A Proposed Stability Characterization and Verification Method for High-Order Single-Bit Delta-Sigma Modulators

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Abstract—A characterization method for verifying stability and measuring the performance quality of arbitrary noise transfer functions for the delta-sigma modulator is presented. The method consists of first obtaining the maximum stable input range sweep in the pass-band. The variance estimate can then be used to make a sharper upper limit for maximum allowed input range. This can then be used to measure the global SNR performance in the pass-band. The proposed method enables us to make a comprehensive comparison between arbitrary noise transfer function designs and it can also be used to automate the design of noise transfer functions.

I. INTRODUCTION

DELTA-SIGMA modulation [1] (DSM) is a very popular technique for making high-resolution analog-to-digital and digital-to-analog converters. These oversampled data converters have several advantages over conventional Nyquist-rate converters. The features are achieved by the concept of noise shaping and utilizing more linear coarse quantizers.

To achieve stable operation for high-order modulation non-monotonic loop-filters are needed. The specification of these loop-filters is normally given by a noise transfer function (NTF).

In literature NTF solutions give often little information about the stability characterization and performance. The solutions are often only specified by a single SNR value, an example fft-plot for a sinusoidal input signal and perhaps the maximum allowed input amplitude (maxU).

The information given is not adequate to understand the whole performance of the DSM solution. It would be profitable with a standardized characterization and verification method which can determine the properties of a modulator in a wider extent and in this way approach a complete characterization of the NTF performance.

This is highly relevant when one wants to compare different arbitrary NTFs. Not only NTFs which are produced by common filter design techniques like Chebychev and Butterworth. Which is the best is a natural question. But this simple question has a problematic answer because of the lack of good standardized characterization methods.

This paper uses the difference equation [1, 14.3] as the simulation model and is therefore a system level view of the DSM operation. But the concept can be transformed to encompass other simulation models, i.e. hardware implementations.

II. DSM FUNDAMENTALS

The basic concept of delta-sigma modulation is to use feedback for improving effective resolution of a coarse quantizer. In this way it aims to predict and correct the next quantization error. A simplified DSM operation can be seen in figure 1. The input is fed to the DSM which outputs a coarse oversampled quantized representation of the input, in this example single-bit. The bit-stream can then be digital or analog filtered for the wanted purpose.

Although it is possibly to gain some intuitive insight of the DSM operation, the easiest way to understand the operation is by mathematical analysis of the Linear Model. Figure 2 shows a general block diagram of a delta-sigma modulator. This diagram is used since it is sufficiently general to encompass the majority of single-quantizer modulator topologies [3]. The modulator is split into two parts, a two-input linear part and the nonlinear quantizer part. To do analysis we replace the quantizer by an addition of a noise signal e. This is known as the Linear Model.
for many topologies. The input transfer functions have been labeled for convenience as

\[
L_0(z) = G(z)/H(z) \quad L_1(z) = [H(z) - 1]/H(z)
\]

With these functions we can write the output of the linear block as the following sum of the inputs

\[
Y(z) = L_0(z)U(z) + L_1(z)V(z)
\]

By defining the error signal \( e \) as \( E(z) = Y(z) - V(z) \) we can rearrange the equation to give a formula for the output of the modulator in terms of its input and the error signal:

\[
V(z) = G(z)U(z) + H(z)E(z)
\]

Equation (1) captures the essence of noise shaping. The modulator output consists of independently filtered signals and noise components. This makes it possible, with a proper loop-filter, to spectrally separate the input signal from the noise introduced by quantization, i.e. noise shaping. We will now rename \( G \) and \( H \) to respectively the signal transfer function (STF) and the noise transfer function (NTF), that is

\[
\text{STF}(z) = \frac{V(z)}{U(z)} \quad \text{and} \quad \text{NTF}(z) = \frac{V(z)}{E(z)}
\]

This gives us the possibility to analytically measure the theoretical performance of a given loop-filter. The noise attenuation in the pass-band can be calculated through the frequency response of the NTF, that is \( |\text{NTF}(z)| \). The NTF is the heart of high-order DSM design.

The only restriction on NTF besides modulator stability, which we will discuss later, is causality. Causality is satisfied when the realizable condition \( \text{ntf}(0) = 1 = \text{NTF}(\infty) = 1 \) is met\(^1\). In practice this means that we must have at least one delay in the feedback loop [2, 4.1].

The NTF should act as a high pass filter for the ordinary delta-sigma modulators. Delta-sigma bandpass modulators, which also exist, have other NTF needs. The general desired ideal magnitude response of the NTF is

\[
|\text{NTF}(z)| = \begin{cases} 
0 & \text{inside the pass-band} \\
1 & \text{outside the pass-band}
\end{cases}
\]

This is the ideal need and is of course not possible to realize.

