Rational inflation forecasts and structural breaks: Does the theory fit the facts?*

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Abstract

Macroeconomic forecasts are expectations about future values of macroeconomic variables. Rational expectations (RE) macroeconomic theory implies almost complete immunity of forecasts to structural breaks in the economy.If the New Keynesian Phillips curve is the correct model of inflation, the forecaster knows this, and the break occurs in the expectations process, the forecast error bias only lasts for one period. The bias is automatically corrected through the updating of the initial conditions in the next forecast round, i.e. in the after-break forecast. Moreover, if the forecaster is fully rational and act on the same information as the price setters, even the before-break prediction errors will be unbiased. The analysis shows that the forecast records of central banks may be difficult to reconcile with the rational use of RE macro models in practical forecasting for inflation targetting.

Keywords: Inflation forecasts; structural breaks; monetary policy models; New Keynesian Phillips curve; inflation targeting;

JEL codes: E5, E6

1 Introduction

Macroeconomic forecasts are expectations about future values of macroeconomic variables. The rational expectations hypothesis represents "the notion that economic decisions makers like households and firms do not make systematic mistakes in forecasting".[1] Clearly, inflation targeting central banks are prime examples of economic decisions makers. This makes it interesting to study the properties of inflation forecasts under the assumption that central banks attempt to produce rational expectations forecasts, and in a realistic forecasting situation where the economies are subject to structural breaks.

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[1]The Royal Swedish Academy of Sciences, 2011.

Regime shifts and structural breaks are the main cause of forecast failures in economics, see Clements and Hendry (1999), Pesaran et al. (2006), Giacomini and Rossi (2009) and Hendry (2011) among others. It is near unavoidable that structural breaks that occur after a forecast has been produced lead to large and systematically errors when the forecast is evaluated. In the case of an *after-forecast* break the task is therefore to detect the nature of the regime shift as quickly as possible, in order to avoid repeated unnecessary forecast failure also after the break has become part of the information set of the (next) updated forecast. Failing to pick up a *before-forecast* break may be due to low statistical power of tests of parameter instability. There are also practical issues that complicate and delay the detection of regime shifts. For example, there is usually uncertainty about the quality of the provisional data for the period that initialize the forecasts, making it difficult to assess the significance of a structural change. Hence both *after* and *post* forecast structural breaks are realistic aspects of real life forecasting situations of the type faced by for example inflation forecasting central banks. In the social sciences generally, much can be said in favour of adaptive forecasting processes, cf. Tetlock (2005). In economics, the literature on forecasting and model evaluation provide several guidelines, see e.g., Bates and Granger (1969), Granger (1999), Hendry (2001) and Rossi and Sekhposyan (2011).

The models and methods that are used in modern macroeconomic forecasting have different degrees of immunity with respect to structural breaks. The forecasts from equilibrium correction models with "backward-looking" expectations are little adaptable to structural changes in the economy. The forecasts from those models are in particular susceptible to breaks in the means of equilibrium relationships, and shifts in autonomous growth rates. By contrasts, the simple random walk model gives automatic adaptation to *pre* forecast breaks, making it a robust forecasting method, see e.g. Clements and Hendry (2008,2011). In order to avoid forecast failure after a structural break, the model based forecasts must be corrected by the user (so called intercept corrections) until the change can be "built into" the model structural equations (by changing the estimation sample or by more fundamental re-specification, see e.g., Pesaran et al. (2011) and Eitrheim et al. (2002)). Intercept correcting forecasts from a multi-equation dynamic macroeconometric model is however, non-trivial. It is difficult in practice to form an early and clear opinion about the permanency of a break and how to represent in a model, see Bårdsen and Nymoen (2009). On the other hand, as analysed in Bårdsen et al. (2011), there are relevant cases where model based forecasts are sheltered from structural breaks.

This note supplements the existing literature by pointing out (in line with the Lucas-critique) that rational expectations models should be more robust to structural breaks than backward-looking models are. For the case of inflation forecasts, and given that the hybrid New Keynesian Phillips curve model (NKPC) is the true model of inflation, the bias in forecast errors, due to a break in the expectations process, may be automatically corrected when the forecast. is updated. This happens in the case were i) the NKPC is the correct model of inflation, ii) a break occurs in the expectations process, and iii) the forecaster is aware of both i) and ii).

In this case, the only difference between the forecaster and the agent is that the forecaster discovers the break in expectations one period after the event. The price setters use period t information to form expectations, while the forecaster uses period

$t-1$ information to "model" expectations. We refer to this as the case of an *almost rational forecaster*. In this case there will be a bias in the 1-step forecast errors that is produced in period $t-1$, if the structural break occurs in period t (an after-forecast structural break). However if there is not another structural break happening in period $t+1$, the updated forecast which is based on period t information, will not be biased. Hence almost rational expectations inflation forecasts will adapt quickly to breaks—they promise to be quite robust. One way of making the forecasts fully rational is to assume that also the price setters use period $t-1$ information. In this version of the model there will never be systematic forecast errors for π_t, not even when the break in the expectations formation process occurs after the forecast has been made. The fully rational forecasts therefore promise to be fully immune to structural breaks in the forcing variable of inflation.

