Fiscal policy coordination and international trade*

Torben M. Andersen
University of Aarhus
CEPR, IZA, and EPRU
January 2005

Abstract

While assertions are often made in the public debate that non-cooperative fiscal policies suffer a contractionary bias, general equilibrium models with an explicit welfare approach have shown that the bias is unambiguously expansionary. In this paper it is argued that the latter result relies on a particular and critical way of modelling international trade, and under a more plausible trade structure it is possible that fiscal policy is insufficiently expansionary in the non-cooperative case. Non-cooperative policy making thus implies that fiscal policies are used too little if they expand private employment, and too much if they contract private employment. Inefficiencies in non-cooperative fiscal policies are shown to be increasing when product markets become more integrated.

JEL: F1, F42, J23, J3
Keywords: Spillovers, coordination, employment

*Comments and suggestions from participants at workshops at CESifo and IZA are gratefully acknowledged, in particular from the discussants Ulrich Hange and Donatella Gatti. Financial support from the Danish Social Science Research Council is gratefully acknowledged.
1 Introduction

Received wisdom among policy makers and commentators is that non-cooperative fiscal policies tend to be insufficiently expansionary, and therefore proposals for coordinated fiscal efforts are often advanced (in particular in periods with increasing unemployment). However, despite the intuitive appeal of this assertion it seems to be a fairly robust finding in general equilibrium models of fiscal policy interactions in open economies that non-cooperative fiscal policies tend to be too expansionary (public consumption is at an inefficiently high level, cf. references below). The paper argues that this result relies on a particular and critical way of modelling international trade, and under a more plausible trade structure it is possible that fiscal policy is insufficiently expansionary in the non-cooperative case. Critical for the bias in fiscal policy is how fiscal policy affects private sector employment (via wage formation). In particular it is found that non-cooperative policy making implies that fiscal policies are used too little if they expand private employment, and too much if they contract private employment.

The issue of interdependencies in employment and social policies has a long history in economic theory. According to traditional Keynesian reasoning there is a tendency that countries choose insufficiently expansionary fiscal policies since the demand leakage reduces the expansionary domestic effects of fiscal policies (see e.g. Cooper (1985)). This line of reasoning has often motivated proposals for coordinated fiscal expansions intended to overcome free rider problems in policies oriented towards output and employment. This view has been contested on two related accounts, namely the usual problems associated with the theoretical foundation of Keynesian models and the fact that policy evaluations are not based on an explicit welfare analysis, but instead rely on arbitrary policy objective functions.

To overcome both of these problems a number of authors have considered international interdependencies in fiscal policy in explicitly formulated general equilibrium models, and have addressed the issue of cooperative vs. non-cooperative policy making from an explicit welfare approach. The standard set-up has featured specialized production, that is, countries specialize (exogenously) in production of specific commodities, which they trade with each other. One surprising, but robust finding in these models is that fiscal policies tend to be too expansionary when comparing the non-cooperative to the cooperative policy outcome (see e.g. Chari and Kehoe (1990), Devereux (1991), Turnovsky (1988) and van der Ploeg (1987, 1988). The reason is a terms-of-trade or “beggar thy neighbour” effect. Fiscal policy in the form of demand for domestically produced goods tends to shift demand from foreign to domestic products, which in turn is perceived to improve the terms of trade and thus the real income of the home country. No such terms-of-trade effect arises in the (symmetric) cooperative case, and therefore non-cooperative policies tend to be too expansionary.2

1 In standard macromodels the direction of the bias in non-cooperative fiscal policies is in general ambiguous depending on the particular structure of the model and policy objectives.

2 Irrespective of whether the policy in absolute terms is expansionary or contractionary.
Even in the presence of distortions implying an inefficient, low level of activity and therefore a potential role for fiscal policy in expanding activity and employment, this result holds and is driven by the terms-of-trade effect (Andersen and Sørensen (1995), Andersen, Rasmussen and Sørensen (1996)). Likewise the terms-of-trade effect implies that the optimal setting of tax rates should aim at twisting demand towards domestically produced goods (see Holmlund and Kolm (2002), Lockwood (2000)).

The crucial terms-of-trade effect for the findings refereed above depends critically on the assumed exogenous specialization across countries. This assumption essentially implies that the fiscal authorities exploit the market power the domestic economy has relative to its trading partners by being the sole producer of a particular set of commodities. Basically, the specialized production model relies on exogenously given product characteristics, which explains the structure of specialized commodities. However, this assumption seems to match the situation for OECD countries less and less. Trade is increasing rapidly, and an increasing share of trade and thus production is in commodities, which in principle can be produced (almost) anywhere, implying that an increasing share of the domestic production can be replaced by foreign production (see e.g. Coppel and Durand (1999), IMF (2002)). These commodities may not be perfect substitutes, but there is a high degree of flexibility – up to the “protection” caused by trade frictions and differences in costs etc. – which in turn have important implications for the interdependencies in policies across countries.

The present paper uses a simplified Ricardian trade model allowing an endogenous determination of specialization and therefore of which commodities become tradeables (importables and exportables) and non-tradeables, that is, the allocation of production between domestic and foreign producers is endogenous. The analysis builds on the modeling framework in Dornbusch, Fischer and Samuelson (1977), which has proved useful for analyses of international product market interactions and the effects of further product market integration. Recent work has extended the basic model to allow for heterogeneity in wage setting, imperfectly competitive product markets, horizontal and vertical specialization, heterogeneity in trade frictions and dynamics (see e.g. Andersen (2002), Andersen and Sørensen (2003), Andersen and Skaksen (2003), Bernard, Eaton, Jensen and Kortum (2001), Yi (2003), Melitz (2002) and Bergin and Glick (2003). Empirical work has documented the importance of these mechanisms, see e.g. Davis and Weinstein (2001), Eaton and Kortum (2002) and Bernard, Jensen and Schott (2003).

The present paper builds on the basic insights from the above-mentioned references concerning international production allocation and specialization. To avoid unnecessary technicalities the model is a workhorse version of the Ricardian trade model capturing the key qualitative aspects of the works referred to. The location of production is flexible up to differences caused by relative wages, comparative advantages and trade frictions. It is endogenous which commodities

3 Usually a CES preference ordering of commodities with different characteristics is assumed, and the production of each variety is exogenously attributed to a particular country. This structure is also well-known from the so-called Armington specification.
are non-tradeables and tradeables (exportables or importables) in equilibrium, i.e. specialization is endogenous. In this setting there are obvious policy interdependencies, and non-cooperative policies are affected by the concern that they may have harmful effects on competitiveness (relative wages) and therefore in turn employment and income creation. If so, it may imply that policy makers end up with fiscal policies with a “contractionary” bias.

The particular form of fiscal policy considered in this paper is public employment (to produce public goods). This is motivated both by the empirical observation that this is the most important component of public consumption, and the fact that it can be modelled in a straightforward way. It is conjectured that the basic qualitative lessons can be generalized to other forms of fiscal instruments.

The paper runs as follows: Section 2 sets up the details of the two-country model, and section 3 considers the mechanisms determining employment. Next section 4 considers employment policies and the differences between non-cooperative and cooperative policies, and how these are affected by further international integration. Section 5 offers a few suggestions for future research.

2 The Model

Consider two fairly similar countries. They produce and trade in commodities of which there is a continuum indexed by $i \in [0, 1]$. Each good in principle be produced either at home or abroad, and can be imported or exported across countries. Denote the price at which commodity $i$ can be produced by domestic producers by $P_i$ and the price at which foreign producers can produce it by $P_i^*$ (the exchange rate is assumed constant, and normalized to one without loss of generality). Trade involves various frictions in the form of explicit and implicit trade costs. Trading one unit of a commodity internationally absorbs $\kappa$ units of the good in frictions (Samuelson’s iceberg costs), that is, if a foreign commodity is imported it costs $(1 + \kappa) P_i^*$ in the domestic market, similarly, if a domestic firm exports a commodity and sells it at the price prevailing in the foreign market it obtains a net-price $(1 + \kappa)^{-1} P_i^*$. Trade frictions are symmetric with respect to the direction of trade. International integration can now easily be captured by a reduction in $\kappa$.

To focus on the basic effects running via the trade structure, both product and labour markets are assumed competitive. Generalizing the framework to include imperfectly competitive product and labour markets is relative straightforward (see e.g. Andersen and Sorensen (2004)). However, since it does not change the basic mechanism related to the endogenous determination of the trade position of various goods, but adds to the expositional complexity, it is left out. For the same reason the model is static leaving out intertemporal

---

4The model structure builds on Dornbusch, Fischer and Samuelson (1977). See also Obstfeld and Rogoff (1996) for a textbook version of the model.

5An earlier version of this paper had labour markets as imperfectly competitive in the form of a standard right-to-manage monopoly model, which gave the same qualitative results.
aspects.

Production
Domestic production of a given good $i$ is possible by use of a technology given as

$$Y_i = A_i l_i$$

where $A_i$ denotes an exogenous productivity parameter, and $l_i$ is the labour input in the production function. Labour is assumed to be homogeneous, and the wage rate is accordingly uniform across producers/firms.

