Optimal contract length in a reputational model of monetary policy

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Received 15 October 1993; revised 15 November 1995

Abstract

We develop a reputational model of monetary policy with endogenous contract length. In choosing contract length private sector agents trade off recontracting costs against the expected costs resulting from trading under contracts based on incorrect inflation expectations. However, due to an externality, the private and socially optimal contract lengths differ. When selecting individual contract length private agents do not consider the effect of contract length on the surprise inflation/output trade-off. In a reputational model of monetary policy this results in an excessive equilibrium inflation rate. The socially optimal outcome may be achieved by public policies that reduce recontracting costs.

JEL classification: E5

Keywords: Monetary policy; Credibility; Contract length

1. Introduction

A lot of recent research into credibility aspects of monetary policy has focused attention on the role of reputation as a solution to the time consistency problem (see the surveys by Fischer (1986), Rogoff (1987), and Blackburn and Christensen (1989)). One line of this literature follows Barro and Gordon (1983), Canzoneri

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(1985), Alesina (1987), and Fischer and Summers (1989) by describing the determination of monetary policy as a strategy played by the government in an infinitely repeated game with discounting (a supergame). In these games the economy is usually modeled by a surprise Phillips curve, providing the government with a short run incentive to create unanticipated inflation, thus raising output. However, such policies will ruin the government’s reputation, and lead to higher inflation in the longer run. The equilibrium ‘best credible’ monetary policy, and hence inflation rate, is then given by the lowest possible inflation rate which is sustainable, that is, a policy promise where the short run gain from a deviation by the government is outweighed by the long run loss of reputation and resulting increase in inflation.

In this literature it is usually assumed that nominal wage contracts constitute the microfoundations for the surprise Phillips curve, 1 thus the duration of any output change associated with a movement along the surprise Phillips curve depends upon the length of contracts. 2 However contract length is endogenous and potentially subject to influence by government policy.

The purpose of this paper is to analyze a model that incorporates endogenous labour contract length in a reputational model of monetary policy determination. The private sector consists of a large number of agents that individually set the contract length at the privately optimal level. However, as each agent has only a negligible effect on aggregate variables he will ignoring that longer contracts increase the persistence of any output deviation from the natural rate. However the more persistent is the output deviation the greater will be the gain for a government that induces an inflationary surprise. This leads to a higher equilibrium rate of inflation. In other words, due to an external effect the private and socially optimal contract lengths differ, leading to an excessive level of inflation in equilibrium. 3 It will also be shown that the government may use negotiation subsidies to achieve contracts that are socially optimal.

While the contribution of this paper is primarily theoretical it explains rather well the stylized facts concerning changes in the nature of the US business cycle in the pre 1939 and post 1945 periods. We show that the higher average rate of inflation and lower amplitude in output cycles in the post 1945 period may be explained very naturally by our model. We shall follow Gray and Kandil (1991) in attributing structural changes in real variables to changes in the economy’s underlying distribution of shocks. However, our analysis will have the added advantage of also being able to explain structural changes in nominal variables.

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1 See Fischer (1977) for the original formulation.


3 We conjecture that this result will also occur in asymmetric information models of monetary policy where private agents gradually learn the policy maker’s preferences (see Cukierman (1992), for an excellent overview of this literature).
The rest of our paper is organized as follows. In Section 2 we present the
general structure of the model. In Section 3 we solve the model to obtain the
privately optimal contract length and the implications for inflation and output. We
also provide a brief discussion of the empirical evidence in this section. In Section
4 we derive the socially optimal contract length, demonstrate that it differs from
the private optimum, and examine the public finance issue of optimal contract
renegotiation subsidies. In Section 5 we give a conclusion. All proofs are in the
appendix.

2. The model

The paradigm used in this paper is a continuous time hybrid of the Barro and
Gordon (1983) monetary policy game model and the Gray (1978) model of
endogenous contract length. The model consists of a government and a large
number of homogeneous private sector agents. The government, which dislikes
inflation but desires output, controls the inflation rate via monetary policy (subject
to a random component). The private agents form price level expectations on the
basis of which they agree on nominal wage contracts of a specified length, \( \tau \).
These nominal wage contracts are uniformly staggered over time, and are based on
rational price level expectations made at the time contracts are fixed. The length of
the contract is set optimally by the private agents, where the costs of frequent
renegotiations are weighed against the costs of expectational errors with respect to
the price level. The economy is described in reduced form by a surprise Phillips
curve which determines output relative to inflation for given price level expecta-
tions and contract length. Formally, all agents are treated as infinitely lived.
However, the discount rates \( \delta \) (government) and \( r \) (private agents) could be