The two implications just mentioned are derived in section 4, after we have introduced the inflation model, its solution and the use of the solution for forecasting in section 2 and the forecasts for the no-break case in section 3. Section 5 shows the relevance of the theoretical analysis for understanding the properties of the forecasts of Norges Banks, a best practice inflation targeting central bank. Section 6 concludes and points to the macroeconomic "forecast record" as a source of information about the relevance of a large class of modern macroeconomic models. In particular, we propose that dynamic stochastic general equilibrium (DSGE) models are less suited for forecasting than the consensus view holds; that central banks recognize this; that they face the task of forecasting under the possibility of structural breaks and without being able to assert that the economy "corrects back" to a pre-break rational expectations equilibrium.

2 The NKPC model and a solution for inflation

The hybrid New Keynesian Phillips curve, NKPC, is well established in modern macroeconomic theory and forecast targeting central banks in particular use medium term forecasting models where the supply-side is in terms of this model. We write the NKPC as

$$\pi_t = \underset{\geq 0}{a^f} E_t[\pi_{t+1}] + \underset{\geq 0}{a^b} \pi_{t-1} + \underset{>0}{b} s_t + \varepsilon_{\pi t}, \tag{1}$$

where π_t is the rate of inflation, $E_t\left[\pi_{t+1}\right]$ is the expected rate of inflation in period $t+1$, given the information available for forecasting at the end of period t. The intercept of the equation has been omitted for simplicity. The variable s_t denotes the logarithm of firms' real marginal costs and $\varepsilon_{\pi t}$ is a disturbance term with zero mean. In many applications, notably Gali and Gertler (1999), the disturbance term is omitted, which suggests a stronger interpretation which is often referred to as the NKPC holding in "exact form". The theory consistent signs are given below the parameters. The 'pure' NKPC is specified without the lagged inflation term ($a^b = 0$). Regarding the sum of the inflation coefficients, it is custom to specify $a^f + a^b \leq 1$ as a restriction, which rules out an explosive solution in the purely backward-looking case.

In the following, the third variable in (1), s_t, is the logarithm of the wage-share, which is the common operational definition of firms' marginal cost of production.

The coefficient b is assumed to be strictly positive, and there are no other economic explanatory variables in this model of inflation dynamics for the closed economy case.

In order to derive the solution for π_t we need to make assumptions about the process of the explanatory variable. There are two branches in the literature. First, Gali and Gertler (1999), GG hereafter, who base their solution on the assumption that s_t is a strongly exogenous variable that Granger causes π_t. Second, Sbordone (2002) makes use of the the definition $s_t = ulc_t - p_t$, where ulc_t is the logarithm of nominal unit labour costs and p_t is the logarithm of the price level, to derive a rational expectations solution that is separate from the GG solution. In Sbordone's solution, unit labour costs, not the wage-share takes the role of forcing variable in the solution[2].

In this note, Gali's and Gertler's solution is used, hence we consider the closed form rational expectations solution when s_t follows an autoregressive process of order k:

$$s_t = c_{s1}s_{t-1} + \cdots + c_{sk}s_{t-k} + \varepsilon_{s,t}\ . \tag{2}$$

Equations (1) and (2) define the NKPC model. For simplicity we assume that the two disturbances $\varepsilon_{\pi,t}$ and $\varepsilon_{s,t}$ are independently normally distributed variables. The two equations define a model of one-way Granger causality between s_t and π_t, hence the name *forcing variable* for s_t is well chosen.[3]

We obtain the solution for π_t as

$$\pi_t = r_1\pi_{t-1} + \frac{b}{a^f r_2}\sum_{i=0}^{\infty}(\frac{1}{r_2})^i E_t s_{t+i} + \frac{1}{a^f r_2}\varepsilon_{\pi,t} \tag{3}$$

where r_1 and r_2 are the two roots of $r^2 - (1/a^f)r + (a^b/a^f) = 0$.[4] $E_t s_{t+i}$ denotes the rational expectation for s_{t+i}, conditional on (2) and information available in period t. We follow custom and find a solution by imposing the transversality condition

$$(\frac{1}{r_2})^i E_t s_{t+i} \to 0 \text{ as } i \to \infty. \tag{4}$$

The closed form solution is discussed in detail in Nymoen et al. (2012). It is:

$$\pi_t = r_1\pi_{t-1} + \frac{b}{a^f r_2}K_{s1}s_t + \cdots + \frac{b}{a^f r_2}K_{sk}s_{t-k+1} + \frac{1}{a^f r_2}\varepsilon_{\pi,t}, \tag{5}$$

where $r_1 \leq 1$ and it is assumed that

$$\left|\frac{r_{sj}}{r_2}\right| < 1 \text{ for } j = 1, 2, \ldots, k \tag{6}$$

[2]Sbordone takes the pure NKPC with $a^b = 0$, as her starting point, but the closed form solution for inflation nevertheless contains π_{t-1} on the right hand side.