Firms are competitive (with free entry), and the output price of domestically produced goods of type $i$ is accordingly linked to the wage $(W)$ as

$$P_i = A_i^{-1} W$$

Commodity $i$ will be imported if foreign producers can supply at a price that is competitive in the domestic market $(i \in I)$, i.e.

$$P_i > (1 + \kappa) P^*_i$$

and exported if domestic firms are competitive in the foreign market $(i \in E)$, i.e.

$$P_i < (1 + \kappa)^{-1} P^*_i$$

The trade frictions imply that commodities are non-tradeables $(i \in NT)$ provided

$$(1 + \kappa)^{-1} P^*_i \leq P_i \leq (1 + \kappa) P^*_i$$

Although there may be price differences for these commodities between the domestic and the foreign market there is no trade, since the trade friction is too large to make trade worthwhile.

Households
There is a continuum of households indexed by $h \in [0, 1]$. Households’ utility functions are given as

$$U(c_h, l_h, g) \ , \ U_c > 0, U_{cc} < 0, U_l < 0, U_{ll} < 0, U_g > 0, U_{gg} < 0,$$

where the first term gives the utility derived from the private consumption bundle $(c_h)$, the second term the disutility from work $(l_h)$ and the final term is

---

6 Allowing for imperfectly competitive product markets will not change the analysis qualitatively, provided that product market power is symmetric across the two countries, see Andersen (2002) and Andersen and Sørensen (2003).
the utility derived from publicly provided goods and services \((g)\). The private consumption bundle is defined over the available commodities as

\[
ch = \left( \int_0^{1 \theta} c_{hi}^{\frac{1}{\theta}} di \right)^{\frac{1}{\theta}}, \ \theta > 1,
\]

where \(c_{hi}\) is the consumption of commodity \(i\) by household \(h\).

It follows straightforwardly that the demand for commodity \(i\) by household \(h\) can be written

\[
c_{hi} = \left( \frac{P_i}{P} \right)^{-\theta}(1 - \tau)r_h
\]

where \((1 - \tau)r_h\) denotes the real disposable income of the household and where the consumer price index \(P\) is defined as

\[
P = \left[ \int_0^{1} P^{1-\theta} di \right]^{\frac{1}{1-\theta}}
\]

For later reference note that aggregate demand for commodity \(i\) is

\[
c_i = \int_0^{1} c_{hi} dh = \left( \frac{P_i}{P} \right)^{-\theta}(1 - \tau)r
\]

where \(r\) is aggregate real income given by

\[
r = \int_0^{1} r_h dh = \int_0^{1} W \frac{P}{P} l_h dh = \frac{W}{P} l
\]

with \(W\) denoting the nominal wage and \(l\) total employment (= sum of private \((c)\) and public \((g)\) employment, see below).

The private consumption bundle for household\(^7\) \(h\) can be written

\[
c_h = (1 - \tau)r_h
\]

\[
= (1 - \tau)\frac{W}{P} l_h
\]

The individual labour supply decision yields the first order condition

\[
(1 - \tau)\frac{W}{P} U'_c((1 - \tau)\frac{W}{P} l_h, l, g) + U'_l((1 - \tau)\frac{W}{P} l_h, l, g) = 0
\]

\(^7\)Note that there is no profit income due to the assumption of constant returns to labour and free entry of firms.
Implying a labour supply relation

\[ l_h^* = l^* \left(1 - \tau \right) \frac{W}{P}, g \]  \hspace{1cm} (5)

Labour supply depends on the after tax real wage and the level of public goods. The latter influence labour supply because public goods may be substitutes or complements to private goods, e.g. Algan, Cahuc and Zylberberg (2002). Hence a change in public activities affect labour supply both through this channel and through the effect on the tax rate, cf. below.

Finally, if we assume that all agents are identical it follows that adopting a utilitarian welfare criterion implies a welfare function (all agents achieve the same allocation in equilibrium)

\[ U((1 - \tau) \frac{W}{P}, l, g) \]  \hspace{1cm} (6)

**Government**

In the present setting with endogenous trade patterns and specialization the specification of public activities needs to be considered carefully. In the specialized production model (cf. discussion in the introduction) the specification is less critical, and it is usually assumed that public demand is demand of the domestically produced commodity.\(^8\) This assumption cannot readily be applied here since it is endogenous whether a commodity is produced at home or foreign. The problem with “endogenous trade patterns” does not arise when considering government demand for labour to produce public goods/services. This also captures the empirical fact that wage expenditures are the dominate fraction of public consumption, i.e. most public sector activities are hiring of people to perform various tasks, cf. e.g. IMF(2001), and therefore public employment is a non-trivial share of overall employment.\(^9\) Denote public demand for labour by \( g \), and assume for simplicity a linear technology linking inputs and outputs in the public sector (output=input). The utility of the goods/services produced is denoted \( V(g) \), cf. the household utility function. The budget balance of the public sector reads

\[ \frac{W}{P} g = \tau r \]  \hspace{1cm} (7)

Note that it is assumed that the wage rate for public employees corresponds to that of private employees, i.e. workers are homogeneous and indifferent between working for the private and public sector.\(^{10}\) It is relatively straightforward to

---

\(^8\) A government interested in increasing domestic activity (in the presence of an inefficiently low level of employment) will always have an interest in biasing demand towards domestic commodities. Hiring workers to perform various tasks is an obvious and empirically very relevant way of inducing such a bias.

\(^9\) For a group of OECD countries Algan, Cahuc and Zylberberg (2002) report that public employment in 2000 constituted close to 20% of total employment ranging from 8% in Japan to 31% in Norway.

\(^{10}\)Holmlund (1997) analyses wage formation in the private and public sector in a union model. He shows that a public expansion lowers total employment if unions are relatively more powerful in the public than in the private sector.
build into the model a wage difference between the private and public sector if the relative wage is constant (but highest in the private sector). Since this does not change anything qualitatively this generalization is skipped.

Using the public sector budget constraint (7) it follows that private disposable income can be written

\[ (1 - \tau) r = (1 - \tau) \frac{W}{P} (e + g) \]

\[ = \frac{W}{P} e \]

Public employment therefore has no income effects, which in turn implies that variations in public activity or employment have no Keynesian aggregate demand effects. The model is classical in structure, and any effects of variations in public employment arises from the way it affects relative prices (real wages) (see below).

Note that the budget constraint (7) implies that the tax rate is \( \tau = \frac{g}{e + g} \).

Using the labour supply relation (5) (all individuals in equilibrium supply the same amount of labour \( l^h = l = e + g \)) yields the following wage relation (inverted labour supply relation)\(^{11}\)

\[ \frac{W}{P} = \lambda(e, g) \] (8)

In general the signs of the partial derivatives of the wage function with respect to private and public employment are ambiguous. Rather than going through a taxonomy of all possible sign constellations, the case where \( \lambda_e > 0 \) is considered, i.e. an increase in private employment leads to an increase in the real wage. Allowing for \( \lambda_g \leq 0 \) spans the possible cases on how a change in public employment affects private employment (leading to either an increase or decrease).

Note that this formulation captures mechanisms that have been discussed in the literature on how public sector activities affect labour supply and thus wages. Public employment leads to taxation, which in turn affects labour supply via substitution and income effects. An increase in the income tax thus tends to lower labour supply via the former effect and increase it via the latter. This leaves an ambiguity on how taxes affect labour supply depending both on preferences and the tax structure.\(^{12}\) It is well-known that the simplest way to ensure that public activities are expansionary is to adopt the so-called “classical” approach in the sense of assuming that income effects dominate substitution

\(^{11}\)We have from (5) that

\[ e + g = l^s \left( \frac{W}{P} \frac{e}{e + g} \right) \]

Implying that the real wage is given by the following implicit function

\[ \frac{W}{P} = \lambda(e, g) \]

\(^{12}\)E.g. lump sum taxation would only involve the income effect, whereas a proportional income tax releases both the substitution and the income effects.
effects in labour supply (see Dixon (1987), Dixon and Rankin (1994), Baxter and King (1993)). This case is thus captured by $\lambda_g < 0$, while the standard case is captured by $\lambda_g > 0$.

It is also well-known that the effects of public sector activities depend on whether they are substitutes or complements to private consumption, see. e.g. Algan, Cahuc and Zylberberg (2002). An important example of this is public provision of child-care, which may lead to an expansion of labour supply of females by targeting people with relatively high labour supply elasticities, see. e.g. Gustafsson and Stafford (1992) and Graafland (2000). This case corresponds thus to $\lambda_g < 0$.

**Equilibrium Conditions**

The equilibrium condition for non-tradeables reads

$$C_i = Y_i \text{ if } i \in NT$$

and for exportables the condition is

$$C_i + (1 + \kappa)C_i^* = Y_i \text{ if } i \in E$$

Similar conditions hold for the foreign country, and since an importable for the domestic country is an exportable for the foreign country this fully characterizes the equilibrium conditions for product markets.

Notice that employment is demand determined in equilibrium given as the sum of private ($e$) and public ($g$) employment, i.e. $l = e + g$, and that trade is balanced (static model).

