TITLES AND ABSTRACTS
ROSENDAL AUGUST 19–25 2005

Vigleik Angeltveit (MIT)

Topological Hochschild cohomology of Morava K-theory

Let E_n be Morava E-theory and $K_n = E_n/(p, u_1, \dots, u_{n-1})$ be 2-periodic Morava K-theory. It turns out that there are infinitely many A_∞ structures on K_n, and even many homotopy classes of multiplications, and that $THH(K_n)$ depends on the A_∞ structure in a way I will try to make precise. Because the canonical map $E_n \to K_n$ is central, it extends to a map $E_n \to THH(K_n)$, and it turns out that this map is an equivalence if a certain matrix that describes the non-commutativity of the multiplication on K_n is invertible.

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Christian Ausoni (Bonn)

Algebraic K-theory of ring spectra and Bott periodicity

One can construct infinite families of classes in the algebraic K-theory of a number ring R by mapping its units into $K_1(R)$ and using Bott periodicity. We show that similar constructions hold also for more general structured ring spectra.

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Maria Basterra (New Hampshire)

Obstruction theory for E_n structures on spectra

Working on the category of S-modules we can describe an obstruction theory to E_n structures on connective spectra in terms of derivations. We use this theory to show that the multiplication on the Brown–Peterson spectrum is at least E_4.

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Andrew Blumberg (Chicago)

A localization sequence for $K(ku)$

In this talk, I will describe a cofiber sequence of spectra $K(\mathbb{Z}) \to K(ku) \to K(KU)$ which arises from the localization $ku \to KU$. This is joint work with Mike Mandell.

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Bob Bruner (Wayne State)

Differentials in the homological homotopy fixed point spectral sequence

We show that for a $\mathbb{T}$-equivariant commutative S-algebra R such as $R = THH(B)$, the differentials in the spectral sequence for the (continuous) homology of the homotopy fixed points are determined in a very strong way by the action of the Dyer–Lashof algebra. All but a finite number of the Dyer–Lashof operations cancel one another, while the finite number remaining persist to E^∞. This permits a complete determination of the spectral sequence in a number of interesting cases, relevant to Rognes' approach to calculating the algebraic K-theory of commutative S-algebras.

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Gunnar Carlsson (Stanford)

Some ideas on the K-theory of fields

In this talk we will discuss how ideas from equivariant K-theory, as well as some specific properties of absolute Galois groups, can be put together to study the algebraic K-theory of fields.

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Daniel G. Davis (Purdue)

Iterated homotopy fixed points for the Lubin–Tate spectrum

Let E be the nth Lubin–Tate spectrum, and let G be the extended nth Morava stabilizer group, which acts on E. Let H and K be closed subgroups of G, with H normal in K. Let $F = K/H$. Then F is a profinite group that acts on E^{hH}, the H-homotopy fixed points of E. When K/H is finite, Devinatz and Hopkins showed that $(E^{hH})^{hF}$ is just E^{hK}. However, when F is not finite, it is not even known how to construct this iterated homotopy fixed point spectrum. We partially remedy this by showing that, when F is not finite, E^{hH} is a continuous F-spectrum, and we construct its F-homotopy fixed point spectrum and an associated descent spectral sequence.

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Ethan Devinatz (Washington)

Homotopy groups of homotopy fixed point spectra associated to E_n

I will discuss a general approach to computing homotopy groups of continuous homotopy G fixed points of E_n, where E_n is the Landweber exact spectrum whose coefficient ring is the Lubin–Tate moduli space of lifts of the height n Honda formal group law over the field of p^n elements, and G is a closed subgroup of the extended Morava stabilizer group $\mathbb{G}_n$. This approach is only practical if the continuous cohomology of G (with coefficients in the prime field) is easily calculable, but I will show how such cases might be used to obtain global information in stable homotopy theory related to finiteness properties of homotopy groups of $K(n)$-local spectra and to Brown–Comenetz duality. I will give explicit calculations for some G.

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Bjørn Dundas (Bergen)

Higher topological cyclic homology (joint with G. Carlsson)

Let A be a commutative $\mathbb{S}$-algebra. Then topological Hochschild homology can be iterated. We consider a model for iterated THH with a torus action and additional structure extending Hesselholt and Madsen's interpretation via the de Rham–Witt complex.

In particular, there are self-maps F_α (Frobenius), R_α (restriction) and V_α (Verschiebung) for each injective linear endomorphism α of $\mathbb{Z}_p^{\times n}$ and differentials d_l for subcircles of the n-torus. These satisfy formulas analogous to, but more interesting than, those in the one-dimensional situation.

The goal is to understand the chromatic red-shift conjecture.

—

Halvard Fausk (Oslo)

Stable equivariant homotopy theory for pro-finite groups

We extend the theory of equivariant orthogonal spectra from finite groups to pro-finite groups. A model structure is given on the category of pro-G-spectra for a pro-finite group G. This model structure is set up so that we get a well behaved Atiyah–Hirzebruch spectral sequence for pro-G-spectra. In particular, we get a homotopy fixed point spectral sequence.

