCM lacks the wage share as a real attractor in the wage-price spiral. Devoid of that coordinating information, the wage share trends.

The analytical results and the numerical properties are specific to the supply-side oriented model. But they illustrate something general within the familiar AS-AD setting. Both qualitative and quantitative properties of a model may change with relatively small changes in specifications and parameters. This does *not* imply that the model or analysis lacks robustness. It rather demonstrates that an interdependent dynamic system — an economic model, or the real economy — can be inherently sensitive to conditions. If we view the model as a tool, then its properties depend on our theoretical assumptions, our modelling and our estimation/calibration procedure. If we view the model as a simplified representation of the data-generating mechanism in the real-world economy, then we have learnt that aggregate behavior in the economy might change qualitatively with (minor) changes in goals and in the dynamic and interdependent interactions of economic agents (causing changes to parameter values in a fixed model structure).

In the present paper we have focused on the supply side, and deliberately kept the model simple in order to manage a thorough — both theoretical and numerical — analysis. From this basis model we plan to include extensions one by one, and build our understanding step by step. The duality — first order stability and second order instability — make us wonder whether inclusion of more variables and mechanisms may prevent real trends or limit the scope of cycles. Finding that cyclical fluctuations can be a typical feature of

is that the wage growth Δw_t (10) and inflation Δq_t (11) might be positive even if pi and a should be stationary ($\Delta pi \approx 0$ and $a \approx const$). Hence, should the import price (1) and productivity (3) *temporarily* stop growing, the domestic wage and prices would keep on trending upwards. But, for domestic wage and prices to be trending variables and equations (8)-(9) to be valid formulations in an economy where the exogenous import price and productivity are stationary variables (permanently, not temporarily), we would need to rationalize the constant terms c_q and c_w. Self-fulfilling expectations is a possibility, which might also rationalize a continued wage and price growth during a temporary stop in foreign inflation and productivity growth.

[8] Zambelli (2007) has an interesting discussion of Frisch's work and macro dynamics in the 1930s — and claims that Frisch's famous 'rocking-horse' model does *not* generate cycles for plausible parameter values!

the solution motivates the inclusion of an exchange rate equation and a reaction function for the interest rate, say a simple Taylor-type rule, in order to study the stabilising role of monetary policy. The interdependence between the exchange rate and the interest rate makes it natural to incorporate both in a model that may represent an inflation targeting regime for example. Then the issues of expectations and varying degrees of foresight also have to be addressed.

A Definitions of variables, parameters and coefficients

Variables, parameters and coefficients are explained in the main text, but they are collected here for convenience. All variables are logarithmically transformed, and are listed in alphabetical order:

a_t	Labour productivity (autonomous/exogenous), eq. (3)
D_t	Step dummy to facilitate an exogenous shift in the unemployment level, in eq. (15)
ecm_t^f	Firms' real-wage gap $rw_t - rw_t^f$, eq. (6)
ecm_t^b	Workers' (bargained) real-wage gap $rw_t - rw_t^b$, eq. (7)
p_t	Consumer price, eq. (2) and (12)
pi_t	Import price in domestic currency, eq. (1)
$p_t - q_t$	Wedge between consumer and producer real wage, in eq. (5) and (11)
q_t	Producer price, eq. (8) and (10)
q_t^f	Price goal of producers in a steady growth economy, eq. (4)
re_t	Real exchange rate $pi_t - q_t$, eq. (19)
rw_t^b	Bargained real wage $w_t^b - q_t$
rw_t^f	Optimal producer real wage $w^f - q$
u_t	Unemployment (endogenous or exogenous), eq. (15)
w_t	Nominal hourly wage, eq. (9) and (11)
ws_t	Wage share $w_t - q_t - a_t$,
w_t^b	Bargained wage (goal), eq. (5)
$\varepsilon_{\text{variable},t}$	Temporary shock (or residual) to nominal 'variable'
$\epsilon_{\text{variable},t}$	Temporary shock (or residual) to real 'variable'

The parameters and coefficients are grouped according to the equation they belong to:

Equation for the producer price inflation (10)

c_q	Constant in the expression for price growth
m_q	Mark-up on marginal labour cost
ψ_{qw}	Elasticity of nominal wage growth
ψ_{qpi}	Elasticity of import price inflation
θ_q	Strength/speed of equilibrium-correction in price setting
μ_q	$= \theta_q \vartheta$ or ς, where:
ϑ	is the effect of unemployment on marginal labour cost
ς	is the effect of unemployment in case of no equilibrium-correction