### 3 Equilibrium - exogenous fiscal policy

It is useful to first consider the determination of the equilibrium allocation for a given level of the policy variables before turning to an analysis of policy choices. Since the equilibrium can be characterized in terms of private sector employment ($e$), the following considers the determination of employment in general equilibrium to the two-country model. It is assumed that two countries are completely symmetric (for productivity this applies to the distribution of productivity, cf. below), and that they both have the structure outlined above. All foreign variables are denoted by $\ast$.

Define the relative productivity of domestic firms relative to foreign firms in producing commodity $i$ as

$$a_i \equiv \frac{A_i}{A_i^*}$$

Productivity is distributed according to the same density function in the two countries, and the sectors are ordered in such a way that $a$ is increasing in $i$, that is, for low values of $i$ the foreign country has a comparative advantage,
and for high values of $i$ the home country has a comparative advantage. The comparative advantage variable $a_i$ is symmetrically distributed with a density function $\psi$ over the interval $[a^{-1}, a]$, $a > 1$. This implies that $a_i = 1$ for $i = 1/2$, that is, for half the sectors the domestic economy has a comparative advantage relative to the foreign country and vice versa. It is assumed that the distribution of relative productivity does not have too much mass at a single point, i.e. $\psi(a_i) \leq \psi$ for all $i$. This assumption ensures monotonicity (see appendix B).

While productivity is assumed exogenous here, the differences across countries can be interpreted as capturing many of the effects associated with the new trade theory (see e.g. Krugman (1995) and Grossman and Helpman (1995)). Note that it is an implication that the average skill levels are the same in the two countries. Moreover, similar trade frictions in all sectors rule out that some low productivity sectors can be protected by high trade frictions so as to maintain a status as non-tradeables.

Finally, define relative wages or wage competitiveness as

$$\omega \equiv \frac{W}{W^*}$$

Since labour is assumed to be the only (variable) input, it follows that relative wages and comparative advantages solely determine international competitiveness.

Trade and domestic activity
A key question is which commodities are traded in equilibrium, and which factors determine the direction of trade for each type of commodity. It follows straightforward from (4) and (1) that a commodity $i$ is a non-tradeable ($i \in NT$) at home if

$$(1 + \kappa)^{-1}A_i^{-1}W^* < A_i^{-1}W < (1 + \kappa)A_i^{-1}W$$

or

$$(1 + \kappa)^{-1}A_i^{-1}W^* < \omega a_i < (1 + \kappa)A_i^{-1}W$$

The commodity is an exportable ($i \in E$) for the home country if

$$(1 + \kappa)^{-1} > \frac{\omega}{a_i}$$

\[13\] The index is thus not necessarily in any way related to the characteristics of the commodities, that is, two commodities with fairly similar characteristics (say French and German cars) can be located quite differently on the unit interval, since one country has a comparative advantage in the production of the one type, and the other country has a comparative advantage in the production of the other type.

\[14\] Assume that $A_i$ is uniformly distributed over the interval $[1 - x, 1 + x]$ and $A_i^*$ is uniformly distributed over the interval $[1 + x, 1 - x]$. Hence $\frac{A_i}{\psi}$ is distributed over the interval $[\frac{1}{1+x}, \frac{1+x}{1-x}]$, with a density function with the property that $\psi(\frac{1}{z}) = \psi(z)$.  

10
and an importable \((i \in I)\) for the home country if
\[
\frac{\omega}{a_i} > (1 + \kappa)
\]
Define the critical import level for comparative advantage as
\[
a_I \equiv \omega(1 + \kappa)^{-1}
\]
and the critical export level for comparative advantage as
\[
a_E \equiv \omega(1 + \kappa)
\]
It follows that exportable sectors are given by
\[
E \equiv \{a_i \mid a_i > a_E\}
\]
the non-tradeable sectors by
\[
NT \equiv \{a_i \mid a_I \leq a_i \leq a_E\}
\]
and the importable sectors by
\[
I \equiv \{a_i \mid a_i < a_I\}
\]
Exports take place for activities for which the domestic economy holds a sufficiently strong comparative advantage relative to wages and trade frictions, while conversely imports take place when the comparative advantage of the foreign country is sufficiently strong. The non-tradeable sectors are made up of activities with "intermediate" levels of comparative advantages (seen relative to relative wages and trade frictions).

Obviously, if a commodity \(i\) is an importable in the home country, it is an exportable in the foreign country \((i \in I \Rightarrow i \in E^*)\), and vice versa.

An increase in the relative wage affects the trade and production structure since
\[
\frac{\partial a_I}{\partial \omega} > 0, \quad \frac{\partial a_E}{\partial \omega} > 0
\]
The higher the relative wage, the higher the required comparative advantage needed to be protected from imports, and to be able to export. An increase in the relative wage will thus increase the number of commodities being imported as well as decrease the number of commodities exported. The net effect is tantamount to a "net-export" of jobs or production.

The lower the trade frictions, the smaller the non-tradeable sector other things being equal since
\[
\frac{\partial a_I}{\partial \kappa} < 0, \quad \frac{\partial a_E}{\partial \kappa} > 0
\]
implying that more goods are imported and exported (more trade), and that specialization among the countries is increasing.
Equilibrium employment

Equilibrium private employment \((e)\) can be written as a function of foreign private employment \((e^\ast)\), the level of public employment at home \((g)\) and abroad \((g^\ast)\), i.e. (cf. Appendix)

\[
e = \pi(e^\ast, g, g^\ast), \quad 0 < \pi_e < 1, \pi_g \geq 0, \pi_g^\ast \geq 0
\]

(9)
a similar expression holds for foreign employment. Note that \(g\) and \(g^\ast\) are exogenous policy variables, and total employment equals \(l = e + g\).

An increase in foreign employment always has a positive effect on domestic employment. The reason is both that higher foreign employment increases foreign income and therefore export demand for home products, and that the induced increase in foreign wages causes a change in competitiveness leading to an expansion of the number of goods that can be exported and a reduction in the number of goods that are imported. This mechanism captures an international “employment multiplier”, and shows how policy can affect the location of jobs across the two countries.

Considering the partial effect of a change in public employment on private employment we have (see Appendix)

\[
sign \pi_g = -sign \lambda_g
\]

and

\[
sign \pi_{g^\ast} = sign \lambda_g^\ast
\]

That is, the impact effect on private employment of an increase in public employment is positive if wages decrease \((\lambda_g < 0)\), and vice versa if wages increase \((\lambda_g > 0)\). The reason is the effects a wage change has on private employment. There is also a fiscal policy spillover effect since a foreign increase in public employment lowers domestic private employment if foreign wages decrease \((\lambda_g^\ast < 0)\) and vice versa if they increase \((\lambda_g^\ast > 0)\). Note that the domestic and foreign impact responses always have opposite signs: if the policy change on impact benefits domestic employment, it harms foreign employment, and vice versa. This is related to the endogenous trade structure, since a domestic wage increase implies that production of some commodities moves to the foreign country (less export and more import), i.e. domestic private employment tends to fall, and foreign private employment to increase (and vice versa for a wage decrease).

Turning to the general equilibrium effect of a change in public employment where the interdependencies between the two countries are taken into account, we find

\[
\frac{de}{dg} = \frac{1}{1 - (\pi_e)^2}[\pi_g + \pi_e \pi_g^\ast]
\]

(10)
while the effect on foreign employment is given as

\[
\frac{de^\ast}{dg} = \frac{1}{1 - (\pi_e)^2}[\pi_g^\ast + \pi_e^\ast \pi_g]
\]

(11)

\[
^{15}\text{Observe that the symmetry assumption implies that } \lambda_g^\ast = \lambda_g.
\]
It can be shown (see appendix)

\[
\text{sign} \frac{de}{dg} = \text{sign} \frac{de^*}{dg} = -\text{sign} \lambda_g
\]

The first part of the equation says that if a change in public employment has an expansionary effect on domestic employment, it also has an expansionary effect on foreign employment, and vice versa. This captures the important spillover effect from trade. Although the impact effects have opposite signs (see above), the total effect is equal in sign due to the fact that variations in employment affect income and therefore via the "international employment" multiplier the levels of demand and trade.\textsuperscript{16} To put it differently, the direct activity effect of a change in fiscal policy always dominates the production switching effect arising from changes in relative wages. The second part of the expression gives the condition under which a change in public employment expands or contracts private employment.

Gains from lower trade frictions
An advantage of this approach to modelling international product markets is that it also makes it possible to consider the effect of further product market integration. It is useful to point out that there are welfare gains from international integration. Lower frictions (κ) imply both that less resources are absorbed by trade frictions and that there are gains from further division of labour or exploitation of the scope for specialization given by differences in comparative advantages. A reduction of trade friction will thus increase employment in symmetric equilibrium for a given fiscal policy (see appendix), i.e.

\[
\frac{\partial e}{\partial \kappa} < 0
\]

it follows that lower trade frictions increase real income and consumption

\[
\frac{\partial r}{\partial \kappa} < 0 ; \frac{\partial c}{\partial \kappa} < 0
\]

Aggregate welfare is improved both due to the consumption gain from lower trade frictions and the fact that employment is increasing (employment is inefficiently low due to labour market distortions), i.e.

\[
\frac{\partial}{\partial \kappa} U(c, l, g) < 0
\]

Finally, note that lower trade frictions lead to more trade, that is, the non-tradeable sector shrinks, and both the importable and the exportable sector increases. The metric for international integration used in the present analysis

\textsuperscript{16}The reason is that a change in employment always has a direct effect (income changes for given wages) as well as an indirect effect (the wage change), while the change in public employment only on impact releases the indirect effect.
is thus consistent with the trend increase in international trade, which has been observed over recent decades (cf. introduction).