[3]Sbordone (2002) provides a solution for a different interpretation of the model, and for the non-hybrid version, where nominal unit-labour costs drive inflation.

[4]See Bårdsen et al. (2005, Appendix A) who build on Pesaran (1987, p. 108-109).

where r_{sj} are the roots $r_s^k - c_{s1}r_s^{k-1} - c_{sk-1}r_s - c_{sk} = 0$, the characteristic equation associated with the process of the forcing variable s_t. When (6) holds, the transversality condition is satisfied, and the coefficients K_{sj} in the solution (5) exist and are given by:

$$K_{s1} = 1/(1 - c_{s1}(\frac{1}{r_2}) - \cdots - c_{sk}(\frac{1}{r_2})^k) \tag{7}$$

$$K_{si} = (c_{si}(\frac{1}{r_2}) + \cdots + c_{sk}(\frac{1}{r_2})^{k-i+1})/(1 - c_{s1}(\frac{1}{r_2}) - \cdots - c_{sk}(\frac{1}{r_2})^k), i = 2, \ldots, k \tag{8}$$

For concreteness we consider the case of $k = 2$, which covers the applications found in the literature. The closed form solution is then

$$\pi_t = r_1\pi_{t-1} + \frac{b}{a^f r_2}K_{s1}s_t + \frac{b}{a^f r_2}K_{s2}s_{t-1} + \frac{1}{a^f r_2}\varepsilon_{\pi,t} \tag{9}$$

The constants K_{s1} and K_{s2} can be expressed as

$$\begin{aligned} K_{s1} &= -\frac{r_2}{r_{s2} - r_{s1}}\left\{\frac{\frac{r_{s1}}{r_2}}{1 - \frac{r_{s1}}{r_2}} - \frac{\frac{r_{s2}}{r_2}}{1 - \frac{r_{s2}}{r_2}}\right\} \\ &= \frac{1}{1 - \frac{1}{r_2}(c_{s1} + \frac{1}{r_2}c_{s2})}, \end{aligned} \tag{10}$$

$$\begin{aligned} K_{s2} &= \frac{r_2}{r_{s2} - r_{s1}}\left\{\frac{r_{s2}\frac{r_{s1}}{r_2}}{1 - \frac{r_{s1}}{r_2}} - \frac{r_{s1}\frac{r_{s2}}{r_2}}{1 - \frac{r_{s2}}{r_2}}\right\} \\ &= \frac{c_{s2}}{r_2}K_{s1}. \end{aligned} \tag{11}$$

r_{sj} $(j = 1, 2)$ are the roots of the characteristic equation associated with (2).

So far we have abstracted from an intercept in the forcing process, which is a limitation when we want to discuss forecasting since it is easy to imagine that regime shifts, before and after the forecast has been made, can affect the mean of the process (the case of $|r_{sj}|^{\max} < 1$ and $|r_2| \leq 1$) or the drift of the process (the case of $|r_{sj}|^{\max} = 1$ and $|r_2| < 1$). To accommodate these possibilities, we also give the solution for the model given by the NKPC (1) and

$$s_t = c_{s0} + c_{s1}s_{t-1} + c_{s2}s_{t-2} + \varepsilon_{s,t} \tag{12}$$

Since s_t is the wage-share in the GG version of the model, we impose stationarity ($|r_{sj}|^{\max} < 1$) and denote the mean of s_t by μ_s. It is given by

$$\mu_s = \frac{c_{s0}}{1 - c_{s1} - c_{s2}}$$

meaning that (12) can be written in terms of deviation from mean

$$s_t - \mu_s = c_{s1}(s_{t-1} - \mu_s) + c_{s1}(s_{t-2} - \mu_s) + \varepsilon_{s,t}$$

The solution (9) is replaced by

$$\pi_t = r_1\pi_{t-1} + \frac{b}{a^f r_2}K_{s1}(s_t - \mu_s) + \frac{b}{a^f r_2}K_{s2}(s_{t-1} - \mu_s) + \frac{b}{a^f}\left(\frac{1}{r_2 - 1}\right)\mu_s + \frac{1}{a^f r_2}\varepsilon_{\pi t}. \tag{13}$$

as the appendix shows.

