UiO : University of Oslo

ECON 4160: Econometrics-Modelling and Systems Estimation
Lecture 1: Introduction

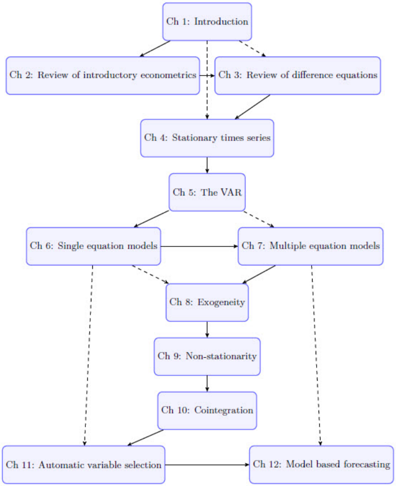
Ragnar Nymoen

Department of Economics University of Oslo

20 August 2018

Lecture plan

- The lectures will follow the main route map of the textbook, see right picture.
- Start today with **Ch. 1**.
- Lecture 2 and 3 (joint with ECON 5106) review the background in econometrics.
- Lecture 4 and 5 cover another background: the maths of difference equations, a main analytical workhorse for this course.
- See posted Lecture plan for more info.



Computer classes, seminars and obligatory

- **Computer classes**: They are listed as *Plenary exercises* on the semester page.
- **Seminars**: There are 8 listed on the semester page. The exercise set to the first seminar has already been posted. The others will follow as the semester progresses.
- **Obligatory assignment**: One of the seminars is an obligatory written assignment. Details about dates come later

The aim of the rest of the Introductory lecture is to give a first, intuitive, introduction to several **key concepts** in dynamic modelling.

By the use of two simple economic models (ie bachelor level).

As noted, the literature reference to this lecture is Chapter 1 in the textbook: *Dynamic Econometrics for Empirical Macroeconomic Modelling*.

Dynamics is typical of economic systems

- A typical trait of real-world macroeconomic systems is that the adjustment to a shock (*ie* impulse) spans several time periods.
- *Impulses* to the macroeconomic system that are *propagated* by the system's internal mechanisms, into effects that lasts for several periods after the occurrence of the shock.
- Remark: "occurrence" is not always unambiguous. Sometimes the effects can start **before** the implementation of a policy change for example.
- In models of such systems, time must play an *essential* role.

Let t denote time and let C denote private consumption and let I denote private income. Which of the following are dynamic model equations?

$$
\begin{aligned}
&\text{a)}\ C = 1.5 + 0.75I \\
&\text{b)}\ C_t = 1.5 + 0.75I_t \\
&\text{c)}\ C_t = 1.5 + 0.75I_{t-1} \\
&\text{d)}\ C_t = 1.5 + 0.25_t + 0.25I_{t-1} + 0.25C_{t-1}
\end{aligned}
$$

In this Lecture we review two simple models that illustrate several concepts of we will use throughout the course.

Cobweb model

Let P_t denote the market clearing price in a product market in time period t, and let Q_t denote equilibrium demand and supply in period t (i.e. $Q_t^D = Q_t^S = Q_t$)) The model consists of the demand equation, (1), and the supply equation (2):

$$Q_t = aP_t + b, \ a < 0 \tag{1}$$

$$Q_t = cP_{t-1} + d, \ c > 0 \tag{2}$$

- The defining trait is that the short-run response of supply with respect to a price change is zero.
- Supply is **perfectly inelastic** in the short run. Lagged supply response with respect to price. Interpretations:
  - Production and/or delivery lags
  - Supply depends on the expected price P_t^e, but expectation formation is:

$$P_t^e = P_{t-1}$$

Cobweb-model

- Initial **stationary state** is (P_{10}, Q_{10}).
- It is defined by the (blue) intersection point between the **static long-run** supply and demand schedules:

$$Q = aP + b \quad \text{D-curve}$$
$$Q = cP + d \quad \text{S-curve}$$

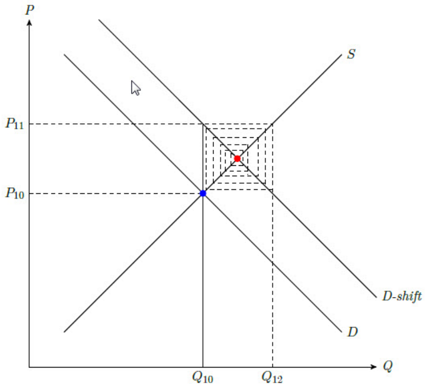


Figure 1.1: Deterministic dynamics: Cobweb model.