4 There is the question of why the agents do not agree fully contingent contracts, which would
remove the need for renegotiation. In our model a fully contingent contract would involve full
indexation to the aggregate price level. Such a contract may not be chosen both because of potential set
up costs, and, if wage rate changes themselves are costly, because of the expense of frequent wage
adjustments. To simplify the exposition we have chosen to ignore indexation completely, but our main
results would not be qualitatively effected by partial indexation. In a straight Barro and Gordon set up
partial indexation will lower the rate of inflation under discretion. This has an ambiguous effect on the
best credible inflation rate since the values both of Temptation and Enforcement fall. In our analysis
optimal contract length would increase with the degree of indexation. We cannot say what will happen
to inflation as two effects are present. Indexation may reduce the government’s incentive to inflate as
discussed for the Barro and Gordon case. However, even if it does so the longer contracts will increase
the government’s incentive to inflate (see proposition 4 below). We cannot conclude that the
introduction of indexation by the government (even if the indexation scheme were itself a time
consistent policy) can push the result of the policy game towards the social optimum. If the indexation
scheme and contract length were chosen simultaneously by private agents they still have no incentive
to consider the external effects of their choices on the government’s trade off between inflationary
surprises and output. That the resultant choices end up being socially optimal seems unlikely.
interpreted as incorporating the probability that the government loses office, or that private agents cease to participate in the economy.

The preferences of the government are described by a quadratic expected cost function of the form

\[
E\left\{ \int_{t=0}^{\infty} e^{-\delta t} \left[ \left( \frac{1}{2} \right) \left( \Pi(t) \right)^2 + b' \left( \bar{Y} - Y(t) \right)^2 \right] dt \right\}
\]

(1)

where \( \Pi(t) \) is the realized inflation rate, \( \bar{Y} \) the natural rate of output and \( b' > 0 \) is a preference parameter (\( Y \) and \( P \) below are measured in natural logarithms). The government’s objective is to minimize this cost function subject to the constraint imposed by a surprise Phillips curve of the form

\[
Y(t) = \bar{Y} + \alpha \left[ P(t) - P^E(t) \right]
\]

(2)

where \( P(t) \) is the aggregate price level and \( P^E(t) = (1/\tau) \int_{t=0}^{t} P^E(t, t-\tau) d\tau \) the aggregate price level expectation that is the basis for contracts agreed at time \( t \), where \( P^E(t, t-I) \) is the expected price level at time \( t \), of all agents who agreed on a contract based on expectations at \( t-I \) (Recall that contracts are revised at intervals of \( \tau \) length, and the starting points of these intervals are distributed uniformly over time). (2) is of course a reduced form representation of the economy. Combining (1) and (2) gives

\[
E\left\{ \int_{t=0}^{\infty} e^{-\delta t} \left[ \left( \frac{1}{2} \right) \Pi(t)^2 - b \left( P(t) - P^E(t) \right)^2 \right] dt \right\}
\]

(3)

where \( b = \alpha b' \). The government’s choice variable is assumed to be the deterministic component of the inflation rate \( \Pi(t) \) which is assumed to be set at discrete intervals (‘at noon each day’). This is a purely technical assumption, to avoid the problems with trigger strategies in continuous time repeated game models. The relationship between the realized price level and \( \Pi(t) \) is

\[
P(t) = P(h) + \int_{i=h}^{t} \Pi(i) di + \int_{i=h}^{t} \epsilon(i),
\]

(4)

where \( \epsilon(t) = \sigma z(t) \), \( z \) is a standard Wiener process (i.e. independent, zero mean and unit variance increments), and \( \sigma \) is the variance parameter. As the Wiener process is not differentiable, the price level will not be differentiable. However, we define the rate of inflation as the average growth in the price level measured over short intervals, assumed to be of length \( dt' \)

\[
\Pi(t) = \frac{1}{dt'} \left( \int_{i=t}^{t+dt'} \Pi(i) di + \int_{i=t}^{t+dt'} \epsilon(i) \right).
\]

(5)

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A possible policy would have the government adjusting the policy component of $\Pi(t)$ to compensate for variations in the stochastic element, so as to keep the realized rate of inflation constant. For a given average rate of inflation this strategy would be optimal. However, as will become clear below, any such compensation will change the length of the contract period, and hence the best credible inflation rate. It is not clear that such a government policy would be optimal. To concentrate attention on the issues we are concerned with here we shall assume that the government may only choose among inflation rates that are non-contingent within intervals of length $\tau$. \footnote{The issue of how much the government should attempt to keep the realized inflation rate constant is part of a much broader debate. We believe this model unsuitable for exploring this issue, and that any model constructed for that purpose should -- at least -- allow for the distinctions between real and nominal shocks.}

