UiO : University of Oslo

ECON 4160: Econometrics-Modelling and Systems Estimation
Lecture 9: Multiple equation models II

Ragnar Nymoen

Department of Economics University of Oslo

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The reference to this lecture is:

- Chapter 7.7-7.11 in the textbook: *Dynamic Econometrics for Empirical Macroeconomic Modelling*.

Identification through Ω restrictions

- The relationships between the parameters in $\mathbf{\Omega}$ of the SEM, and $\mathbf{\Sigma}$ (the reduced form), are not taken into account when rank and order conditions for identification are used.
- Can say that rank and order conditions are "safe to use" when the SEM error-variance matrix $\mathbf{\Omega}$ is *unrestricted*.
- However, care must be taken when the theoretical model specifies restrictions on $\mathbf{\Omega}$ (one covariance set to zero for example).
- In such cases, one or more equations of the SEM can be identified, even if they are not identified on the rank and order conditions.

- Example: In the same way as in Lecture 8, obtain the reduced form of the SEM:

$$Q_t + a_{12}P_t = a_{10} + \epsilon_{1t} \text{ (demand, } a_{12} > 0) \tag{1}$$

$$a_{21}Q_t + P_t = a_{20} + \epsilon_{2t} \text{ (supply, } \; a_{21} < 0). \tag{2}$$

- From the reduced form: calculate expectations and variances of Q_t and P_t and the covariance. This is done in (7.20)-(7.24) in the book.
- Assume, for example that $a_{21} = 0$ **and** $\omega_{12} = 0$ (a restriction on $\mathbf{\Omega}$).
- The equations (7.20)-(7.24) then becomes a determined equation system: There is enough information in the reduced form to identify the parameters of the SEM. Check it out !
- This is the simplest example of a **recursive model**

General definition of recursive model

- From Lecture 8, the general notation of a SEM (eq. (7.8) in book):

$$\mathbf{A}\mathbf{y}_t = \sum_{i=1}^{p} \mathbf{A}\mathbf{\Phi}_i \mathbf{y}_{t-i} + \sum_{i=0}^{q} \mathbf{A}\mathbf{\Gamma}_i \mathbf{x}_{t-i} + \mathbf{A}\mathbf{Y}\mathbf{D}_t + \mathbf{A}\boldsymbol{\varepsilon}_t. \qquad (3)$$

  The SEM is a *recursive model* when:

  1. $\mathbf{A}$ is a (upper or lower) *triangular matrix* with ones along the main diagonal, and:
  2. $\mathbf{\Omega}$ is a diagonal covariance matrix.

- All equations of a recursive model are identified.
- Note that the definition does not involve any restrictions on the matrices with lagged coefficients $\mathbf{A}\mathbf{\Phi}_i$.

Unidentified VAR impulse respones

- A main objective when VARs are estimated in macro economics, it to calculate *impulse responses* with respect to the VAR disturbances in $\boldsymbol{\varepsilon}_t$
- We have seen that the impulse responses are always well defined mathematically.
- However, the economic interpretation of VAR impulse responses is generally not clear.
- Again, look at the marked demand and supply example. From the reduced form, the VAR disturbances are:

$$\varepsilon_{1t} = \underbrace{\frac{\epsilon_{1t} - a_{12}\epsilon_{2t}}{1 - a_{12}\beta_{21}}}_{\varepsilon_{1t}} \tag{4}$$

$$\varepsilon_{2t} = \underbrace{\frac{\epsilon_{2t} - a_{21}\epsilon_{1t}}{1 - a_{12}a_{21}}}_{\varepsilon_{2t}}. \tag{5}$$

“Imaginary shocks” critique of modern macro

Macroeconomics is a controversial field, and an interesting debate was rekindled by Paul Romer’s (until recently World Bank Chief Economist) critique of the dominance of impulse responses in modern macro, which he dubbed as responses to imaginary shocks. One of the underlying issues is the identification of VAR shocks.
Paul Romer was awarded the “Nobel Price” on 7 October, for endogenous growth theory

Structural VARs

- A VAR that has been restricted in such a way that the impulse response functions can be identified with changes in the SEM distrurbances are called a *Structural VAR*, SVAR.
- The most used "identification scheme" historically for SVARs is known by the technical name *Cholesky decompositon*.
- To choose a Cholesky decomposition to identify VAR impulse responses is however equivalent to specifying a recursive model.
- In the economics of monetary policy in particular, other identification schemes have been developed.
- One famous approach is to impose restrictions in the form restricted long-run dynamic multipliers.
- Chapter 7.8 includes a few references to the literature.