3 Forecasting inflation in the stationary, no-break, case

The above solution was derived for the constant parameter, no-break, case, i.e. the stationary case. We assume that the true DGP is given by (12) and (13) and that the forecaster knows this. We first consider one step ahead forecasts, and let T_1 represent the period when the forecast is made. The solution for π_{T_1+1} will be

$$\pi_{T_1+1} = r_1\pi_{T_1} + \frac{b}{a^f r_2}K_{s1}(s_{T_1+1} - \mu_s) + \frac{b}{a^f r_2}K_{s2}(s_{T_1} - \mu_s) + \frac{b}{a^f}\left(\frac{1}{r_2 - 1}\right)\mu_s + \frac{1}{a^f r_2}\varepsilon_{\pi,\, T_1+1}$$

while the forecast $\widehat{\pi}_{T_1+1|T_1}$ is

$$\widehat{\pi}_{T_1+1|T_1} = r_1\pi_{T_1} + \frac{b}{a^f r_2}K_{s1}(\widehat{s}_{T_1+1|T_1} - \mu_s) + \frac{b}{a^f r_2}K_{s2}(s_{T_1} - \mu_s) + \frac{b}{a^f}\left(\frac{1}{r_2 - 1}\right)\mu_s \tag{14}$$

$\widehat{s}_{T_1+1|T_1}$ is obtained from (12) as

$$\widehat{s}_{T_1+1|T_1} = c_0 + c_1 s_{T_1} + c_2 s_{T_1-1} \tag{15}$$

The No-Beak forecast error is

$$f^{NB}_{T_1+1|T_1} = \pi_{T_1+1} - \hat{\pi}_{T_1+1} = \frac{1}{a^f r_2}\varepsilon_{\pi,\, T_1+1} + \frac{b}{a^f r_2}K_{s1}\varepsilon_{s,T_1+1} \tag{16}$$

The multi-step forecasts are derived form the usual forward progression based on (13), giving the forecast errors

$$f^{NB}_{T_1+J|T_1} = \frac{1}{a^f r_2}\sum_{i=0}^{J-1}(r_1)^i\varepsilon_{\pi T_1+J-i} + \frac{b}{a^f r_2}K_{s1}\sum_{i=0}^{J-1}(r_1)^i\varepsilon_{sT_1+J-i}, \quad J = 1, 2, ... \tag{17}$$

which has (16) as a special case, and which are all unbiased in terms of E_{T_1}.

4 Forecasting inflation with breaks in the s_t process

The main distinction is between forecasts that are made before a break occurs, i.e., an after-forecast break, and the case of before-forecast breaks, i.e. after-break forecasts that conditions of the period where the break occurs.

4.1 After-forecast break

Assume that a break in the s_t process occurs in period $T_1 + 1$, so that instead of c_0, the intercept becomes $c_0 + c_0^*$, giving:

$$\mu_s^* = \frac{c_0 + c_0^*}{1 - c_1 - c_2} \tag{18}$$

Applying the solution in (13) to inflation in period $T_1 + 1$ gives

(19)

$$\pi_{T_1+1} = r_1\pi_{T_1} + \frac{b}{a^f r_2}K_{s1}(s_{T_1+1} - \mu_s^*) + \frac{b}{a^f r_2}K_{s2}(s_{T_1} - \mu_s^*) + \frac{b}{a^f}\left(\frac{1}{r_2 - 1}\right)\mu_s^* + \frac{1}{\gamma_f r_2}\varepsilon_{\pi,T_1+1},$$

The forecast for π_{T_1+1} that is conditioned on T_1 is given by (14) also in this case, but the forecast error becomes

$$\begin{aligned} f_{T_1+1|T_1}^{AB} &= \frac{b}{a^f r_2}K_{s1}(s_{T_1+1} - \widehat{s}_{T_1+1|T_1}) \\ &+ \frac{b}{a^f r_2}(K_{s1} + K_{s2} - r_2(r_2 - 1)^{-1})(\mu_s^* - \mu_s) \\ &+ \frac{1}{a^f r_2}\varepsilon_{\pi T_1+1} \end{aligned} \tag{20}$$

Since

$$(s_{T_1+1} - \widehat{s}_{T_1+1|T_1}) = (1 - c_{s1} - c_{s2})(\mu_s^* - \mu_s) + \varepsilon_{s,T_1+1} \tag{21}$$

there is a bias in the 1-step inflation forecast errors in this case of an after-forecast structural break in the forcing variable s_t. It follows that also the dynamic forecasts errors $\pi_{T_1+J|T_1} - \pi^e_{T_1+J|T_1}$ will be biased, since the forecasted mean of the forcing variable is μ_s rather than μ_s^*. The forecasts errors $s_{T_1+J|T_1} - s^e_{T_1+J|T_1}$ are of course also biased.

4.2 Before-forecast break

We now consider period $T_1 + 2$, conditional on period $T_1 + 1$ information. The solution for π_{T_1+2} is exactly as (19), with the relevant changes of subscripts. If the forecasts are to be immune to the Lucas critique, it must be assumed that the forecaster knows that agents' expectations have changed and they use

$$\widehat{s}_{T_1+2|T_1+1} \equiv E_{T_1+1}(s_{T_1+2}) = c_0^* + c_1 s_{T_1+1} + c_2 s_{T_1} \tag{22}$$

to produce the forecast $\widehat{\pi}_{T_1+1|T_1}$. Hence if the forecasters are rational and make efficient use of the information available on time $T_1 + 1$, in particular they know about the regime shift of that period, we are back to the case where the forecasts uses the correct mean of the forcing variable, meaning that we have:

$$f^{BF}_{T_1+1+J|T_1+1} = f^{NB}_{T_1+1+J|T_1+1}, \quad J = 1, 2, ... \tag{23}$$

which, in terms of E_{T_1+1} is unbiased for all J.

