UiO : University of Oslo

ECON 4160: Econometrics-Modelling and Systems Estimation

Lecture 6: The VAR

Ragnar Nymoen

Department of Economics University of Oslo

18 September 2018

The reference to this lecture is:

- Chapter 5 in the textbook: *Dynamic Econometrics for Empirical Macroeconomic Modelling*.

- In Lecture 5 we looked at OLS as a method to estimate the coefficients of the stationary AR(1) model:

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \varepsilon_t, \;\; |\phi_1| < 1, t = 1, \ldots, T \qquad (1)$$

  where $\varepsilon_t \sim IIN\left(0, \sigma_\varepsilon^2\right) \;\forall\, t$.
- Found that in particular the OLS-estimator $\hat{\phi}_1$ is consistent, but that it is biased in finite samples (Hurwitz-bias).
- The t-value was however *not* biased away from zero, even in small samples.
- We can regard (1) as the simplest possible VAR: a "single equation VAR" and hope that the good news about estimation extends to genuine VARs, with two of more endogenous variables.
- This turns out to be correct.

2-variable VAR

The simplest example of a genuine dynamic **system of variables** is the bivariate VAR(1):

$$\begin{pmatrix} Y_t \\ X_t \end{pmatrix} = \begin{pmatrix} \phi_{10} \\ \phi_{20} \end{pmatrix} + \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix} \begin{pmatrix} Y_{t-1} \\ X_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{yt} \\ \varepsilon_{xt} \end{pmatrix}. \quad (2)$$

When the assumption of independent and identically Gaussian disturbances is explicit:

$$\begin{pmatrix} \varepsilon_{yt} \\ \varepsilon_{xt} \end{pmatrix} \sim IIN \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \underbrace{\begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{pmatrix}}_{\mathbf{\Sigma}} \right), t = 1, ..., T. \quad (3)$$

it it custom to refer to (2) as a Gaussian VAR.

Stationarity condition

- From Ch 4.8 and Ch. 3.8 we have that the condition for joint stationarity (stability) of Y_t and X_t is that the two roots of

$$\left| \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right| = 0$$

  are are both less than one in magnitude.
- This is a special case of the condition that eigenvalues of the companion matrix **B** are less than one, *ie*:

$$|\mathbf{B} - \lambda \mathbf{I}| = 0.$$

  have roots less than one in magnitude.
- This condition also applies to the general VAR with n endogenous variables and p lags in each row, with notation (5.4) in the book, reproduced in the next frame.

The Y_{it} , $i = 1, 2, ...n$ endogenous variables are collected in the vector:

$$\underset{(n\times 1)}{\mathbf{y}_t} = (Y_{1t}, Y_{2t}, \ldots, Y_{nt})',$$

and the general notation for the VAR:

$$\mathbf{y}_t = \mathbf{\Phi}_1\mathbf{y}_{t-1} + \mathbf{\Phi}_2\mathbf{y}_{t-2} + ... + \mathbf{\Phi}_p\mathbf{y}_{t-p} + \mathbf{YD}_t + \varepsilon_t \qquad (4)$$

where $\mathbf{\Phi}_i$ $(i = 1, 2, \ldots, p)$ are $(n \times n)$ matrices and:

$$\varepsilon_t \sim IIN(\mathbf{0}, \mathbf{\Sigma}). \qquad (5)$$

- The companion matrix **B** is constructed from the $\mathbf{\Phi}_i$'s in the way explained in Chapter 3.8
- In (4), the non-homogenous part $\mathbf{YD}_t$ shall be interpreted broadly as including: constant term, trend, seasonals and other dummies.

Estimation of VAR

- Theorem 5.1 and Box 5.2 tell us what we hoped for: OLS estimation of each row in the VAR gives approximate ML estimators for the coefficients
- Testing of individual coefficients can be based on conventional t-values.
- For joint hypothesis testing (several parameters, restrictions across equations), the *Likelihood ratio* test can be used:

$$LR = -2(\mathcal{L}_R^* - \mathcal{L}_U^*),$$

  it has an asymptotic Chi-square distribution with degrees of freedom equal to the number of restrictions being tested.
- $\mathcal{L}_U$ is the likelihood value *without* imposing the restrictions being tested. $\mathcal{L}_R$ is the *restricted* likelihood, with the restrictons imposed on the VAR

