UiO : University of Oslo

ECON 4160: Econometrics-Modelling and Systems Estimation

Lecture 10: Exogeneity

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The reference to this lecture is:

- Chapter 8 in the textbook: *Dynamic Econometrics for Empirical Macroeconomic Modelling*.

Exogeneity paradox

- We have seen in this course that the variable X_t in the ADL model equation

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \beta_0 X_t + \beta_1 X_{t-1} + \epsilon_t, t = 1, 2, ..., T, \quad (1)$$

can be either exogenous *or* endogenous in the econometric sense of the term:
  - Exogenous (or at least pre-determined): uncorrelated with ϵ_t (but maybe not with ϵ_{t-1})
  - Endogenous: Correlated with ϵ_t.
- Hence we have the paradox that the same variable can be both endogenous and exogenous in one and the same model equation.
- A solution of the paradox: Define exogeneity relative to which *parameters of interest* we are focusing on in our econometric investigation.

Weak exogeneity I

- With reference to the ADL in (1): X_t is *Weakly Exogenous*, WE , if consistent and efficient estimation of the *parameter of interest* β_0 does not make use of information contained in the marginal process that generates X_t.
  - If Y_t and X_t are generated by a Gaussian VAR, and (1) is the conditional model of Y_t that we have derived from the VAR, then X_t is WE for β_0,
  - OLS of β_0 is FIML in this case.
  - No information is lost by not taking the marginal model of X_t into account in the estimation.
- For X_t to be WE there cannot be any direct or indirect (cross-equation restrictions) links between the parameters of interest ($\boldsymbol{\theta}_1$ in the notation of Chapter 8), and the parameters of the marginal model ($\boldsymbol{\theta}_2$).

Weak exogeneity II

- If consistent and efficient estimation of β_0 in (1), is not possible without taking the marginal model into account, X_t is *not* WE for β_0.
- Hence if β_0 is a parameter in a SEM model equation, X_t is *not* WE.

Other parameters of interest and WE of X

- Assume that the parameters of interests are the characteristic roots that determine whether Y_t is stationary or not.
  - If X_t is WE for those parameters, they can be estimated from (1) without taking the rest of the system into account.
  - Can they?
- Assume that the parameters of interest are the dynamic multipliers of Y_t with respect to a change in X_t. Is X_t WE ?
- Assume that the parameters of interest are the impulse responses of Y_t with respect to a change in ϵ_t. Is X_t WE?

Weak exogeneity and the possibility of estimation

- Weak exogenity is a fundamental property, almost a premise for estimating empirical models
- A WE variable needs not be a regressor variable
- It can be an instrumental variable.
- One way of of thinking about an exactly identifying variable $(Z_t)$is that it is weakly exogenous for the parameters of interest in the model equation.

Granger causality

- Granger's concept of causality builds on the idea that cause comes before effect
- Easiest to define with reference to a 2-variable VAR(1):
  - If $\phi_{12} \neq 0$ and $\phi_{21} = 0$, there is one-way causation from X_t to Y_t
  - X is *Granger causing* Y.

Strong exogeneity

- If X_t is weakly exogenous in the ADL in (1) and X_t is not Granger caused by Y_t, then X_t is *strongly exogenous* , SE.
- Generally, WE plus Granger non-causality generates SE
- While WE is about exogeneity with respect to estimation, SE is needed to do valid forecasting of $Y_T + h$ based on a given future path for X_{T+h}.
- Another terminology for Granger non-causality is “no feedback”, *ie* from lagged Ys on X
- In general, economic systems are characterized by joint feed-back, so Granger non-causality is a strong assumption.