Simultaneous equations bias of OLS

- If an equation of a SEM is identified, at least one consistent estimator exists for the model equation's coefficients.
- Is OLS a consistent estimator ?
- Without any loss of generality, we can answer this question in the simplest case with no dynamics and a single structural equation.
- Reproduce (7.34)-(7.36) here:

$$C_t = a + b(GDP_t) + \epsilon_{Ct}, 0 < b < 1, \tag{6}$$

$$GDP_t = C_t + J_t, \tag{7}$$

$$J_t = J^* + \epsilon_{Jt}, \tag{8}$$

- Since J_t is an exogenous variable we set $Cov(\epsilon_{Ct}, \epsilon_{Jt}) = 0$.

- Follow the steps in Chapter 7.9.1 and show that the probability limit of the OLS estimator of b can be expressed as:

$$plim(\hat{b}_{OLS} - b) = (1 - b)\left(\frac{\sigma_C^2}{\sigma_C^2 + \sigma_I^2}\right) > 0. \quad (9)$$

  showing that OLS is overestimating the SEM parameter b.
- The size of the bias depends on the ratio between consumption variance and GDP variance.
- This results carries over to a SEM where the consumption function is a dynamic model equation.
- In Chapter 6.2.3 we showed an example where the ratio was very low.
- In this chapter, (7.43) shows a different example, with $\sigma_C^2 = 50$ and $\sigma_J^2 = 50$. Biases are then notable.

Indirect least squares (ILS)

- ILS uses the OLS estimates of the Reduced Form (RF) coefficients to "solve out" the estimates for the SEM coefficients.
- Since the OLS gives consistent estimation of reduced form coefficients, and the SEM parameters are determined by knowledge of the RF, ILS gives consistent estimation of SEM coefficients.
- See Chapter 7.9.2
- This works fine for exactly identified SEM equations.
- But cumbersome for larger models, hence in practice, IV estimation is used.

Instrumental variables (IV)

- If a reminder about basic IV estimation in needed, consult for example Chapter 2.5.
- In the book we write the first (structural) equation of a SEM as:

$$\mathbf{y}_1 = \mathbf{X}_{1X}\boldsymbol{\beta}_{11} + \mathbf{X}_{1Y}\boldsymbol{\beta}_{21} + \boldsymbol{\epsilon}_1, \tag{10}$$

  where
  - The matrix $\mathbf{X}_{1X}$ holds the data of the exogenous variables (including lags) in the SEM that are included in the first equation.
  - $\mathbf{X}_{1Y}$ holds the data for the included endogenous variables.
- The Generalized IV estimator (GIV) is given as:

$$\hat{\boldsymbol{\beta}}_{1,GIV} = (\widehat{\mathbf{Z}}_1'\mathbf{X}_1)^{-1}\widehat{\mathbf{Z}}_1'\mathbf{y}_1. \tag{11}$$

- If the equation is exactly identified on the order condition, $\widehat{\mathbf{Z}}_1$ is a matrix with valid IVs.
- What is the IV estimator for b in the simple Keynesian model?

GIV and 2SLS

- If the equation is over-identified, $\widehat{\mathbf{Z}}_1$, contains the included exogenous variables (as before), and the best linear predictions of the included endogenous variables.
- Those predictions are obtained as the fitted values from OLS estimation of the Reduced Form, and is referred to as the first stage of 2SLS estimation
- The second stage is the OLS estimation of the structural equation, taking the predictions from the first stage as regressors.
- 2SLS gives an estimator which is equivalent to the GIV estimator in equation (11).
- The algebra for this is in Ch. 7.9.3. and is included in the manuscript for completeness. You are not expected to show the equivalence of GIV and 2SLS using matrix algebra
- Concentrate instead of understanding why 2SLS solves the "luxury problem" of having more than one consistent estimator!

