Monetary Policy in a Small Open Economy with a Preference for Robustness

Richard Dennis
Federal Reserve Bank of San Francisco
Kai Leitemo
Norwegian School of Management (BI)
Ulf Söderström*
IGIER

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Abstract

We use robust control techniques to study the effects of model uncertainty on monetary policy in an estimated, semi-structural, small-open-economy model of the U.K. Compared to the closed economy, the presence of an exchange rate channel for monetary policy not only produces new trade-offs for monetary policy, but it also introduces an additional source of specification errors. We find that exchange rate shocks are an important contributor to volatility in the model, especially when policy is conducted with discretion, and that the gain to commitment is very large. We also find that the exchange rate equation is particularly vulnerable to model misspecification, along with the equation for domestic inflation. How a robust policymaker should guard against these sources of model misspecification depends to a large extent on whether it sets policy with commitment or discretion.

Keywords: Model uncertainty, Robust control, Commitment, Discretion.

JEL Classification: E52, E61, F41.

*phDennis: Economic Research Department, Mail Stop 1130, Federal Reserve Bank of San Francisco, 101 Market Street, San Francisco, CA 94105, USA, richard.dennis@sf.frb.org; phLeitemo: Department of Economics, Norwegian School of Management (BI), 0442 Oslo, Norway; kai.leitemo@bi.no; phSöderström: Department of Economics and IGIER, Bocconi University, Via Salasco 5, 20136 Milan, Italy, ulf.soderstrom@uni-bocconi.it. We are grateful for comments from Maria Demertzis, Juha Kilponen, and seminar participants at the Finnish National Economist Meeting, Sveriges Riksbank, the University of Brescia, the Bank of Italy, and De Nederlandsche Bank. The views expressed in this paper do not necessarily reflect those of the Federal Reserve Bank of San Francisco or the Federal Reserve System.
1 Introduction

Although the canonical New Keynesian model (Goodfriend and King, 1997; Clarida, Gali, and Gertler, 1999; and Woodford, 2003) is used extensively to analyze monetary policy, important question about its structure remain unresolved. For instance, there are ongoing debates about the role of forward-looking inflation expectations, and the nature of the driving variable—real marginal costs or an output gap—in the New Keynesian Phillips curve, and about the importance of habit formation and consumption smoothing in the forward-looking "IS" curve. More generally, it is widely understood among practitioners that monetary policy affects the economy with "long and variable lags" in ways that many models do not acknowledge.

Of course, these debates about the appropriate structure of closed-economy New Keynesian models apply equally to open-economy specifications. After all, the transmission mechanisms and the monetary policy channels that operate in open-economy models are often similar to those operating in closed-economy specifications. However, unlike in the closed economy, in the open economy there can be concerns about the level of exchange rate pass-through, concerns centered around whether pass-through is full or partial, and about the extent to which imports are consumed or employed as intermediate inputs in the production of domestic goods. Similarly, exchange rate dynamics are difficult to model at the best of times and from an empirical standpoint there is good reason to view uncovered interest parity with suspicion. Importantly, these concerns extend beyond parameter uncertainty, amounting to a concern about the very structure of the model used to describe the economy.

Faced with uncertainty about its model, a central bank can react in several ways. It could choose to simply take the model at face value, essentially setting policy while ignoring the possibility that its model might be misspecified. However, such an approach could easily give rise to unwelcome or even disastrous outcomes. An alternative approach, one better suited to a prudent central bank, would be to take the possibility of model misspecification into account when formulating its policy and the literature offers several ways of going about this.\(^1\) One way is for the central bank to build several models and to use these models to develop a policy that produces reasonable, if not optimal, outcomes in all of the models, as described in Levin, Wieland, and Williams (1999, 2003). Although this approach is intuitive and simple to implement, it is not necessarily the most attractive. After all, the approach does not allow the central bank to address any concerns it may have about parameter uncertainty, it does not accommodate the possibility that agents in addition to the central bank may also be concerned about model uncertainty, and it

\(^1\)Dennis (2005) provides an accessible survey of these different methods.
assumes that each of the models provides an equally plausible description of the economy. An alternative is for the central bank to take a Bayesian approach, using Bayesian methods to estimate a range of models and Bayesian model averaging to evaluate competing policies (see Brock, Durlauf, and West, 2004; and Batini, Justiniano, Levine, and Pearlman, 2005). The Bayesian approach does not assume that all of the models are equally plausible and it readily accommodates both parameter and model uncertainty, but it still does not allow all agents in the model to be concerned about model uncertainty. A third approach, the approach pursued in this paper, is for the central bank to formulate policy using robust control methods, methods that are specifically designed to minimize the economic consequences of the worst case model misspecification (Hansen and Sargent, 2006). Some advantages of the robust control approach are that the policymaker need only develop a single model, that it can allow all the agents in the model to be concerned about model misspecification, and that the specification errors can reflect both model and parameter uncertainty.2

We study the conduct of monetary policy using a version of the Monacelli (2005) dynamic stochastic general equilibrium small-open-economy model that we estimate on U.K. data. Unlike previous papers that have considered the design of monetary policy in open-economy contexts, we introduce a concern for model misspecification on the part of the central bank and focus on policy rules that have been formulated purposefully to be robust to model misspecification. In the tradition of Hansen and Sargent (2006), we assume that the central bank possesses a benchmark model of the economy, which it is concerned may be misspecified, but that it is unwilling to posit a probability distribution over possible specification errors. The central bank allows for specification errors that lie within a neighborhood of its benchmark specification and conducts monetary policy to guard against the worst-case specification error. In taking this approach the central bank recognizes that its policy will be suboptimal if its benchmark model is actually specified correctly, but it still conducts policy this way, gaining comfort from the knowledge that by doing so it is insuring against catastrophic outcomes.

Unlike the closed economy, the open-economy model that we consider allows households to consume goods produced both domestically and abroad, with domestic goods prices subject to a Calvo-style (Calvo, 1983) price rigidity, and imported goods prices subject to incomplete exchange rate pass-through. The model allows a portfolio allocation choice between domestic and foreign bonds, giving rise to an uncovered interest parity condition and making the exchange rate an important channel for monetary pol-

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2 A weakness with the robust control approach that we readily acknowledge is that it can only help the central bank (and other agents) guard against specification errors that lie in a neighborhood of the chosen “reference” model. Moreover, although different agents can have varied attitudes toward misspecification, they all share the same reference model.
icy and risk premium shocks an important source of economic volatility. As we show, the exchange rate channel introduces additional trade-offs that the central bank must acknowledge when designing policy and it introduces an additional location for possible model misspecification.

Starting with the case where the central bank has complete confidence in its model, we first establish that the exchange rate is a crucial variable for monetary policy. Using a form of variance decomposition, we demonstrate that exchange rate shocks account for a large fraction of the volatility in the economy, especially when monetary policy is conducted under discretion. It follows that the gains to commitment are large in our estimated model. After introducing a concern for robustness, we show that the uncovered interest parity condition is a particularly damaging venue for model misspecification, partly because the uncovered interest parity condition is subject to large risk premium shocks and partly because the exchange rate presents the central bank with a challenging trade-off when responding to shocks. Due to this trade-off, the central bank cannot mitigate specification errors that reside in the uncovered interest parity without adversely affecting inflation and output.