Equation for the nominal wage growth (11)

c_w	Constant in the expression for wage growth
m_w	Constant in the expression for bargained wage
ψ_{wq}	Elasticity of producer (domestic) price inflation
ψ_{wp}	Elasticity of consumer-price inflation
θ_w	Strength/speed of equilibrium-correction in wage formation
μ_w	$= \theta_w \varpi$ or φ, where:
ϖ	is the impact of unemployment on bargained wage
φ	is the effect of unemployment in case of no equilibrium-correction
ω	Elasticity of price wedge

Equation for the consumer price inflation (12)

ϕ	Degree of closeness of the economy

Equation for the rate of unemployment (15)

c_u	Constant in the reduced-form expression
α	Degree of persistence in unemployment
ρ	Degree of feedback from the (lagged) real exchange rate

Exogenous processes

g_a	Constant underlying growth in productivity, eq. (3)
g_{pf}	Constant underlying foreign inflation, eq. (1)

Standard deviation of shocks (residuals) to variable z :

σ_z	$z \in \{q, w, u, pi, a\}$

B Model analysis

Structural form

The structural form of the model is given by equations (10)-(12). We want to transform producer price q_t, nominal wage w_t, import price pi_t into the real exchange rate $re_t \equiv pi_t - q_t$ and the productivity corrected real wage $ws_t \equiv w_t - q_t - a_t$. From (2) we get $p_t - q_t = (1-\phi)\, re$. After some manipulations we arrive at the following structural form equations for the real variables:

$$\begin{aligned}
(1-\psi_{qw})\, re_t + \psi_{qw}\, ws_t &= (1-\psi_{qw})\, re_{t-1} + (\psi_{qw} - \theta_q)\, ws_{t-1} + \mu_q\, u_{t-1} \\
&\quad + (1-\psi_{qw} - \psi_{qpi})\, \Delta pi_t - \psi_{qw}\, \Delta a_t - (\theta_q\, m_q + c_q) - \varepsilon_{q,t}, \\
(1-\psi_{wq} - \phi\, \psi_{wp})\, re_t - ws_t &= (1-\psi_{wq} - \phi\, \psi_{wp} + \theta_w\, \omega(1-\phi))\, re_{t-1} - (1-\theta_w)\, ws_{t-1} + \mu_w\, u_{t-1} \\
&\quad + (1-\psi_{wq} - \psi_{wp})\, \Delta pi_t + \Delta a_t - (\theta_w\, m_w + c_w) - \varepsilon_{q,t}.
\end{aligned}$$

According to (13)-(14), $\mu_q = \theta_q\, \vartheta$ or ς, and $\mu_w = \theta_w\, \varpi$ or φ. After solving for re and ws, the nominal variables can be reconstructed as follows: $q_t = pi_t - re_t$, $p_t = (1-\phi)\, re_t + q_t$, and $w = ws_t + q_t + a_t$.

The unemployment rate (15) is already a real variable. The structural form of the model with the transformed variables and unemployment can be written as a vector equation $\mathbf{A}\,\mathbf{y}_t = \mathbf{B}\,\mathbf{y}_{t-1} + \mathbf{C}\,\mathbf{x}_t + \boldsymbol{\varepsilon}_t$,where $\mathbf{y}'_t = (re_t, ws_t, u_t)$ is a vector of current endogenous real variables, and $\mathbf{y}'_{t-1}$is a vector of the same variables, but lagged. The vector $\mathbf{x}'_t = (\Delta pi_t, \Delta a_t, 1)$ contains the autonomous/exogenous variables. $\mathbf{A}$ and $\mathbf{B}$ are both 3×3 matrices of structural coefficients for the current and the lagged endogenous real variables:

$$\mathbf{A} = \begin{pmatrix} 1-\psi_{qw} & \psi_{qw} & 0 \\ 1-\psi_{wq} - \phi\, \psi_{wp} & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 1-\psi_{qw} & \psi_{qw} - \theta_q & \mu_q \\ 1-\psi_{wq} - \phi\, \psi_{wp} + \theta_w\, \omega(1-\phi) & -(1-\theta_w) & \mu_w \\ -\rho & 0 & \alpha \end{pmatrix}.$$

$\mathbf{C}$ is a 3×3 matrix of structural coefficients of the exogenous variables:

$$\mathbf{C} = \begin{pmatrix} 1-\psi_{qw} - \psi_{qpi} & -\psi_{qw} & -(c_q + \theta_q m_q) \\ 1-\psi_{wq} - \psi_{wp} & 1 & -(c_w + \theta_w\, m_w) \\ 0 & 0 & c_u \end{pmatrix}.$$

