The New Keynesian Phillips Curve: A meta-analysis*

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Abstract

Several influential papers report evidence which support the specification and theory of the New Keynesian Phillips curve, NPC. We show that the same evidence, when viewed jointly, does not support the NPC as an internally consistent theory of inflation with a high degree of relevance for e.g., inflation targeting central banks. Our analysis does not refute generally that the theory that inflation is forward-looking. It does however strongly indicate that a better research strategy than the one followed so far, is to test that hypothesis within an inflation model which explains the joint evidence, and which encompasses existing empirical models.

Keywords: New Keynesian Phillips Curve, forward-looking price setting, rational expectations, econometric methodology.

JEL classification: B41, C22, E31, E52

1 Introduction

The New Keynesian Phillips Curve, hereafter NPC, has become the standard model of the supply side in the macro models used for monetary policy analysis. This position is due to its theoretical underpinnings, laid out in Clarida et al. (1999), and to the supportive empirical results in the studies of Galí and Gertler (1999, henceforth GG) on US data, and Galí, Gertler and López-Salido (2001, henceforth GGL (2001)) on euro-area data. Rudd and Whelan (2005b) and Linde (2005) criticize several aspects of the estimation and inference procedures used by GGL (2001), but this line of critique is rebutted in a recent paper by GGL (2005), who re-assert

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that the NPC, in particular the dominance of forward-looking behavior, is robust to choice of estimation procedure and specification bias.

In this paper we perform a meta-analysis of the empirical status of the NPC, based on the assumption that although estimation methods and operational definitions always need careful consideration, the existing evidence is relevant. We show that even though the individual pieces of evidence are supportive of the NPC, which we refer to as sequential corroboration, the joint implication of the same evidence may still entail refutation of the hypothesis that the NPC really captures the important determinants of inflation.

The methodological principle that an explanation needs to be able to account for observable facts other than the facts that the hypothesis was first intended to explain, is well established, see Elster (2007). This covers the case of being able to explain the joint evidence and not just each single piece of evidence in isolation, or sequentially, see Ericsson and Hendry (1999). Specifically, we show, with the aid of a VAR representation of the inflation system, that the joint evidence cannot be explained by the NPC and its underlying stationary economic theory. For example, the two findings that the forward and lagged inflation terms have coefficients that are significantly different from zero, corroborate the hybrid NPC. However if the two coefficients sum to unity, which they typically do in the studies that report supportive evidence, the implication may be that the inflation has a unit-root. Several aspects of the economic interpretation of the NPC becomes problematic if a unit-root is implied by the evidence reported in favour of the NPC.

The paper is organized as follows: In section 2 we review briefly the NPC model and the associated rational expectations solution. In section 3 we review the empirical results on the NPC, from a consensus perspective. With one modification, this results in the same set of ‘stylized facts’ about the NPC as listed in GGL (2005). In section 4, we embed the NPC in the vector autoregressive model, VAR, which provides the natural statistical model for linear dynamic relationships in economics. The purpose is to investigate joint implication of the stylized facts about the NPC within the framework of the VAR. In section 2 the question of congruence between the rational expectations solution and a (long-run) unit-root in the data generating VAR is briefly discussed. In section 5 we use our framework to re-assess the evidence on US and euro area data. Section 6 concludes.

2 The NPC and the rational expectations solution

The hybrid NPC which has become standard is given as

\[
\pi_t = a^0 E_t [\pi_{t+1}] + a^1 \pi_{t-1} + b s_t + \varepsilon_{\pi_t},
\]

where \(\pi_t\) is the rate of inflation, \(E_t [\pi_{t+1}]\) is the expected rate if inflation in period \(t + 1\), given the information available for forecasting at the end of period \(t\). \(s_t\) is the time series of firms’ real marginal costs. \(\varepsilon_{\pi_t}\) is a disturbance term with zero mean. In many applications, notably GG and GGL, the disturbance term is omitted, which suggests a stronger interpretation which is often referred to as the NPC holding in “exact form”. We will also consider the exact form, in section 4 below, but for the time being we keep the more traditional formulation with a disturbance term.
The ‘pure’ NPC is specified without the lagged inflation term \( (\alpha^b = 0) \). In the case of the pure NPC, Roberts (1995) has shown that several New Keynesian models with rational expectations have (1) as a common representation — including the models of staggered contracts developed by Taylor (1979, 1980) and Calvo (1983), and the quadratic price adjustment cost model of Rotemberg (1982).\(^1\)

The rationale for allowing \( \alpha^b > 0 \) is that the theory applies to a (significant) portion of optimal price adjustments in period \( t \), but not to all. Hence, in each period, a share of the overall rate of inflation is determined by last period’s rate of inflation, for example because of backward-looking expectations. In the following, the third variable in (1), \( s_t \), is the logarithm of the wage-share, which is the operational definition of firms’ marginal costs of production.