In addition to the uncovered interest parity condition, our results highlight that a central bank worried about model misspecification should also be concerned about the equation for domestic inflation—the open-economy Phillips curve—echoing results for the closed economy. Our analysis also shows that specification errors located in other equations, such as those for imported goods inflation and the output gap, pose a much smaller problem for the central bank. A challenge for central banks in small open economies, therefore, is to develop better empirical models for domestic inflation and the exchange rate.

Although model uncertainty—particularly uncertainty concerning exchange rate determination—is clearly important for central banks in small open economies, surprisingly few studies have examined the issue. Lees (2004) analyzes a stylized small-open-economy model and finds that robust policies are generally more aggressive in response to shocks and that they imply less interest rate inertia. For his calibration, Lees (2004) concludes that although the exchange rate is an important source of specification errors, the consequences of these specification errors outweigh the benefits to the central bank from exploiting the exchange rate channel to better stabilize the economy. Leitemo and Söderström (2005a) study the robustness of simple policy rules to uncertainty about exchange rate determination in a calibrated, stylized, small-open-economy model, concluding that a standard Taylor rule that responds to CPI inflation and the output gap performs well. They also conclude that the Taylor rule is more robust to uncertainty about the formation of exchange rate expectations than are rules that respond to ex-
change rate movements. Leitemo and Söderström (2005b) present an analytic treatment of robust control in a small-open-economy model. They show that by guarding against specification errors in the supply side of the model the central bank raise the volatility of output while by guarding against specification errors in the demand side of the economy the central bank raises the volatility of inflation. Finally, Batini, Justiniano, Levine, and Pearlman (2005) study the effects of Bayesian model uncertainty on monetary policy in an estimated two-country model. Unlike our study, they focus on large open economies and investigate the gains to policy coordination.

The remainder of the paper is organized as follows. In Section 2 we present the theoretical model and its empirical counterpart, which is estimated using U.K. data. In Section 3 we discuss the methods we use to construct the optimal policy when the central bank has a preference for robustness. Section 4 applies these methods to the empirical model and discusses the results. Section 5 concludes.

2 The model: theory and empirics

We study the New Keynesian small-open-economy model developed by Monacelli (2005), which builds on Clarida, Galí and Gertler (2001, 2002). A key feature of the model is that households can consume goods produced both domestically and abroad, with the prices of both goods characterized by price rigidity. With imported goods subject to price rigidity and with importers pricing to market, the model can reproduce the incomplete exchange rate pass-through widely found to characterize the behavior of imported goods’ prices following exchange rate shocks. Since there is ample evidence supporting incomplete exchange rate pass-through, allowing for sticky imported goods prices seems reasonable, especially since it is likely to be important for the design of monetary policy. An important feature of the model is that it is not possible to achieve full price stability by setting the output gap to zero. The interest rate policy required to generate a zero output gap destabilizes inflation through its influence on imported goods prices.

2.1 The model

Domestic firms sell their product in a monopolistically competitive environment, hiring labor in a perfectly competitive labor market, and setting prices to maximize the expected discounted value of the firm, subject to a CES demand schedule and a linear-in-labor production technology. Following Calvo (1983), prices are set in a staggered manner and only domestic inputs are used in production. Goods produced domestically are assumed

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3See, for instance, Campa and Goldberg (2005).
to be tradeable. In this economic environment, the equation for domestic price inflation is given by a Phillips curve with the form

$$\pi_t^H = \beta E_t \pi_{t+1}^H + \kappa_x x_t + \kappa_\psi \psi_t,$$

(1)

where $\beta \in (0, 1)$ is the subjective discount factor, $E_t$ is a conditional expectations operator, $\pi_t^H \equiv p_t^H - p_{t-1}^H$ is the inflation rate for goods produced in the domestic economy, $x_t$ is the output gap, measured as the percent deviation of domestic output from its flexible-price level, and $\psi_t$ is the percent deviation from the law of one price. Equation (1) differs from the standard closed-economy New Keynesian Phillips curve through the inclusion of $\psi_t$, which enters here because the domestic good can be exported.

The Law-of-One-Price (LOP) variable, $\psi_t$, is the percent deviation of world market prices (measured in terms of domestic currency) from the domestic price of foreign goods:

$$\psi_t \equiv e_t + p_t^* - p_t^F = q_t - (1 - \gamma)s_t,$$

(2)

where $e_t$ is the nominal exchange rate, $p_t^*$ is the price of the foreign aggregate good measured in foreign currency, $p_t^F$ is the domestic price of the foreign aggregate good, $q_t \equiv e_t + p_t^* - p_t$ is the real exchange rate, $s_t \equiv p_t^F - p_t^H$ is the terms of trade, and $\gamma \in [0, 1)$ is the share of imported goods in consumption.

If import prices were flexible, then the law of one price would hold, implying $p_t^F = e_t + p_t^*$ and $\psi_t = 0$. However, in the model imported goods prices are not flexible. Instead, while imports are purchased from abroad in a perfectly competitive market, the importing firms sell the imported goods in a monopolistically competitive market, pricing-to-market subject to their demand schedule and a Calvo-style price rigidity. The imported goods, now priced in domestic currency, are then combined (according to a Dixit-Stiglitz aggregator) to create an aggregate imported good, which is sold to households for consumption purposes. Pricing-to-market, together with Calvo-pricing, gives rise to incomplete exchange rate pass-through and implies that imported-goods prices adjust gradually in response to movements in world market prices. The Phillips curve for import price inflation is given by

$$\pi_t^F = \beta E_t \pi_{t+1}^F + \lambda_\psi \psi_t,$$

(3)

In this last respect the model differs importantly from McCallum and Nelson (1999), who assume that imported goods are an intermediate input into the production of domestic goods.
Aggregate CPI inflation is a weighted average of domestic and imported goods inflation:

$$\pi_t^C = (1 - \gamma)\pi_t^H + \gamma\pi_t^F. \quad (4)$$

On the demand side, the economy is populated by infinitely-lived households that consume an aggregate domestic good and an aggregate imported good and that save by holding domestic and foreign one-period nominal bonds. The household intertemporal consumption choice leads to an expression for the output gap of the form

$$x_t = E_t x_{t+1} - \chi(r_t - E_t \pi_{t+1}^H - \pi_t) + \zeta E_t \Delta \psi_{t+1}, \quad (5)$$

where $r_t$ is the one-period nominal interest rate and $\pi_t^H$ is the natural real interest rate, which evolves over time according to

$$\pi_t = \phi E_t \Delta y_{t+1}^* + \theta z_t, \quad (6)$$

where $z_t$ is a domestic productivity shock and $\Delta y_{t+1}^*$ is the growth rate of world output.