Reduced form

Tedious matrix calculations $\mathbf{y}_t = \mathbf{A}^{-1}\mathbf{B}\,\mathbf{y}_{t-1} + \mathbf{A}^{-1}\mathbf{C}\,\mathbf{x}_t + \mathbf{A}^{-1}\boldsymbol{\varepsilon}_t \equiv \mathbf{R}\,\mathbf{y}_{t-1} + \mathbf{P}\,\mathbf{x}_t + \boldsymbol{\epsilon}_t$.yield the reduced form (19), where the coefficients of the real exchange rate are

$$\begin{aligned}
l &= 1 - \theta_w\, \omega\, \psi_{qw}\, (1-\phi)/\chi, \\
k &= (\theta_q - \theta_w \psi_{qw})/\chi, \\
e &= 1 - (\psi_{qpi} + \psi_{qw}\, \psi_{wp}\, (1-\phi))/\chi, \quad = 0 \text{ if dynamic homogeneity}, \\
n &= (\mu_q + \mu_w\, \psi_{qw})/\chi, \\
d &= (m_q\, \theta_q + c_q + (m_w\, \theta_w + c_w)\, \psi_{qw})/\chi.
\end{aligned}$$

and the denominator is $\chi = 1 - \psi_{qw}(\phi\psi_{wp} + \psi_{wq}) > 0$. The reduced-form coefficients of the productivity-corrected real-wage are

$$\begin{aligned}
\lambda &= \theta_w\, \omega\, (1-\psi_{qw})(1-\phi)/\chi, \\
\kappa &= 1 - (\theta_w\, (1-\psi_{qw}) + \theta_q\, (1-\psi_{wq} - \phi\, \psi_{wp}))/\chi, \\
\xi &= (\psi_{wp}\, (1-\psi_{qw})(1-\phi) - \psi_{qpi}\, (1-\psi_{wq} - \phi\, \psi_{wp}))/\chi, \qquad = 0 \text{ if dynamic homogeneity} \\
\eta &= (\mu_w\, (1-\psi_{qw}) - \mu_q\, (1-\psi_{wq} - \phi\, \psi_{wp}))/\chi, \\
\delta &= ((m_w\, \theta_w + c_w)(1-\psi_{qw}) - (m_q\, \theta_q + c_q)(1-\psi_{wq} - \phi\, \psi_{wp}))/\chi.
\end{aligned}$$

The error terms of the real variables are linear combinations of the error terms of their constituent nominal variables: $\epsilon_{re,t} = (\varepsilon_{q,t} + \psi_{qw}\, \varepsilon_{w,t})/\chi$ and $\epsilon_{prw,t} = (\varepsilon_{q,t}\, (1-\psi_{wq} - \phi\, \psi_{wp}) - \varepsilon_{w,t}\, (1-\psi_{qw}))/\chi$.

Stability

The unemployment equation (15) is a reduced form. The equation alone suggests that $\alpha = 1$ makes unemployment a random walk, and thus *destabilises* the system. But, the question of stability is not decided by the properties of a single equation. When the equation is an integral part of a system of equations, *(in)stability is a system property.* There is feedback between unemployment and the real exchange rate. Substituting the reduced form expression for the real exchange rate in (19) into the unemployment equation (15) allows us write unemployment as a lag polynomial:

$$(1 - \alpha\, L + \rho\, n\, L^2)\, u_t = const - \rho\, l\, re_{t-2} + \rho\, k\, ws_{t-2} - \rho\, e\, \Delta pi_{t-1} + residual_t,$$

where $const = c_u + \rho\, d$, $residual_t = \epsilon_{u,t} - \rho\, \epsilon_{re,t-1}$ and the lag operator $L^s : u_t \to u_{t-s}$. We see directly that $\alpha = 1$ *and* $\rho = 0$ make unemployment a random walk and autonomous, i.e. effectively exogenous. Alternatively, $\alpha = 1$ and $n = 0$ would cause instability, but $n > 0$ always. With unemployment depending on the real exchange rate re, and the real exchange rate depending on the wage-price spiral and on unemployment, we cannot infer (in)stability from the unemployment equation alone.