The root locus [1, 4.2.2][2, 4.2] suggests that in order for a modulator to be stable the input \( Y \) to the quantizer must not be allowed to become too large [4], [5], [6]. By “not” stable we mean that the modulator exhibits large, although not necessarily unbounded, states and poor SNR compared with the predicted. Since the input to the quantizer is given by \( (Y - V)\text{NTF}(z) \) or \( E(z)\text{NTF}(z) \), modulator stability is dependent on both the maximum allowed input level \((\text{max}_U)\) and the feedback level gain of the NTF. This requirement leads to the conclusion that stability is dependent on a restricted NTF gain, or the Out of Band Gain (OBG), and that there is a restriction on the maximum allowed input amplitude range \((\text{max}_U)\).

A typical NTF characterization can be seen in figure 3.

The goal when designing the NTF is to obtain a high as possible noise attenuation in the pass-band while maintaining low enough OBG to sustain stability. These two values are a trade-off.

![Fig. 3. Typical NTF frequency response](image)

When a good performing NTF is found it is just a matter of transforming the NTF to filter coefficients for the chosen topology/architecture. And though topology and architecture widely diverse the DSM behavior is principally the same for one NTF. It determines both the theoretical performance and the stability of the underlying modulator [3]. The only restriction is that a hardware implementation of a modulator can introduce coefficient errors that will naturally distort the result to some degree [1, 5.11].

### III. THE PROPOSED METHOD

To give a performance measurement of an NTF is a somewhat problematic quest since it’s not possible to give a straightforward measurement of the stability and quality. The quantizer ruins for the analysis and since the input to modulator is arbitrary it is not possible to test a modulator for every possible input sequence, since there are an infinite number of different combinations.

But if we assume that a given modulator input sinusoid, given by amplitude and frequency, reveals some of the modulator properties, we can systematically use this assumption to make a characterization of the system. We will see that in this way it is possible to converge to a stability characterization without the need of infinite number of input signals. The following is our proposal.

**Proposition 1:** When a higher order modulator is to be verified and characterized, the \( \text{max}_U \) should be swept over the signal-band of interest. Not only will this plot tell us how the maximum stable input threshold varies over the signal-band,
but its variance will also give a better understanding of the expected input range and hence help us to give a more strict maximum allowed input range. This global maximum allowed input range should then be used to make a similarly sweep for the simulated SNR + distortion (SINAD).

Instability in practice is characterized by a low oscillation frequency, producing long alternating strings of 1's and 0's [1, 4.6]. The maximum allowed input range can therefore be found effectively by the use of the computational fast difference equation implemented to detect these long strings.

The maximum stable input limit sweep reflects in our opinion more of the stability properties of a modulator. Several simulations have shown that if the modulator is stable for the sinusoidal $x = a \sin(f)$ it is not necessarily stable for amplitudes less than $a$. But this sweep and its curve variance can help us to give a reasonable sharper upper limit for maximum allowed input range.

The $max_U$ sweep reveals some stochastic properties of the modulators stability as you can see in figure 5. Note also that a more aggressive\(^2\) NTF has a higher variance in the $max_U$ sweep. It is difficult to determine a sharp upper limit for the $max_U$ of these aggressive NTFs without our proposed method.

The characterization of two NTF designs is given in figure 4. Both fourth-order designs have an OBG of approximately 1.5 and share the approximately same $max_U$ range of 0.6 over the pass-band. But the Chebychev design benefits from the spread zeros in the pass-band which give it a higher SNR performance (99.5dB versus 88dB). The 128-points characterization figures in this paper, that is a $max_U$ and a SINAD sweep, took approximately three minutes to compute on a normal desktop computer.

\(^2\)An aggressive NTF is in literature referred to NTF with a high OBG.

Note that many designs have a ripple in the SINAD sweep from $f = 0$ to $\frac{1}{2}$. This occurs for designs with “low” OBG ($< 1.6$) and is an effect of the third harmonic component of the input signal, see figure 6. There is therefore a connection...
between the shape of the ripple and the frequency pass-band response to the NTF as illustrated by figure 7. Also note that all the $max_U$ sweep shapes coincide well with the $max_U$ characterization of aggressiveness shapes of figure 5.

This method can also be used to automate the process of designing NTFs. A function can take an NTF as input and return the $max_U$ and the estimated SNR performance. A program can then use this function to automatically adjust an NTF to the designers needed specifications.

IV. EXAMPLES

Let us now characterize two different designs from literature. A sixth-order design from [1, 10.3.1] is specified by a $max_u$ of above 0.5 and peak SNR of approximately 110 dB. This coincides well with our characterization method in figure 8. Another design which specifies a Matlab simulated $max_u$ of 0.92 and an SNR performance of 122 dB is given in figure 9. Here our characterization is more conservative and gives the design a $max_u$ below 0.9 and an estimated SNR performance of about 114 dB. The last example can have been simulated in another way and therefore arrives at other specification numbers. Our point here is to call the need for a standardized characterization method.

V. CONCLUSIONS

We have proposed a method to characterize arbitrary NTF for DSM. This proposed method ensures that the peak SNR is not just a good local result in a noisy stability image, but that the stability and performance are distributed over the pass-band. The method gives us the possibility to compare arbitrary NTFs which again enables us to automate the NTF design process.

VI. FURTHER STUDIES

Standardizing the method further and producing a complete DSM characterization and verification tool-kit as an extension to [8].

REFERENCES