Finally, with perfect capital mobility, the portfolio choice between holding domestic and foreign bonds implies that the nominal exchange rate is determined by the uncovered interest parity (UIP) condition

$$e_t = E_t e_{t+1} - r_t + r_t^*, \quad (7)$$

where $r_t^*$ is the foreign one-period nominal interest rate.

### 2.2 The empirical specification

The theoretical specification outlined above provides a simple microfounded description of private-sector behavior in an economy where goods prices are subject to stickiness. However, the model abstracts from the information and decision lags that can give rise to gradual adjustments and inertial responses following shocks. On the supply side, such inertial responses may be rationalized by firms using rule-of-thumb pricing (for example, Galí and Gertler, 1999) or price indexation (Christiano, Eichenbaum, and Evans, 2005) and, on the demand side, by consumers being subject to habit formation (Fuhrer, 2000).

To accommodate the possibility that there may be information and/or decision lags, we adopt the empirical specification of the Monacelli (2005) model estimated by Leitemo (2006), who allows the data to influence the model’s lead-lag structure while retaining the theoretical model’s steady-state properties. Following Rudebusch (2002), Leitemo (2006) uses the expected annual inflation over the coming year to represent the forward-looking
component of inflation in the Phillips curve, and, following Rotemberg and Woodford (1997) and Christiano, Eichenbaum, and Evans (2005), assumes that decisions by households and firms are subject to a one-period implementation lag.

2.2.1 Data

The equations in the model are estimated using GMM applied to U.K. data over the period 1980Q1 to 2001Q4. All data are observed at the quarterly frequency and were obtained from either the U.K. national accounts, the IMF, or the OECD. With respect to prices and inflation, domestic inflation, $\pi_t^H$, is constructed according to $\pi_t^H \equiv 4(p_t^H - p_{t-1}^H)$, where $p_t^H$ is the GDP price deflator. Similarly, imported-goods inflation, $\pi_t^F$, is measured using the price of imported goods according to $\pi_t^F \equiv 4(p_t^F - p_{t-1}^F)$. The quarterly rate of consumer price inflation, $\pi_t^C$, is given by equation (4). Four-quarter rates for domestic inflation, imported goods inflation, and consumer price inflation are easily obtained according to the general expression $\bar{\pi}_t^i \equiv \frac{1}{4} \sum_{j=0}^{3} \pi_{t-j}^i$, where $i$ denotes the particular inflation measure. Following Batini and Haldane (1999), the share of imported goods in the consumer basket, $\gamma$, is set to 0.25.

Regarding quantities and relative prices, the output gap, $x_t$, is calculated by applying an HP filter to log real GDP,\(^5\) the effective real exchange rate, $q_t$, is measured by deflating the effective trade-weighted nominal exchange rate by the corresponding relative CPI indices, and the law-of-one-price gap (LOP), $\psi_t$, is generated from equation (2), using the detrended effective real exchange rate and terms of trade. With respect to interest rates, the domestic interest rate, $r_t$, is the 3-month U.K. interest rate expressed at an annual rate and the foreign interest rate, $r_{r_t}^*$, is the 3-month OECD average, also expressed at an annual rate. The U.K. and the foreign 3-month interest rates expressed as quarterly rates are then given by $r_{q,t} \equiv \frac{1}{4} r_t$ and $r_{q,t}^* \equiv \frac{1}{4} r_{r_t}^*$, respectively. Finally, the foreign output, $y_t^*$, is approximated by the OECD output gap, with $\Delta y_t^*$ representing the quarterly growth rate of foreign GDP.

\(^5\)The smoothing parameter in the HP filter was set to 1,600, the standard value for quarterly data.
2.2.2 Equations

Domestic inflation is modeled according to (with standard errors in parentheses below the estimated coefficients)

\[
\pi_t^H = 0.58 E_{t-1} \bar{\pi}_{t+3}^H + 0.42 \left( -0.39 \pi_{t-1}^H + 0.22 \pi_{t-2}^H + 0.72 \pi_{t-3}^H + 0.45 \pi_{t-4}^H \right) + 0.28 E_{t-1} x_t + 0.038 E_{t-1} \psi_t + \varepsilon_t^H,
\]

\[
\sigma = 0.021,
\]

in which, reflecting the one-period implementation lag, all expectations are formed using a one-period lagged information set. The null-hypothesis that dynamic homogeneity holds could not be rejected and is imposed, so the coefficients on (leads and lags of) inflation sum to one, ruling out any permanent trade-off between domestic output and domestic inflation. While lags of inflation clearly play an important role in the domestic inflation Phillips curve, it is notable that the forward-looking component of inflation receives a larger weight than the backward-looking component, consistent with the Galí, Gertler and López-Salido (2001) and Smets (2003) estimates for the euro area, and with the Galí and Gertler (1999) estimates for the U.S., but assigning a larger weight to future inflation than either Fuhrer (1997) or Rudebusch (2002). The coefficient on the output gap, at 0.28, indicates relative rapid price adjustment, but is broadly consistent with Smets (2003) and Batini and Haldane (1999). The slope coefficient on the LOP gap is relatively small, suggesting relatively low substitutability in foreign consumption between domestic goods and foreign goods.

Turning to the Phillips curve for imported inflation, the empirical implementation of equation (3) is

\[
\pi_t^F = 0.78 E_{t-1} \bar{\pi}_{t+3}^F + 0.22 \left( 1.11 \pi_{t-1}^F - 0.11 \pi_{t-4}^F \right) + 0.56 E_{t-1} \psi_t + \varepsilon_t^F,
\]

\[
\sigma = 0.058.
\]

Similar to the Phillips curve for domestic inflation, in equation (9) the forward-looking inflation component receives greater weight than the lagged inflation component and dynamic homogeneity could not be rejected and is imposed. The coefficient on the LOP variable, the driving variables in the specification, is large, implying that movements in import costs are passed reasonably quickly into the domestic price of imported goods. Together, equations (8) and (9) establish that domestic prices are subject to greater price rigidity than imported goods prices. The shock to imported goods inflation also has a somewhat larger standard error than the shock to domestic inflation, suggesting that
shocks to imported goods prices are likely to be an important source of volatility in the economy, especially because offsetting these shocks forces a trade-off between stabilizing output and stabilizing inflation.

On the demand-side, the equation for the output gap is estimated to be

\[ x_t = 0.53 E_{t-1} x_{t+1} + 0.47 \left( 1.36 x_{t-1} - 0.36 x_{t-2} \right) - 0.066 (r_{t-1} - E_{t-1} \bar{\pi}_{t+2}^H) \]
\[ + 0.11 E_{t-1} \Delta \psi_t + 0.25 E_{t-1} \Delta y_t^* + \varepsilon_t, \]
\[ \sigma = 0.0041. \]

The output gap responds importantly to the expected future output gap, reflecting standard consumption smoothing behavior, but also to two lags of the output gap. By responding to two lags of the output gap, this open-economy IS curve shares much in common with closed-economy specifications estimated for the U.S., see Fuhrer and Moore (1995) and Rudebusch (2002). Interestingly, the coefficients on the two lags of the gap imply that the gap depends on the level of the lagged gap, consistent with the presence of habit formation, but also on the growth rate of the lagged gap, which cannot be associated easily with habit formation. Because equation (10) is estimated using output data rather than consumption data the two lags of the gap may reflect not just habit formation, but the joint effects of habit formation and investment dynamics (Dennis, 2004). It is worth noting, first, that the coefficient on the ex-ante real interest rate is relatively small and, as a consequence, that reasonably large interest rate movements would be required to completely stabilize the gap, and, second, that the standard error of the demand shock is small relative to the shocks to the two Phillips curves.