If the forecasts, as opposed to the solution, is not fully rational, i.e.,λc_0^* with $(\lambda \neq 1)$ is used in the forecasts , instead of c_0^*, there will however be a non-zero expected forecast error.

4.3 Comparison with non-RE model forecasts

Interestingly, under the assumptions we stated above, the almost rational inflation forecasts have the same robustness property as the differencing and double-differencing procedures that take out the effect of breaks in means and growth rates, and without importing the unwanted increase in forecast uncertainty in periods of no breaks, Clements and Hendry (2008).

We see that there is a clear distinction between *before* forecast breaks and *after* forecasts breaks. The theory implies that the effects of a structural break on NKPC based forecast will only affect the forecast that is made in a period preceding a break in the process governing the wage-share. As soon as the forecasts can be made conditional on the period of the shock, they become immune to the break, and there is no bias.

The property of the NKPC forecasts is different from the forecasts from a conditional econometric model of inflation, or from a VAR model of inflation. In these models the bias lingers on also in forecasts that are produced conditional on the period of the break. In order to avoid, or at least reduce, damage to forecasts after a structural break, the model forecasts need to be corrected by the user until the change can be finally "build into" the model structure, see Clements and Hendry (1996), Bårdsen and Nymoen (2009).

Hence, there is an interesting difference between the two broad type of models. Based on the assumption that the NKPC model corresponds to the true data generating process, this model has the implication that forecast failures are quickly corrected, as expectations become updated. A non-RE model of inflation will in general predict differently, namely that several rounds of inflation forecasting will be affected after a break, unless the forecast is successfully intercept corrected, but see Bårdsen et al. (2011) for relevant exceptions from this rule in a small open economy macroeconometric model.

4.4 The importance of the information set

Above we have maintained the assumption used in the NKPC literature, namely that agents and the forecasters can condition on period t information about the forcing variable, i.e., $E_t s_t = s_t$. If we instead, assume that conditioning is with respect to period $t-1$, it is reasonable to write the NKPC as:

$$\pi_t = a^f E_{t-1}[\pi_{t+1}] + a^b \pi_{t-1} + b E_{t-1}[s_t] + \varepsilon_{pt}, \tag{24}$$

and the forcing process (12). Despite the conditioning on $t-1$, much of the derivation goes through as before, in particular we obtain the closed from solution correspond-

ing to (13) as
(25)
$$\pi_t = r_1\pi_{t-1} + \frac{b}{a^f r_2}K_{s1}(s_{t-1}-\mu_s) + \frac{b}{a^f r_2}K_{s2}(s_{t-2}-\mu_s) + \frac{b}{a^f}\left(\frac{1}{r_2-1}\right)\mu_s + \frac{1}{a^f r_2}\varepsilon_{\pi t}.$$

where the only differences are that $(s_{t-1}-\mu_s)$ appears in place of $(s_t-\mu_s)$, and that $(s_{t-2}-\mu_s)$ replaces $(s_{t-1}-\mu_s)$. This is because the solution for π_t is fundamentally driven by the expectations process for s_t, so if the expectations about s_t are conditioned on $t-1$, then the solution reflects that exactly.

Assume again that there is a break in the s_t process in period T_1+1, so that instead of c_0, the intercept becomes $c_0+c_0^*$ in period T_1+1. We now have $E_{T_1}s_{T_1+1} = c_1 s_{T_1} + c_2 s_{T_1-1} + c_0$, so the one step-ahead forecast error for s_{T_1+1} is c_0^* $+\eta_{T_1+1}$. This structural break in the forcing process is not reflected in the inflation forecast $\widehat{\pi}_{T_1+1|T_1}$, but neither is it reflected in the solution for inflation in period T_1+1, $\pi_{T_1+1,}$ (conditional on T_1)! Hence in this case the one-step forecast error is the same as in the No-Break case:

$$\pi_{T_1+1|T_1} - \widehat{\pi}_{T_1+1|T_1} = f^{NB}_{T_1+1|T_1}$$

which is unbiased despite the structural break in the s_t process taking place in period T_1+1. It appears that because the DGP is fundamentally expectations driven in the rational equilibrium case, there will "never" be systematic forecast errors for π_t (that are due to breaks in the forcing process) if the conditional expectation in the NKPC is in terms of lagged information rather than in terms of current information.

