ECON 4160: Econometrics-Modelling and Systems Estimation

Lecture 1: Introduction

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Lecture plan

▶ The lectures will follow the main route map of the textbook, see right picture.
▶ Start today with Ch. 1.
▶ Lecture 2 and 3 (joint with ECON 5106) review the background in econometrics.
▶ Lecture 4 and 5 cover another background: the maths of difference equations, a main analytical workhorse for this course.
▶ See posted Lecture plan for more info.
Computer classes, seminars and obligatory assignments

- **Computer classes**: They are listed as *Plenary exercises* on the semester page.
- **Seminars**: There are 8 listed on the semester page. The exercise set to the first seminar has already been posted. The others will follow as the semester progresses.
- **Obligatory assignment**: One of the seminars is an obligatory written assignment. Details about dates come later.

The aim of the rest of the Introductory lecture is to give a first, intuitive, introduction to several **key concepts** in dynamic modelling.

By the use of two simple economic models (i.e., bachelor level).

As noted, the literature reference to this lecture is Chapter 1 in the textbook: *Dynamic Econometrics for Empirical Macroeconomic Modelling.*
Dynamics is typical of economic systems

- A typical trait of real-world macroeconomic systems is that the adjustment to a shock \((ie\) impulse) spans several time periods.

- \textit{Impulses} to the macroeconomic system that are \textit{propagated} by the system’s internal mechanisms, into effects that lasts for several periods after the occurrence of the shock.

- Remark: “occurrence” is not always unambiguous. Sometimes the effects can start \textbf{before} the implementation of a policy change for example.

- In models of such systems, time must play an \textit{essential} role.
Let $t$ denote time and let $C$ denote private consumption and let $I$ denote private income. Which of the following are dynamic model equations?

a) $C = 1.5 + 0.75I$

b) $C_t = 1.5 + 0.75I_t$

c) $C_t = 1.5 + 0.75I_{t-1}$

d) $C_t = 1.5 + 0.25t + 0.25I_{t-1} + 0.25C_{t-1}$

In this Lecture we review two simple models that illustrate several concepts of we will use throughout the course.
Cobweb model

Let $P_t$ denote the market clearing price in a product market in time period $t$, and let $Q_t$ denote equilibrium demand and supply in period $t$ (i.e. $Q^D_t = Q^S_t = Q_t$). The model consists of the demand equation, (1), and the supply equation (2):

1. $Q_t = aP_t + b, \ a < 0$  \hspace{1cm} (1)
2. $Q_t = cP_{t-1} + d, \ c > 0$  \hspace{1cm} (2)

- The defining trait is that the short-run response of supply with respect to a price change is zero.
- Supply is **perfectly inelastic** in the short run. Lagged supply response with respect to price. Interpretations:
  - Production and/or delivery lags
  - Supply depends on the expected price $P^e_t$, but expectation formation is:
    $$P^e_t = P_{t-1}$$
Cobweb-model

- Initial **stationary state** is \((P_{10}, Q_{10})\).
- It is defined by the (blue) intersection point between the **static long-run** supply and demand schedules:

  \[
  Q = aP + b \quad \text{D-curve} \\
  Q = cP + d \quad \text{S-curve}
  \]
Cobweb-model dynamics

- Assume an exogenous and **permanent** demand increase.
- Formally: a positive change in the parameter, $b$
- In the static analysis, only the long-run effect can be analysed.
- In the dynamic analysis, the short-run, and all “intermediate-run” effects can be analysed.
- The **stability** (or not) of the process can be analysed.
- All this, and more, means that the dynamic model offers more content and analysis than the static model.
Cobweb-model dynamic effects

The system is in a stationary state in period 10, meaning that \( P_{10} = P_9 \) and \( Q_{10} = Q_9 \).
The Demand-shift hits the system in period 11.

- Effect in the period of shock:
  \( P_{11} > P_{10}, Q_{11} = Q_{10} \).
- It is custom to call this an impact effect or short run effect.
Cobweb-model dynamic effects

- Effect in period of shock: 
  \( P_{11} > P_{10}, Q_{11} = Q_{10} \)

- Effect one period after the shock: 
  \( P_{12} < P_{11}, Q_{12} > Q_{11} \).

Figure 1.1: Deterministic dynamics: Cobweb model.
Cobweb: Final form and autoregressive parameter

▶ A final form equation of a model expresses an endogenous variable as function of its own lags and of current and lagged values of exogenous variables.

▶ Final form equations are important for understanding the solution of dynamic models, and for analysing the effects of shocks.