The estimated coefficients on the two open-economy variables in equation (10) are revealing. First, the coefficient on the LOP variable, at 0.11, is somewhat larger than its coefficient of 0.038 in the Phillips curve for domestic inflation. Clearly, deviations from the law of one price, which can be exploited for profit through international trade in the domestic good, generate much bigger effects on quantities than prices, further evidence supporting the rigidity of domestic goods prices. At the same time, the coefficient on \( \Delta y_t^* \) reveals that increases in foreign GDP growth generate greater demand for the domestic good, suggesting, perhaps, that the U.K. is demand constrained in its ability to export at the world price.

Allowing the foreign real interest rate to be autoregressive, the “risk-adjusted” uncov-

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6Because the theory model assumes away the existence of capital, any investment dynamics are naturally associated with mechanisms that provide a source of dynamics, such as habit formation.
The interest parity condition is estimated to be

\begin{equation}
q_t = E_t q_{t+1} + r r^*_{q,t} - (r_{q,t} - E_t \pi^C_{t+1}), \tag{11}
\end{equation}

\begin{equation}
rr^*_{q,t} = 0.50 r r^*_{q,t-1} + 0.19 r r^*_{q,t-2} + 0.11 r r^*_{q,t-3} + \varepsilon^q_t, \tag{12}
\end{equation}

\[ \sigma = 0.037, \]

where the parameter restrictions associated with UIP were tested and could not be rejected at usual significance levels. Equation (11) implies that the real exchange rate will “jump” following shocks to ensure that any real interest rate differential is offset by an expected change in the real exchange rate. Because the real exchange rate jumps following shocks it is likely to be highly volatile, particularly given the large standard error of the risk premium shock, and this volatility has obvious implications for consumer price inflation, through imported goods inflation. Note that expectations in the UIP equation are formed using period \( t \) rather than period \( t-1 \) information, reflecting the fact that there is no implementation lag with respect to portfolio allocation decisions.

Finally, the OECD output growth is modeled according to a first-order autoregressive process as

\begin{equation}
\Delta y^*_t = 0.51 \Delta y^*_{t-1} + \varepsilon^y_t, \tag{13}
\end{equation}

\[ \sigma = 0.0050. \]

Equation (13) illustrates clearly that shocks to foreign real GDP growth have a reasonably small standard error and that, following such shocks, movements in real foreign growth decay quickly.

Viewed as a system, two features of equations (8) to (13) are worth highlighting. First, the model does not allow a permanent trade-off between inflation and output, a knife-edge result that could easily be overturned if either equation (8) or equation (9) were misspecified. Second, it is movements in the LOP variable that are critical for output and inflation, not movements in the real exchange rate and the terms-of-trade themselves. As a consequence, the model does not uniquely pin-down steady-state values for either the real exchange rate or the terms-of-trade. The real UIP condition has important implications for the change in the real exchange rate, but no implications for the level. Similarly, equation (2) shows that many combinations of the real exchange rate and the terms of trade are consistent with any given value of the LOP variable. Therefore, depending on how monetary policy is conducted, transitory shocks may have permanent effects on the real exchange rate and the terms of trade, reflecting a form of hysteresis.
3 The robust control approach

Equations (8) to (13) represent the central bank’s “reference model,” the model the central bank believes best characterizes the data-generating process. To accommodate the central bank’s fears that this reference model may be misspecified the central bank is assumed to formulate monetary policy using robust control methods. As emphasized in Hansen and Sargent (2006), robust control allows the central bank to purposefully design a policy that guards against misspecifications, or distortions, to the reference model that are “small” in the sense that the distorted model lies in a neighborhood “close” to the reference model. To solve the central bank’s robust control problem, we deviate slightly from the approach developed by Hansen and Sargent (2006) and others, and employ the structural-form solution methods described in Dennis, Leitemo, and Söderström (2006).

To formulate the central bank’s robust control problem, we begin by distorting the reference model by including specification errors. These specification errors reflect the central bank’s concern for misspecification and give rise to a “distorted model” that can be written in the form

$$A_0y_t = A_1y_{t-1} + A_2E_{t+1} + A_3u_t + A_4(v_t + \varepsilon_t), \quad (14)$$

where $y_t$ is the vector of endogenous variables, $u_t$ is the vector of policy instrument(s), $v_t$ is a vector of specification errors, $\varepsilon_t$ is a vector of innovations, and $A_0$, $A_1$, $A_2$, $A_3$, and $A_4$ are matrices with dimensions conformable with $y_t$, $u_t$, and $\varepsilon_t$ that contain the parameters of the model. The matrix $A_0$ is assumed to be nonsingular and the elements of $A_4$ are determined to ensure that the shocks are distributed according to $\varepsilon_t \sim \text{iid} [0, I_s]$. The dating convention is such that any variable that enters $y_{t-1}$ is predetermined, known by the beginning of period $t$. The specification errors, $v_t$, are intertemporally constrained to satisfy the “budget constraint”

$$E_0 \sum_{t=0}^{\infty} \beta^t v_t' v_t \leq \eta, \quad (15)$$

where $\eta \in [0, \bar{\eta})$ represents the total budget for misspecification. When $\eta$ equals zero, then equation (15) implies that $v_t = 0$ for all $t$, in which case the distorted model, equation (14), collapses to the reference model.

[Add note on difference between structural and state-space methods?]
The central bank’s objective function is assumed to take the form

$$E_0 \sum_{t=0}^{\infty} \beta^t [y_t'Wy_t + u_t'Qu_t],$$

(16)

where $W$ and $Q$ are matrices containing policy weights and are assumed to be symmetric positive-semi-definite, and symmetric positive-definite, respectively.

The central bank sets policy so as to guard against the worst case misspecification, formulating policy subject to the distorted model with the view that the misspecification will be as damaging as possible. Private sector agents form expectations with the same view. The central bank’s fear that the misspecification will be as damaging as possible is operationalized through the metaphor that the specification errors in $v_t$ are chosen by an evil agent whose objectives are diametrically opposed to those of the policymaker. Hansen and Sargent (2006) show that the problem of minimizing equation (16) with respect to $u_t$ and maximizing with respect to $v_t$ subject to equations (14) and (15) can be replaced with an equivalent multiplier problem in which

$$E_0 \sum_{t=0}^{\infty} \beta^t [y_t'Wy_t + u_t'Qu_t - \theta v_t'v_t],$$

(17)

is minimized with respect to $u_t$ and maximized with respect to $v_t$, subject to equation (14). The multiplier $\theta \in [\theta, \infty)$ is inversely related to the budget for misspecification, $\eta$, and represents the shadow price of a marginal relaxation in equation (15).