It is instructive to consider the closed form solution when \( s_t \) is regarded as a forcing variable which is not Granger caused by \( \pi_t \). Mavroeidis (2005) shows that identification of the parameters of the PCM requires that \( s_t \), follows a \( k’ \)th order autoregressive process for \( s_t \):

\[
s_t = c_{s1}s_{t-1} + \cdots + c_{sk}s_{t-k} + \varepsilon_{s,t}
\]

(1) and (2) define the NPC model. For simplicity we assume that the two disturbances \( \varepsilon_{\pi,t} \) and \( \varepsilon_{s,t} \) are independently normally distributed variables.

Following Bårdsen et al. (2005, Appendix A) we obtain the partial solution for \( \pi_t \) as

\[
\pi_t = r_1\pi_{t-1} + b\sum_{i=0}^{\infty}(r_2^i - r_1^i)E_t\pi_{t+i} + \frac{1}{a^f r_2} \varepsilon_{\pi,t}
\]

where \( r_1 \) and \( r_2 \) are the two roots:

\[
r_i = \frac{1}{2a^f} \pm \frac{1}{2} \sqrt{\left( \frac{1}{a^f} \right)^2 + 4(-\frac{\alpha^b}{a^f})}, \quad i = 1, 2.
\]

We define the root \( r_1 \) (often called the “stable” root) as

\[
r_1 = \frac{1 - \sqrt{1 - 4a^f\alpha^b}}{2a^f}.
\]

To find the full (closed form) solution for inflation we use (2) to obtain the rational expectation solution for \( E_t\pi_{t+i} \). If we define \( ss_t = (s_t, \ldots, s_{t-k+1})' \), \( \varepsilon_t = (\varepsilon_t, 0, \ldots, 0)' \) and the companion matrix

\[
C_s = \begin{pmatrix}
    c_{s1} & c_{s2} & \cdots & c_{sk} \\
    1 & 0 & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & \cdots & 1 & 0
\end{pmatrix},
\]

(2) can be expressed as

\[
ss_t = C_s ss_{t-1} + \varepsilon_t
\]

\(^1\)The overlapping wage contract model of sticky prices is also attributed to Phelps (1978).
Hence,
\[ E_t(s_{t+i}) = C_s's_t \quad \text{and} \quad E_t(s_{t+i}) = e'C_s's_t, \]
where \( e = (1, 0, \ldots, 0)' \). If all the eigenvalues of \( \frac{1}{r_2}C_s \) have modulus less than 1,
\[ \sum_{i=0}^{\infty} \left( \frac{1}{r_2} \right)^i E_t s_{t+i} = \sum_{i=0}^{\infty} e' \left( \frac{1}{r_2}C_s \right)^i s_t = e' \left( \sum_{i=0}^{\infty} \left( \frac{1}{r_2}C_s \right)^i \right) s_t = e' (I - \frac{1}{r_2}C_s)^{-1} s_t. \]
The solution (3) becomes:
\[ \pi_t = r_1 \pi_{t-1} + \frac{b}{a' r_2} K_{s1} s_t + \cdots + \frac{b}{a' r_2} K_{sk} s_{t-k+1} + \frac{1}{a' r_2} \varepsilon_{s,t}, \]
where \( K_{s1} = 1/(1-c_{s1}(\frac{1}{r_2})-\cdots-c_{s1}(\frac{1}{r_2})^k) \) and \( K_{si} = (c_{si}(\frac{1}{r_2})+\cdots+c_{sk}(\frac{1}{r_2})^{k-i+1})/(1-c_{s1}(\frac{1}{r_2})-\cdots-c_{s1}(\frac{1}{r_2})^k) \), \( i = 2, \ldots, k \).
For concreteness we consider the special case of \( k = 2 \), which is also sufficient for identification. In the case of \( k = 2 \), the constants \( K_{s1} \) and \( K_{s2} \) can be expressed as
\[ K_{s1} = -\frac{r_2}{r_{s2} - r_{s1}} \left\{ \frac{r_{s1}}{r_2} - \frac{r_{s2}}{r_2} \right\} \]
\[ = \frac{1}{1 - \frac{1}{r_2}(c_{s1} + \frac{1}{r_2} c_{s2})}, \]
\[ K_{s2} = \frac{r_2}{r_{s2} - r_{s1}} \left\{ \frac{r_{s2} r_{s1}}{r_2} - \frac{r_{s1} r_{s2}}{r_2} \right\} \]
\[ = \frac{c_{s2}}{r_2} K_{s1}. \]
\( r_{si} (i = 1, 2) \) are the roots of the characteristic equation associated with (2). It is important to note that these expressions are based on the assumption
\[ \left| \frac{r_{si}}{r_2} \right| < 1 \quad \text{for} \quad i = 1, 2, \]
see Nymoen (2009), which therefore is also a necessary and sufficient condition for the closed form Rational Expectations (RE) solution in (4), i.e., for the case of \( k = 2 \). Because \( r_{s1}, \ldots, r_{sk} \) are the eigenvalues of \( C_s \), the assumption (7) is in the general case the condition that the eigenvalues of \( \frac{1}{r_2}C_s \) have modulus less than 1.
If we assume \( |r_{s1}| < 1 \) and \( |r_{s2}| < 1 \) so that \( s_t \) is stationary, integrated of degree zero, \( s_t \sim l(0) \), and condition (7) holds, the RE solution for \( \pi_t \) exists. We now consider two cases. First, if \( r_1 < 1 \) in (4), the rational expectations driven inflation rate is integrated of degree zero, \( \pi_t \sim l(0) \). Second, if \( r_1 = 1 \), the RE solution implies \( \pi_t \sim l(1) \). In this case, the inflation rate is non-stationary while the forcing variable is stationary. Hence in this case there cannot be cointegration between \( s_t \) and \( \pi_t \), even though \( s_t \) Granger causes \( \pi_t \).
As we shall see below, the unbalanced case \( \pi_t \sim l(1) \) and \( s_t \sim l(0) \) is of empirical relevance since many estimated NPC models are homogenous and obey the
restriction \( a^f + a^b = 1 \). In this case the NPC-roots become:

\[
\begin{align*}
    r_1 &= 1 \\
    r_2 &= \frac{1}{a^f - 1}.
\end{align*}
\]

Since the largest \( s \)-root \( (|r_{si}|_{\text{max}} = \max_{1 \leq i \leq 2} |r_{si}|) \) is less than 1 in the case of \( s_t \sim l(0) \), \( |r_2| < 1 \) does not necessarily violate requirement (7). However, the size of the largest \( s \)-root defines a limit for \( a^f \):

\[
|a^f| < \frac{1}{|r_{si}|_{\text{max}} + 1}, \text{ when } a^f + a^b = 1,
\]

so when for example, \( |r_{si}|_{\text{max}} = 0.95 \), we need \( a^f < 0.51 \) for consistency with (7).

The above discussion generalises to the case of a \( k \)th order \( s_t \)-process in (2).

In the general case, \( |r_{si}|_{\text{max}} \) is defined as \( |r_{si}|_{\text{max}} = \max_{1 \leq i \leq k} |r_{si}| \).

(10) also applies to the case where \( |r_{si}|_{\text{max}} = 1 \) and therefore \( s_t \sim l(1) \). That there is unique rational expectations solution also in the case when both \( \pi_t \) and \( s_t \) are non-stationary is consistent with the seminal paper by Blanchard and Kahn (1980). In this sense we have rational expectation theory of a non-stationary rate of inflation. However, we need \( |a^f| < 0.5 \) in order for the homogenous NPC to be consistent with the RE solution in (4). As we shall see below, the hypothesis of homogeneity is often tested in the literature, and with non-rejection as a result. By itself that finding is consistent with RE solution. However, the necessary requirement of \( a^f < 0.5 \) is not a typical finding in the empirical NPC literature.

### 3 Empirical results about the NPC

The main references supporting the NPC are the mentioned papers by GG and GGL (2001, 2005) who assert that the following three results are proven characteristics of NPC for all data sets:

1. **Forward dominance** The two null hypotheses of \( a^f = 0 \) and \( a^b = 0 \) are rejected both individually and jointly. The coefficient on expected inflation exceeds the coefficient on lagged inflation substantially. Typically, researchers conclude along the line of:
   
   \[ a^f > 0.5 > a^b > 0. \]

2. **Homogeneity** The hypothesis of \( a^f + a^b = 1 \) is typically not rejected at conventional levels of significance, although the estimated sum is usually a little less than one numerically.

3. **Strong forcing variable** When real marginal costs are proxied by the log of the wage-share, the coefficient \( b \) is positive and significantly different from zero at conventional levels of significance.

Critics of the NPC have challenged the robustness of all three Results, but with different emphasis and from different perspectives. The inference procedures and estimation techniques used by GG and GGL (2001) have been criticized by Rudd
and Whelan (2005,2007), and others, but GGL (2005) show that their initial Results 1 and 2 remain robust to these objections. The homogeneity restriction seems to be accepted as typical in both camps, see e.g., Rudd and Whelan (2006) and Chao and Swanson (2009).