For the forecast vector $(\widehat{\pi}_{T_1+1|T_1}, \hat{s}_{T_1+1|T_1})$, there will be a bias though, but this is solely due to errors in forecasting of $s_{T_1+1|T_1}$.

5 Actual inflation forecast performance

The above analysis suggests that rational inflation forecasts should be relatively good, even in the case of structural break in the forecast period. But what is the experience from actual inflation forecasting. In this section I briefly review the actual forecast record of the Norges Bank (The Central Bank of Norway), a recognized "best practice" inflation targeting central bank, see Woodford (2007). On 29 March 2001 Norway formally introduced an inflation targeting monetary policy regime. Like several other central banks, Norges Bank started the inflation targetting epoch with a (partly public) "position debate" about macroeconomic models for the purpose of inflation targetting. Following the principles for modelling laid down in Svensson et al. (2002) Norges Bank quickly established a rational expectations forward-looking model which was subsequently replaced by a full blown DSGE model dubbed Norwegian Economic Model, *NEMO* Brubakk et al. (2006).

As documented in Juel et al. (2008, Ch. 4), *NEMO* and its predecessor represent the main conceptual reference framework for Norges Banks forecasting and policy evaluation.[5] More recently, the role of *NEMO* for the short-term forecasts

[5]Tthis is representative of forecast targeting central banks. Hammond (2010) http://www.bankofengland.co.uk/education/ccbs/handbooks/ccbshb29.htmTable C shows that 20 out of 27 inflation targeting central banks either use or are developing a DSGE type model for

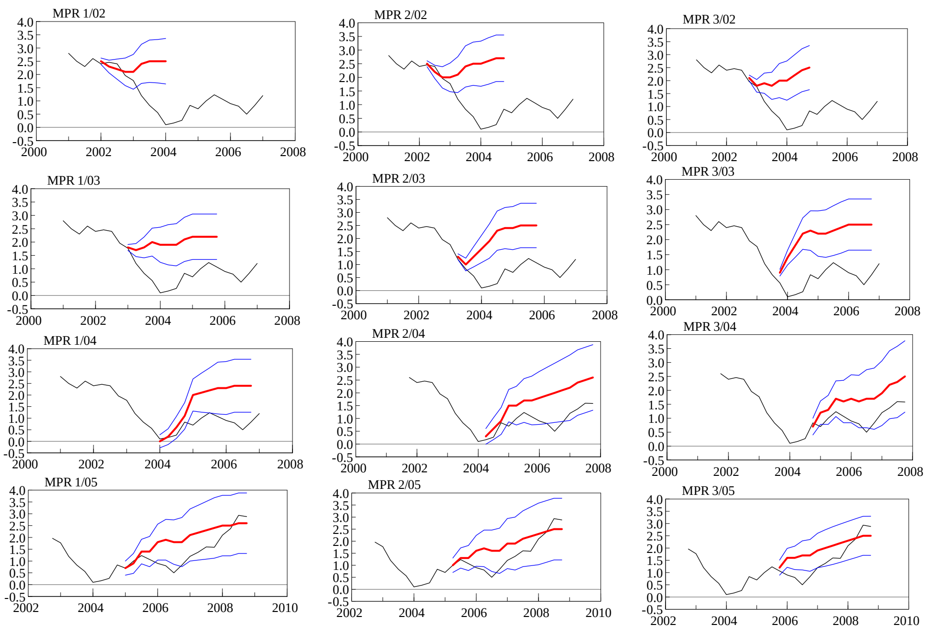

Figure 1: Actual CPI-ATE inflation rate and Norges Bank's inflation forecasts (thicker line) and the 90% confidence regions. MPR 1/02 to MPR 3/05.

(defined as 1-4 quarters ahead) has been reduced, as Norges Bank now relies on ensemble forecasts for these horizons, see Gerdrup et al. (2009). The ensemble used for forecasting core inflation contains 167 models, *NEMO* being one of them. However, for horizons 1 to 4 years ahead, NEMO the forecasts carry full weight, see Olsen (2011).

As discussed by Akram and Nymoen (2009), the horizon for monetary policy is relevant for optimal interest rate setting in an inflation targeting regime. Since interest rate setting is linked to the inflation forecasts, and vice versa, any changes in the policy horizon is also likely to affect the inflation forecasts. Specifically, since all end-of-period forecasts are 2.5 %, a short policy horizon will imply forecasts that converge more quickly to the target than will be the case for a longer policy horizon. Initially Norges Bank operated inflation targeting with a 2-year forecast horizon. In the summer of 2004, the policy horizon was changed to 1-3 years. Judging from the graphs, this instigated a change in the following MPR inflation forecasts, which seem to have been geared towards 2.5% over a longer period than before.

Norges Bank's forecasts are published in fan-charts where the wideness of the bands represents 30%, 60% and 90% probabilities for future inflation rates. The forecasted uncertainty is particularly relevant when assessing forecast performance. Actual inflation rates outside the 90% bands represent *forecast failures*, since they are inflation outcomes that were judged to be highly unlikely in Norges Bank's analysis of the Norwegian inflation mechanism. As noted above, forecast failures are not uncommon in economics and can sometimes be used constructively to increase knowledge, see e.g., Eitrheim et al. (2002). That said, in a practical forecasting situation there is a premium on avoiding forecast failures.

