ECON 4160: Econometrics-Modelling and Systems Estimation
Lecture 10: Exogeneity

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The reference to this lecture is:

► Chapter 8 in the textbook: *Dynamic Econometrics for Empirical Macroeconomic Modelling*. 
Exogeneity paradox

- We have seen in this course that the variable $X_t$ in the ADL model equation

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \beta_0 X_t + \beta_1 X_{t-1} + \epsilon_t, t = 1, 2, ..., T, \quad (1)$$

can be either exogenous or endogenous in the econometric sense of the term:

- Exogenous (or at least pre-determined): uncorrelated with $\epsilon_t$ (but maybe not with $\epsilon_{t-1}$)
- Endogenous: Correlated with $\epsilon_t$.

- Hence we have the paradox that the same variable can be both endogenous and exogenous in one and the same model equation.

- A solution of the paradox: Define exogeneity relative to which parameters of interest we are focusing on in our econometric investigation.
Weak exogeneity I

- With reference to the ADL in (1): $X_t$ is Weakly Exogenous, WE, if consistent and efficient estimation of the parameter of interest $\beta_0$ does not make use of information contained in the marginal process that generates $X_t$.
  - If $Y_t$ and $X_t$ are generated by a Gaussian VAR, and (1) is the conditional model of $Y_t$ that we have derived from the VAR, then $X_t$ is WE for $\beta_0$,
  - OLS of $\beta_0$ is FIML in this case.
  - No information is lost by not taking the marginal model of $X_t$ into account in the estimation.

- For $X_t$ to be WE there cannot be any direct or indirect (cross-equation restrictions) links between the parameters of interest ($\theta_1$ in the notation of Chapter 8), and the parameters of the marginal model ($\theta_2$).
Weak exogeneity II

- If consistent and efficient estimation of $\beta_0$ in (1), is not possible without taking the marginal model into account, $X_t$ is not WE for $\beta_0$.

- Hence if $\beta_0$ is a parameter in a SEM model equation, $X_t$ is not WE.
Other parameters of interest and WE of X

▶ Assume that the parameters of interests are the characteristic roots that determine whether \( Y_t \) is stationary or not.
  ▶ If \( X_t \) is WE for those parameters, they can be estimated from (1) without taking the rest of the system into account.
  ▶ Can they?

▶ Assume that the parameters of interest are the dynamic multipliers of \( Y_t \) with respect to a change in \( X_t \). Is \( X_t \) WE?

▶ Assume that the parameters of interest are the impulse responses of \( Y_t \) with respect to a change in \( \epsilon_t \). Is \( X_t \) WE?
Weak exogeneity and the possibility of estimation

- Weak exogeneity is a fundamental property, almost a premise for estimating empirical models.
- A WE variable needs not be a regressor variable.
- It can be an instrumental variable.
- One way of thinking about an exactly identifying variable \((Z_t)\) is that it is weakly exogenous for the parameters of interest in the model equation.
Granger causality

- Granger’s concept of causality builds on the idea that cause comes before effect
- Easiest to define with reference to a 2-variable VAR(1):
  - If $\phi_{12} \neq 0$ and $\phi_{21} = 0$, there is one-way causation from $X_t$ to $Y_t$
  - $X$ is Granger causing $Y$. 
Strong exogeneity

- If $X_t$ is weakly exogenous in the ADL in (1) and $X_t$ is not Granger caused by $Y_t$, then $X_t$ is **strongly exogenous**, SE.
- Generally, WE plus Granger non-causality generates SE.
- While WE is about exogeneity with respect to estimation, SE is needed to do valid forecasting of $Y_{T+h}$ based on a given future path for $X_{T+h}$.
- Another terminology for Granger non-causality is “no feedback”, *ie* from lagged $Y$s on $X$.
- In general, economic systems are characterized by joint feed-back, so Granger non-causality is a strong assumption.
Invariance