▶ For the cobweb-model, the two final form equations become: (see Exercise 1.1 in the book):

\[ P_t = \phi_1 P_{t-1} + \frac{d - b_t}{a}, \quad (3) \]
\[ Q_t = \phi_1 Q_{t-1} + \frac{da - cb_{t-1}}{a}, \quad (4) \]

\( \phi_1 \) is the autoregressive parameter.

\[ \phi_1 = \frac{c}{a}. \quad (5) \]

▶ Note: \( \phi_1 < 0 \) is implied by the assumptions of the model.
Cobweb: Stable, unstable and explosive dynamics

- $\phi_1 < 0$ is known as **negative autocorrelation**
- A closer characterization of cobweb dynamics requires additional assumptions:
  - Stable: $-1 < \phi_1 < 0 \iff c < -a$
  - Instable: $\phi_1 = -1 \iff c = -a$
  - Explosive: $\phi_1 < -1 \iff c > -a$
- What has been assumed in the graphs above?
Computer simulated effects

- By using numbers for the parameters, we can computer-simulate the dynamic responses.
- The responses are oscillating,
- But the oscillations become dampened over time.
- The graphs demonstrate the phenomenon of stable dynamics with negative autocorrelation,
Stationary state and equilibrium correction

- We can formalize the stationary state \((P^*,Q^*)\) as the solution of the static model:

\[
Q^* = aP^* + b \quad \text{D-curve}
\]
\[
Q^* = cP^* + d \quad \text{S-curve}
\]

and obtain:

\[
Q^* = \frac{ad - bc}{a - c} \tag{6}
\]
\[
P^* = \frac{d - b}{a - c} \tag{7}
\]

- The stationary state, i.e. equilibrium state, is (globally asymptotically) stable if and only if:

\[-1 < \phi_1 < 1,
\]

- Note, again, only \(\phi_1 < 1\) is implied by the assumption of the model.

- Stability of the equilibrium \((P^*,Q^*)\), secured by \(-1 < \phi_1\) is therefore a separate assumption.
Stationary state and equilibrium correction

Following Ch. 1.4 in the book, we can re-express the two final form equations to become:

\[ \Delta P_t = (\phi_1 - 1) [P_{t-1} - P^*], \]  
\[ \Delta Q_t = (\phi_1 - 1) [Q_{t-1} - Q^*], \]  

where \( \Delta P_t \) and \( \Delta Q_t \) denote the differences from period \( t - 1 \) to \( t \). For example

\[ \Delta P_t = P_t - P_{t-1} \]

- In the stable case of \( \phi_1 > -1 \), cobweb model dynamics is equilibrium correcting.
- As we have seen, after the demand shock, \( Q_t \) and \( P_t \) are oscillating around the new equilibrium
- The equilibrium correction associated with stable dynamics will play a central role in the course.
- A much used acronym is ECM, for Error Correction Model.
Random fluctuations (random variables)

- So far, the dynamics have been deterministic, as in dynamic economic theoretical analysis.
- However, to build econometric models, by the use of data and statistics, the endogenous economic variables need to be represented as random (stochastic) variables.
- We therefore introduce two random shocks: $\epsilon_{dt}$ (demand) and $\epsilon_{st}$ (supply):

$$Q_t = aP_t + b_t + \epsilon_{dt}, \ a < 0, \quad (10)$$

$$Q_t = cP_{t-1} + d + \epsilon_{st}, \ c > 0. \quad (11)$$

$(\epsilon_{dt}, \epsilon_{st})$ are specified with zero means, and with fixed variances.

In econometrics, it is custom to refer to (10) and (11) as **structural equations**, and to assume that the two **structural disturbances** are uncorrelated.
Reduced form and final form (stochastic versions)

- A reduced form equation expresses a current endogenous variable in terms of exogenous variables, and of lags of itself and of other endogenous variables. In our case:

\[
P_t = \frac{c}{a}P_{t-1} + \frac{d - b_t}{a} + \frac{\epsilon_{st} - \epsilon_{dt}}{a}, a < 0 \tag{12}
\]

\[
Q_t = cP_{t-1} + d + \epsilon_{st}, c > 0 \tag{13}
\]

- The stochastic version of the two final form equations become (use the same steps as in Exercise 1.1 in book):

\[
P_t = \phi_1 P_{t-1} + \frac{d - b_t}{a} + \frac{\epsilon_{st} - \epsilon_{dt}}{a}, \tag{14}
\]

\[
Q_t = \phi_1 Q_{t-1} + \frac{da - cb_{t-1}}{a} + \frac{a\epsilon_{st} - c\epsilon_{dt-1}}{a}. \tag{15}
\]
Computer simulated effects

- By using numbers for the parameters, and the computer’s random number generator, we can simulate the dynamic responses also for the random variables case.
- Demand-shift occurring in period 11.
Estimation of cobweb dynamics

- Since the plots show computer generated data, using $a = -1.3$ and $c = 1$ we know what the true value of $\phi_1$ is.
- But what if we draw “a veil of ignorance” over it all, and imagine that we would have to rely on an estimate of $\phi_1$?
- Would OLS give a reliable estimate?
- Not a trivial question to answer. For example: Does the IID assumption hold?