The private agents incur costs if their price level expectations are incorrect, when they renegotiate contracts or if they have to experience inflation. Their expectations of the price level are formed rationally, formally $P^E(t, j\tau) = E[P(t)|I(j\tau)]$, where $I(j\tau)$ is the information set at time $j\tau$. The duration of the wage contract, $\tau$, is given by minimizing the following expected cost function:

$$
E\left(\sum_{j=0}^{\infty} \int_{t=j\tau}^{(j+1)\tau} e^{-rt}\left[ P(t) - P^E(t, j\tau) \right]^2 dt \right) + \sum_{j=0}^{\infty} e^{-rj\tau} c + \int_{t=0}^{\infty} e^{-rt} \left( \frac{1}{2} \right) \Pi(t)^2 dt,
$$

(6)

where $c > 0$ is the cost of renegotiating the nominal contract. The private agents treat their expectations of government policy as certain. In equilibrium, there will be no deviation by the government, so the private agents' expectations of government policy will be correct. We have

**Proposition 1.** There exists a unique privately optimal contract length, $\tau^* \in (0, +\infty)$ which minimizes expression (6). $\tau^* = \tau(\sigma, r, c)$ has the following properties: $\tau_1(\cdot) < 0$, $\tau_2(\cdot) \geq 0$, $\tau_3(\cdot) > 0$.

The proof of this and all subsequent propositions may be found in the appendices.

These are standard conclusions, the privately optimal contract length is increasing in the renegotiation cost, decreasing in the variance of the price level, $\sigma^2$, and has a complex relationship to the discount factor $r$. Our next step is to relate $\tau^*$ to the outcome of a policy game between the government and private agents.
3. The effects of contract length in a policy game

In this section we set out a continuous time version of the Barro and Gordon (1983) model. In Barro and Gordon’s model discretionary policy is defined as the policy maker’s optimal rate of inflation for one period given that he treats current and future inflationary expectations as exogenous. In effect, the policy maker neglects any consequences beyond the first period when setting the rate of inflation. Barro and Gordon suggest that the period length in their model should correspond to “the length of time over which the policy maker can enjoy the results of his cheating”. In a continuous time model, as in the real world, things are a bit more complicated. As contract revision is distributed over time, some agents will adapt more quickly to the cheating than others. Our aim is to define discretionary policy in the continuous time setting so that it corresponds as closely as possible to the discrete time formulation (we provide further discussion of this at the end of this section). So we assume that the government sets the optimal rate of inflation over a period \([0, \tau]\) equal to the length of a contract, it neglects any consequences beyond \(\tau\). In the optimization problem the government takes into account that new contracts are being continuously set, and that at the time they are being determined these new contracts incorporate the correct price level and rate of inflation. As shown in the appendix, if \(\overline{\Pi}\) is the policy maker’s announced policy, which private agents believe (rationally in equilibrium) will be followed, then the total expectational error at time \(t \in [0, \tau]\) is

\[
P(t) - P^E(t) = (\overline{\Pi} - \hat{\Pi}) t \left(1 - \frac{t}{\tau}\right) + U(t, \tau), \tag{7}
\]

where the cumulative stochastic error for the interval \([t - i, t]\) is \(u(t, t - i) \equiv \int_{t-i}^{t} e^{-i\delta} \, d\epsilon(j)\), and \(U(t, \tau) \equiv (1/\tau) \int_{0}^{\tau} u(t, t - i) \, di\). The first term corresponds to the errors in policy expectations (i.e. the accumulated policy error adjusted for the proportion of the private agents that make the error) and the second term is the total error due to the stochastic component. Government policy under discretion is given by

\[
\min \mathbb{E}\left\{\int_{t=0}^{\tau} e^{-\delta t} \left[\left(\frac{1}{2}\right)(\overline{\Pi}(t))^2 - b(P(t) - P^E(t))\right] \, dt\right\}, \tag{8}
\]

where the relationships between \(\overline{\Pi}\) and \(\Pi(t)\) and \(P(t) - P^E\) are as defined in (5) and (7) respectively.