- The concept of parameter *invariance* addresses how a parameter of interest, *eg* $\beta_0$ in the ADL (1), “reacts” to a structural break elsewhere in the system.
- In the general notation of Chapter 8, we say that $\theta_1$ is invariant with respect to a break in a parameter of the marginal part of the system $\theta_2$, if $\theta_1$ stays unchanged when $\theta_2$ breaks.
- In the simplest case, we have

$$\beta_0 = \frac{\sigma_{XY}}{\sigma_X^2},$$

so if there is a break in $\sigma_X^2$, $\beta_0$ will also break unless there is a proportional change in $\sigma_{XY}$.
- There is nothing that guarantees that kind of invariance. But nothing hindering it either. So we should test (see below)
Super exogeneity, SuE

- WE plus invariance generates Super Exogeneity (SuE)
- As just noted, invariance is a possible property of conditional models.
- If SuE is a model trait, it validates the use of the conditional models to analyze the effects of policy changes.
- Then “refutes” the Lucas critique
Summary

1. Can we estimate our parameters of interest efficiently without specifying the process that generated $X_t$? If the answer is “yes”, $X_t$ is weakly exogenous.

2. Can we forecast $Y$ efficiently by conditioning on a forecast for $X$ that does not involve the forecasted $Y$ values? If the answer is “yes”, and the answer to 1. is also “yes”, $X_t$ is strongly exogenous.

3. Can we do valid policy analysis based on the conditional model? If the answer is “yes”, and the answer to 1. is also “yes”, $X_t$ is super exogenous.
Remarks

▶ As noted above, weak exogeneity applies to instrumental variables as well as to regressors.

▶ Invariance is also general: Model equations in a SEM can have coefficients that are invariant (or not) to structural breaks elsewhere in the multivariate system of equations (or in the generating processes of the instrumental variables).

▶ However, in the following we continue to focus on conditional models.

▶ It should also be made clear that invariance is a relative property: Useful empirical econometric models can be invariant to certain regime changes and structural breaks, but not all (thinkable) breaks.

▶ Econometrics models are products of civilization and, as such, will break down sooner or later.
Testing Weak Exogeneity

To obtain a test, we can focus on the difference between two estimators of the coefficient vector $\beta$ in:

$$y = X\beta + \varepsilon,$$

where one is the OLS estimator $\hat{\beta}_{OLS}$, and another is consistent both with exogeneity and without it, ie the IV estimator, $\hat{\beta}_{IV}$. The test situation can be written as:

$$H_0 : \text{plim}(\hat{\beta}_{IV} - \hat{\beta}_{OLS}) = 0 \text{ against } H_1 : \text{plim}(\hat{\beta}_{IV} - \hat{\beta}_{OLS}) \neq 0.$$

But where should any significant difference between $\hat{\beta}_{OLS}$ and $\hat{\beta}_{IV}$, come from?
The answer must be: From the rest of the system, from the marginal models of the variables in $X$ in (2).

This means that we can perform the test without actually doing IV estimation, which of course is a convenient simplification.

We can therefore test $H_0$ by estimating the marginal models for $X$ by OLS, calculate residuals from the set of marginal models and then test if those residuals are significant when added to the original model (estimated by OLS) as regressors.

This test often called the Durbin-Wu-Hausman (DWH) test.
Testing Granger non-causality and SE

- This is done by testing the relevant zero-restrictions on the coefficient matrices of the VAR.
- Think of testing Strong Exogeneity of DLPAW in the obligatory!
Testing invariance and SuE

- Again, the testing procedure is quite intuitive:
- If there is evidence of breaks in the marginal models of the regressors in a conditional model:
- Represent these breaks by indicator variables (break-dummies)
- Test whether the break-dummies are significant when added to the conditional model.
- If significant, the $H_0$ of invariance is rejected.
How do we “find” breaks to test for?

Know your presence and your past:

If a law, or a market (de)regulation etc, happened in the sample period, in a way that affected $X_t$ it is almost always worth testing the invariance of the model with respect to such known breaks.

Can also identify breaks in an objective way using a method called Impulse Indicator Saturation, IIS, see Lecture 15.

Recursive estimation and plots are also very revealing about lack of invariance.