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Imma Galvez (London)

Witten genus and quasimodular forms

We are going to consider the Witten genus as originally defined in the 80's. We observe that each of its coefficients makes sense whenever it is computed for a Spin manifold. We consider its image in the ring of quasimodular forms as defined by Zagier. This is a very interesting differential ring and has many good properties. The values of the genus have been studied in other works, and moreover we have recently found some more nice combinatorics associated to it. Questions about the relation between this invariant and other invariants taking values in the ring of quasimodular forms will be raised.

Along the lines of Landweber, Ravenel and Stong, a generalized cohomology theory associated to this genus can be constructed using some localization. Of course, this is to be closely related to complex oriented modified versions of topological modular forms spectra. The "theorem of the cube" holds as well for this genus. Cooperations are obtained as integral divided congruences, and their generating series satisfies differential equations, in particular a very simple PDE in terms of the modular parameter. Adams and Hecke operations make sense, and there is more room for "Ramanujan" operators than in elliptic cohomology . . .

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John Greenlees (Sheffield)

Brave new commutative algebra and modular representation theory

(joint with Benson, Dwyer and Iyengar)

Work of Dwyer, Greenlees and Iyengar has shown that if one formulates the Gorenstein condition in a homotopically invariant fashion it applies to so-called brave new commutative rings, and that it often implies a duality statement which includes many known interesting examples in topological contexts, together with some new ones.

The cohomology ring of a finite group has a Gorenstein-like duality property first established in the almost Cohen–Macaulay case by Benson and Carlson. This property is inherited by localizations of the cohomology ring, where it suggests an underlying structural property of complexity quotient categories investigated by Benson and Krause. The talk will describe how the framework of Dwyer, Greenlees and Iyengar can be adapted to apply to these complexity quotient categories to give a conceptual formulation of the conjectured structural properties, and how a homotopically invariant form of local duality gives a proof.

The talk may go on to discuss the role of the notion of complete intersections in brave new commutative algebra and modular representation theory.

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Mark Hovey (Wesleyan)

Comments on Freyd's generating hypothesis

In this informal presentation, I will discuss some results related to the generating hypothesis. I will give the proof that if the generating hypothesis holds, then the Brown-Comenetz dual of the sphere is the quotient of a self-map of a wedge of spheres. I will also prove that one result of the generating hypothesis, that the homotopy of a finite torsion spectrum is never finitely generated as a module over the homotopy of the sphere, is in fact true. And I will discuss some K-local results related to Devinatz' work on the generating hypothesis.

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Sverre Lunøe–Nielsen (Oslo)

A homological approach to topological cyclic theory

The affirmed Segal conjecture for groups of prime order can be phrased as a homotopy limit problem, namely whether the $\mathbb{Z}/p$-fixpoints of the equivariant sphere spectrum is p-adically equivalent to the homotopy fixpoints.

In the talk I will explain how we can generalize the homological calculations originally carried out by Lin ($p = 2$) and Gunawardema ($p > 2$) in the case of the Segal conjecture. The result is that the Segal conjecture for $\mathbb{Z}/2$ holds for $THH(BP)$.

The machinery developed can be applied to show that for the Johnson–Wilson spectra $BP\langle m\rangle$, fixpoints and homotopy fixpoints of $THH(BP\langle m\rangle)$ agree in homotopy above some degree after introducing suitable finite coefficients. This generalizes results by Bökstedt–Madsen ($H\mathbb{Z}_{(p)} = BP\langle 0\rangle$), Hesselholt–Madsen ($H\mathbb{F}_p = BP\langle -1\rangle$) and Ausoni–Rognes ($\ell = BP\langle 1\rangle$).

The work presented is joint with John Rognes and is part of a programme to calculate topological cyclic homology of commutative S-algebras using homological methods.

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Jacob Lurie (Harvard)

Elliptic cohomology and derived algebraic geometry

In this talk, we will review the classical point of view on elliptic cohomology, leading up to the construction of the spectrum of topological modular forms (tmf) by Hopkins and Miller. Then we will introduce the language of derived algebraic geometry and explain how it can be used to produce a new (and easier) construction of tmf. If time permits, we will describe how equivariant elliptic cohomology arises naturally in this context.

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Mike Mandell (Cambridge)

THH **of Waldhausen categories**

Definitions and basic properties of THH for Waldhausen categories. Work in progress with Andrew Blumberg.

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J. Peter May (Chicago)

Ancient history and Thom Thom spectra

I'll recall the starting point of this area of mathematics with some reminiscences about the invention of E_∞ ring spaces (sadly neglected these days) and E_∞ ring spectra. The original motivations concerned calculational understanding on the infinite loop space level of geometric manifestations of chromatic level one phenomena, leading, for example, to calculations in topological cobordism. The approach and results may be relevant to the current focus on chromatic level two phenomena. Some of the basic constructions apply to String, at least with its 2-categorical definition.