The solution to this problem returns decision rules for the policy instrument $u_t$ and the specification errors $v_t$ that are functions of the predetermined variables $y_{t-1}$ and the shocks $\varepsilon_t$. There are two distinct equilibria that we are interested in. The first is the “worst-case” equilibrium, which is the equilibrium that pertains when the policymaker and private agents design policy and form expectations based on the worst-case misspecification and the worst-case misspecification is realized. The second is the “approximating” equilibrium, which is the equilibrium that pertains when the policymaker and private agents design policy and form expectations based on the worst-case misspecification, but the reference model transpires to be specified correctly. Solving equation (14) with the optimal decision rules values for the instrument and the distortions produces the worst-case outcomes for $y_t, u_t$ and $v_t$. To construct the approximating equilibrium, we set $v_t = 0$, while retaining the equations for $E_t y_{t+1}$ and $u_t$ generated by the worst case equilibrium, and substitute these into equation (14) to solve for $y_t$. The solution

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8When the central bank conducts monetary policy with commitment the solution is in fact a function of the complete histories of $y_{t-1}$ and $\varepsilon_t$ (Currie and Levine, 1993).
procedures we use are described in detail in Dennis, Leitemo, and Söderström (2006).

3.1 Detection error probability and $\theta$

Following Hansen and Sargent (2006) we set the shadow price, $\theta$, in order to generate a particular “detection error probability,” the probability that an econometrician would infer incorrectly whether the approximating equilibrium or the worst-case equilibrium generated the data. Let model $A$ denote the approximating model and model $B$ denote the worst-case model, then the probability of making a detection error is given by

$$ p(\phi) = \frac{\text{prob}(A|B) + \text{prob}(B|A)}{2}, $$

(18)

where $\text{prob}(A|B)$ (prob$(B|A)$) represents the probability that the econometrician erroneously chooses model $A$ (model $B$) when in fact model $B$ (model $A$) generated the data. To calculate a detection-error probability we assume that the selection of one model over another is based on the likelihood ratio principle. Therefore, with $\{z^B_t\}_T$ denoting a finite sequence of economic outcomes generated by the worst-case equilibrium, model $B$, and let $L_{AB}$ and $L_{BB}$ denote the likelihood associated with models $A$ and $B$, respectively, then the econometrician chooses model $A$ over model $B$ if $\log(L_{BB}^m/L_{AB}^m) < 0$. Generating $M$ independent sequences $\{z^B_t\}_T$, prob$(A|B)$ can be calculated according to

$$ \text{prob}(A|B) \approx \frac{1}{M} \sum_{m=1}^{M} \left[ \text{I} \left[ \log \left( \frac{L_{BB}^m}{L_{AB}^m} \right) < 0 \right] \right], $$

(19)

where $\text{I}[\log (L_{BB}^m/L_{AB}^m) < 0]$ is the indicator function that equals one when its argument is satisfied and equals zero otherwise; prob$(B|A)$ is calculated analogously using draws generated from the approximating model. Let

$$ z_t = H_A z_{t-1} + G_A \varepsilon_t, $$

(20)

$$ z_t = H_B z_{t-1} + G_B \varepsilon_t $$

(21)

govern equilibrium outcomes under the approximating equilibrium and the worst-case equilibrium, respectively. Using the QR decomposition we decompose $G_A$ according to $G_A = Q_A R_A$ and $G_B$ according to $G_B = Q_B R_B$. By construction, $Q_A$ and $Q_B$ are orthogonal matrices ($Q_A Q_A' = Q_B Q_B' = I_s$) and $R_A$ and $R_B$ are upper triangular. Let

$$ \hat{\varepsilon}_{ij}^t = R_i^{-1} Q_i' (z_i^t - H_i z_{i-1}^t), \quad \{i, j\} \in \{A, B\} $$

(22)
represent the inferred innovations in period $t$ when model $i$ is fitted to data $\{z_t\}_1^T$ that are generated according to model $j$ and let $\hat{\Sigma}^{ij}$ be the associated estimates of the innovation variance-covariance matrices. Then

\[ \log \left( \frac{L_{AA}}{L_{BA}} \right) = \log |R_A^{-1}| - \log |R_B^{-1}| + \frac{1}{2} \text{tr} \left( \hat{\Sigma}^{B|A} - \hat{\Sigma}^{A|A} \right), \tag{23} \]

\[ \log \left( \frac{L_{BB}}{L_{AB}} \right) = \log |R_B^{-1}| - \log |R_A^{-1}| + \frac{1}{2} \text{tr} \left( \hat{\Sigma}^{A|B} - \hat{\Sigma}^{B|B} \right), \tag{24} \]

where \( \text{tr} \) is the trace operator.

For a given value of $\theta$, the associated detection error probability to calculated by simulation using equations (23), (24), (19), and (18). Going in the other direction, $\theta$ can be determined by selecting a detection error probability and (numerically) inverting equation (18).

4 Robust monetary policy in the estimated model

In this section we apply our methodology to the small open economy described by equations (8) to (13). The central bank’s objectives are assumed to be of a standard quadratic form, so monetary policy is directed toward stabilizing CPI inflation, the output gap and the interest rate around their long-run levels, which are normalized to zero, as per

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left[ \left( \pi_t^C \right)^2 + \lambda x_t^2 + \nu r_t^2 \right], \tag{25} \]

where we set $\lambda = 1$, $\nu = 0.05$ and $\beta = 0.99$. We begin with the case where the central bank has complete confidence in its model, and thus no preferences for robustness, and characterize the equilibrium when policy is set with commitment and discretion. We then introduce a preference for robustness on the part of the central bank and discuss the worst-case and approximating equilibria. The robustness parameter $\theta$ is set to generate a detection error probability of 0.10, using 10,000 draws of a sample of 200 observations, allowing the distortions to be of a reasonable magnitude, but not so large to make it inconceivable that they would not have previously been detected.\(^9\)

4.1 The rational expectations equilibrium

The rational expectations equilibrium is characterized by both private agents and the central bank having full confidence in the model. Table 1 shows the unconditional

\(^9\)In our model, a detection error probability of 0.10 is obtained by setting $\theta$ to 0.0145 under commitment and 0.0455 under discretion.
Table 1: Volatility and loss in the rational expectations equilibrium

<table>
<thead>
<tr>
<th>( \pi_H )</th>
<th>( \pi^F )</th>
<th>( \pi^C )</th>
<th>( x_t )</th>
<th>( \Delta q_t )</th>
<th>( \psi_t )</th>
<th>( r_t )</th>
<th>Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Commitment</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.76</td>
<td>150.08</td>
<td>6.66</td>
<td>3.32</td>
<td>101.77</td>
<td>88.15</td>
<td>17.10</td>
<td>10.42</td>
</tr>
<tr>
<td><strong>Discretion</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.00</td>
<td>515.37</td>
<td>31.88</td>
<td>19.92</td>
<td>672.81</td>
<td>947.84</td>
<td>139.35</td>
<td>56.73</td>
</tr>
</tbody>
</table>

Note: The table shows the unconditional variances of key variables and expected loss in the rational expectations equilibrium. The loss function is given by equation (25), with \( \beta = 0.99, \lambda = 1, \nu = 0.05 \).