Now, the expectational error is

\[
\int_{t=0}^{\tau} e^{-\delta t} (P(t) - P^E(t)) = (\overline{\Pi} - \hat{\Pi}) H(\tau, \delta) + Z(\tau, \delta), \tag{9}
\]

where

\[
H(\tau, \delta) \equiv \int_{t=0}^{\tau} e^{-\delta t} t \left(1 - \frac{t}{\tau}\right) \, dt \quad \text{and} \quad Z(\tau, \delta) \equiv \int_{t=0}^{\tau} e^{-\delta t} U(t, \tau) \, dt. \tag{10}
\]
$H(\tau, \delta)$ is a complex expression that indicates how the policy error is reflected in an accumulated gain in output above the natural rate. Note that the expected values of the Wiener process is zero, while the variance is a constant independent of government policy, $\Pi$. Thus, here and below we can safely ignore the stochastic component of the price level in the derivation of government policy. Let $K(\tau, \delta) = \int_0^\tau e^{-\delta t} dt$. The minimization problem (8) is equivalent to

$$\min_{\Pi} \left( \frac{K(\tau, \delta)}{2} \right) \Pi^2 - bH(\tau, \delta)(\Pi - \Pi^D)$$

We may now derive

**Proposition 2.** The government's optimal discretionary monetary policy is $\Pi^D = bH(\tau, \delta)/K(\tau, \delta)$. Furthermore, $\partial \Pi^D / \partial \tau > 0$.

This result differs from Barro and Gordon (1983)'s in two important respects. First, the optimal discretionary policy is increasing in the contract length. The intuition here is that with longer contracts it takes longer for private agents to adapt to the actual price level. Thus, an increase in the rate of inflation will have a larger effect on output. This induces the government to choose a higher inflation rate. Second, the optimal discretionary policy depends on the discount factor. This is due to our use of a continuous time model, where an increase in the rate of inflation involves instantaneous costs from higher inflation, whereas it takes some time ($\tau$) until the benefit from higher output reaches its maximum level.

From Proposition 2, it is clear that if the government neglects any consequences of it's policy beyond the contract period, then the only possible equilibrium when the private agents form rational price expectations involves expected inflation equal to $\Pi^D$.

Clearly any policy with $\Pi \in [0, \Pi^D]$ yields a lower expected cost for both the government and private sector agents than $\Pi^D$. Thus, if the government announces and follows some policy $\Pi \in [0, \Pi^D]$ and this is believed by private sector agents, then this would be superior to the policy outcome under discretion, $\Pi^D$. However the problem faced is that this $\Pi^D$ must be credible, i.e., the government must find it optimal to actually set $\Pi = \Pi^D$ ex post. If this is not the case, the private agents will not believe in the promise. To obtain such a solution we introduce the notion of reputational effects. Barro and Gordon assume that if the government breaks the promise in one period, it is 'punished' in the next with inflation expectations equal to the discretionary policy $\Pi^D$. We assume that private agents will believe a governments policy promise provided the government had kept it's promises for the prior interval of length $\tau$. However, once a policy promise is broken, inflationary expectations will be $\Pi^D$ for an interval of length $2\tau$. This consists of the period $\tau$ it takes all private agents to adjust, plus a further
period $\tau$ of punishment. \footnote{There are two key problems associated with trigger strategy equilibria in models of this class. Firstly, the equilibria are not unique. Secondly, it is not clear how private agents coordinate on a strategy and hence an equilibrium. This analysis does suggest a possible solution to these problems. The trigger strategy is selected by the agent or agents renegotiating contracts at that $t$ when the government cheats. Other agents follow the initial lead when they renegotiate.} (With very long punishment periods a promise of zero inflation will be credible for reasonable contract lengths. When zero inflation is credible, Proposition 4 below is modified in an obvious way: The best credible policy promise is zero irrespective of the contract length).

Consider now the implications of the government cheating on the interval $[0, 2\tau]$, by announcing an inflation rate policy $\tilde{\Pi} < \Pi^D$, but playing $\tilde{\Pi} = \Pi^D$. From Proposition 2, $\Pi^D$ is indeed the optimal policy if the government is to cheat, as long as the private agents expect $\Pi^D$ whenever the government deviates (below we consider an alternative assumption where expectations are contingent on $\tilde{\Pi}$). From (3) and (7) we may write the expected cost over the period $[0, 2\tau]$ experienced by a government that cheats as (here and below we omit the stochastic component, as this cancels out in the further analysis; moreover note that $\int_{t=0}^{2\tau} e^{-\delta t} dt = K(\tau, \delta)(1 + \beta)$, where $\beta = e^{-\delta\tau} < 1$ – the discount factor over the length of the contract period)

$$\left(\frac{K(\tau, \delta)(1 + \beta)}{2}\right) (\Pi^D)^2 - bH(\tau, \delta)(\Pi^D - \tilde{\Pi}).$$