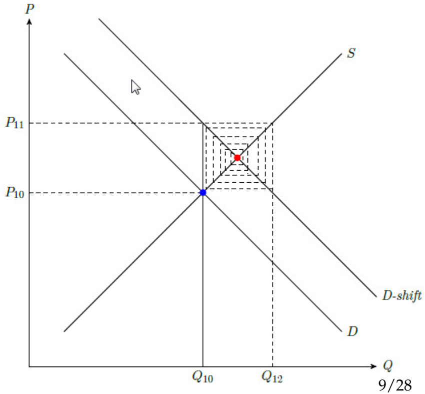
Cobweb-model dynamics

- Assume an exogenous and **permanent** demand increase.
- Formally: a positive change in the parameter, b
- In the static analysis, only the long-run effect can be analysed.
- In the dynamic analysis, the short-run, and all "intermediate-run" effects can be analysed.
- The **stability** (or not) of the process can be analysed.
- All this, and more, means that the dynamic model offers more content and analysis than the static model.

Cobweb-model dynamic effects

The system is in a stationary state in period 10, meaning that $P_{10} = P_9$ and $Q_{10} = Q_9$.
The Demand-shift hits the system in period 11.

- Effect in the period of shock:
  $P_{11} > P_{10}, Q_{11} = Q_{10}$.
- It is custom to call this an **impact effect** or **short run** effect.



Cobweb-model dynamic effects

- Effect in period of shock: $P_{11} > P_{10}, Q_{11} = Q_{10}$
- Effect one period after the shock: $P_{12} < P_{11}, Q_{12} > Q_{11}$.

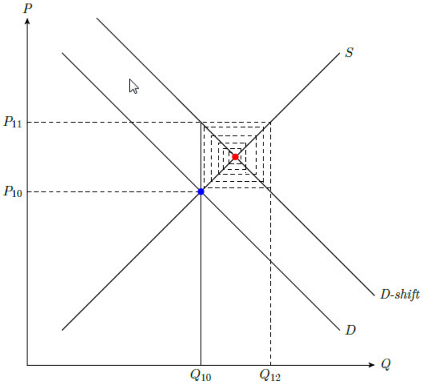


Figure 1.1: Deterministic dynamics: Cobweb model.

Cobweb: Final form and autoregressive parameter

- A **final form equation** of a model expresses an endogenous variable as function of its own lags and of current and lagged values of exogenous variables.
- Final form equations are important for understanding the solution of dynamic models, and for analysing the effects of shocks.
- For the cobweb-model, the two final form equations become: (see Exercise 1.1 in the book):

$$P_t = \phi_1 P_{t-1} + \frac{d - b_t}{a} \tag{3}$$

$$Q_t = \phi_1 Q_{t-1} + \frac{da - cb_{t-1}}{a}, \tag{4}$$

ϕ_1 is the **autoregressive parameter**.

$$\phi_1 = \frac{c}{a}. \tag{5}$$

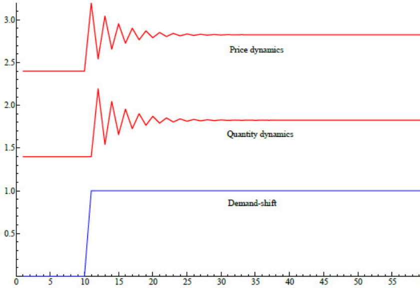
- Note: $\phi_1 < 0$ is implied by the assumptions of the model.

Cobweb: Stable, unstable and explosive dynamics

- $\phi_1 < 0$ is known as **negative autocorrelation**
- A closer characterization of cobweb dynamics requires additional assumptions:
  - Stable: $-1 < \phi_1 < 0 \Leftrightarrow c < -a$
  - Instable: $\phi_1 = -1 \Leftrightarrow c = -a$
  - Explosive: $\phi_1 < -1 \Leftrightarrow c > -a$
- What has been assumed in the graphs above?

Computer simulated effects

- By using numbers for the parameters, we can computer-simulate the dynamic responses.
- The responses are oscillating,
- But the oscillations become dampened over time.
- The graphs demonstrate the phenomenon of **stable dynamics** with **negative autocorrelation**,



Stationary state and equilibrium correction

- We can formalize the stationary state (P^*,Q^*) as the solution of the static model:

$$\begin{aligned} Q^* &= aP^* + b \quad \text{D-curve} \\ Q^* &= cP^* + d \quad \text{S-curve} \end{aligned}$$

and obtain:

$$Q^* = \frac{ad - bc}{a - c} \tag{6}$$

$$P^* = \frac{d - b}{a - c} \tag{7}$$

- The stationary state, i.e. equilibrium state, is (globally asymptotically) stable if and only if:

$$-1 < \phi_1 < 1,$$

- Note, again, only $\phi_1 < 1$ is implied by the assumption of the model.
- Stability of the equilibrium (P^*,Q^*), secured by $-1 < \phi_1$ is therefore a separate assumption.