The volatility for all variables is fairly high, a result that can be explained partly by the fact that the model is estimated over a period with relatively high volatility, as evidenced in the high regression standard errors. An alternative reason for the high volatility could be that the model is misspecified and fails to include all channels of adjustment in a correct manner, a possibility that motivates our analysis of robust policy below. In any case, the rational expectations equilibrium provides a natural baseline against which to compare the effects of robust policy.

Independently of whether policy is conducted with commitment or discretion, the variables specific to the open economy—imported goods inflation, the real exchange rate, and the law-of-one-price gap—are more volatile than the domestic rate of inflation and the output gap, suggesting that external shocks are an important driving force. This impression is confirmed in Table 2, which shows the contribution of each shock to the unconditional variances and to loss reported in Table 1. It is clear from Table 2 that exchange rate shocks account for most of the variability in all variables other than domestic inflation. To some extent this result arises because exchange rate shocks have a higher variance than the other shocks, but it also reflects the fact that these shocks pose a difficult policy trade-off for the central bank and are difficult to offset.

Table 1 also shows that the value of commitment is large in the model: central bank loss is more than 80% lower with commitment than with discretion.\(^{10}\) These benefits to commitment arise primarily because commitment allows the central bank to better stabilize the exchange rate. Because the exchange rate is a highly forward-looking variable, managing exchange rate expectations is especially important in an open economy.

\(^{10}\) These results are also demonstrated for the theoretical model by Monacelli (2005).
Table 2: Volatility and loss decomposition in the rational expectations equilibrium

<table>
<thead>
<tr>
<th>Shock</th>
<th>$\pi^H_t$</th>
<th>$\pi^F_t$</th>
<th>$\pi^C_t$</th>
<th>$x_t$</th>
<th>$\Delta q_t$</th>
<th>$\psi_t$</th>
<th>$r_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commitment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\varepsilon^H_t$</td>
<td>49.15</td>
<td>1.12</td>
<td>44.35</td>
<td>18.42</td>
<td>2.51</td>
<td>2.87</td>
<td>9.42</td>
</tr>
<tr>
<td>$\varepsilon^F_t$</td>
<td>0.02</td>
<td>23.54</td>
<td>32.98</td>
<td>0.22</td>
<td>0.58</td>
<td>0.92</td>
<td>0.83</td>
</tr>
<tr>
<td>$\varepsilon^x_t$</td>
<td>0.07</td>
<td>0.03</td>
<td>0.10</td>
<td>7.79</td>
<td>0.04</td>
<td>0.03</td>
<td>2.52</td>
</tr>
<tr>
<td>$\varepsilon^q_t$</td>
<td>50.73</td>
<td>75.22</td>
<td>22.48</td>
<td>72.88</td>
<td>96.85</td>
<td>96.15</td>
<td>84.61</td>
</tr>
<tr>
<td>$\varepsilon^{y*}_t$</td>
<td>0.03</td>
<td>0.09</td>
<td>0.09</td>
<td>0.69</td>
<td>0.03</td>
<td>0.03</td>
<td>2.62</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Discretion</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon^H_t$</td>
<td>75.35</td>
<td>0.57</td>
<td>13.32</td>
<td>2.05</td>
<td>0.01</td>
<td>0.03</td>
<td>1.42</td>
</tr>
<tr>
<td>$\varepsilon^F_t$</td>
<td>0.86</td>
<td>7.73</td>
<td>7.46</td>
<td>1.21</td>
<td>0.14</td>
<td>0.96</td>
<td>0.80</td>
</tr>
<tr>
<td>$\varepsilon^x_t$</td>
<td>0.28</td>
<td>0.00</td>
<td>0.06</td>
<td>1.40</td>
<td>0.01</td>
<td>0.00</td>
<td>0.35</td>
</tr>
<tr>
<td>$\varepsilon^q_t$</td>
<td>23.12</td>
<td>91.69</td>
<td>79.07</td>
<td>95.04</td>
<td>99.82</td>
<td>98.99</td>
<td>96.90</td>
</tr>
<tr>
<td>$\varepsilon^{y*}_t$</td>
<td>0.38</td>
<td>0.01</td>
<td>0.09</td>
<td>0.31</td>
<td>0.03</td>
<td>0.02</td>
<td>0.54</td>
</tr>
</tbody>
</table>

Note: The table shows the percentage of the unconditional variances and central bank loss in the rational expectations equilibrium that is due to each shock.

Additional results (available upon request) suggest that the benefit to commitment rises as imported goods prices become more flexible, because exchange rate volatility then has an even greater impact on prices in this situation. Furthermore, the gain to commitment is negligible in the closed-economy version of this model.\footnote{Dennis and Söderström (2005) examine the gains to commitment in a variety of estimated closed-economy models. They show that the gain depends not only on the degree of forward-looking behavior, but also on the existence of implementation and decision lags, which tend to reduce the gains to commitment. Our estimated open-economy model includes multi-period lags in all equations as well as one-period decision lags. Without these lags, the gain to commitment is likely to be even larger.}

Figures 1–10 show impulse responses to unit-sized shocks under commitment and discretion. (For now focus on the solid lines representing the rational expectations equilibrium.) These figures illustrate the difficult trade-off caused by exchange rate shocks, which have a stronger impact on the economy than other shocks, and therefore require a more forceful response by monetary policy. With commitment, the central bank is able to manage expectations to better stabilize the economy and the initial shocks are often followed by reversals. Interestingly, the optimal policy makes the real exchange rate non-stationary. As noted earlier, the real exchange rate affects the economy only through the LOP gap, which depends also on the terms of trade (see equation (2)). With imperfect pass-through, a real exchange rate that is non-stationary, but cointegrated with the terms of trade, allows the central bank to better stabilize the LOP gap and the broader economy. But this requires the central bank to be able to commit to future policies: discretionary
policy cannot manage the real exchange rate in this manner.\textsuperscript{12} Due to the central bank’s inability to manage expectations under discretion, the interest rate must respond more vigorously than under commitment, especially following external shocks.

### 4.2 The worst-case equilibrium

We now introduce a lack of confidence in the model on the part of the central bank and allow an evil agent to distort the reference model by choosing the specification errors, $\mathbf{v}_t$ in equation (13), to maximize central bank loss. The central bank chooses policy so as to minimize the impact of these distortions.

Table 3 reports the unconditional variances and cross-correlations among the specification errors in the worst-kind equilibrium when the central bank and the evil agent act under commitment and discretion. The distribution of the distortions tells us to which equation a specification error has the greatest impact on central bank loss. Complementing Table 3, Table 4 displays the unconditional variances of key variables under the worst-case and approximating equilibria, along with the value of the central bank loss function.