(12)

The expected costs of a government that does not cheat over the same time period is

$$\left(\frac{K(\tau, \delta)(1 + \beta)}{2}\right) (\tilde{\Pi})^2.$$

(13)

The lowest credible inflation rate policy is then the lowest $\tilde{\Pi}$ such that (13) $\leq$ (12). It is found by equating (12) to (13) and simplifying:

$$\left(\frac{K(\tau, \delta)(1 + \beta)}{2}\right) \left[ (\Pi^D)^2 - (\tilde{\Pi})^2 \right] - bH(\tau, \delta)(\Pi^D - \tilde{\Pi}) = 0.$$

(14)

Expression (14) may be solved for the lowest credible inflation rate policy, i.e. the lowest inflation rate promise the government will not fink on.

Proposition 3. Let $\tilde{\Pi}(\tau, \delta)$ be the lowest credible policy promise, that is, the lowest possible $\tilde{\Pi}$ for which (14) holds. Then, $\tilde{\Pi}(\tau, \delta) = \Pi^D(2/(1 + \beta) - 1) > 0$.

This corresponds to the Barro and Gordon (1983) result, but also includes an important extra element. The best credible inflation rate policy depends upon contract length; specifically
Proposition 4. \( \frac{\partial \Pi(t)}{\partial \tau} > 0 \). The best credible inflation rate is increasing in the contract length.

Longer contracts provide both a greater temptation for the government to cheat and inflate, as they can enjoy greater accumulated output effects, and greater enforcement, as the punishment period is extended. However, longer contracts also push the punishment period further into the future where it is discounted more. There are thus opposing effects on the best credible inflation rate. We find that the effects on temptation are greater and thus longer contracts imply a higher best credible inflation rate.

3.1. Robustness

It might appear that our result so far are sensitive to the assumptions made about how private agents form their expectations. We wish to briefly argue that they are quite robust. \(^8\) Suppose now that private sector price level expectations are contingent, specifically that they expect that when the government deviates it will continue with this policy. If the government cheats at \( t = 0 \) and sets \( \Pi > \bar{\Pi} \), then the private agents will expect \( \bar{\Pi} \) for all \( t \in [0, 2\tau] \). The optimal level of cheating \( \Pi^c \) is found by minimizing

\[
\min_{\Pi^c} \left\{ \frac{K(\tau, \delta)(1 + \beta)}{2} \Pi^2 - bH(\tau, \delta)(\bar{\Pi} - \Pi) \right\}.
\]

(15)

solving the first-order condition gives

\[
\Pi^c = \frac{bH(\tau, \delta)}{K(\tau, \delta)(1 + \beta)} > 0.
\]

(16)

(1 + \beta) > 1 implies that \( \Pi^c < \Pi^D \). If the government were to deviate, it would set an inflation rate below the discretionary policy \( \Pi^D \) as this would reduce the long-run costs of a rise in inflation. If the government sticks to it’s promised \( \bar{\Pi} \), the costs are \( K(\tau, \delta)(1 + \beta)\bar{\Pi}^2/2 \). The lowest credible policy promise, \( \bar{\Pi}^c \) when the alternative government policy is to cheat and play \( \Pi^c = bH(\tau, \delta)/[K(\tau, \delta)(1 + \beta)] > 0 \), is found using the same approach as above. It is then easily found that no promised inflation rate below \( \bar{\Pi}^c = bH(\tau, \delta)/[K(\tau, \delta)(1 + \beta)] \) will ever be credible. The intuition is simple but striking. For given expectations the optimal rate of inflation when cheating solves for the best possible constant rate over the interval \([0, 2\tau]\). Any policy promise will also involve a constant rate

\(^8\) In the working paper version of the model we analyze contingent price level expectations in greater detail, and also provide analysis of how the results are affected by allowing for renegotiation. We find that our conclusions are qualitatively unaffected if we require contracts to be Weakly Renegotiation Proof or Pearce Renegotiation Proof. This analysis is available on request to the interested reader.
of inflation over this interval, combined with certain inflationary expectations. By
definition the promised rate of inflation cannot give lower costs than the optimal.

Proposition 5. When private sector price level expectations are contingent the
following hold.
(i) The optimal level of cheating by the government is $II^c = bH(\tau, \delta)/
[K(\tau, \delta)(1 + \beta)]$.
(ii) The lowest possible policy promise is $\tilde{II}^c = \tilde{II} = bH(\tau, \delta)/[K(\tau, \delta)(1 +
\beta)]$.
(iii) The inflation level is increasing in the contract length $-\partial II^c/\partial \tau =
\delta \tilde{II}^c/\partial \tau > 0$.
The intuition behind these results is exactly as before. The effect of contract length
on the outcome of the game is an effect external to the individual private agent’s
choice problem.