Stationary state and equilibrium correction

Following Ch. 1.4 in the book, we can re-express the two final form equations to become:

$$\Delta P_t = (\phi_1 - 1)\left[P_{t-1} - P^*\right], \tag{8}$$

$$\Delta Q_t = (\phi_1 - 1)\left[Q_{t-1} - Q^*\right], \tag{9}$$

where ΔP_t and ΔQ_t denote the differences from period $t-1$ to t. For example

$$\Delta P_t = P_t - P_{t-1}$$

- In the stable case of $\phi_1 > -1$, cobweb model dynamics is **equilibrium correcting**.
- As we have seen, after the demand shock, Q_t and P_t are oscillating around the new equilibrium
- The equilibrium correction associated with stable dynamics will play a central role in the course.
- A much used acronym is ECM, for **Error Correction Model**.

Random fluctuations (random variables)

- So far, the dynamics have been deterministic, as in dynamic economic theoretical analysis
- However, to build econometric models, by the use of data and statistics, the endogenous economic variables need to be represented as **random (stochastic) variables**.
- We therefore introduce two random shocks: ϵ_{dt} (demand) and ϵ_{st} (supply):

$$Q_t = aP_t + b_t + \epsilon_{dt},\ a < 0, \tag{10}$$
$$Q_t = cP_{t-1} + d + \epsilon_{st},\ c > 0. \tag{11}$$

(ϵ_{dt},ϵ_{st}) are specified with zero means, and with fixed variances.

In econometrics, it is custom to refer to (10) and (11) as **structural equations**, and to assume that the two **structural disturbances** are uncorrelated.

Reduced form and final form (stochastic versions)

- A *reduced form equation* expresses a current endogenous variable in terms of exogenous variables, and of lags of itself and of other endogenous variables. In our case:

$$P_t = \frac{c}{a}P_{t-1} + \frac{d - b_t}{a} + \frac{\epsilon_{st} - \epsilon_{dt}}{a}, a < 0 \tag{12}$$

$$Q_t = cP_{t-1} + d + \epsilon_{st}, c > 0 \tag{13}$$

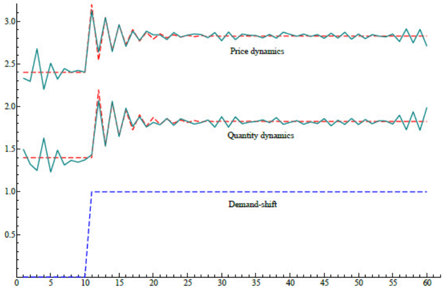
- The stochastic version of the two final form equations become (use the same steps as in Exercise 1.1 in book):

$$P_t = \phi_1 P_{t-1} + \frac{d - b_t}{a} + \frac{\epsilon_{st} - \epsilon_{dt}}{a}, \tag{14}$$

$$Q_t = \phi_1 Q_{t-1} + \frac{da - cb_{t-1}}{a} + \frac{a\epsilon_{st} - c\epsilon_{dt-1}}{a}. \tag{15}$$

Computer simulated effects

- By using numbers for the parameters, and the computer's random number generator, we can simulate the dynamic responses also for the random variables case.
- Demand-shift occurring in period 11.



Estimation of cobweb dynamics

- Since the plots show computer generated data, using $a = -1.3$ and $c = 1$ we know what the true value of ϕ_1 is.
- But what if we draw "a veil of ignorance" over it all, and imagine that we would have to rely on an estimate of ϕ_1?
- Would OLS give a reliable estimate?
- Not a trivial question to answer. For example: Does the IID assumption hold?

We can formulate (12) as an regression model equation:

$$P_t = \beta_0 + \beta_1 P_{t-1} + \beta_2 S11_t + u_t, \tag{16}$$

$S11_t$ is a **step-dummy** which captures the shift in the demand function:

$$S11_t = \begin{cases} 1 \text{ if } t \geq 11, \\ 0 \text{ otherwise.} \end{cases} \tag{17}$$

Estimation of cobweb dynamics

OLS estimation of (16) by the use of the 59 observations gives:

$$P_t = -\underset{(0.053)}{0.755}\, P_{t-1} + \underset{(0.128)}{4.22} + \underset{(0.027)}{0.745}\, S11_t \tag{18}$$

- The numbers in parentheses are standard errors of the estimated coefficients (see Exercise 1.3 in the book)
- How close is the estimated ϕ_1 to the true value?
- Note that P_{t-1} is not a **strictly exogenous** regressor (as defined in introductory econometrics),
- Instead P_{t-1} is **pre-determined**: uncorrelated with current and future disturbances, $u_t, u_{t+1}, ...$, but correlated with lagged (past) disturbances $u_{t-1}, u_{t-2}, ...$
- Apparently, estimation of the parameter ϕ_1 is possible even without exogenous regressors, (and without the use of instrumental variables)?
- Or was it just a stroke of good luck?
- The course will develop the econometric theory needed to give general answers to these questions.