However, there is still an issue about the robustness of Result 3, and GGL (2005) seem to overlook the fact that several researchers have found it difficult to confirm the view that the wage-share is a robust explanatory variable in the NPC. Already Bårdsen et al. (2004) showed that the significance of the wage share in the GGL (2001) model is fragile, as it depends on the exact implementation of the GMM estimation method used, thus refuting that Result 3 is a robust feature of NPC estimated on euro-area data. Fanelli (2008) using a vector autoregressive regression model on the euro-area data set, find that the NPC is a poor explanatory model. On the US data, Mavroeidis (2006) have shown that real marginal costs appears to be an irrelevant determinant of inflation, confirming the view in Fuhrer (2006) about the difficulty of developing a sizeable coefficient on the forcing variable in the US NPC.

It appears therefore that researchers ought to be able to agree that Results 1 and 2 are robust features of the NPC, while a consensus version of Result 3 might be:

### 3* Fragile forcing variable

The numerical and statistical significance of $b$ will vary, depending on the operational definition of real marginal costs, the estimation method and the sample.

In the next section we show that these facts about the NPC have interesting joint implications. Specifically, we show that the joint of occurrence of rejection of $b = 0$ (as covered by Result 3*), and non-rejection of the hypotheses covered by Results 1 and 2 may imply that the NPC is without economic predictive power for inflation, since the model then reduces to a random walk model. Another case where this occurs is when the (strong) version of Result 3 is granted ($b > 0$), and $a^f + a^b = 1$ holds, but $s_t$ is a strongly exogenous variable.

## 4 VAR implications for stationarity and cointegration

We make use of the VAR representation of the time series of inflation, $\pi_t$, and the wage-share, $s_t$, to define the possible parameter constellations of the NPC in (1). Without loss of generality, we assume that any deterministic shifts (cause of non-stationarity) have been removed from the two variables, hence the discussion is based on the assumption that the vector of time series $Y_t = (\pi_t, s_t)'$ is either I(1) or I(0), and represented by a VAR of finite order $p$. Hence we consider the model

$$Y_t = A_1 Y_{t-1} + \ldots + A_p Y_{t-k} + \varepsilon_t$$

for fixed values of $Y_{-k+1}, \ldots, Y_0$, and a zero-mean disturbance $\varepsilon_t$.

Our premise is that (1) implies that the parameter vector $(a^f + a^b - 1, b)'$ is in the cointegration space defined by (11), which in turn is determined by the rank of the matrix $\Pi$:

$$\Pi = \sum_{i=1}^{p} A_i - I$$

6
where $I$ denotes the 2 dimensional identity matrix.

1. $\text{rank} = 2$. In this case $(\pi_t, s_t)'$ is stationary, $l(0)$. There are two separate long-run means, which we denote $m_\pi$ and $m_s$, corresponding to the steady-state solution of the system. In this case $(a^f + a^b - 1, b)'$ defines a linear combination of the two long-run means, i.e., by taking the unconditional mean on both sides of (1) we obtain:

\[ (a^f + a^b - 1) m_\pi - b m_s = 0. \]

2. $\text{rank} = 1$. There is only one cointegration vector, i.e., the cointegration space is spanned by $(a^f + a^b - 1, b)'$.

(a) $a^f + a^b \neq 1$ and $b \neq 0$. $\pi_t$ and $s_t$ are $l(1)$ and cointegrated.

(b) $a^f + a^b = 1$ and $b \neq 0$. $\pi_t$ is $l(1)$, $s_t$ is $l(0)$.

(c) $a^f + a^b \neq 1$ and $b = 0$. $\pi_t$ is $l(0)$, $s_t$ is $l(1)$.

3. $\text{rank} = 0$. $(\Delta \pi_t, \Delta s_t)'$ is stationary, $l(0)$

Case 1, where both inflation and the forcing variable are stationary, $l(0)$, hence $\text{rank} = 2$, may be seen as the reference case. As noted by Fanelli (2008) this is how the variables are treated when (1) is estimated by GG and GGL(2001), and it is part of the rationale for inflation targeting regimes which takes the stationarity of inflation as a premise. This does not rule that inflation can be highly persistent over many samples. Case 1 accommodates Result 1 above, and both versions of Result 3 as well. Regarding Result 2, care must be taken to distinguish between the case of strongly exogenous $s_t$, and the case of Granger causation between lags of inflation and $s_t$. In the case of strong exogeneity of $s_t$, Result 2: $a^f + a^b = 1$ becomes inconsistent with $\text{rank} = 2$. The reason is that $a^f + a^b = 1$ implies that the characteristic equation associated with the matrix $II$ has one unit root, as the appendix shows.

If Result 2, $a^f + a^b = 1$, is granted, the only way to maintain the idea of $l(0)$ inflation is that the wage-share $s_t$ is Granger caused by inflation, see Bårdsen et al. (2004). This is of course nothing in the theory which rules out this possibility, but on the other hand it is not implied by the theory either. In any case, econometric studies which estimate the NPC, GG and GGL (2001) are examples, do usually not consider this possibility, in any other way than through the use of GMM estimation with a wide instrument set. Another, remark is that the RE solution, obtained in section 2, is based on one-way causation with $s_t$ as a forcing variable.