3.2. Some empirical evidence

At this juncture it is interesting to note how well this model explains some of
the key stylized fact about the US business cycle in the pre 1939 and post 1945
eras. To this end we summarize the conclusions provided by Taylor (1986) and

Taylor in comparing the two periods 1910–1940 and 1952–83 concludes that
the latter period has been characterized by: 1. Higher average levels of wage and
price inflation. 2. Lower variances of the output gap, wage and price inflation. 3.
A much lower variance of shocks to inflation. 4. More persistent inflation and
output fluctuations. Taylor’s findings agree with those of DeLong and Summers
(1986) who in comparing 1923–1940 and 1949–1982 conclude that in the latter of
these two periods: 1. Output cycles have lower amplitude. 2. Output shocks have
greater persistence. 3. Aggregate demand shocks are smaller. 4. Wage contracts
are longer. 5. Average inflation is higher (4–5% higher). Further support for these
conclusions may be found in Sachs (1980), and Gordon (1980). A brief inspection
of our preceding model reveal that it is consistent with all five of these stylized
facts. In our model the lower is the variance of the exogenous price shocks (as
characterized by $\sigma^2$), the longer will be the contract length, the more persistent

\footnote{It was the apparent conflict of facts 1 and 4 that spawned the literature on whether or not wage
flexibility is destabilizing. Our analysis agrees with Gray and Kandil (1991) that facts 1 and 4 are both
endogenous consequences of fact 3. However, our analysis improves on Gray and Kandil’s since they
cannot explain the difference in the average inflation rates across the two periods.
\footnote{Our model is neither designed for nor equipped to analyse the effects of oil price shocks. We do
point out that this time period was part of the Taylor and DeLong and Summers data sets that produced
the empirical results we claim support our theory. One interpretation might be that despite the unusual
events of the 70s, the features our theory explains are still strong enough to emerge from the data.}
will be output deviations and the greater will be the inflation rate (see Proposition 4). It should be noted that our model does not appear to be consistent with the cross-country correlation between mean inflation and nominal demand variability of 0.92 found by Ball et al. (1988). (We note that this inconsistency only arises if it is assumed there is a significant positive correlation between nominal demand variability and the variance of unanticipated shocks to inflation as the theoretical model specifies). Our view is that there are other country specific factors and institutions that may account for the findings by Ball et al. (1988). The preferences of governments (and how they are selected in different economic environments), monetary and labour market institutions may all vary across countries. Thus we believe that the direct time series evidence on the US business cycle presented by Taylor (1986), DeLong and Summers (1986) and others (with which our model is consistent) is a more suitable test for our model than the cross-country evidence of Ball et al..

4. Public policy and contract length

In the preceding sections we have established the relationship between contract length and the inflation rate determined as an equilibrium solution to a policy game. In this section we investigate whether the privately optimal contract length is socially optimal, and the potential role of public policy in determining contract length. First we wish to know if $\tau^*$ is socially optimal, where the socially optimal $\tau$ is defined as that which minimizes the costs of the private agents, when it is taken into consideration that the inflation rate is endogenous. \[11\]

**Proposition 6.** There exists a $\tau^{**}$ which is socially optimal. Furthermore, $\tau^* > \tau^{**}$, the privately optimal contract is longer than the socially optimal contract whenever zero inflation is incredible $\tilde{\Pi}(t) = \Pi(\tau, \delta) > 0$. \[12\]

The intuition behind Proposition 6 is that to each agent the equilibrium inflation rate is exogenous (as each agent has a negligible effect on the aggregate economy), so when setting the contract length, the private agents will not take into consideration that long contracts result in greater persistence for any deviation in output from the natural rate and thus in a higher equilibrium rate of inflation. However, that the choice of contract length sets the parameters for the policy game

\[11\] Given the analysis that follows the term privately collectively optimal might be superior to socially optimal since we have not discussed how social optimality relates to the government objective function.

\[12\] Our analysis in this section is for the case of non-contingent private sector expectations as in the development of Proposition 3. The same result may be proven for contingent private sector price level expectations as in Proposition 5. We do not provide this result here, it is, however, available on request.
is known by both the government and private sector and provides a rationale for public policy.