The dynamic properties of the model depend on the eigenvalues of the recursion matrix $\mathbf{R}$. Unfortunately, their expressions are too large and complicated for algebraic analysis. We have to resort to a mix of algebraic analysis and numerical investigations and simulations. Let us *presume* a stable system in which all real variables converge to constant values $\mathbf{y} = \mathsf{E}\mathbf{y}_t = (re, ws, u) = (\mathbf{I} - \mathbf{R})^{-1}\, \mathbf{P}\, \mathbf{g}$ when the exogenous variables grow at constant rates $\mathbf{g} = \mathsf{E}\boldsymbol{\Delta}\mathbf{x}_t = (g_{pf}, g_a, 1)$ and all shocks are switched off: $\boldsymbol{\epsilon} = \mathsf{E}\boldsymbol{\epsilon}_t = \mathbf{0}$. A necessary although not sufficient requirement for stability is

$$\det(\mathbf{I} - \mathbf{R}) = (1 - \alpha)[(1 - l)(1 - \kappa) + k\,\lambda] + \rho\,[n\,(1 - \kappa) + k\,\eta] \neq 0.$$

The determinant is zero only when $(1 - \alpha)[(1 - l)(1 - \kappa) + k\,\lambda] = 0$ *and* $\rho\,[n\,(1 - \kappa) + k\,\eta] = 0$. The first equality holds if $\alpha = 1$ or $\kappa = 1$ and/or $l = 1$ *and* $k\,\lambda = 0$. The second equality holds if $n\,(1 - \kappa) = k\,\eta = 0$. The PCM restrictions $\{\theta_q = \theta_w = 0\} \Rightarrow \{\kappa, l = 1 \text{ and } k = \lambda = 0\} \Rightarrow \{\det(\mathbf{I} - \mathbf{R}) = 0\}$. The resulting real unit root $r = [\kappa + l \pm \sqrt{(\kappa - l)^2 - 4\,k\,\lambda}]/2 = 1$ causes a trend in the wage share ws. No trend is possible in the ECM.

Steady-state in the ECM

More tedious matrix calculations $\mathbf{y} = (\mathbf{I} - \mathbf{R})^{-1}\, \mathbf{P}\, \mathbf{g}$ yield the (presumed) steady state (20)-(22) of the ECM, with final-form coefficients of re:

$$\begin{aligned}
e_{ss} &= (1 - \alpha)[\theta_q(1 - \psi_{wq} - \psi_{wp}) + \theta_w(1 - \psi_{qw} - \psi_{qpi})]/(\theta_q\, \theta_w\, \Omega),\\
b_{ss} &= (1 - \alpha)(\theta_q - \theta_w\, \psi_{qw})/(\theta_q\, \theta_w\, \Omega),\\
d_{ss} &= (c_u(\varpi + \vartheta) - (1 - \alpha)[m_w + m_q + c_w/\theta_w + c_q/\theta_q])/\Omega,
\end{aligned}$$

with $\Omega = \omega\,(1 - \phi)\,(1 - \alpha) + \rho\,(\varpi + \vartheta)$, because $\mu_w = \theta_w\, \varpi$ and $\mu_q = \theta_q\, \vartheta$ in the ECM. The final-form coefficients of ws are

$$\begin{aligned}
\xi_{ss} &= [\theta_w(1 - \psi_{qw} - \psi_{qpi})(\omega\,(1 - \phi)(1 - \alpha) + \rho\,\varpi) - \vartheta\,\rho\,\theta_q(1 - \psi_{wq} - \psi_{wp})]/(\theta_q\, \theta_w\, \Omega),\\
\beta_{ss} &= [\theta_w\, \psi_{qw}\,(\omega\,(1 - \phi)(1 - \alpha) + \rho\,\varpi) + \vartheta\,\rho\,\theta_q]/(\theta_q\, \theta_w\, \Omega),\\
\delta_{ss} &= [(\omega\,(1 - \phi)(1 - \alpha) + \rho\,\varpi)(m_q + c_q/\theta_q) - \vartheta\,\{\rho\,(m_w + c_w/\theta_w) + c_u\,\omega\,(1 - \phi)\}]/\Omega.
\end{aligned}$$

The final-form coefficients of u are

$$\begin{aligned}
\mathfrak{e}_{ss} &= \rho\,(\theta_q(1 - \psi_{wq} - \psi_{wp}) + \theta_w(1 - \psi_{qw} - \psi_{qpi}))/(\theta_q\, \theta_w\, \Omega),\\
\mathfrak{b}_{ss} &= \rho\,(\theta_q - \theta_w\, \psi_{qw})/(\theta_q\, \theta_w\, \Omega),\\
\mathfrak{d}_{ss} &= (c_u\, \omega\,(1 - \phi) + \rho\,[m_w + m_q + c_w/\theta_w + c_q/\theta_q])/\Omega.
\end{aligned}$$

Dynamic homogeneity $\{\psi_{wq} + \psi_{wp} = \psi_{qw} + \psi_{qpi} = 1\} \Rightarrow \{e_{ss} = \mathfrak{e}_{ss} = \xi_{ss} = 0\}$, hence there is no long-run effect of foreign inflation (g_{pf}) on re, ws and u.