When $\text{rank} = 1$, as in Case 2, there are several possibilities which are of interest. First, in Case 2a, with $a^f + a^b \neq 1$ and $b \neq 0$, the NPC equation can be interpreted as a cointegration equation between the two $l(1)$ variables $\pi_t$ and $s_t$. In an econometric perspective, identified cointegration relationships represent partial structure, because they are invariant to omitted stationary variables, and as such they are usually regarded as interesting entities. We do not know of economic theoretical work on the NPC which covers the case of integration and cointegration from first principles. In any case, it is important to note that Case 2a hinges on non-homogeneity of the hybrid NPC, which is contradicted by Result 2 above. Fanelli and Palomba (2010) discuss the cointegrating implications of homogeneity.
and non-homogeneity in the NPC without attaching any structural interpretation to the implied restrictions.

Case 2b is the constellation of \( a^I + a^b = 1 \), and \( b \neq 0 \). The rate of inflation is \( I(1) \), contradicting the underlying assumption of a stationary inflation rate.\(^2\) Case 2c is the last constellation which is consistent with stationary inflation. However, Case 2c cannot be reconciled with the idea that marginal cost is the explanatory variable of inflation, since \( b = 0 \) in the cointegrating vector.

Case 2 is consistent with the following important insight which is due to Fuhrer (2006): The typical NPC fails to deliver the result intended by its inventors, namely that inflation persistence is ‘inherited’ from the persistence of the forcing variable. Instead, the derived inflation persistence, using estimated NPCs, turns out to be completely dominated by ‘intrinsic’ persistence (due to the accumulation of disturbances of the NPC equation). Fuhrer gives the following succinct summary: the lagged inflation rate is not a ‘second order add on to the underlying optimizing behaviour of price setting firms, it is the model’. Case 2 matches this analysis exactly: In Case 2a and 2b, inflation is \( I(1) \) and therefore follows a stochastic trend — persistence is intrinsic. In Case 2c inflation is \( I(0) \) but \( s_t \) has a zero coefficient in the “NPC”, so persistence is again due to lagged inflation.

In Case 3, with \( \text{rank} = 0 \), the vector hypothesized by the NPC takes the form \((a^I + a^b - 1, b)^\prime = (0, 0)^\prime\), hence the economic content/interpretation of the NPC has no counterpart in the properties of the VAR. Note that this constellation is consistent with the empirical results 1, 2 and 3*.

In sum it emerges that the stylized facts about the NPC in most cases contradicts any intial assumption about stationarity of the rate of inflation. the economic theory underlying the NPC equation (1). The difficulty of reconciling the empirical facts 1-3 of the NPC with stationarity of inflation is also relevant for the new generation of macro models called dynamic stochastic general equilibrium (DSGE) models. DSGE models represent inflation and real marginal costs as stationary, since both variables are modelled as deviations from their respective steady states (see Del Negro and Wouters (2006)), thus implying that a stationary solution always exists.

The above analysis also suggests that a coherent testing procedure for NPC could be based on a VAR which as a minimum should contain inflation and, for example, the wage-share (as an operational definition of the forcing variable), see Bårdsen et al. (2004) and Fanelli (2007). Two important studies using such an approach, by Fanelli and Ch 3(JuseM07), show that the stationarity assumption is difficult to maintain for euro-area and US data, a point that we return to below. More generally, it appears that the system perspective is underplayed in the NPC literature, which is dominated by single equation estimation by GMM. Though robust in a formal statistically sense, the use of GMM on a single NPC equation obscures the issue about the endogeneity or exogeneity of \( s_t \), which we have seen is all-important for interpretation. Moreover, as noted by Mavroeidis (2006) endogeneity of \( s_t \) is an important issue for estimation since this enhances identification, because it provides more strong instruments for identifying expected inflation.

\(^2\)It is quite common to investigate the time series properties of \( \pi_t \) and \( s_t \) separately, and to conclude that \( \pi_t \sim I(1) \) and \( s_t \sim I(0) \). In our analysis this correspond to having \( s_t \) as exogenous and finding that the \( \text{VAR} \) has one unit-root, corresponding to \( a^I + a^b = 1 \).
GMM, as routinely applied, requires stationary variables for the statistical inference to be valid. Evidence of non stationary variables, e.g., in the form of unit root tests, should therefore lead to a reconsideration of the available evidence, and the framework for inference on which it is based. The use of “t-values” from GMM, which assumes a stationary statistical model is then formally unwarranted. A related problem is that, when the Calvo framework is used to derive NPC, a first order linear approximation around a steady state inflation is used. This represents a poor approximation when inflation is non stationary on the basis of the evidence presented, this highlights the importance of developing interaction between theory and empirical results both ways.