We first note that the variance of the distortions are of a larger magnitude in the commitment equilibrium. The evil agent’s ability to commit to future distortions allows him to have a greater impact on central bank loss. This finding is also illustrated by the impulse responses in Figures 1–10. Under commitment the distortions typically have a more persistent effect on the economy, introducing more volatility than under discretion. Thus, central bank loss in the worst-case equilibrium increases by more under commitment, where it is 75\% larger than in the rational expectations equilibrium, than under discretion, where it is 25\% larger than in the rational expectations equilibrium.

A second important observation from Table 3 concerns the relative magnitudes of the distortions. Although the evil agent puts most emphasis on distorting the exchange rate and the domestic inflation equations, under both commitment and discretion, there are important differences. Under discretion, the variance of the distortion to the exchange rate equation is 15 times greater than the second largest distortion, which is to the domestic inflation equation. Under commitment, by comparison the exchange rate distortions are less important relative to the other distortions, and are smaller than the distortions to the domestic inflation equation. This reflects the fact that the central bank under discretion is very vulnerable to exchange rate disturbances, as shown earlier for the rational expectations equilibrium.

\textsuperscript{12}The non-stationarity of the real exchange rate is not due to the specification of the empirical model, but is due to the presence of imperfect exchange rate pass-through. The same mechanism is also present in the theoretical model, see Monacelli (2005).
### Table 3: Unconditional variances and cross correlations of specification errors

<table>
<thead>
<tr>
<th>Specification error</th>
<th>$v_t^H$</th>
<th>$v_t^F$</th>
<th>$v_t^y$</th>
<th>$v_t^q$</th>
<th>$v_t^{y*}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Commitment:</strong> $\theta = 0.0145$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v_t^H$</td>
<td>201.70</td>
<td>−0.52</td>
<td>0.87</td>
<td>−0.72</td>
<td>0.73</td>
</tr>
<tr>
<td>$v_t^F$</td>
<td>13.97</td>
<td>−0.76</td>
<td>0.68</td>
<td>−0.70</td>
<td></td>
</tr>
<tr>
<td>$v_t^x$</td>
<td>0.94</td>
<td>0.94</td>
<td>−0.67</td>
<td>0.89</td>
<td></td>
</tr>
<tr>
<td>$v_t^{y*}$</td>
<td></td>
<td></td>
<td></td>
<td>132.99</td>
<td>−0.99</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.61</td>
</tr>
<tr>
<td><strong>Discretion:</strong> $\theta = 0.0455$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v_t^H$</td>
<td>15.63</td>
<td>0.18</td>
<td>−0.18</td>
<td>0.59</td>
<td>−0.12</td>
</tr>
<tr>
<td>$v_t^F$</td>
<td>2.75</td>
<td>−0.39</td>
<td>−0.18</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>$v_t^x$</td>
<td>0.94</td>
<td>0.94</td>
<td>−0.67</td>
<td>0.89</td>
<td></td>
</tr>
<tr>
<td>$v_t^{y*}$</td>
<td></td>
<td></td>
<td></td>
<td>242.37</td>
<td>−0.83</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.40</td>
</tr>
</tbody>
</table>

Note: The table shows the unconditional variances (along the diagonal) and cross correlations of the worst-case specification errors (off the diagonal). The parameter $\theta$ is chosen so as to produce a detection error probability of 0.10.

A third important feature is that there is a high degree of co-movement between the disturbances. The absolute values of the correlation coefficients are all above 0.5 in the commitment case and of a slightly smaller magnitude in the discretionary case. The reason for this co-movement between the disturbances is of course that it generates shock-combinations that are difficult for the policymaker to counteract.

The correlation patterns are perhaps easier to understand for the discretion case than they are for the commitment case. Under discretion, neither of the agents can commit to future policies and must respond instantaneously to shocks as a consequence. Under commitment, the instantaneous correlation is of lesser importance as the evil agent can create a mix of disturbances that is distributed over time and is difficult for the monetary policymaker to counteract. This produces sophisticated responses by the evil agent that are less intuitive. It is frequently the case that the direction of the co-movement is reversed under commitment relative to discretion.

Under discretion, the correlation between the domestic inflation and exchange rate model disturbances large and positive (0.59). Intuitively, it makes sense for the evil agent to distort both the domestic inflation and the exchange rate equations with both distortions pushing CPI inflation in the same direction. This generates a greater need for the central bank to stimulate or contract aggregate demand to offset the change in inflation.
Table 4: Volatility and loss under the robust policy

<table>
<thead>
<tr>
<th>Variable</th>
<th>Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta q_t$</td>
<td>138.04</td>
</tr>
<tr>
<td>$\psi_t$</td>
<td>24.86</td>
</tr>
<tr>
<td>$r_t$</td>
<td>18.18</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Commitment</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Worst-case equilibrium</td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.0145</td>
</tr>
<tr>
<td>$\pi^H$</td>
<td>12.71</td>
</tr>
<tr>
<td>$\pi^F$</td>
<td>240.14</td>
</tr>
<tr>
<td>$\pi^C$</td>
<td>9.60</td>
</tr>
<tr>
<td>$x_t$</td>
<td>8.40</td>
</tr>
<tr>
<td>$\Delta q_t$</td>
<td>146.19</td>
</tr>
<tr>
<td>$\psi_t$</td>
<td>138.04</td>
</tr>
<tr>
<td>$r_t$</td>
<td>24.86</td>
</tr>
<tr>
<td>Approximating equilibrium</td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.0145</td>
</tr>
<tr>
<td>$\pi^H$</td>
<td>11.26</td>
</tr>
<tr>
<td>$\pi^F$</td>
<td>194.66</td>
</tr>
<tr>
<td>$\pi^C$</td>
<td>8.03</td>
</tr>
<tr>
<td>$x_t$</td>
<td>6.53</td>
</tr>
<tr>
<td>$\Delta q_t$</td>
<td>144.76</td>
</tr>
<tr>
<td>$\psi_t$</td>
<td>127.42</td>
</tr>
<tr>
<td>$r_t$</td>
<td>21.22</td>
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</table>

<table>
<thead>
<tr>
<th>Discretion</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Worst-case equilibrium</td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.0455</td>
</tr>
<tr>
<td>$\pi^H$</td>
<td>8.72</td>
</tr>
<tr>
<td>$\pi^F$</td>
<td>609.13</td>
</tr>
<tr>
<td>$\pi^C$</td>
<td>37.64</td>
</tr>
<tr>
<td>$x_t$</td>
<td>27.93</td>
</tr>
<tr>
<td>$\Delta q_t$</td>
<td>823.85</td>
</tr>
<tr>
<td>$\psi_t$</td>
<td>1173.23</td>
</tr>
<tr>
<td>$r_t$</td>
<td>155.13</td>
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<tr>
<td>Approximating equilibrium</td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.0455</td>
</tr>
<tr>
<td>$\pi^H$</td>
<td>7.82</td>
</tr>
<tr>
<td>$\pi^F$</td>
<td>512.85</td>
</tr>
<tr>
<td>$\pi^C$</td>
<td>31.80</td>
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<td>$x_t$</td>
<td>21.43</td>
</tr>
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<td>$\Delta q_t$</td>
<td>835.73</td>
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<tr>
<td>$\psi_t$</td>
<td>974.01</td>
</tr>
<tr>
<td>$r_t$</td>
<td>120.95</td>
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</tbody>
</table>

Note: The table shows the unconditional variances of key variables in the worst-case and approximating equilibria. The parameter $\theta$ is chosen so as to produce a detection error probability of 0.10. The central bank loss function is given by equation (25), with $\beta = 0.99$, $\lambda = 1$, $\nu = 0.05$.