We can formulate (12) as an regression model equation:

$$P_t = \beta_0 + \beta_1 P_{t-1} + \beta_2 S_{11_t} + u_t,$$

(16)

$S_{11_t}$ is a **step-dummy** which captures the shift in the demand function:

$$S_{11_t} = \begin{cases} 
1 & \text{if } t \geq 11, \\
0 & \text{otherwise.}
\end{cases}$$

(17)
Estimation of cobweb dynamics

OLS estimation of (16) by the use of the 59 observations gives:

\[ P_t = -0.755 P_{t-1} + 4.22 + 0.745 S11_t \]  
(18)

- The numbers in parentheses are standard errors of the estimated coefficients (see Exercise 1.3 in the book)
- How close is the estimated \( \phi_1 \) to the true value?
- Note that \( P_{t-1} \) is not a strictly exogenous regressor (as defined in introductory econometrics),
- Instead \( P_{t-1} \) is pre-determined: uncorrelated with current and future disturbances, \( u_t, u_{t+1}, \ldots \), but correlated with lagged (past) disturbances \( u_{t-1}, u_{t-2}, \ldots \)
- Apparently, estimation of the parameter \( \phi_1 \) is possible even without exogenous regressors, (and without the use of instrumental variables)?
- Or was it just a stroke of good luck?
- The course will develop the econometric theory needed to give general answers to these questions.
The Fulton data set

- As first example of real world time series, we look at the Fulton Fish Market Data set, Graddy (2006).

- Downloadable from the the Lecture 1 module,

- where you also find the cobweb model data set used above.
Negative and positive autocorrelation

- The cobweb model’s negative autocorrelation, with dominant short-run oscillations for both price and quantity, can be relevant for markets and sectors of the economy where supply is inelastic in the short run.

- Agricultural products are examples of this (although in modern economies supply can be replenished).

- Housing supply is necessarily fixed in the short-run (it is a stock), but net housing demand can change a great deal, and rapidly. The implication is that housing prices have a potential for volatility.

- The market for foreign exchange, under a floating exchange rate regime. Since the net supply of domestic currency is fixed, variations in the net demand for kroner will determine the exchange rate.

- However, in markets for manufactured goods and services, the main principle may be that short-run output is largely demand driven. And positive autocorrelation may be more typical than negative autocorrelation.
A macro model with positive autocorrelation

A Keynesian model (Appendix A shows a real Business Cycle Model, requires more symbols and derivations):

\[ C_t = a + bGDP_t + cC_{t-1} + \epsilon_C t, \]  \hspace{1cm} (19)
\[ GDP_t = C_t + J_t, \]  \hspace{1cm} (20)
\[ J_t = J^* + \epsilon_J t, \]  \hspace{1cm} (21)

- \( C_t \) private consumption in period \( t \).
- \( GDP_t \) and \( J_t \) represent gross domestic product, and capital formation (ie investment).
- The three variables are defined in real terms.
- \( a \) - \( c \) are parameters.
- \( \epsilon_C t \) and \( \epsilon_J t \) represent random shocks to consumption and investment, with zero means (ie mathematical expectation). For simplicity, we assume that they are uncorrelated.
A macro model with positive autocorrelation

▶ Final form equation for private consumption (Exercise 1.4 in book):

\[ C_t = \frac{c}{1-b} C_{t-1} + \frac{1}{1-b} \left[ a + bJ^* + \epsilon_{Ct} + b\epsilon_{jt} \right]. \]  

(22)

▶ We see that:

\[ \phi_1 = \frac{c}{1-b} \]  

(23)

is the autoregressive coefficient.

▶ With \( c > 0 \), and \( 0 < b < 1 \), \( \phi_1 \) is positive, \( \phi_1 > 0 \), and private consumption is therefore positively autocorrelated in this model.

▶ Positive autocorrelation can be stable, unstable or explosive.

▶ Stable dynamics: \( \phi_1 < 1 \)
A macro model with positive autocorrelation

The stationary equilibrium state defined by:

\[ C^* = a + bGDP^* + cC^*, \]
\[ GDP^* = C^* + J^*, \]
\[ J = J^*. \]

is stable if \( 0 \leq \phi_1 < 1 \).

Plot shows simulated dynamics, using parameters \( b = 0.25 \) and \( c = 0.65 \), hence: \( \phi_1 = 0.713 \).

ECM for \( C_t \):

\[ \Delta C_t = (\phi_1 - 1) [C_{t-1} - C^*] + \frac{\epsilon_{Ct} + b\epsilon_{Jt}}{1 - b}, \]
Casuality, correlation and invariance

- Real world macroeconomic systems are complex and changing.
- Although we can learn about the common autocorrelation from estimation of a single equation, the main interest is usually relationships between variables.
- Therefore, methods for estimation of multiple-equation models for mutual dependencies will be central.
- Models of systems can include *causes* in the form of exogenous variables, as in the picture.
Casuality, correlation and invariance

► As we know: Correlation should not be mistaken for causality.
► Correlation is also different from regression.
► A certain form of stability of coefficient, called invariance, can sometimes aid causality discussions in dynamic econometrics.
► Invariance captures the idea that parts of a system (e.g., the mutual dependence between $X$ and $Y$ conditional on $Z$) can be stable even if there is a regime change in the process that generate $Z$.
► Different concepts of econometric exogeneity will be central in this course. Chapter 8 in the book.
References