Inserting these expressions into the steady-state equations(20)-(22) and rearranging lead to the following steady-state expressions as functions of all model parameters and the exogenous growth rates g_{pf} and g_a:

$$\begin{aligned}
re &= [(1 - \alpha)(F_w - m_w + F_q - m_q) + c_u(\varpi + \vartheta)]\,/\Omega,\\
ws &= [(\Omega - \vartheta\,\rho)\,(F_q - m_q) - \vartheta\,\rho\,(F_w - m_w) + \vartheta\, c_u\, \omega\,(1 - \phi)]\,/\Omega,\\
u &= [-\rho\,[F_w - m_w + F_q - m_q] + c_u\, \omega\,(1 - \phi)]\,/\Omega,
\end{aligned}$$

where $F_w = (g_{pf}(1 - \psi_{wq} - \psi_{wp}) + g_a - c_w)/\theta_w$, $F_q = (g_{pf}(1 - \psi_{qw} - \psi_{qpi}) - g_a\, \psi_{qw} - c_q)/\theta_q$ and $\Omega = \omega\,(1 - \phi)\,(1 - \alpha) + \rho\,(\varpi + \vartheta)$.

Long run in the PCM

The PCM restrictions $\theta_q = \theta_w = 0$ simplify the recursion matrix to

$$\mathbf{R} = \begin{pmatrix} 1 & 0 & n \\ 0 & 1 & -\eta \\ -\rho & 0 & \alpha \end{pmatrix}. \tag{24}$$

The matrix $\mathbf{R}$ has the three eigenvalues 1 and $\frac{1}{2}\left(1 + \alpha \pm \sqrt{(1 - \alpha)^2 - 4\,n\,\rho}\right)$. The unit eigenvalue makes the wage share ws a random walk process. Constant terms induce drift, which produces a visible trend in the wage share. Depending on the parameterisation, it might be positive or negative. An additional trend in the real exchange rate and/or in the unemployment rate (which would be economically less meaningful) requires $n = 0$ or $\rho = 0$. Neither complies with the model. That rules out any trend in re or u.

We see from its second column of $\mathbf{R}$ in (24) that ws does not affect re nor u. It can thus be uncoupled from the latter two, and we can split the system in two. The dynamics of re and u have the following reduced form:

$$\underbrace{\begin{pmatrix} re_t \\ u_t \end{pmatrix}}_{\mathbf{z}_t} = \underbrace{\begin{pmatrix} 1 & n \\ -\rho & \alpha \end{pmatrix}}_{\mathbf{R}} \underbrace{\begin{pmatrix} re_{t-1} \\ u_{t-1} \end{pmatrix}}_{\mathbf{z}_{t-1}} + \underbrace{\begin{pmatrix} e & -d \\ 0 & c_u \end{pmatrix}}_{\mathbf{P}} \underbrace{\begin{pmatrix} \Delta pi_t \\ 1 \end{pmatrix}}_{\mathbf{x}_t} + \underbrace{\begin{pmatrix} \epsilon_{re,t} \\ \epsilon_{u,t} \end{pmatrix}}_{\boldsymbol{\epsilon}_t}, \tag{25}$$

while the trending behavior of ws depends on lagged u and exogenous growth:

$$\Delta ws_t = -\eta\, u_{t-1} + \begin{pmatrix} \xi & -1 & \delta \end{pmatrix} \begin{pmatrix} \Delta pi_t \\ \Delta a_t \\ 1 \end{pmatrix} + \epsilon_{prw,t}. \tag{26}$$

Presuming a stable subsystem, we can solve the steady-state equation $\mathbf{z} = (\mathbf{I} - \mathbf{R})^{-1}\, \mathbf{P}\, \mathbf{g}$, with $\mathbf{g} = (g_{pf}, 1)$:

$$\underbrace{\begin{pmatrix} re \\ u \end{pmatrix}}_{\mathbf{z}} = \underbrace{\begin{pmatrix} 0 & -n \\ \rho & 1-\alpha \end{pmatrix}^{-1}}_{(\mathbf{I}-\mathbf{R})^{-1}} \underbrace{\begin{pmatrix} e & -d \\ 0 & c_u \end{pmatrix}}_{\mathbf{P}} \underbrace{\begin{pmatrix} g_{pf} \\ 1 \end{pmatrix}}_{\mathbf{g}} = \underbrace{\begin{pmatrix} e'_{ss} & d'_{ss} \\ -\mathfrak{e}_{ss} & \mathfrak{d}_{ss} \end{pmatrix}}_{(\mathbf{I}-\mathbf{R})^{-1}\,\mathbf{P}} \underbrace{\begin{pmatrix} g_{pf} \\ 1 \end{pmatrix}}_{\mathbf{g}}.$$