Lower trade frictions also change the spillover effects between the two countries, and we have (see appendix)

\[
\frac{\partial \pi^*}{\partial \kappa} < 0, \quad \frac{\partial |\pi_g|}{\partial \kappa} > 0, \quad \frac{\partial |\pi^*_g|}{\partial \kappa} < 0
\]

that is, the lower the trade friction, the higher the “international employment multiplier”, and the less does a change in domestic public employment affect domestic private employment numerically, and the more it affects foreign employment numerically.

Considering how integration affects the sensitivity of private employment to public employment, we find (see Appendix B) that

\[
|\frac{\partial \delta e}{\partial \kappa \delta g}| > 0
\]

\[
|\frac{\partial \delta e^*}{\partial \kappa \delta g}| < 0
\]

The smaller the trade friction, the less do variations in domestic public employment affect domestic private employment, and the more do they affect foreign private employment. To put it differently, lower trade frictions imply that expansionary employment policies have a smaller effect on domestic employment, whereas the effect on foreign employment increases (and vice versa). The spillover effects thus become stronger the smaller the trade frictions. This can also be interpreted as capturing the fact that with further integration of product markets, the domestic policy maker’s control over the domestic economy decreases, and the sensitivity to foreign policies increases.

4 Optimal fiscal policies

Consider next the determination of fiscal policy. Assume the policy decision to be derived from a utilitarian criterion maximizing the utility of the representative household, cf. (6). Both the case of non-cooperative and cooperative policy making are considered to assess the importance of the interdependencies in policies between the two countries, and how they are affected by further international integration. In both cases symmetric equilibria are considered.

The optimal policy \((g)\) thus maximizes (6), which is equivalent to

\[
Max_{\delta g} \Gamma(e, g) \equiv U(\lambda(e, g)e, e + g, g)
\]

subject to private employment being determined by (9). The non-cooperative solution is found by taking foreign public employment \((g^*)\) for given, and the cooperative solution is found by imposing the condition \(g = g^*\) on the optimization problem.
The optimal level of public employment is determined by the condition

\[ V_g = -U_l - U_c\lambda_ge + [U_c(\lambda_ge + \lambda) + U_l]\left(-\frac{de}{dg}\right) \]

(12)

where it has been used from (2) that \( U_c\lambda + U_l = 0 \).

The LHS of (12) gives the direct marginal benefit of a change in public sector activities. The RHS captures the social marginal costs of a change in public activity in three terms. First there is the direct effect in the form of more work (\(-U_l\)). Second the effect on private consumption (\(-U_c\lambda_ge\)). The final term gives the marginal utility effect of a change in employment (\(U_c\lambda_ge(-\frac{de}{dg})\)) given by the product of the marginal utility of consumption times the income gain obtained by a change in private employment. Note that this is negative if employment increases (\(\lambda_g < 0\)), and vice versa if employment decreases. If an increase in public employment increases private employment (\(\lambda_g < 0\)) implies that the marginal costs of public activities are lowered, and vice versa. A difference between the non-cooperative and the cooperative case may thus arise because the perceived effects on private employment differ in the two cases.

### 4.1 Non-cooperative and cooperative fiscal policies

For both the non-cooperative and cooperative case the optimal level of public employment is determined by balancing marginal benefits and marginal costs (12). The two cases differ due to a difference in the perceived effect of a change in public employment on private employment (\(\frac{de}{dg}\)). In the cooperative case eventual interdependencies or externalities in employment policies between the two countries are taken into account.

To compare the two cases note first that a change in public employment in the non-cooperative case is perceived to affect private employment less than in the cooperative case, i.e. (see appendix)

\[ \left| \frac{dc^{\text{coop}}}{dg} \right| > \left| \frac{dc^{\text{non-coop}}}{dg} \right| \]

This difference can be interpreted in terms of a demand and a cost spillover effect. Single countries do not take into account that a change in domestic public employment and income will increase the demand for foreign products (demand spillover), whereas they perceive that wage competitiveness will be affected (cost spillover). In the cooperative case the demand spillover effect is taken into account while there is no cost spillover effect to take into account (relative wage competitiveness is not affected in symmetric equilibrium).

---

17 Observe that the consumption and employment effects are always oppositely signed. If the consumption effect is positive, the employment effect is negative, and vice versa.
Foreign policy changes affect domestic welfare since

\[ \frac{d\Gamma(e,g)}{dg^*} = \frac{\partial\Gamma}{\partial e} \frac{de}{dg^*} \]

Since \( \frac{d\Gamma}{de} > 0 \), it follows that there is a positive spillover effect if the employment effect is positive (\( \frac{de}{dg^*} > 0 \)) and a negative spillover effect if the employment effect is negative (\( \frac{de}{dg^*} < 0 \)). It is well-known that the sign of these spillover effects determine the direction in which non-cooperative policies are biased (see Cooper and John (1988)). Accordingly, if an increase in public employment has an expansionary effect on domestic employment we have, cf. appendix, that

\[ g^{\text{coop}} > g^{\text{non-coop}} \text{ if } \frac{de}{dg} > 0 \] (13)

Note that \( \text{sign} \frac{de}{dg} = \text{sign} \frac{de}{dg^*} \).

Non-cooperative policy making implies that policies that can increase private employment are at an inefficiently low level. Non-cooperative policy making does not take into account that the policy benefits employment for the trade partners, but fears that competitiveness is deteriorated. More could be done to improve aggregate activity and employment if countries coordinated their fiscal policies.

In the opposite case we have

\[ g^{\text{coop}} < g^{\text{non-coop}} \text{ if } \frac{de}{dg} < 0 \] (14)

If public employment has a contractionary effect on employment, fiscal policy is at an inefficiently high level. Non-cooperative policy makers do not take into account that the policy is harming employment for its trade partners, and they perceive that competitiveness can be improved by use of this policy instrument.

In sum, non-cooperative policy making implies that fiscal policies are used too little if they are expansionary and too much if they are contractionary. It is an implication that in comparing private employment in the non-cooperative and the cooperative case we have unambiguously that

\[ e^{\text{coop}} > e^{\text{non-coop}} \] (15)

Private sector employment is thus unambiguously larger in the cooperative case compared to the non-cooperative case. In this sense non-cooperative policies have a contractionary bias. There are two mechanisms generating this result. Non-cooperative policy makers disregard the demand spillover and over-estimate the cost spillover.

Comparing the two cases in terms of total employment is less straightforward. If the policy is expansionary we have unambiguously that

\[ l^{\text{coop}} > l^{\text{non-coop}} \text{ if } \frac{de}{dg} > 0 \]
which follows directly from (13) and (15). Whereas for the contractionary case we have

\[ l^{coop} \geq l^{non-coop} \text{ if } \frac{de}{dq} < 0 \]

This result is interesting since it brings out that the inefficiency caused by non-cooperative policy making cannot be gauged from the total employment level, but only from private employment.

To interpret the present finding of a possible contractionary bias to the expansionary bias found in models with an exogenous specialized production structure it is worth noting the following difference between the two ways of modeling international trade. With exogenous specialization an increase in the demand for domestic products translates into an improvement in the terms of trade. In the present model with endogenous specialization a policy expanding private employment is perceived to increase wages and thus deteriorating wage competitiveness counteracting the expansionary effect. Therefore the expansionary effect is underestimated (relative) to the cooperative case, and accordingly non-cooperative policies end up being insufficiently expansionary (and vice versa if \( \lambda_g < 0 \)).

### 4.2 International integration

Would further international integration of product markets affect the interdependencies in policies, and if so in what direction? Is there reason to be more or less concerned about these externalities in the face of further integration than in the past with less integration?

The model with endogenous specialization allows a straightforward way to address one aspect of further market integration, namely a reduction in barriers to trade captured by a reduction in the friction \( \kappa \). To see the effects involved it is convenient first to solve for the symmetric cooperative equilibrium since it is the simplest. The optimal level of public employment is determined by the condition (12) balancing marginal benefits and costs of public sector activities. International integration may affect the optimal level of public employment by affecting the marginal costs.