The negative correlation between the output gap and exchange rate distortions (−0.67) arises due to the fact that as the domestic currency depreciates, putting upward pressure on inflation, the interest rate needs to be raised, which has a depressing effect on output. It is then convenient for the evil agent to put downward pressures on output to limit the policymakers ability to neutralize the disturbance by raising interest rates.

Similar arguments can be made for the correlation between the disturbance to foreign output and output gap (0.89), on the one hand, and the exchange rate (−0.83), on the other. As the evil agent drives foreign demand down, it is welfare reducing to lower output further by either disturbing the output equation directly or by producing a domestic currency appreciation. In either case, the interest rate needs to be cut to offset the decline in output and inflation then rises on the expectation of a greater depreciation of the domestic currency in the medium term.

What are the reasons for why the evil agent distorts the domestic inflation and exchange rate equations? The finding that the exchange rate equation is vulnerable to misspecification has a strong intuition. Views regarding the success for exchange rate modeling and forecasting have not changed markedly since the pessimistic results reported by Meese and Rogoff (1983). But it is striking how strong the effects of misspecification are, in particular under discretion. In the model, the exchange rate influences the LOP gap which directly influences both output and inflation. Firms set prices to reflect expected future marginal costs, so domestic and imported goods inflation both depend on
the expected future sum of the LOP gap. Aggregate domestic demand depends similarly on the expected current LOP gap as consumers substitute between foreign and domestic goods in their consumption. The exchange rate is therefore a key avenue through which the evil agent can increase volatility in all equations, thereby increasing central bank loss. In addition, exchange rate movements present difficult trade-offs for the central bank, making it even more attractive for the evil agent to distort the exchange rate equation. Since the exchange rate channel forces output and inflation in different directions, exploiting the exchange rate channel allows the evil agent to simultaneously increase the volatility of both output and inflation.

A third reason for why the UIP condition is an attractive venue for distortions is the high variance of shocks to the (risk-premium corrected) foreign interest rate. As the robust control problem is formulated, greater noise provides the evil agent greater latitude to hide its distortions.

The imported inflation equation is also subject to a high residual variance and has a direct impact on the target variable. Nevertheless, the distortions to this equation are of a fairly small magnitude because they distortions can be offset relatively easily through exchange rate movements brought about by small interest rate adjustments. The exchange rate has a strong impact on imported inflation (through the LOP gap) and the required exchange rate movements (and interest rate movement) induce only small changes in the other variables.

With regard to domestic inflation, it is known that the Phillips curve in a closed economy is very vulnerable to specification errors, as such distortions create a more difficult trade-off for the central bank than other distortions (Leitemo and Söderström, 2004). This result holds also in the open economy, although here the exchange rate equation is even more vulnerable to misspecification.

### 4.3 Robust policy and the approximating equilibrium

While the worst-case equilibrium reveals what specification errors are most damaging for the central bank, the approximating equilibrium provides information on the effects of the central bank’s preference for robustness on monetary policy and the economy in the situation where there turns out to be no misspecification. Comparing the interest rate volatility for rational expectations in Table 1 with that for the approximating equilibrium in Table 4, we see that the robust monetary policy is more volatile under commitment but less volatile under discretion than the non-robust policy. The central bank’s desire to guard against model misspecification increases volatility in almost all variables, especially under commitment, because commitment allows the evil agent to do more damage.
Compared to the rational expectations equilibrium, loss is more than 40% larger in the approximating equilibrium with commitment, while it is essentially unchanged with discretion. The large increase in loss for the commitment case again reflects the greater damage caused by the evil agent when he can commit to future specification errors, which, of course, necessitate stronger policy responses.

This result is confirmed by Figures 1–10, where the impulse responses are more volatile in the approximating equilibrium than in the rational expectations equilibrium. Surprisingly, the smallest effect of the central bank’s preference for robustness lies in response to exchange rate shocks when policy is set with discretion (see Figure 8), where the effects are almost negligible.

5 Conclusion

We set out to study the effects of model uncertainty on monetary policy in a small open economy. We first showed that exchange rate shocks were an important source of volatility in the open economy, even without taking model uncertainty into account. We showed that exchange rate shocks are especially important when policy is set with discretion, and that, as a consequence, the gain to commitment was very large in the model.

Introducing a preference for robustness by the central bank, we showed that monetary policy was particularly sensitive to distortions to the exchange rate and to the domestic inflation equations, while distortions to the other equations were of relatively minor importance. When policy is set with discretion, the exchange rate equation was most sensitive to misspecification, while with commitment, the domestic inflation equation was more vulnerable. Since exchange rate specifications are often perceived to be fragile from an empirical modeling perspective, the sensitivity of the economy to exchange rate distortions pose a major challenge for monetary policy.

The policy implications of our results are immediate. To improve monetary policy in the open economy, a better understanding of the behavior of domestic inflation and the exchange rate is crucial. Reducing the scope for misspecification in other equations, such as those for imported goods inflation or the output gap, appears to be of much less importance, at least in our model. At the same time, it is just as important, or perhaps even more important, for central banks in open economies to enhance their ability to commit to future policies. The importance of commitment may explain why countries with small open economies have been more willing to introduce formal targets for inflation, to publish forecasts, and to improve on transparency than many countries with larger, less open, economies.
References


Figure 1: Impulse responses to a domestic inflation shock with commitment

Figure 2: Impulse responses to a domestic inflation shock with discretion
Figure 3: Impulse responses due to an imported goods inflation shock with commitment

Figure 4: Impulse responses due to an imported goods inflation shock with discretion
Figure 5: Impulse responses to a domestic demand shock with commitment

Figure 6: Impulse responses to a domestic demand shock with discretion
Figure 7: Impulse responses to an exchange rate shock with commitment

Figure 8: Impulse responses to an exchange rate shock with discretion
Figure 9: Impulse responses to a foreign output growth shock with commitment

(a) Domestic inflation
(b) Imported inflation
(c) CPI inflation
(d) Output gap
(e) Real exchange rate
(f) Law−of−one−price gap
(g) Interest rate
(h) Foreign output growth

Figure 10: Impulse responses to a foreign output growth shock with discretion

(a) Domestic inflation
(b) Imported inflation
(c) CPI inflation
(d) Output gap
(e) Real exchange rate
(f) Law−of−one−price gap
(g) Interest rate
(h) Foreign output growth