The Fulton data set

- As first example of real world time series, we look at the Fulton Fish Market Data set, Graddy (2006).
- Downloadable from the the **Lecture 1 module**,
- where you also find the cobweb model data set used above.

Negative and positive autocorrelation

- The cobweb model's **negative autocorrelation**, with dominant short-run oscillations for both price and quantity, can be relevant for markets and sectors of the economy where supply is inelastic in the short run.
- *Agricultural products* are examples of this (although in modern economies supply can be replenished)
- *Housing supply* is necessarily fixed in the short-run (it is a stock), but net housing demand can change a great deal, and rapidly. The implication is that housing prices have a potential for volatility.
- The market for *foreign exchange*, under a floating exchange rate regime. Since the net supply of domestic currency is fixed, variations in the net demand for kroner will determine the exchange rate.
- However, in markets for *manufactured goods and services*, the main principle may be that short-run output is largely demand driven. And **positive autocorrelation** may be more typical than negative autocorrelation

A macro model with positive autocorrelation

A Keynesian model (Appendix A shows a real Business Cycle Model, requires more symbosl and derivations):

$$C_t = a + bGDP_t + cC_{t-1} + \epsilon_{Ct}, \tag{19}$$

$$GDP_t = C_t + J_t, \tag{20}$$

$$J_t = J^* + \epsilon_{Jt}, \tag{21}$$

- C_t private consumption in period t.
- GDP_t and J_t represent gross domestic product, and capital formation (*ie* investment).
  The three variables are defined in real terms.
- a - c are parameters.
- ϵ_{Ct}, and ϵ_{Jt} represent random shocks to consumption and investment, with zero means (*ie* mathematical expectation). For simplicity, we assume that they are uncorrelated.

A macro model with positive autocorrelation

- Final form equation for private consumption (Exercise 1.4 in book):

$$C_t = \frac{c}{1-b}C_{t-1} + \frac{1}{1-b}\left[a + bJ^* + \epsilon_{Ct} + b\epsilon_{Jt}\right]. \qquad (22)$$

- We see that:

$$\phi_1 = \frac{c}{1-b}, \qquad (23)$$

  is the autoregressive coefficient.
- With $c > 0$, and $0 < b < 1$, ϕ_1 is positive, $\phi_1 > 0$, and private consumption is therefore positively autocorrelated in this model.
- Positive autocorrelation can be stable, unstable or explosive.
- Stable dynamics: $\phi_1 < 1$

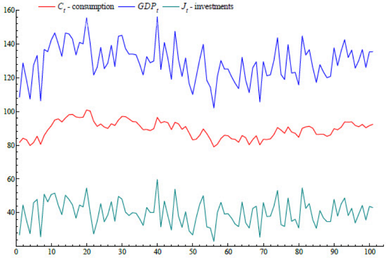
A macro model with positive autocorrelation

- The stationary equilibrium state defined by:

$$C^* = a + bGDP^* + cC^*,$$
$$GDP^* = C^* + J^*,$$
$$J = J^*.$$

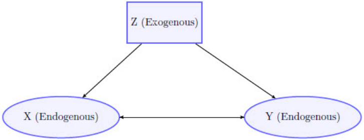
- is stable if $0 \leq \phi_1 < 1$.
- Plot shows simulated dynamics, using parameters $b = 0.25$ and $c = 0.65$, hence: $\phi_1 = 0.713$.
- ECM for C_t:

$$\Delta C_t = (\phi_1 - 1)\left[C_{t-1} - C^*\right] + \frac{\epsilon_{Ct} + b\epsilon_{Jt}}{1-b},$$



Casuality, correlation and invariance

- Real world macroeconomic systems are complex and changing.
- Although we can learn about the common autocorrelation from estimation of a single equation, the main interest is usually relationships between variables
- Therefore, methods for estimation of multiple-equation models for mutual-depedencises will be central.
- Models of systems can include *causes* in the form of exogenous variables, as in the picture.



Casuality, correlation and invariance

- As we know: Correlation should not be mistaken for causality.
- Correlation is also different from regression.
- A certain form of stability of coefficient, called **invariance**, can sometimes aid causality discussions in dynamic econometrics.
- Invariance captures the idea that parts of a system (eg the mutual dependence between X and Y conditional on Z) can be stable even if there is a **regime change** in the process that generate Z.
- Different concepts of econometric **exogeneity** will be central in this course. Chapter 8 in the book.

References

Graddy, K. (2006). The Fulton Fish Market. *Journal of Economic Perspectives*, *20*, 207–220.

Nymoen, R. (2018). *Dynamic Econometrics for Empirical Macroeconomic Modelling*. Manuscript.