The effect of international integration on the optimal level of public employment turns out to be ambiguous (see appendix).

\[ \frac{\partial g^{coop}}{\partial \kappa} \geq 0 \]

The reason for this ambiguity is that a reduction in the trade friction implies – other things being equal – an increase in the real income and employment, which in turn affects the marginal utility of consumption and leisure. Since the latter affects the marginal costs of public activities the effect is in general ambiguous. An interesting implication of this finding is that there is no unambiguous conclusion as to whether more product market integration leads to a shrinking or an expanding public sector.
Although the level effect is ambiguous, the difference between the cooperative and non-cooperative level of public employment increases when the trade friction is reduced. It can be shown that
\[
\frac{\partial}{\partial \kappa} | g^{\text{coop}} - g^{\text{non-coop}} | < 0
\]
i.e. the inefficiencies implied by non-cooperative policy decisions are larger the more integrated the product markets. The intuition for this result is that
\[
\frac{\partial}{\partial \kappa} \left| \frac{d e^{\text{coop}}}{d g} \right| - \frac{\partial}{\partial \kappa} \left| \frac{d e^{\text{non-coop}}}{d g} \right| < 0
\]
The lower the trade friction, the larger are the spillover effects, and therefore the larger (numerically) the difference between the effect on employment of changes in public employment perceived in the non-cooperative case compared to the cooperative case. Intuitively, the larger the spillover effects, the larger the difference between the cooperative and non-cooperative case, since this difference is driven by the spillover effects.

5 Concluding remarks

Product market structures matter for the interdependencies in fiscal policies. In models with exogenous specialization, non-cooperative fiscal policies tend to have an expansionary bias, whereas with endogenous specialization there is a tendency to a contractionary bias (measured in terms of private employment). This points both to the need to be careful about the specification of product market structures in analyses of fiscal policy interdependencies, but also the need for further work on the role of product market structures. The formulation adopted in the present paper is obviously very stylized and an important topic for future research is to consider various mechanisms affecting product market interdependencies.

An attraction of the present framework is that it also makes it possible to analyse the consequences of further international product market integration. Since this is an important part of the ongoing “globalization” process, this is important for its effect on how policy works and the need for policy coordination. The present analysis lends support to the view that domestic instruments become less effective, and policy spillovers increase the more integrated markets are. This suggests that the gains from policy coordination are increasing alongside the integration process. An interesting issue for further research would be to quantify these gains, and evaluate whether integration may significantly affect the gains from policy cooperation.

The present analysis shares the problem arising when analysing fiscal policy in an explicit general equilibrium framework that “Keynesian” aggregate demand effects do not arise, and that fiscal policy mainly works by affecting labour supply/wage formation. While one important channel through which
fiscal policy works, it is important in future work also to consider the aggregate demand effects that usually shape policy views and debates on the issue of fiscal policy coordination.

Appendix

Employment

Define $\omega \equiv \frac{W}{W^*} = \frac{\lambda(e,g)}{\lambda(e^*,g^*)}$ and $a_i \equiv \frac{A_i}{A_i^*}$. It follows that commodity $i$ is produced domestically if $a_i > a_i^* \equiv \omega(1 + \kappa)$, while there is no domestic production, and the commodity is imported if $a_i < a_i^* \equiv \omega(1 + \kappa)^{-1}$.

Equilibrium private employment can now be written as the sum of employment in the production of non-tradeables, the employment needed to serve the domestic market for the exportable, and the employment needed to serve the foreign demand for the exportable, i.e.

$$e = \int_{a_i}^{a_i^{AE}} A_i^{-1} \left( \frac{A_i^{-1}W}{P} \right)^{-\theta} \frac{W}{P} e h(a_i) da_i$$

$$+ \int_{a_i}^{\pi} A_i^{-1} \left( \frac{A_i^{-1}W}{P} \right)^{-\theta} \frac{W}{P} e h(a_i) da_i$$

$$+ \int_{a_i}^{\pi} (1 + \kappa)A_i^{-1} \left( \frac{A_i^{-1}W(1 + \kappa)}{P^*} \right)^{-\theta} \frac{W^*}{P^*} e^* h(a_i) da_i$$

where $h(a_i)$ gives the density function for the relative productivity. Note that the disposable income determining demand equals $(1 - \tau) \frac{W}{P} l = W \frac{e + g}{P} - \tau \frac{W}{P} l = \frac{W}{P} e$. Using the wage equation (8) we get

$$e = \int_{a_i}^{a_i^{AE}} A_i^{-1} \left( A_i^{-1}\lambda(e,g) \right)^{-\theta} \lambda(e,g) e h(a_i) da_i$$

$$+ \int_{a_i}^{\pi} A_i^{-1} \left( A_i^{-1}\lambda(e,g) \right)^{-\theta} \lambda(e,g) e h(a_i) da_i$$

$$+ \int_{a_i}^{\pi} (1 + \kappa)A_i^{-1} \left( A_i^{-1}\lambda(e,g)(1 + \kappa) \right)^{-\theta} \lambda(e^*,g^*) e^* h(a_i) da_i$$

where it has also been used that

$$P \equiv \left[ \int_0^{P_i^{1-\theta}} d\frac{1}{P_i^{1-\theta}} \right]^{1/\theta} = \left[ \int_0^{a_j} ((1 + \kappa)W A_i^{s-1})^{1-\theta} h(a_i) da_i + \int_{a_j}^{\pi} (W A_i^{s-1})^{1-\theta} h(a_i) da_i \right]^{-1/\theta}$$

$$= [v^{*} W^{s-1} + v W^{1-\theta}]^{\frac{1}{1-\theta}}$$

where

$$v^* \equiv (1 + \kappa)^{1-\theta} \int_0^{a_i} (A_i^{s-1})^{1-\theta} h(a_i) da_i$$

19
\[ v \equiv \int_{a_i} (A_i^{-1})^{1-\theta} h(a_i) da_i \]

Hence, the relative price can be written as

\[
\frac{P}{P^*} = \frac{[v^* W^{1-\theta} + v W^{1-\theta}]^{\frac{1}{1-\theta}}}{[v^* W^{1-\theta} + v W^{1-\theta}]^{\frac{1}{1-\theta}}} = \frac{[v + v \omega^{1-\theta}]^{\frac{1}{1-\theta}}}{[v^* \omega^{1-\theta} + v]^{\frac{1}{1-\theta}}} \equiv \rho(\omega), \quad \rho'(\omega) > 0
\]

In symmetric equilibrium \( \omega = 1 \), and \( \rho(1) = 1 \). Finally, it has been used that

\[
W \frac{P}{P^*} = W \frac{P}{P^*} = \lambda(e, g) \rho(\omega)
\]

The employment relation (16) can be rewritten

\[
1 = \lambda(e, g)^{1-\theta} \left[ \int_{a_i} A_i^{\theta-1} h(a_i) da_i 
+ \int_{a_E} A_i^{\theta-1} h(a_i) da_i 
+ \int_{a_E} (1 + \kappa)^{1-\theta} A_i^{\theta-1} \rho(\omega)^{-\theta} \lambda(e^*, g^*) e^* \lambda(e, g)e \ h(a_i) da_i \right]
\]

or in more compact form as

\[
1 = \lambda(e, g)^{1-\theta} [\Gamma_1 + \Psi \Gamma_2]
\]

where

\[
\Gamma_1 \equiv \int_{a_i} A_i^{\theta-1} h(a_i) da_i + \int_{a_E} A_i^{\theta-1} h(a_i) da_i > 0
\]

\[
\Gamma_2 \equiv (1 + \kappa)^{1-\theta} \int_{a_E} A_i^{\theta-1} h(a_i) da_i > 0
\]

\[
\Psi \equiv \rho(\omega)^{-\theta} \lambda(e^*, g^*) e^* \lambda(e, g)e > 0
\]

For later reference note that

\[
\Gamma_{1e} = -A_i^{\theta-1} h(a_i) \frac{\partial a_i}{\partial e} < 0, \quad \Gamma_{1g} = -A_i^{\theta-1} h(a_i) \frac{\partial a_i}{\partial g} \leq 0
\]

\[
\Gamma_{1e^*} = -A_i^{\theta-1} h(a_i) \frac{\partial a_i}{\partial e^*} > 0, \quad \Gamma_{1g^*} = -A_i^{\theta-1} h(a_i) \frac{\partial a_i}{\partial g^*} \leq 0
\]

\[
\Gamma_{2e^*} = -(1 + \kappa)^{1-\theta} A_i^{\theta-1} h(a_E) \frac{\partial a_E}{\partial e^*} < 0, \quad \Gamma_{2g^*} = -(1 + \kappa)^{1-\theta} A_i^{\theta-1} h(a_E) \frac{\partial a_E}{\partial g^*} \leq 0
\]
\(\Gamma_{e^*} = -(1 + \kappa)^{1 - \theta} A_E^{\theta - 1} h(a_E) \frac{\partial a_E}{\partial e^*} > 0, \Gamma_{2g^*} = -(1 + \kappa)^{1 - \theta} A_E^{\theta - 1} h(a_E) \frac{\partial a_E}{\partial g^*} \leq 0\)

\[\Psi_e = -\theta \rho(\omega)^{-\theta - 1} \rho \frac{\lambda(e^*, g^*)e^*}{\lambda(e, g)e} - \rho(\omega)^{-\theta} \frac{\lambda(e^*, g^*)e^*}{(\lambda(e, g)e)^2} (\lambda e + \lambda) < 0\]

\[\Psi_g = -\theta \rho(\omega)^{-\theta - 1} \rho \frac{\lambda(e^*, g^*)e^*}{\lambda(e, g)e} - \rho(\omega)^{-\theta} \frac{\lambda(e^*, g^*)e^*}{(\lambda(e, g)e)^2} (\lambda g) \geq 0\]

\[\Psi_{e^*} = \theta \rho(\omega)^{-\theta - 1} \rho \frac{\lambda(e^*, g^*)e^*}{\lambda(e, g)e} + \rho(\omega)^{-\theta} \frac{\lambda(e^*, g^*)e^*}{\lambda(e, g)e} > 0\]

\[\Psi_{g^*} = \theta \rho(\omega)^{-\theta - 1} \rho \frac{\lambda(e^*, g^*)e^*}{\lambda(e, g)e} + \rho(\omega)^{-\theta} \frac{\lambda(e^*, g^*)e^*}{\lambda(e, g)e} \leq 0\]