The privately optimal choice of contract length depends on the variance of inflation, \( \sigma^2 \), the private discount factor, \( r \), and the renegotiation cost, \( c \). This renegotiation cost may take several forms, it may be time spent in the negotiation process, an agency or legal fee paid to some third party, or even the risk of lost production due to an industrial dispute. Clearly there are public policies which affect the incidence of these costs. For example the provision of an arbitration service may significantly reduce time spent in the negotiation process. Suppose now that the renegotiation cost \( c \) is rewritten \( c(1 - k) \) where \( k \in [0, 1] \) represents the government’s share in the cost, to be financed by lump-sum taxes on the private agents. Can this \( k \) be chosen such that private agents choose the socially optimal contract length?

**Proposition 7.** There exists a unique \( k \in [0, 1] \) such that \( \tau^* = \tau^{**} \).

Notice that while a policy of \( k = 1 \) would reduce contract length \( \tau^* \) to zero, and therefore make the labour market a spot market, this would imply continuous renegotiations and infinite renegotiation costs, thus this would not be socially optimal.

### 5. Conclusion

The purpose of this paper has been to analyze the consequences for equilibrium inflation of endogenizing the length of private sector contracts in a reputational model of monetary policy. The model developed has the merit of facilitating the investigation of these theoretical issues and of explaining very well some of the stylized differences between the pre 1939 and post 1945 business cycle. We find that the privately optimal contract length is not socially optimal. Contracts are too long because individual contract setters do not take into account the external implications of contract length for the determination of monetary policy. The long contracts increase the temptation of the government to raise inflation in an attempt to raise output. The consequence of the increase in temptation is that the government must set a higher rate of inflation in order to make it credible.

We also show that the problem of sub-optimal private contract length can be solved if the government adopts policies such as providing arbitration services that effectively subsidize contract renegotiation. Such subsidies provide incentives that may cause the private and socially optimal contract lengths to correspond. There are of course other externalities of contract length, \(^{13}\) yet our analysis suggests in which way policies that affect renegotiation costs should be directed.

\(^{13}\) Ball (1987) shows that there is an externality of contract length on the variability of aggregate demand.
Acknowledgements

Financial support from the Leif Johansen foundation and the Research Council of Norway is gratefully acknowledged. This paper is part of the research project ‘Unemployment, institutions and economic policy’ at SNF-Oslo. We wish to thank Geir Asheim and Alex Cukierman for very helpful discussions, and Jo Anna Gray and two anonymous referees for this journal for many useful comments.

Appendix A

Proof of Proposition 1. As the private agents’ expectations of government policy will be correct, the only source to expectational errors concerning the price level is the random component, so the expected cost function can be reformulated to

\[
\text{Min} \sum_{j=0}^{\infty} \int_{t=j\tau}^{(j+1)\tau} e^{-r(t)}(t-j\tau)\sigma^2 \, dt + \sum_{j=0}^{\infty} e^{-r\tau}c + \int_{t=0}^{\infty} e^{-r(t)} \left( \frac{1}{2} \right) \Pi(t)^2 \, dt
\]

(6')

where we have used the stochastic integral

\[
E\left[ P(t) - P^E(t, j\tau) \right]^2 = E\left[ \int_{t=j\tau}^{T} (d\epsilon(i) - E[d\epsilon(i)|I(j\tau)]) \right]^2 = (t-j\tau)\sigma^2.
\]

(A.1)

Consider the first integral in (6'):

\[
\int_{t=j\tau}^{(j+1)\tau} e^{-r(t)}(t-j\tau)\sigma^2 \, dt = \sigma^2 e^{-r\tau} \int_{0}^{\tau} e^{-rt} \, dt
\]

(A.2)

Now,

\[
\int_{t=0}^{\tau} e^{-rt} \, dt = -\left[ \frac{1}{r} e^{-rt} + \frac{1}{r^2} e^{-rt} \right]
\]

(A.3)

Using (A.3), the minimand gives

\[
\text{Min} \sum_{j=0}^{\infty} \left\{ \left( \frac{\sigma^2}{r} \right) \left[ \left( \frac{1}{r} e^{-r\tau} - \left( \frac{1}{r} e^{-r(j+1)\tau} \right) + e^{-r\tau}c \right) \right] + \int_{t=0}^{\infty} e^{-r\tau} \frac{1}{2} \Pi(t)^2 \, dt \right\}
\]

(A.4)

Now solving the geometric series

\[
\sum_{j=0}^{\infty} e^{-r\tau} = \frac{1}{1 - e^{-r\tau}} \quad \text{and} \quad \sum_{j=0}^{\infty} e^{-r(j+1)\tau} = \frac{e^{-r\tau}}{1 - e^{-r\tau}}.
\]