5 Re-interpreting the evidence for US and euro area NPCs

Above we have showed that although sequential testing, as summarized in Result 1-3, can result in a positive assessment of the model, care must be taken to also evaluate the joint evidence.

As a first example we reconsider the claim that the NPC is a success when tested on US data. A representative estimate of GG’s (1999) pure NPC, referred to by for example Kurmann (2005), is

\[
\pi_t = E\pi_{t+1} + 0.035s_t
\]

where standard errors are in parentheses. \(a^f\) has been restricted to 1 on the basis that the typical estimate is very close to one (Result 1 and 2). The estimate of \(b = 0.035\) is positive, and significantly different from zero (Result 3). Moreover, the fit of this model, when a VAR in \(\pi_t\) and \(s_t\) of the 4th order is used to generate forecasts of the wage-share, is very impressive as documented by figure 1 in Kurmann (2005).

However, as shown above, the joint evidence may entail refutation of stationarity. Equation (13) imposes a unit-root on the system and there is little evidence of a inflation causing the \(s_t\): Kurmann (2005) calculates the Granger causality tests and finds that the null hypothesis that inflation does not Granger cause \(s_t\) can only be rejected at a marginal confidence level of 0.22. Jointly, the homogeneity of (13), and strong exogeneity of \(s_t\) implies refutation of the NPC as a model of stationary inflation.

This conclusion carries over to the GG’s US hybrid NPC, since \(a^f + a^b = 1\) is a restriction that all authors find to be acceptable (see e.g., GaliGertLope05, and because Kurmann’s causality test covers the 2nd order VAR in \(\pi_t\) and \(s_t\) that is implied by the hybrid version of the NPC.

In their recent study Boug et al. (2010), tests the restriction that the rational expectations solution implies for a VAR in \(\pi_t\) and \(s_t\). Unlike the single equation estimation by GMM, this methodology, as just noted, makes it more easy to appreciate the joint evidence. They find that for the Gali and Gertler data set, statistical tests allow that the rank of \(\Pi\) can be set to 2, which taken at face value is consistent

\footnote{Specifically, the model “Restricted \(\beta\) (1)” in Table 1 in Gali and Gertler (1999). The sample period is 1960(1)-1997(4).}
with the existence of an long-run mean and stationarity of US inflation. However, the choice between full rank and \(rank(\Pi) = 1\) is a marginal one. Moreover, using maximum likelihood estimation, Boug et al. (2010) obtain \(\hat{a}^I + \hat{a}^s = 1.03\) and an estimate of \(b\) which is numerically low and statistically insignificant. Taken, together the implication is that the VAR models US inflation need to be replaced by a VAR in differences, refuting the underlying economic theory of the NPC and any associated stationay rational expectations solution.\(^4\)

Gali et al. (2001) present evidence for *euro area inflation* which they interpret as supportive of the NPC model, i.e. the evidence is in line with Result 1., 2. and 3. above. Bårdsen et al. (2004) contain the following replication:

\[
\pi_t = 0.68 \pi_{t+1} + 0.28 \pi_{t-1} + 0.019 s_t
\]

which is different from Gali et al. (2001) in that the statistical (as well as numerical) significance of the wage-share is not apparent. In fact, Bårdsen et al. (2004) do a sensitivity analysis that show that the sign of the estimated coefficient of \(s_t\) becomes negative when the weighting method used in GMM estimation is changed, and back again to positive when there is seemingly small change in the list of instrumental variables. Hence, our suggestion above, that Result 3\(^*\) gives a more balanced summary of the evidence than the unconditional statement found in Results 3.

The analysis in Bårdsen et al. (2004) show that Result 1 and 2 apply to the euro area. In particular, the sensitivity analysis shows that the sum \(\hat{a}^I + \hat{a}^s\) ranges from 0.96 to 1.07, and the results entail that \(a^I + a^s = 1\) cannot be rejected except with very high p-values. Since the significance of the wage share is fragile (suggesting \(b \approx 0\)), the joint evidence shows that there is only a weak argument for \(rank(\Pi) = 2\) for the euro area data. The case of \(rank(\Pi) = 1\) as we have seen, refutes the NPC model: Either inflation is stationary, but with \(b = 0\), so inflation may cause the labour share, but not vice versa. Alternatively, \(b \neq 0\) but then \(\pi_t = I(1)\), meaning that the maintained assumption of stationary inflation dynamics is contradicted by the evidence.

Boug et al. (2010) also contains estimates VAR for the euro-area, using the GGL (2001) data set. In this case the formal statistical tests gives \(rank(\Pi) = 1\) as the conclusion. The maximum likelihood estimation gives \(a^I + a^s = 1.05 \ b = 0.002\). If 1.05 is regarded as significantly different from one, while the null of \(b = 0\) cannot be rejected, we have then formal support for case 2c above, with stationary inflation, but with non-stationarity wages share. Based on these results, the euro-area NPC reduces to an autoregressive model in differences.