Note also that in symmetric equilibrium we have

\[\Psi_{e^*} = -\Psi_e ; \ \Psi_{g^*} = -\Psi_g, \ \Gamma_{ie} = -\Gamma_{ie^*} ; \ \Gamma_{ig} = -\Gamma_{ig^*} \ \ i = 1, 2\]

and

\[\frac{\partial a_i}{\partial e} = \frac{\lambda_e}{\lambda_g} \frac{\partial a_i}{\partial g} ; \ \frac{\partial a_E}{\partial e} = \frac{\lambda_e}{\lambda_g} \frac{\partial a_E}{\partial g}\]

Domestic and foreign employment can now be summarized by the implicit functions

\[e = \pi(e^*, g, g^*), \ \ e^* = \pi(e, g^*, g, \kappa)\]  

(18)

(19)

where

\[\frac{\partial e}{\partial e^*} = \pi_e = \frac{-\frac{\lambda^1\theta[\Gamma_{1e^*} + \Gamma_2 \Psi_{e^*} + \Psi \Gamma_{2e^*}]}{(1 - \theta)\lambda^\theta \lambda e [\Gamma_1 + \Psi \Gamma_2] + \lambda^{1 - \theta}[\Gamma_1e + \Gamma_2 \Psi e + \Psi \Gamma_2e]} > 0\]

\[\frac{\partial e}{\partial g} \equiv \pi_g = \frac{-(1 - \theta)\lambda^\theta \lambda g [\Gamma_1 + \Psi \Gamma_2] + \lambda^{1 - \theta}[\Gamma_{1g} + \Gamma_2 \Psi g + \Psi \Gamma_{2g}]}{(1 - \theta)\lambda^\theta \lambda e [\Gamma_1 + \Psi \Gamma_2] + \lambda^{1 - \theta}[\Gamma_{1e} + \Gamma_2 \Psi e + \Psi \Gamma_{2e}] \geq 0\]

\[\frac{\partial e}{\partial g^*} \equiv \pi_{g^*} = \frac{-(1 - \theta)\lambda^\theta \lambda e [\Gamma_1 + \Psi \Gamma_2] + \lambda^{1 - \theta}[\Gamma_{1g^*} + \Gamma_2 \Psi g^* + \Psi \Gamma_{2g^*}]}{(1 - \theta)\lambda^\theta \lambda g [\Gamma_1 + \Psi \Gamma_2] + \lambda^{1 - \theta}[\Gamma_{1g} + \Gamma_2 \Psi g + \Psi \Gamma_{2g}] \geq 0\]

It is easily seen that \(sign(\pi_g) = -sign \lambda_g\), and \(sign(\pi_{g^*}) = sign \lambda_g\), and hence \(sign(\pi_g) = -sign(\pi_{g^*})\), and moreover \(|\pi_e| < 1\).

**Employment multipliers**

It follows straightforward from (18) and (19) that

\[\frac{de}{dg} = \frac{1}{1 - (\pi_e)^2} [\pi_g + \pi_e \pi_{g^*}]\]

21
and
\[ \frac{de}{dg^*} = \frac{1}{1 - (\pi_e^*)^2} [\pi_g^* + \pi_e^* \pi_g^*] \]

To analyse the employment multipliers in further detail it is useful to define
\[
\begin{align*}
\Theta_1 & \equiv (1 - \theta) \lambda^{-\theta} \lambda_e \left[ \Gamma_1 + \Psi \Gamma_2 \right] < 0 \\
\Theta_2 & \equiv \lambda^{1-\theta} \left[ \Gamma_2 \Psi_e + \Gamma_1 e + \Psi \Gamma_2 e \right] < 0 \\
\Theta_3 & \equiv \lambda^{1-\theta} \left[ \Gamma_2 \Psi_g + \Gamma_1 g + \Psi \Gamma_2 g \right] \geq 0
\end{align*}
\]

Note that \( \text{sign} \Theta_3 = -\text{sign} \lambda_g \), and moreover.
\[
\begin{align*}
\Theta_3 &= \frac{\lambda_g}{\lambda_e} \Theta_2 + \frac{\lambda_g^{1-\theta} \Gamma_2 \rho - \theta \rho}{e} \\
&= \frac{\lambda_g}{\lambda_e} \left[ \Theta_2 + \lambda^{1-\theta} \Gamma_2 \rho - \theta \rho \frac{1}{e} \right] = \frac{\lambda_g}{\lambda_e} \Theta_3
\end{align*}
\]

where \( \tilde{\Theta}_3 \equiv \Theta_2 + \lambda^{1-\theta} \Gamma_2 \rho - \theta \rho \frac{1}{e} < 0 \).

It follows that
\[
\begin{align*}
\pi_e^* &= \frac{\Theta_2}{\Theta_1 + \Theta_2} \\
\pi_g &= -\frac{\lambda_g}{\lambda_e} \frac{\Theta_1 + \Theta_3}{\Theta_1 + \Theta_2} \\
&= -\frac{\lambda_g}{\lambda_e} \frac{1 + \lambda^{1-\theta} \Gamma_2 \rho - \theta \rho \frac{1}{e}}{\Theta_1 + \Theta_2} \\
\pi_g^* &= \frac{\Theta_3}{\Theta_1 + \Theta_2}
\end{align*}
\]

and therefore
\[
\pi_g + \pi_g^* = -\frac{\lambda_g}{\lambda_e} \frac{\Theta_1}{\Theta_1 + \Theta_2}
\]
hence \( \text{sign}(\pi_g + \pi_g^*) = -\text{sign} \lambda_g \). Moreover
\[
\frac{\pi_g^*}{\pi_g} = -\frac{\Theta_3}{\lambda_g \Theta_1 + \Theta_3} = -\frac{\tilde{\Theta}_3}{\Theta_1 + \tilde{\Theta}_3}
\]

implying that
\[
|\pi_g| > |\pi_g^*|
\]

Turning to the expressions determining the employment multiplier we have
\[
\begin{align*}
\pi_g + \pi_e^* \pi_g^* &= -\frac{\lambda_g}{\lambda_e} \frac{\Theta_1 + \Theta_3}{\Theta_1 + \Theta_2} + \frac{\Theta_3}{\Theta_1 + \Theta_2} \frac{\Theta_2}{\Theta_1 + \Theta_2} \\
&= \frac{1}{\Theta_1 + \Theta_2} \left[ -\frac{\lambda_g}{\lambda_e} \Theta_1 + \Theta_3 + \Theta_3 \frac{\Theta_2}{\Theta_1 + \Theta_2} \right] \\
&= \frac{-\Theta_1}{\Theta_1 + \Theta_2} \left[ \frac{\lambda_g}{\lambda_e} + \frac{\Theta_3}{\Theta_1 + \Theta_2} \right]
\end{align*}
\]
hence \( \text{sign} (\pi_g + \pi_e \pi_g^*) = -\text{sign} \lambda_2 \). Using that

\[
\pi_g^* + \pi_e \pi_g = \frac{\Theta_3}{\Theta_1 + \Theta_2} \left( \frac{\lambda_e \Theta_1 + \Theta_3}{\Theta_1 + \Theta_2} \right)
\]

\[
= \frac{1}{\Theta_1 + \Theta_2} \left( \Theta_3 - \Theta_2 \frac{\lambda_e \Theta_1}{\Theta_1 + \Theta_2} \right)
\]

\[
= \frac{1}{\Theta_1 + \Theta_2} \left( \Theta_3 - \Theta_2 \frac{\lambda_e \Theta_1}{\Theta_1 + \Theta_2} \right)
\]

\[
= \frac{\Theta_1}{(\Theta_1 + \Theta_2)^2} \left( \Theta_3 - \Theta_2 \frac{\lambda_e \Theta_1}{\Theta_1 + \Theta_2} \right)
\]

\[
= \frac{\Theta_1}{\Theta_1 + \Theta_2} \left( \frac{\lambda_e \Theta_1}{\Theta_1 + \Theta_2} \right)
\]

\[
= \frac{\lambda_e \Theta_1}{\Theta_1 + \Theta_2}
\]

\[
\text{it follows that} \quad \text{sign}(\pi_g^* + \pi_e \pi_g) = -\text{sign} \lambda_2, \quad \text{and therefore} \quad \text{sign} \frac{dc}{dg} = \text{sign}(\pi_g^* + \pi_e \pi_g) = \text{sign} \pi_g^* = -\text{sign} \lambda_2. \quad \text{Finally, note that}
\]

\[
\frac{dc}{dg} + \frac{de}{dg^*} = \frac{\pi_g + \pi_e \pi_g^*}{1 - (\pi_e^e)^2} + \frac{\pi_g^* + \pi_e \pi_g}{1 - (\pi_e^e)^2}
\]

\[
= \frac{\pi_g^* + \pi_e \pi_g}{1 - (\pi_e^e)^2}
\]

\[
= \frac{\lambda_e \Theta_1}{\Theta_1 + \Theta_2}
\]

\[
= \frac{-\lambda_2}{\lambda_e}
\]

\[
(20)
\]