Invariance

- The concept of parameter *invariance* addresses how a parameter of interest, *eg* β_0 in the ADL (1), "reacts" to a structural break elsewhere in the system.
- In the general notation of Chapter 8, we say that $\boldsymbol{\theta}_1$ is invariant with respect to a break in a parameter of the marginal part of the system $\boldsymbol{\theta}_2$, if $\boldsymbol{\theta}_1$ stays unchanged when $\boldsymbol{\theta}_2$ breaks.
- In the simplest case, we have

$$\beta_0 = \frac{\sigma_{XY}}{\sigma_X^2},$$

  so if there is a break in σ_X^2, β_0 will also break unless there is a proportional change in σ_{XY}
- There is nothing that guarantees that kind of invariance. But nothing hindering it either. So we should test (see below)

Super exogeneity, SuE

- WE plus invariance generates Super Exogeneity (SuE)
- As just noted, invariance is a possible property of conditional models.
- If SuE is a model trait, it validates the use of the conditional models to analyze the effects of policy changes.
- Then "refutes" the Lucas critique

Summary

1. Can we estimate our parameters of interest efficiently without specifying the process that generated X_t?
   If the answer is "yes", X_t is weakly exogenous.
2. Can we forecast Y efficiently by conditioning on a forecast for X that does not involve the forecasted Y values?
   If the answer is "yes", and the answer to 1. is also "yes", X_t is strongly exogenous.
3. Can we do valid policy analysis based on the conditional model?
   If the answer is "yes", and the answer to 1. is also "yes", X_t is super exogenous.

Remarks

- As noted above, weak exogeneity applies to instrumental variables as well as to regressors.
- Invariance is also general: Model equations in a SEM can have coefficients that are invariant (or not) to structural breaks elsewhere in the multivariate system of equations (or in the generating processes of the instrumental variables).
- However, in the following we continue to focus on conditional models.
- It should also be made clear that invariance is a relative property: Useful empirical econometric models can be invariant to certain regime changes and structural breaks, but not all (thinkable) breaks.
- Econometrics models are products of civilization and, as such, will break down sooner or later.

Testing Weak Exogeneity

To obtain a test, we can focus on the difference between two estimators of the coefficient vector $\boldsymbol{\beta}$ in:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \tag{2}$$

where one is the OLS estimator $\hat{\boldsymbol{\beta}}_{OLS}$, and another is consistent both with exogeneity and without it, *ie* the IV estimator, $\hat{\boldsymbol{\beta}}_{IV}$. The test situation can be written as:

$$H_0 : plim(\hat{\boldsymbol{\beta}}_{IV} - \hat{\boldsymbol{\beta}}_{OLS}) = 0 \text{ against } H_1 : plim(\hat{\boldsymbol{\beta}}_{IV} - \hat{\boldsymbol{\beta}}_{OLS}) \neq 0.$$

But where should any significant difference between $\hat{\boldsymbol{\beta}}_{OLS}$ and $\hat{\boldsymbol{\beta}}_{IV}$, come from?

- The answer must be: From the rest of the system, from the marginal models of the variables in **X** in (2).
- This means that we can perform the test without actually doing IV estimation, which of course is a convenient simplification.
- We can therefore test H_0 by estimating the marginal models for **X** by OLS, calculate residuals from the set of marginal models and then test if those residuals are significant when added to the original model (estimated by OLS) as regressors.
- This test often called the Durbin-Wu-Hausman (DWH) test.

Testing Granger non-causality and SE

- This is done by testing the relevant zero-restrictions on the coefficient matrices of the VAR.
- Think of testing Strong Exogeneity of *DLPAW* in the obligatory!

Testing invariance and SuE

- Again, the testing procedure is quite intuitive:
- If there is evidence of breaks in the marginal models of the regressors in a conditional model:
- Represent these breaks by indicator variables (break-dummies)
- Test whether the break-dummies are significant when added to the conditional model.
- If significant, the H_0 of invariance is rejected.

- How do we “find” breaks to test for?
  - Know your presence and your past:
  - If a law, or a market (de)regulation etc, happened in the sample period, in a way that affected X_t it is almost always worth testing the invariance of the model with respect to such known breaks.
  - Can also identify breaks in an objective way using a method called Impulse Indicator Saturation, IIS, see Lecture 15
- Recursive estimation and plots are also very revealing about lack of invariance.