The real exchange rate equation has the final-form coefficients

$$\begin{aligned} e'_{ss} &= (1-\alpha)[1 - \psi_{qpi} - \psi_{qw}(\psi_{wq} + \psi_{wp})]/[\rho\,(\varsigma + \varphi\,\psi_{qw})] \quad \text{and} \\ d'_{ss} &= c_u/\rho - (1-\alpha)(c_q + c_w\,\psi_{qw})/[\rho\,(\varsigma + \varphi\,\psi_{qw})]. \end{aligned}$$

The unemployment equation has the final-form coefficients

$$\mathfrak{e}_{ss} = [1 - \psi_{qpi} - \psi_{qw}(\psi_{wq} + \psi_{wp})]/(\varsigma + \varphi\,\psi_{qw}) \quad \text{and} \quad \mathfrak{d}_{ss} = (c_q + c_w\,\psi_{qw})/(\varsigma + \varphi\,\psi_{qw}), \tag{27}$$

Dynamic homogeneity cancels any long-run effect of g_{pf} ($e'_{ss} = \mathfrak{e}_{ss} = 0$).

Equation (26) determines the constant expected growth-rate of ws :

$$\begin{aligned} \Delta ws &= -\eta \underbrace{\begin{pmatrix} -\mathfrak{e}_{ss} & \mathfrak{d}_{ss} \end{pmatrix} \begin{pmatrix} g_{pf} \\ 1 \end{pmatrix}}_{u} + \begin{pmatrix} \xi & -1 & \delta \end{pmatrix} \begin{pmatrix} g_{pf} \\ g_a \\ 1 \end{pmatrix} \\ &= (\eta\,\mathfrak{e}_{ss} + \xi)\, g_{pf} - g_a + (\delta - \eta\,\mathfrak{d}_{ss}) \;\equiv\; \xi'_{ss}\, g_{pf} - g_a + \delta'_{ss}, \end{aligned}$$

where

$$\xi'_{ss} = [\varphi\,(1 - \psi_{qw} - \psi_{qpi}) - \varsigma\,(1 - \psi_{wq} - \psi_{wp})]/(\varsigma + \varphi\,\psi_{qw}) \quad \text{and} \quad \delta'_{ss} = (\varsigma\, c_w - \varphi\, c_q)/(\varsigma + \varphi\,\psi_{qw}).$$

Dynamic homogeneity makes $\xi'_{ss} = 0$, in which case g_{pf} has no effect on the trend in ws. With a constant growth-rate, the level of ws at time point $t \geq 1$ is

$$ws(t) = ws_0 + t \times \Delta ws = ws_0 + t\,(\xi'_{ss}\, g_{pf} - g_a + \delta'_{ss}).$$

Depending on the parameterisation, ws might have a negative or positive trend. A constant ws is theoretically and numerically possible, but unlikely.

Endogenous cycles

The recursion matrix $\mathbf{R}$ has three eigenvalues. They might be real and/or complex. In all parameterisations we have investigated, the eigenvalues are either all real or two of the eigenvalues are complex conjugates. The latter is a pair of complex numbers $\{a + b\,i, a - b\,i\}$, where a is the real part and $b\,i$ is the imaginary part. A pair of complex conjugate eigenvalues makes all interacting endogenous variables in the system cyclical. Their oscillations might be obvious, or be hard to see, depending on the numerical values of the elements of $\mathbf{R}$. If a real eigenvalue $r \geq 1$, then one of the real variables is trending. If $r = 1$ a non-zero constant term provides the random walk with a drift. If a pair of complex conjugate eigenvalues have magnitude $\|r\| = \sqrt{a^2 \pm b^2} < 1$, the oscillations are damped. The oscillations keep or increase in amplitude if $\|r\| \geq 1$. A model without a trend and without oscillations is stable. A model without a trend and with damped oscillations is still asymptotically stable, but less than without oscillations. Model simulations displayed in Figure 1 and 2 show that damped oscillations might dominate the dynamics for a very long time (more than 200 periods/quarters). If the oscillations persist, the model can hardly be called stable.