**Product Market Integration**

In symmetric equilibrium the key term in the employment relation can be written (17) as

\[
\Gamma_1 + \Psi_1 = \int_{a_f} A_{iE}^{\theta-1} h(a_i) da_i + \int_{a_E} A_{i}^{\theta-1} h(a_i) da_i + (1 + \kappa)^{-\theta} \int_{a_E} A_{i}^{\theta-1} h(a_i) da_i
\]

\[\text{We have that}\]

\[
\frac{\partial \Gamma_1 + \Psi_2}{\partial \kappa} = -\frac{\partial a_f}{\partial \kappa} \left(\frac{A_{E}^{\theta-1} h(a_f)}{A_{I^*}^{\theta-1} h(a_I)}\right)^{\theta-1} - \frac{\partial A_{iE}}{\partial \kappa} \int_{a_E} A_{i}^{\theta-1} h(a_i) da_i + (1 + \theta)(1 + \kappa)^{-\theta} \int_{a_E} A_{i}^{\theta-1} h(a_i) da_i
\]

\[
= \left[-(1 + \kappa)^{-\theta} \frac{\partial A_{iE}}{\partial \kappa} \left(\frac{A_{E}^{\theta-1} h(a_f)}{A_{I^*}^{\theta-1} h(a_I)}\right)^{\theta-1} - \frac{\partial a_f}{\partial \kappa} \left(\frac{A_{E}^{\theta-1} h(a_f)}{A_{I^*}^{\theta-1} h(a_I)}\right)^{\theta-1}\right] + (1 + \theta)(1 + \kappa)^{-\theta} \int_{a_E} A_{i}^{\theta-1} h(a_i) da_i
\]

\[
= \left[1 + \left(\frac{A_{I^*}^{\theta-1} h(a_I)}{A_{E}^{\theta-1} h(a_f)}\right)^{\theta-1}\right] + (1 + \theta)(1 + \kappa)^{-\theta} \int_{a_E} A_{i}^{\theta-1} h(a_i) da_i < 0
\]

\[23\]
Where it has been used that $\Psi = 1$, $I = E^*$ and $E = I^*$. Moreover, symmetry across the two countries implies that $A_E = A_F(1 + \kappa)$, and $\frac{A_{E}}{A_{F}} = 1$. It now follows from (17) directly that
\[ \frac{\partial e}{\partial \kappa} < 0 \]
and therefore
\[ \frac{\partial r}{\partial \kappa} < 0; \quad \frac{\partial c}{\partial \kappa} < 0 \]
Moreover,
\[ \frac{\partial}{\partial \kappa} U(c, l, g) = [U(c, \lambda_c \epsilon) \frac{\partial e}{\partial \kappa}] < 0 \]
Employment multipliers and product market integration
Consider first the $\Theta$-coefficients. It follows from (17) that in symmetric equilibrium
\[ \Theta_1 = (1 - \theta)\lambda^{-\theta}\lambda_c \lambda^{\theta - 1} \]
and therefore
\[ \frac{\partial \Theta_1}{\partial \kappa} = 0 \]
Using that
\[ \Theta_2 = \lambda^{1-\theta}[\Gamma_2\Psi_e + \Gamma_1 + \Gamma_2] \]
where
\[ \Gamma_1 + \Gamma_2 = - A_{E}^{\theta - 1} h(a_I) \frac{\partial a_I}{\partial e} - (1 + \kappa) A_{E}^{\theta - 1} h(a_E) \frac{\partial a_E}{\partial e} \]
and
\[ \frac{\partial (\Gamma_1 + \Gamma_2)}{\partial \kappa} = - \left[ 1 - \frac{1}{(1 + \kappa)^2} \right] \lambda_c A_{F}^{\theta - 1} h(a_I) < 0 \]
it follows by observing that $\Psi_e \frac{\partial \Theta_2}{\partial \kappa} > 0$, $\exists \Gamma_2 : h(a) \leq \Gamma_2$ for all $a$ such that
\[ \frac{\partial \Theta_2}{\partial \kappa} = \lambda^{1-\theta}[\Psi_e \frac{\partial \Gamma_2}{\partial \kappa} + \frac{\partial (\Gamma_1 + \Gamma_2)}{\partial \kappa}] > 0 \]
Turning to $\Theta_3$ we have
\[ \Theta_3 = \lambda^{1-\theta}[\Gamma_2 \Psi g + \Gamma_1 + \Psi \Gamma_2] \]
and therefore
\[ \frac{\partial \Theta_3}{\partial \kappa} = \lambda^{1-\theta}[\frac{\partial}{\partial \kappa} (\Gamma_2 \Psi g) + \frac{\partial (\Gamma_1 + \Gamma_2)}{\partial \kappa}] \]
where
\[
\Gamma_{1g} + \Psi \Gamma_{2g} = -A_I^{-1} h(a_I) \frac{\partial a_I}{\partial g} - (1 + \kappa)^{1-\theta} A_E^{-1} h(a_E) \frac{\partial a_E}{\partial g}
\]
\[
= - \left[ \frac{1}{1 + \kappa} + 1 + \kappa \right] \lambda_g A_I^{-1} h(a_I)
\]
and
\[
\frac{\partial (\Gamma_{1g} + \Psi \Gamma_{2g})}{\partial \kappa} = - \left[ 1 - \frac{1}{(1 + \kappa)^2} \right] \lambda_g A_I^{-1} h(a_I)
\]
Note that
\[
\Psi g = -\theta \rho_o \frac{\lambda_g}{\lambda(e^*, g^*)} - \frac{1}{\lambda(e, g)} \lambda_g
\]
where
\[
\rho_o = \frac{v - v^*}{v + v^*}
\]
and
\[
\frac{\partial \rho_o}{\partial \kappa} = \frac{2 \left( \frac{\partial v^*}{\partial \kappa} - \frac{\partial v^*}{\partial \kappa} \right)}{(v + v^*)^2} > 0
\]
since
\[
\frac{\partial v}{\partial \kappa} = (A_I^{-1})^{1-\theta} h(a_I) \frac{1}{(1 + \kappa)^2} > 0
\]
\[
\frac{\partial v^*}{\partial \kappa} = (1 - \theta)(1 + \kappa)^{-\theta} \int_{a_I}^{a_I} \lambda_e (A_I^{-1})^{1-\theta} h(a) da - \frac{1}{(1 + \kappa)^2} < 0
\]
In symmetric equilibrium it follows that $\Gamma_1 = v$ and $\Gamma_2 = v^*$. This follows from the definition of the terms and the symmetry property of the underlying distribution of productivity (and $I = E^*$ and $E = I^*$).

Using that
\[
\Psi g \Gamma_2 = v^* \left[ -\theta \rho_o - 1 \right] \frac{\lambda_g}{\lambda}
\]
\[
= v^* \left[ -\theta \frac{v - v^*}{v + v^*} - 1 \right] \frac{\lambda_g}{\lambda}
\]
\[
= \frac{v^*}{v + v^*} \left[ -\theta(v - v^*) - v - v^* \right] \frac{\lambda_g}{\lambda}
\]
\[
= - \frac{v^*}{v + v^*} [(1 + \theta)v + (1 - \theta)v^*] \frac{\lambda_g}{\lambda}
\]
we have that
\[
\frac{\partial}{\partial \kappa} \Psi g \Gamma_2 = \left[ - \frac{\partial v^*}{\partial \kappa} \left[(1 + \theta)v + (1 - \theta)v^* \right] - \frac{v^*}{v + v^*} \left[(1 + \theta) \frac{\partial v}{\partial \kappa} + (1 - \theta) \frac{\partial v^*}{\partial \kappa} \right] \right] \frac{\lambda_g}{\lambda}
\]
since \( \frac{\partial v^*}{\partial \kappa} < 0 \) it follows that \( \exists \tilde{h}: h(a) \leq \tilde{h} \) for all \( a \) such that

\[
sign \frac{\partial \Theta_3}{\partial \kappa} = sign \lambda_g
\]

recall that \( sign \Theta_3 = -sign \lambda_g \)

It is an implication of the above findings that for \( h(a) \leq \tilde{h} \equiv \min \{ \bar{h}_1, \bar{h}_2 \} \) for all \( a \) that

\[
\frac{\partial \pi_e}{\partial \kappa} = \frac{\partial}{\partial \kappa} \left[ \frac{\Theta_2}{\Theta_1 + \Theta_2} \right] = \frac{\partial \Theta_2}{\partial \kappa} \left( \Theta_1 + \Theta_2 \right) - \Theta_2 \frac{\partial \Theta_2}{\partial \kappa} \left( \Theta_1 + \Theta_2 \right)^2 = \frac{\partial \Theta_2}{\partial \kappa} \Theta_1 \left( \Theta_1 + \Theta_2 \right)^2 < 0
\]

moreover

\[
\frac{\partial \pi_g}{\partial \kappa} = -\frac{\lambda_g}{\lambda_c} \lambda^{1-\theta} \rho^{-\theta} \frac{1}{e} \frac{\partial \kappa}{\partial \kappa} \frac{\Gamma_2}{\Theta_1 + \Theta_2}
\]

therefore

\[
sign \frac{\partial \pi_g}{\partial \kappa} = -sign \lambda_g
\]