A specification test

- Several modern textbook derive the GIV estimator by the use of the IV-criterion function, see ch. 7.9.4. But the exam will not require any details or derivations from this sub-chapter
- Chapter 7.9.3 contains *Specification test* and J-test. The important aim here is the intuitive interpretation: Namely that valid over-identifying instruments should not have any explanatory power for the residuals from GIV/2SLS estimation of the SEM equation.
- The instruments should not have any explanatory power for $\mathbf{y}_{1t}$ beyond what they have as instruments for the included endogenous variables in the equation.

Estimation of the complete model

- GIV/2SLS can be used to estimate all model equations that are identified (not just the first one!)
- If all equations are identified, modern software programs have algorithms for FIML estimation of the complete multiple equation model.
- We do not go into the algebra of FIML for multiple equations models, but intuitively we can understand that an iterative method (Newton or other) is needed to maximize, since the log-likelihood function is complicated/non-linear.
- Historically, 3SLS was important because in older times FIML was too complicated to apply.
- In principle: 3SLS is a GLS method, it uses estimates of the $\mathbf{\Omega}$ matrix to give "weighted 2SLS".
- **Q** What is the difference between IV, GIV, 2SLS, 3SLS and FIML for a SEM which is exactly identified?

RE model equations

- Another type of motivation, apart from simultaneous equations, for the use of GIV/2SLS is when we want to estimate a model equation where on the right hand side is the mathematical expectation of X_t or X_{t+1}.
- Ch. 7.10 contains a model with expected lead: X_{t+1}.
- Substitution of $E(X_{t+1}|\mathcal{I}_{t-1})$ by X_{t+1} and estimation by OLS gives inconsistent estimation, which is due to measurement error, as the chapter shows.
- Technically speaking, the solution is to estimate by IV/2SLS.
- But where does the IVs come from? The logical answer is: "from the rest of the model", which should therefore be specified. The chapter shows a simple example. If the rank condition is satisfied, the order condition is automatically satisfied.

Future dependent models

- Consider the first order process with $\phi_1 > 1$:

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \varepsilon_t, \quad |\phi_1| > 1, \tag{12}$$

  where ε_t $(t = 0, \pm 1, \pm 1, ...)$ is white noise.
- The solution that conditions on a given historical initial value is explosive, and not stable.
- Nevertheless (12) defines stationary time series.
- To obtain a stationary solution we can re-arrange the model equation as:

$$Y_t = \frac{-\phi_0}{\phi_1} + \frac{1}{\phi_1} Y_{t+1} + \frac{1}{\phi_1} \varepsilon_t$$

  and consider repeated *forward insertion* on the right-hand side in this process.
- Solution on next slide

after H-1 insertions:

$$Y_t = \frac{-\phi_0}{\phi_1}\sum_{i=0}^{H-1}\left(\frac{1}{\phi_1}\right)^i + \left(\frac{1}{\phi_1}\right)^H Y_{t+H} + \frac{1}{\phi_1}\sum_{i=0}^{H-1}\left(\frac{1}{\phi_1}\right)^i \varepsilon_{t+i} \quad (13)$$

- $H \rightarrow \infty$ gives a stationary solution with
- $$E(Y_t) = \frac{\phi_0}{(1-\phi_1)}$$
  $$Var(Y_t) = \frac{\sigma_\varepsilon^2}{(1-\phi_1^2)}$$

New Keynesian Phillips Curve, NKPC

- A popular model equation in modern macro which combines the above features is the NKPC.
- Let Δp_t denote (price) inflation, and let s_t be a so called *forcing variable*, (the wage-share, the unemployment rate and GDP-gap are popular choices)

$$\Delta p_t = \underset{\geq 0}{a^f} E_t[\Delta p_{t+1}] + \underset{\geq 0}{a^b} \Delta p_{t-1} + \underset{>0}{b} s_t + \varepsilon_{\pi t}, \tag{14}$$

- To find a solution for Δp_t a process for s_t must be specified.
- That process can also be used to motivate IVs to allow GIV/2SLS estimation of (14)