The (unrestricted) recursion matrix **R** has eight elements. The six reduced form coefficients in the upper two rows are functions of nine structural parameters in the wage-price spiral, as shown earlier in this appendix. In the bottom row are two parameters of the unemployment equation (15). In our numerical investigations, model dynamics and (in)stability appear to be more sensitive to the value of any one of these two unemployment parameters than to any other single structural parameter in the system. That is not surprising considering that α and ρ are reduced-form coefficients that govern the behavior of the unemployment rate. A single structural parameter is always only an element in the reduced-form coefficient in **R**. The parameter values are delimited to the unit interval [0,1]. Hence, the effect of a (non-zero) value of a structural parameter on the real exchange rate, the wage share or the unemployment rate might be diluted by the other parameters in the expressions of the reduced-form coefficients in **R**.

C Parameterisations and simulations

The parameters and coefficients take values on a wide domain. The first two rows in Table 1 delimit the domain. We select a large number of parameterisations from the domain in a fairly space filling way. Each parameterisation is a unique set of values. The ECM and the PCM are simulated with every selected parameterisation. A single parameterisation is the same in both models except for the zero-restrictions that define the PCM.

Four parameterisations are selected and presented in Table 1. They are denoted by the subscripts b for basis, h for dynamic wage and price homogeneity, 3 and 4 for different sets of values. They are selected because they are close to econometric estimates in open economy models, cf. Bårdsen et al. (2005, Ch 5), 'realistic', and illustrate the range of dynamics possible in each model. The constants c_q, m_q, c_w, m_w and c_u are left out of the presentation since they do not affect the dynamics of the model. The constant $m_q = 0.31$ and $m_w = 0.46$ are the same in all simulations. The constants c_q, c_w and c_u are not structural, but rather 'econometric'. In each simulation their values are set so that re, ws and u always start at the same values, i.e. $re_0 = -2.4$, $ws_0 = -0.22$ and $u_0 = 1.38$. The permanent exogenous shift to the unemployment rate is always $\Delta c_u = 0.1$. In the PCM $\varphi = \varpi\ \theta_w$ and $\varsigma = \vartheta\ \theta_q$, where the values of the left parameter in the PCM equals the product of the right parameters in the ECM.

In each of the 8 parameterised models $E_b - P_4$ we perform two simulations. Each simulation generates a single time series for each variable in the model. In the first simulation the solution for each variable at each period t is perturbed by a temporary shock: $\varepsilon_{\text{variable},t} \sim IN(0, \sigma_{\text{variable}})$, where $\sigma_q = \sigma_w = 0.001$, $\sigma_a = 0.0005$, $\sigma_{pf} = 0.01$. In the second simulation, the parameters have the same values, but all temporary shocks are switched off: $\varepsilon_{q,t} = \varepsilon_{w,t} = \varepsilon_{a,t} = \varepsilon_{pf,t} = 0$ for all t. Because the shocks are additive to the linear(ised) model, the deterministic simulation approximates the mean stochastic simulation. In Figure 1 and 2 the ragged graphs are the stochastic simulations and the smooth graphs are the steady-state simulations. The number of simulation periods is 300. In periods $t > 50$ the unemployment rate is subject

	ϑ	θ_q	ς	ψ_{qw}	ψ_{qpi}	ϖ	θ_w	φ	ω	ψ_{wq}	ψ_{wp}	ϕ	α	ρ
min	0	0	0	0	0	0	0	0	0	0	0	0	0	0
max	.65	.5	.1	.6	.7	1	.5	.2	1	.8	.5	1	1	1
E_b	.065	.13	0	.40	.40	.10	.12	0	.5	.5	.2	.6	.85	.10
E_h					.60		.				.5			
E_3	.060	.12		.35	.65	.18	.11		.4	.7	.3	.7	.95	.20
E_4	.060	.10		.30	.70	.22	.10		.3	.7	.3	.7	.98	.20
P_z	-	**0**	.00845			-	**0**	.0120	-					
P_3	-	**0**	.00720			-	**0**	.0198	-					
P_4	-	**0**	.00600			-	**0**	.0220	-					

Table 1: Parameterizations of the models. In the first column: min = minimum value, max = maximum value, E = ECM and P = PCM. The subscripts are: b = *basis* values of the parameters, h = basis values + dynamic *homogeneity*, 3-4 = models with parameters and/or coefficients different from the basis values. To economize on space we let subscript x represent parameterization b, h, 3 and 4, while subscript z represents only b and h. For the PCM only the parameter values different from their values in the corresponding ECM are shown.

The first row lists the parameters and coefficients. When unemployment is targeted, $\rho = 0$ in all models. The second and third rows delimit the domain of each parameter or coefficient. The fourth row (E_b) shows the basis values. The following rows show only values that differ from the basis values.