We started in 1972 by understanding classical Thom spectra as E_∞ ring spectra. In retrospect, we wrote them down as FSP's and saw that FSP's give rise to E_∞ ring spectra. Equivalently, we saw them as orthogonal ring spectra. Combining that perspective with work of Sigurdsson and myself on parametrized orthogonal spectra leads to a new understanding and generalization of Thom spectra. Since some of this conference will focus on the K-theory of K-theory, I'll describe Thom spectra of Thom spectra. This is a new and as yet unexplored construction. It may turn out to be just an idle curiosity, but it illustrates a new way of constructing orthogonal ring spectra from orthogonal ring spectra.

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Jack Morava (Johns Hopkins)

Towards a motivic fundamental group of S^0

In the 1970's, work of Quillen and Sullivan on the Adams conjecture showed that the abelianization of the absolute Galois group of $\mathbb{Q}$ plays a fundamental role in geometric topology. More recently, ideas from the theory of motives have suggested new ways of thinking about the rest of this Galois group.

I believe this work has close connections to Rognes' suggestion that the H-space BU [more precisely, its dual in a certain sense] plays the role of an absolute Galois group for the sphere spectrum, via Waldhausen's proposal (paraphrased), that we regard the map from the sphere spectrum to the Eilenberg-MacLane spectrum of the integers as a kind of universal unfolding

$$\mathrm{Spec}\ \mathbb{Z} \to \mathrm{Spec}\ S^0\ .$$

A preliminary set of notes for these talks has been posted on the Hopf Topology archive, as "Toward a fundamental groupoid for the stable homotopy category".

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Birgit Richter (Hamburg)

A lower bound for coherences on the Brown–Peterson spectrum

We provide a lower bound for the coherence of the homotopy commutativity of the Brown–Peterson spectrum BP at a given prime and prove that it is at least $(2p^2+2p-2)$-homotopy commutative. The proof uses Alan Robinson's obstruction theory for E_∞-structures; the resulting structure is given in explicit terms of a filtered E_∞-operad. Such structures give rise to Dyer–Lashof operations. With the help of these we show that BP cannot be the Thom spectrum of an infinite loop map to BSF.

—

Christian Schlichtkrull (Bergen)

Cyclic algebraic K-theory and the cyclotomic trace

The definition of cyclic algebraic K-theory is similar to the definition of algebraic K-theory, except that instead of considering the usual classifying space of a category one uses Waldhausen's cyclic version. We shall analyze the cyclic algebraic K-theory in various cases and show how it relates to topological cyclic homology.

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Neil Strickland (Sheffield)

Power operations, and buildings for formal groups

Power operations for Morava E-theory are closely related to finite subgroups of formal groups, and to isogenies between formal groups. There is an algebra D of power operations, and one expects that the homological algebra of D-modules should give the initial term for various spectral sequences that compute the homotopy groups of spaces of maps between suitable strictly commutative ring spectra. Some years ago, Hopkins wrote down a chain complex inspired by the theory of buildings for groups such as $GL_n(\mathbb{Q}_p)$, and conjectured that the complex computes Ext groups over D. We will explain all this background, and some recent progress towards a proof of the conjecture.

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Bertrand Toën (Tolouse)

Moduli of objects in a stable homotopy theory

This talk will report on a recent work in collaborations with M. Vaquié, concerned with constructing geometric moduli of (pseudo-)compact objects in a stable model category M. I will start to show that the classifying space of such objects (i.e. the nerve of the category of weak equivalences between them) is only the space of global points of a more general object, called an "S-stack", classifying algebraic families of compact objects parametrised by connective ring spectra. The main theorem of this talk states that under certain finiteness conditions on M (e.g. existence of a compact generator), this S-stack is "algebraic". Both the statement and the proof of this result are based on "brave new algebraic geometry", a theory whose fundations has been settled down recently in collaboration with G. Vezzosi.

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Sarah Whitehouse (Sheffield)

On the $K_{(p)}$-local stable homotopy category

A reformulation of Bousfield's work on the $K_{(p)}$-local stable homotopy category will be presented. We use results about the ring A of degree zero operations in p-local complex K-theory. Bousfield worked with a certain full subcategory of the category of modules over the group ring $\mathbb{Z}_{(p)}[\mathbb{Z}_{(p)}^{\times}]$ and we show that this category is isomorphic to the category of discrete continuous A-modules. This leads to new, often conceptually simpler, formulations of many results and constructions, including cofree objects and a basic four-term exact sequence.

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Samuel Wuethrich (Sheffield)

Multiplicative structures on I-adic towers

Consider the I-adic tower of E-module spectra

$$T = (\cdots \to I^{s+1} \to I^s \to \cdots \to I \to E)$$

for a finite regular quotient $K = E/I$ of a commutative S-algebra E. Assume that E is K-local and that its homotopy groups are concentrated in even degrees. A multiplicative structure on T is a family of maps $I^s \wedge_E I^t \to I^{s+t}$ which are compatible and associative in a strict sense. Such a structure gives rise to an action of the associated graded ring $\mathrm{gr}_I^*(E^*)$ on the Higher Bockstein spectral sequence

$$E_1^{*,*} = \mathrm{gr}_I^*(E^*) \otimes_{K^*} K^*(X) \Longrightarrow E^*(X).$$

Starting from an E-algebra structure on K, we show how Lazarev's theory of derivations for not necessarily commutative S-algebras can be used to construct a multiplicative structure on T.

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