Finally, using

\[
\pi_{g^*} = \frac{\lambda_g}{\lambda_c} \left[ \frac{\Theta_1}{\Theta_1 + \Theta_2} + \lambda^{1-\theta} \rho^{-\theta} \frac{1}{e} \frac{\Gamma_2}{\Theta_1 + \Theta_2} \right]
\]

and

\[
\frac{\partial}{\partial \kappa} \frac{\Theta_1}{\Theta_1 + \Theta_2} = \frac{-\Theta_1 \frac{\partial \Theta_2}{\partial \kappa}}{(\Theta_1 + \Theta_2)^2} > 0
\]

if follows that

\[
sign \frac{\partial \pi_{g^*}}{\partial \kappa} = sign \lambda_g
\]

In summary, we have

\[
\frac{\partial \pi_{e^*}}{\partial \kappa} < 0, \frac{\partial | \pi_g |}{\partial \kappa} > 0, \frac{\partial | \pi_{g^*} |}{\partial \kappa} < 0
\]

For later reference note that

\[
\frac{\partial}{\partial \kappa} \frac{\Gamma_2}{\Theta_1 + 2 \Theta_2} = \frac{\partial \Gamma_2}{\partial \kappa} \left( \Theta_1 + 2 \Theta_2 \right) - \Gamma_2 \frac{\partial \Theta_2}{\partial \kappa} \left( \Theta_1 + 2 \Theta_2 \right)^2 = \frac{\partial \Gamma_2}{\partial \kappa} \Theta_1 + 2 \left[ \Theta_2 \frac{\partial \Gamma_2}{\partial \kappa} - \Gamma_2 \frac{\partial \Theta_2}{\partial \kappa} \right]
\]

using that

\[
\frac{\partial \Gamma_2}{\partial \kappa} \Theta_1 > 0
\]
and
\[
\frac{\partial \Gamma_2}{\partial \kappa} \Theta_2 - \Gamma_2 \frac{\partial \Theta_2}{\partial \kappa} = \frac{\partial \Gamma_2}{\partial \kappa} \left( \lambda^{1-\theta} [\Gamma_2 \Psi_e + \Gamma_1 e + \Psi \Gamma_2 c] \right) - \Gamma_2 \lambda^{1-\theta} \left[ \Psi_e \frac{\partial \Gamma_2}{\partial \kappa} + \frac{\partial (\Gamma_1 e + \Gamma_2 c)}{\partial \kappa} \right]
= \frac{\partial \Gamma_2}{\partial \kappa} \left( \lambda^{1-\theta} [\Gamma_1 e + \Psi \Gamma_2 c] \right) - \Gamma_2 \lambda^{1-\theta} \left[ \frac{\partial (\Gamma_1 e + \Gamma_2 c)}{\partial \kappa} \right] > 0
\]

it follows that
\[
\frac{\partial}{\partial \kappa} \frac{\Gamma_2}{\Theta_1 + 2\Theta_2} > 0
\]

The fiscal employment multiplier is given as
\[
de = \frac{\lambda_g}{\lambda_e} \frac{\Theta_1 + \Theta_2}{\Theta_1 + 2\Theta_2} - \frac{\Theta_3}{\Theta_1 + 2\Theta_2}
= -\frac{\lambda_g}{\lambda_e} \left[ \frac{\Theta_1 + \Theta_2 - \Theta_3}{\Theta_1 + 2\Theta_2} \right]
= -\frac{\lambda_g}{\lambda_e} \left[ \frac{\Theta_1 + \Theta_2 - \left( \Theta_2 + \frac{\lambda^{1-\theta} \Gamma_2 \rho^{-\theta} \Gamma_2}{\Theta_1 + 2\Theta_2} \right) }{\Theta_1 + 2\Theta_2} \right]
= -\frac{\lambda_g}{\lambda_e} \left[ \frac{\Theta_1 + \lambda^{1-\theta} \Gamma_2 \rho^{-\theta} }{\Theta_1 + 2\Theta_2} \right]
= -\frac{\lambda_g}{\lambda_e} \left[ \frac{\Theta_1 + \lambda^{1-\theta} \rho^{-\theta} \Gamma_2}{\Theta_1 + 2\Theta_2} + \lambda^{1-\theta} \rho^{-\theta} \frac{\Gamma_2}{e \Theta_1 + 2\Theta_2} \right]
\]

It follows that
\[
\frac{\partial}{\partial \kappa} \left[ \frac{de}{dg} \right] = -\text{sign} \lambda_g
\]

Since (20) implies
\[
\frac{\partial}{\partial \kappa} \left[ \frac{de}{dg} \right] + \frac{\partial}{\partial \kappa} \left[ \frac{de}{dg^*} \right] = 0
\]

it follows that
\[
\frac{\partial}{\partial \kappa} \left[ \frac{de}{dg^*} \right] = \text{sign} \lambda_g
\]

In summary we therefore have
\[
\begin{align*}
| \frac{\partial}{\partial \kappa} \frac{de}{dg} | & > 0 \\
| \frac{\partial}{\partial \kappa} \frac{de^*}{dg} | & < 0
\end{align*}
\]

Cooperative policy
The symmetric equilibrium employment is determined by the condition
\[
1 = \lambda (e, g)^{1-\theta} \int_{a_E}^{a_I} A_i^{\theta-1} h(a_i) da_i + \int_{a_E}^{e} A_i^{\theta-1} h(a_i) da_i + \int_{a_E}^{(1+\kappa)} A_i^{\theta-1} h(a_i) da_i
\]
where \( a_E = (1 + \kappa) \) and \( a_I = (1 + \kappa)^{-1} \). Accordingly
\[
\frac{de}{dg} = \frac{-\lambda_g}{\lambda_e} < 0
\]
The condition determining optimal public employment reads
\[
\Gamma_g = V_g + [U_c \lambda_c e] \left( \frac{de}{dg} \right) + U_l + U_c \lambda_g e = 0
\]
The second order condition is
\[
\Gamma_{gg} < 0
\]
It follows that
\[
\frac{dg}{d\kappa} = -\frac{\Gamma_{gs}}{\Gamma_{gg}}
\]
Hence \( \text{sign} \left( \frac{dg}{d\kappa} \right) = \text{sign} \Gamma_{g\kappa} \)
Using that \( \frac{de}{dg} = -\frac{\lambda_g}{\lambda_e} \) it follows that
\[
\Gamma_g = V_g + U_l
\]
Therefore
\[
\Gamma_{g\kappa} = U_{lc} \frac{\partial c}{\partial \kappa} + U_{ll} \frac{\partial l}{\partial \kappa} \geq 0
\]
Hence,
\[
\frac{\partial g^{coop}}{\partial \kappa} \geq 0
\]
Comparison of \( \frac{de^{non-coop}}{dg} \) and \( \frac{de^{coop}}{dg} \)
Note that the results derived above directly give the perceived effects of policy in the non-cooperative case. Using from (20) that
\[
\frac{de}{dg} + \frac{de^{* non-coop}}{dg} = \frac{de^{coop}}{dg}
\]
it follows that
\[
\frac{de^{non-coop}}{dg} = \frac{de^{coop}}{dg} = \frac{de^{non-coop}}{dg}
\]
and therefore
\[
\frac{de^{non-coop}}{dg} = \frac{de^{coop}}{dg} - \frac{de^{* non-coop}}{dg}
\]
In this case $\frac{dg}{\lambda g} < 0$, $\frac{dg}{\lambda g} < 0$, $\frac{dg}{\lambda g} < 0$ hence

$$0 > \frac{dg}{\lambda g} > \frac{dg}{\lambda g}$$

In this case $\frac{dg}{\lambda g} > 0$, $\frac{dg}{\lambda g} > 0$, $\frac{dg}{\lambda g} > 0$ hence

$$0 < \frac{dg}{\lambda g} < \frac{dg}{\lambda g}$$

It follows that

$$\frac{|dg|}{\lambda g} > \frac{|dg|}{\lambda g}$$

Using from above that

$$\frac{\partial |dg|}{\lambda g} < 0$$

it follows that

$$\frac{\partial |dg|}{\lambda g} < 0$$

References


Andersen, T.M. and A. Sørensen, 2003, Product market integration, rents and wage formation, CEPR Working paper.


Cooper, R., and A. John, 1988, Coordinating Coordination Failures in Keynesian Models, Quarterly Journal of Economics, 103, 441-463.


Holmlund, B., 1997, Macroeconomic Implications of Cash Limits in the Public Sector, Economica, 64, 49-62.


IMF, 2002, World Economic Outlook, September.


Lockwood, B., 2000, Commodity Taxation and Tax Coordination under Destination and Origin Principle, CEPR DP 2556.