The boldface zeros are the restrictions defining the PCM. Dashes in the ϑ-column and in the ϖ-column mark that the values are irrelevant because of the zero-restriction in the adjacent column. The non-zero basis value of ς is set equal to the product of the basis values of ϑ and θ_q. The nonzero basis value of φ is set equal to the product of the basis values of ϖ and θ_w.

	r_1	r_2	r_3	$\|\|r\|\|_{\max}$	re	ws	u	Figure
E_b	.942	.901	.833	.942	s	s	s	1
E_h	.927+.035 i	.927-.035 i	.838	.928	d.o	d.o	d.o	
E_3	.981+.085 i	.981-.085 i	.861	.984	d.O	d.O	d.O	1
E_4	.997+.079 i	.997-.079 i	.873	1.000	O	O	O	1
P_b	1	.987	.863	1	s	t	s	2
P_h	1	.986	.864	1	s	t	s	
P_3	1	.975+.056 i	.975-.056 i	1	d.O	t+d.O	d.O	
P_4	1	.990+.057 i	.990-.057 i	1	d.O	t+d.O	d.O	2

Table 2: Dynamic properties of the models. The first column lists the different models. Column 2-4 contain the eigenvalues of the recursion matrix **R** and the magnitude of the largest eigenvalue. The three next columns show the dynamic behaviour of each of the three real variables: s = stability, d.o = damped small oscillations, not always visible, d.O = damped large oscillations, clearly visible, O = persistent oscillations of constant amplitude, i.O = increasing oscillations, and t = trend. The simulated dynamics for certain models are displayed in Figure 1 and 2. The final column shows which figures display which models.

to a permanent positive shock Δc_u. The large exogenous increase in the unemployment level is chosen just to make the figures more clear and easy to read.

The parameterisations in Table 1 span a very small part of the parameter space delimited by the min and max values. But the selection suffice for the numerical simulations of the models with those parameterisations to display a full range of dynamics: stability, damped cycles, persistent cycles and trend, as seen in Figure 1 and 2. The numerical eigenvalues in Table 2 justify the limited selection of parameterisations in Table 1.

Table 2 summarises the dynamic properties of the parameterised models in Table 1. The E_b row shows that the ECM with the basis parameterisation is stable. The E_h row refers to the same model with dynamic homogeneity imposed by increasing ψ_{wp} from 0.2 to 0.5. Note that the increase of ψ_{qpi} from 0.4 to 0.6 has no affect on **R**. Its composite coefficients do not contain ψ_{qpi}. Foreign inflation (Δpi) affects re and ws only additively, cf. equation (19). Increasing ψ_{wp} increases the sensitivity of wage growth to inflation. That makes two eigenvalues a pair of complex conjugates, with magnitude $||r||_{\max} = 0.928 < 1$, which causes all three real variables to display damped cycles around stable levels. E_3 and E_4 are two other parameterisations of the ECM with dynamic homogeneity (of different composition) imposed, and with all parameters and coefficients different from their basis values. It is not possible to see from the parameterisation in Table 1 that E_3 displays damped cycles, while E_4 displays constant cycles. But Table 2 reveals it, and Figure 1 in Section.?? shows it clearly. Comparing model E_4 to E_3, the weaker equilibrium-correction in the wage and price formation, and a more responsive unemployment rate no longer stabilise the short-run dynamics over time. Due to the interdependence, all endogenous variables are cyclical.

Without equilibrium-correction in the wage-price spiral, all PCMs (P_x) are unstable due to a trend in the wage share. That is shown analytically in Appendix B. Both u and re remain stable in the basis parameterisation P_b and the PCM with dynamic homogeneity P_h. Increasing the moderating effect of u on Δw and decreasing the moderating effect of u on Δq cause model P_3 and P_4 to display damped oscillations in all variables. Even ws oscillates around its trend, as seen in Figure 2 in Section.??.

It is not feasible to establish general dynamic properties of the models analytically, as functions of the parameter values. If we plot r_i and $||r_i||$ for $i = 1, 2, 3$, as univariate functions of any single parameter and coefficient conditional on the values of the other parameters and coefficients in **R**, no general pattern emerges. In some models and parameterisations an eigenvalue and/or its magnitude is a strictly increasing or decreasing marginal function of a parameter. In other models and/or parameterisations it displays the opposite monotonicity, or an n- or u-shaped dependence. However, the parameterisations in Table 1 and the eigenvalues in Table 2 shows that unbalanced effects on wage growth and on inflation contribute to cyclical behavior.

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