### 6 Summary and discussion

It has become a widely held belief that the NPC represents a significant advance in inflation modelling which finally substantiates the theory of dominating forward-looking behaviour in price adjustment. Even though pieces of empirical evidence

\(^4\)Barkbu and Batini (2005) use the same method, due to Johansen and Swensen (1999), for Canadian data. Their full sample results give a single cointegration relationship with \(a^I + a^s = 1\), which fits into category 2b in our typology. Hence: \(\pi_t \sim I(1)\) and \(s_t \sim I(0)\), or \(\pi_t \sim I(2)\) if \(s_t \sim I(1)\) is assumed from the outset.
individually may corroborate an economic theory, their joint existence may refute that same theory. Hence, when taking toll of the evidence, one need to have an awareness of the possibility that some of the findings might be internally inconsistent, or that their joint existence may entail refutation of important aspects or assumption in the economic theory.

Above we have seen that the joint evidence on the NPC in the euro area and in the US, suggest that inflation has a unit-root. This is because dynamic price homogeneity and an strongly exogenous process for the wage share imply non stationary inflation, which refutes an initial assumption about stationarity, as seen for example in the nature of the solution of the DSGE models which gives stationary solution for inflation and real marginal costs.

In a weaker interpretation, one may allow a unit root in the system determining inflation and real marginal costs for statistical purposes, and regard cointegration between inflation and the wage-share as an interesting finding in its own right. However, whether an economic theory can be established in support for the NPC as an cointegration relationship remains an open issue. We did however, find one special case where the rational expectation solution implied non-stationarity for inflation, with or without a corresponding non-stationarity in the forcing variable. However, that solution hinges on “backward dominance” in the NPC, which again is the opposite of the stylized facts reported as supportive of the NPC.

To the extent that the issues discussed in this paper represent a problem for the status of the NPC, the explanation seems to lie in three distinct aspects of the methodology used in the leading papers in the NPC literature. First, a strong a priori belief in the theory of a single variable forcing inflation. This marks a break with the earlier literature on inflation modelling, which typically included a wide set of explanatory variables, motivated both by theory and by economic and political history, see e.g., Sargan (1980), Nymoen (1991), Rowlatt (1992) and Gordon (1997). Second, the most influential NPC papers have used single equations models, while many aspects of inflation are probably better modelled with the use of a system approach. Simultaneity and/or joint Granger causation between inflation and the forcing variable(s) are two examples. Third, there has been little interest in testing the new inflation model’s econometric performance compared to pre-existing models. Potentially, the NPC represents an important example of, a mainly theoretically specified, model which encompasses a whole class of earlier models—but such a claim have not been substantiated so far.

Models which have been developed parallel to the NPC, shows that inflation can be non-stationary due to regime-shifts rather than unit-roots (so \(\pi_t\) is \(I(0)\) conditional on such breaks). These models avoid the internal inconsistencies of the NPC and they have been shown to be empirically successful, see e.g., Bårdsen and Nymoen (2003). The framework used in that program also allows the specification of testable hypotheses about lead-variables, see e.g. Bjørnstad and Nymoen (2008). At the same time, a second generation of NPC models have now arrived, see Blanchard and Gali (2007). These models augment the Phillips curve with “new” explanatory variables, e.g., the rate of unemployment, and are also used together with bargaining models of wage setting, e.g., Rossi and Fabrizio (2008). Assessment of these developments within in a common econometric framework is an interesting area for future research.
A The homogeneity restriction

Lemma 1 Consider the vector auto-regressive (VAR) model of the form

\[ Y_t = A_{t-1}Y_{t-1} + \cdots + A_{t-k}Y_{t-k} \epsilon_t \]

where \( \epsilon_{k+1}, \ldots, \epsilon_T \) are independent variables with covariance \( \Omega \). Let \( A(z) \) be the associated characteristic polynomial. Assume that the solutions of \( \det[A(z)] = 0 \) are equal to \( z = 1 \) or have modulus larger than 1.

The two conditions

\[ i) \quad \pi_t = a^f E_t[\pi_{t+1}] + a^b \pi_{t-1} + b \epsilon_t, \text{where } a^f + a^b = 1, \ a^f \neq 0 \]

and

\[ ii) \ \pi_t \text{ does not Granger-cause } s_t, \]

imply that the determinant of the characteristic polynomial of the VAR-model equals zero at \( z = 1 \).

If furthermore \( \Gamma = -\frac{d}{dz} A(z)|_{z=1} - \Pi \), then a necessary and sufficient condition for \( \{Y_t\} \) to be integrated of order 1 is that \( \Gamma_{22} \neq 0 \).