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forecasting and policy analysis.

Figure 1 and 2 shows the inflation forecasts from the 21 Monetary Policy Reports published in the period 2002-2008.[6] In each panel there are graphs for the dynamic inflation forecasts together with the 90% forecast confidence bounds, and also the actual inflation rate.

In Figure 1 there are several examples of forecast failure. For example in MPR 2/02, the first four inflation outcomes are covered by the forecast confidence interval, but the continued fall in inflation in 2003 (the second year of the forecast horizon) led to a forecast failure. The forecast failure became even more evident in the two other forecasting rounds from 2002, and also the three forecasts produced in 2003 predicted significantly higher inflation than the actual outcome. Specifically, the forecast confidence interval of MPR 3/03 did not even cover the actual inflation in the first forecast period.

The seventh panel shows that the forecasted zero rate of inflation for 2004(1) in MPR 1/04 turned out to be very accurate. The change from the MPR 3/04 forecast is evident, and can be seen as an adaptation to a lower inflation level. That process continued in MPR 2/04, where the effect of the lengthening of the forecasting and policy horizon mentioned above is clearly visible. Although also the MPR 2/04 forecasts are too high, only one of them represents (formally) a forecast failure. The last four panels in Figure 1 show many of the same features. The 1-step, and sometimes also the 2-step forecasts are accurate, but otherwise the forecasted inflation rate is too high. The MPR 1/05-3/05 forecasts for the end-of-horizon are accurate though, as actual inflation was a little higher than 2.5 %.

Norges Bank's inflation forecasts from 2006, 2007 and 2008 are shown in Figure 2. Compared to the first group of forecasts, these graphs show a more balanced picture with positive and negative forecast errors. The last panel suggest that even short-term forecasting become more difficult to get right when the credit crisis hit the Norwegian economy late in 2008 though.

The point here is however not to assess the relative accuracy of these forecast, but to illustrate that best practice Central Bank has experienced difficulties in meeting the expectations of forecasting without intermittent large and persistent failures.[7] Economists may have paid too little attention to the evidence represented by the forecast record produced by these resourceful forecasting institutions.

6 Discussion

Central banks have used New Keynesian DSGE models with rational expectations to aid monetary policy decisions, and as part of that process, to produce conditional inflation forecast which is the operational target for monetary policy.[8] Based on

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[6]The last Monetary Policy Report in Figure 1 is MPR 3/08. The reports from 2009 are omitted because the forecasts there cover only a short horizon, and the prediction intervals are also incomplete or missing. This reflects Norges Bank's change to a new operational definiton of inflation, as explained in the main text. The available CPI-ATE forecasts from MPR 1/09, MPR 2/09 and MPR 3/09 are used in the comparison of mean forecast errors and mean square forecast errors though.

[7]See ee Falch and Nymoen (2011) for an evaluation of the accuracy of these forecasts..

[8]We keep it simple and think in terms of one single model. In reality, inflation targeting is aided by 'layers' of models where the practical forecasting may be done in terms an augmented

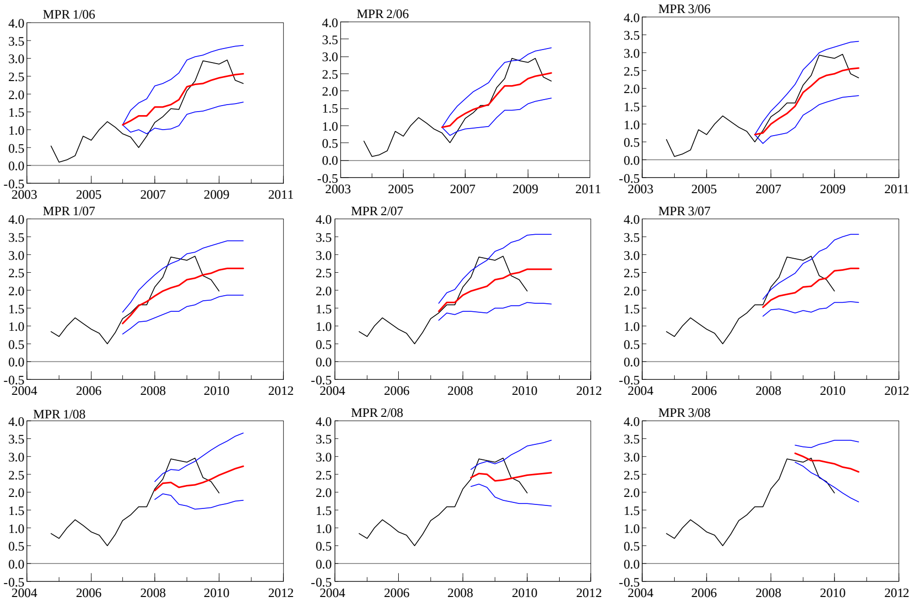

Figure 2: Actual CPI-ATE inflation rate and Norges Bank's inflation forecasts (thicker line) and the 90% confidence regions. MPR 1/06 to MPR 3/08.

this observation, as well as on the analysis above, we made a simple deduction: If central banks have mainly used DSGE models in the forecasting process, large and persistent forecasts errors for inflation should not be found in the forecast records they have produced.