VAR analysis of the cobweb model data

	P_t		Q_t	
	Coeff.	St. error	Coeff.	St. error
P_{t-1}	-0.716	0.039	0.935	0.049
Q_{t-1}	0.021	0.032	-0.029	0.040
1_t	4.094	0.123	-0.834	0.154
$S11_t$	0.723	0.029	0.062	0.036

OLS, Sample $5-100$, $T=96$ no.of parameters $=8$		
	$\mathcal{L}_U = 523.805$	$\frac{T}{2}\ln\left|\hat{\mathbf{\Sigma}}_U\right| = 796.242$
Residual correlations:	$\hat{\sigma}_P = 0.0270$, $\hat{\sigma}_{PQ} = -0.958$	$\hat{\sigma}_Q = 0.0338$
Information criteria:	$AIC = -16.421$ $HQ = -16.34$ $SC = -16.208$	

- Ch. 5.3 explains the Information criteria
- Solve Exercise 5.3 to produce the results, and test that Q_{t-1} can be omitted from both rows due to insignificance:

$$R = -2(\mathcal{L}_R^* - \mathcal{L}_U^*) = -2 \cdot (523.533 - 523.805) = 0.544.$$

VAR impulse response

- Lecture 4: Impulse responses are the partial derivatives of (a VAR variable) with respect to a VAR disturbance.
- For example the response of Q_{t+j} to a change in "it's own disturbance" ε_{2t} is:

$$\delta_j = \frac{\partial Q_{t+j}}{\partial \varepsilon_{2t}}, j = 0, 1, 2, ...$$

  and from Lecture 4 we remember that the responses are strongly influenced by the characteristic roots of the VAR.
- In practice, the estimated responses can be calculated by a program like PcGive after VAR estimation
- However, the VAR responses are not interpretable as responses to *structural* supply or demand shocks. In that sense they are unidentified. Chapter 7 will explain more, but note ''Imaginary shock'' critique of Paul Romer already here.

Forecasting from VAR

- *Forecasting* is a later topic of the course. However, it is natural to mention it at this point as well, as forecasting is a main objective for VAR model users.
- Assume that $Y_{i,t}$, $i = 1, 2, ..., n$ are endogenous variables in a VAR
- Focus on the $Y_{1,t}$, the first variable in the VAR.
- The *conditional expectation* $Y_{1,T+h}$ is the usual choice of h-period predictor of $Y_{1,T+h}$ (T is the period where the forecast is prepared and published)

- From Lecture 4 and 5 we have that the solution for $Y_{1,T+h}$ takes the form of expression (5.21) in the book, and that the conditional expectation of Y_{1T+h} is:

$$E(Y_{1T+h} \mid \mathcal{I}_T) = b_{11}^{(h)} Y_{1T} + b_{12}^{(h)} Y_{1T-1} + \gamma_1 (b_{11}^{(h-1)} + b_{11}^{(h-2)} + \ldots + 1) \tag{6}$$

  since all the future shocks (disturbances) have expectation zero. $b_{11}^{(h)}$ and $b_{12}^{(h)}$ are elements (1,1) and (1,2) in the matrix $\mathbf{B}^h$.
- (6) is a *forecast function*, with h as the argument.
- If $E(Y_{1T+h} \mid \mathcal{I}_T)$ has been chosen as the predictor, it follows that the forecast errors $(Y_{1T+h} - E(Y_{1T+h}))$ are random variables, with expectation zero and with variances that depend on on h.
- Forecasts can presented as fan-charts, as illustrated for cobweb model data on the next slide.

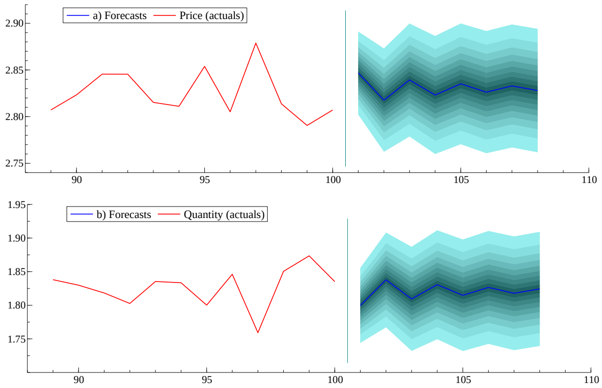

Figure 1: Actual values shown as line graphs, and forecasts as fan charts for price (panel a)) and quantity (panel b)), for the cobweb VAR estimated in slide 8. In this example, the forecasts are conditional on period $T = 100$, and the forecast periods are $T + h = 101, ..., 108$, *ie* $H = 8$.