Proof. Assume a well modelled bivariate VAR with \( k \) lags

\[ Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \cdots A_k Y_{t-k} + \epsilon_t, \]

where \( \epsilon_t, \ t = 1, \ldots \) are independently distributed errors, with mean zero. The associated characteristic polynomial is given by

\[ A(z) = I - A_1 z - A_2 z^2 - \cdots - A_k z^k. \]

For convenience let \( k = 3 \), without loss of generality, in the following. Granger non-causality for \( s_t \) implies

\[ A(z) = I - \begin{pmatrix} a_{11}^{(1)} & a_{12}^{(1)} \\ 0 & a_{22}^{(1)} \end{pmatrix} z - \begin{pmatrix} a_{11}^{(2)} & a_{12}^{(2)} \\ 0 & a_{22}^{(2)} \end{pmatrix} z^2 - \begin{pmatrix} a_{11}^{(3)} & a_{12}^{(3)} \\ 0 & a_{22}^{(3)} \end{pmatrix} z^3. \]

By introducing the homogeneity restriction we can write the exact form of (14) as

\[ (a^f + a^b) \pi_t = a^f E_t[\pi_{t+1}] + a^b \pi_{t-1} + b \epsilon_t \]

\[ a^f \Delta E_t[\pi_{t+1}] = a^b \Delta \pi_t - b \epsilon_t, \]

where \( a^f \neq 0 \). Note that we follow GG and GGL and use the “exact form” of the NPC here, which is not a trivial simplification and we will investigate the NPC with disturbance term at a later stage.
Leading (15) one period and taking expectation conditioned on the information set in period $t$, we get

$$E_t Y_{t+1} = A_1 Y_t + A_2 Y_{t-1} + A_3 Y_{t-2}.$$ 

Subtracting by $Y_t$ on both sides yields

$$E_t \Delta Y_{t+1} = (A_1 - I) Y_t + A_2 Y_{t-1} + A_3 Y_{t-2}.$$ \hfill (18)

The first line in (18) is given by

$$E_t \Delta \pi_{t+1} = \begin{pmatrix} a_{11}^{(1)} - 1 \end{pmatrix} \pi_t + a_{11}^{(2)} \pi_{t-1} + a_{11}^{(3)} \pi_{t-2} + \ldots$$ \hfill (19)

Comparing (17) and (18), gives the following parameter restrictions

$$-a_{11}^{(2)} = a_{11}^{(1)} - 1, \quad a_{11}^{(3)} = 0.$$ 

This puts further restrictions on the characteristic polynomial

$$A(z) = I - \begin{pmatrix} a_{11}^{(1)} & a_{12}^{(1)} \\ 0 & a_{22}^{(1)} \end{pmatrix} z - \begin{pmatrix} a_{11}^{(2)} & a_{12}^{(2)} \\ 0 & a_{22}^{(2)} \end{pmatrix} z^2 - \begin{pmatrix} 0 & a_{12}^{(3)} \\ 0 & a_{22}^{(3)} \end{pmatrix} z^3$$

$$= \begin{pmatrix} 1 - a_{11}^{(1)} z - a_{11}^{(2)} z^2 & \begin{pmatrix} 1 - a_{12}^{(1)} z - a_{12}^{(2)} z^2 - a_{12}^{(3)} z^3 \\ 0 & 1 - a_{22}^{(1)} z - a_{22}^{(2)} z^2 - a_{22}^{(3)} z^3 \end{pmatrix} \\ 0 & \begin{pmatrix} 1 - a_{11}^{(1)} z - \begin{pmatrix} 1 - a_{11}^{(1)} z^2 & A_{12}(z) \\ 0 & A_{22}(z) \end{pmatrix} \end{pmatrix}$$

The determinant of $A(z)$ is given by

$$\det[A(z)] = \det \left[ \begin{pmatrix} 1 - a_{11}^{(1)} z - \begin{pmatrix} 1 - a_{11}^{(1)} z^2 & A_{12}(z) \\ 0 & A_{22}(z) \end{pmatrix} \end{pmatrix} \right].$$

Because $1 - a_{11}^{(1)} z - 1 + a_{11}^{(1)} = 0$, it follows that $\det(A(1)) = 0$, which implies that $\det A(1) = 0$ has a unit root at $z = 1$ and hence $Y_t$ is not $I(0)$.

It remains to show that $Y_t$ is $I(1)$. From (17) it follows that the vector $\beta = (0, b)'$ belongs to the cointegration space. The error correction coefficients are the elements of $\alpha = (1, 0)'$ due to the Granger non-causality. Therefore $\beta_\perp = (1, 0)'$ and $\alpha_\perp = (0, 1)'$, and $\alpha_\perp' \Gamma \beta_\perp = \Gamma_{22}$, which is not zero, proving the result. 

**References**