However, the evidence shows that forecasts failures have continued to be common in the period of inflation targetting, as illustrated by the forecast record of Norges Bank. There are two interpretations of this observation. First, it is possible that inflation targeting central banks are not, after all, rational forecasters in the meaning given above, namely that they know that the DSGE model is the correct model and they are aware when a break has occurred in the expectations process. Quite possibly, it is difficult even for a professional forecasting institution to attain this kind of rationality. A central bank that uses the inflation forecast as its policy target must nevertheless be regarded as better equipped than most other agents to turn knowledge of a true model into an advantage when forecasting.

This leads to a second possibility, namely that DSGE models are less suited for forecasting than the consensus view holds; that central banks recognize this; that they face the task of forecasting under the possibility of structural breaks and without being able to assert that the economy "corrects back" to a pre-break rational expectations equilibrium. This interpretation fits better with the picture we now have about the relative quality of model based inflation forecasts, Faust and Wright (2012) .Unlike the first interpretation, it also gives encouragement to the strive for

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and data-adjusted version of the theoretically pure core model, see Pagan (2003, 2005) (Bank of England), or by averaging over a large number of theoretically and statisticaly very different models, see Gerdrup and Nicolaysen (2011) (Norges Bank).

more adaptive forecasting models for monetary policy. There is a third possibility as well, namely that the analysis of the two-equation model above does not generalize to the full scale DSGE models that are used in practice. This will be addressed in future work, for the time being we note that the expectation about no systematic mistakes in forecasting is not conditioned by the size or degree of complexity of the rational expectations model.

A Non-zero mean in s_t.

In the case of (12) the expectations in $\sum_{i=0}^{\infty}(\frac{1}{r_2})^i E_t s_{t+i}$ becomes:

$$\begin{aligned}
E_t s_t &= s_t \\
E_t s_{t+i} &= k_{s1} r_{s1}^{i+1} + k_{s2} r_{s2}^{i+1} + \mu_s,\ i = 1, 2, ..
\end{aligned}$$

from the solution of the difference equation. Where

$$\mu_s = \frac{c s_0}{1 - c_{s1} - c_{s2}}$$

The long-run mean μ_s is also a particular solution when we assume $1 - c_{s1} - c_{s2} \neq 0$. The constants $k_{s1}$and k_{s2} are determined form the known initial conditions s_t and s_{t-1}:

$$\begin{aligned}
s_{t-1} &= k_{s1} + k_{s2} + \mu_s \\
s_t &= k_{s1} r_{s1} + k_{s2} r_{s2} + \mu_s
\end{aligned}$$

$$\begin{aligned}
k_{s1} &= \frac{r_{s2}}{r_{z2} - r_{z1}}(s_{t-1} - \mu_s) - \frac{1}{r_{z2} - r_{z1}}(s_t - \mu_s) \\
k_{s2} &= \frac{-r_{s1}}{r_{s2} - r_{s1}}(s_{t-1} - \mu_s) + \frac{1}{r_{s2} - r_{s1}}(s_t - \mu_s)
\end{aligned}$$

The progression $\sum_{i=o}^{\infty}(\frac{1}{r_2})^i E_t s_{t+i}$ now becomes:

$$\begin{aligned}
\sum_{i=0}^{\infty}(\frac{1}{r_2})^i E_t s_{t+i} = s_t &+ \sum_{i=1}^{\infty}(\frac{1}{r_2})^i k_{s1} r_{s1}^{i+1} + \sum_{i=1}^{\infty}(\frac{1}{r_2})^i k_{s2} r_{s2}^{i+1} \\
&+ \mu_s \frac{r_2}{r_2 - 1}
\end{aligned}$$

Where the only difference from the derivation without a constant in the forcing variable is that k_{s1} and k_{s2} are functions of the de-meaned s_t and s_{t-1}, and that there there an additional term $\mu_s \frac{r_2}{r_2 - 1}$. The same derivations that underlie (9) go through, leading to (13):

$$\pi_t = r_1\pi_{t-1} + \frac{b}{a^f r_2}K_{s1}(s_t - \mu_s) + \frac{b}{a^f r_2}K_{s2}(s_{t-1} - \mu_s) + \frac{b}{a^f}\left(\frac{1}{r_2 - 1}\right)\mu_s + \frac{1}{a^f r_2}\varepsilon_{\pi t}.$$

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