(A.5)
The minimization problem reduces to
\[
\begin{align*}
\text{Min} & \left\{ \frac{\sigma^2}{r^2} + \frac{c}{1 - e^{-r\tau}} - \left( \frac{\sigma^2}{r} \right) \left( \frac{e^{-r\tau}}{1 - e^{-r\tau}} \right) \right. \\
& 
\left. + \int_0^\tau \frac{e^{-r\tau}}{2} \Pi(t)^2 \, dt \right\}.
\end{align*}
\] (A.6)

The first-order condition to this optimization problem is
\[
\begin{align*}
\frac{-c \sigma e^{-r\tau}}{(1 - e^{-r\tau})^2} - \frac{\sigma^2}{r} \left( \frac{e^{-r\tau}}{1 - e^{-r\tau}} \right) \\
+ \frac{\sigma^2 \tau}{r} \left[ \frac{re^{-r\tau} (1 - e^{-r\tau}) + re^{-r\tau}e^{-r\tau}}{(1 - e^{-r\tau})^2} \right] = 0
\end{align*}
\] (A.7)

where the left-hand side can be simplified to yield
\[
\sigma^2 \left[ \tau - \frac{(1 - e^{-r\tau})}{r} \right] - rc = 0. \tag{A.8}
\]

Now note that
\[
\frac{d}{d\tau} \left[ \tau - \frac{(1 - e^{-r\tau})}{r} \right] = 1 - \frac{1}{e^{r\tau}} \geq 0 \quad \forall \tau \geq 0 \tag{A.9}
\]
so the first term in (A.8) is monotonically increasing in \( \tau \). Furthermore, the left-hand side of (A.8) is continuous in \( \tau \geq 0 \), approaches \(-rc < 0\) when \( \tau \) approaches zero and converges to infinity when \( \tau \) converges to infinity. Thus there exists a unique \( \tau = \tau^* \) which satisfies (A.8).

\( \square \)

**Derivation of (7).** The expectational error at \( t \in (0, \tau] \) made by agents forming expectations at \( t - i \) will be
\[
P(t) - P^E(t, t - i) = \int_{j=t-i}^t \left( \bar{\Pi}(j) - \bar{\Pi}(j, t - i) \right) dj + u(t, t - i),
\] (A.10)

where \( \bar{\Pi}(j, t - i) \) is the inflation rate expected at time \( j \) by agents forming expectations at time \( t - i \). (A.10) simply states that the actual error is the sum of the policy error (the difference between the expected inflation rate and the deterministic component of the realized rate) and the cumulative stochastic error. The policy error of the private agents is \( t - i < 0 \) indicates that the contract is made before time 0, so the duration of the error at time \( t \) is \( t; t - i > 0 \) indicates that the contract is made after time 0, for these contracts there is no policy error
\[
\int_{j=t-i}^t \left( \bar{\Pi}(j) - \bar{\Pi} \right) dj = \begin{cases} 
(\bar{\Pi} - \bar{\Pi}) t & \text{for } t - i \leq 0, \\
0 & \text{for } t - i > 0.
\end{cases}
\] (A.11)
Now the total error – across all individuals – at $t \in [0, \tau]$ may be expressed as

$$\frac{1}{\tau} \int_{i=0}^{\tau} \left[ P(t) - P^E(t, t-i) \right] di$$

$$= \frac{1}{\tau} \int_{i=0}^{t} \left[ P(t) - P^E(t, t-i) \right] di + \left( \frac{1}{\tau} \right) \int_{i=t}^{\tau} \left( P(t) - P^E(t, t-i) \right) di$$

$$= \left( \frac{1}{\tau} \right) \int_{i=t}^{\tau} (\bar{P} - \bar{P}) t di + \frac{1}{\tau} \int_{i=0}^{\tau} u(t, t-i) di$$

$$= (\bar{P} - \bar{P}) t \left( 1 - \frac{t}{\tau} \right) + \frac{1}{\tau} \int_{i=0}^{\tau} u(t, t-i) di \quad \text{(A.12)}$$

where we use $\int_{i=t}^{\tau} t di = \tau t - t^2$.

**Proof of Proposition 2.** The first-order condition to (11) is $K(\tau, \delta) \Pi^D - bH(\tau, \delta) = 0$, which solves for $\Pi^D = bH(\tau, \delta)/K(\tau, \delta)$. To prove that $\partial \Pi^D / \partial \tau > 0$, we solve the integrals and define $x = \delta \tau$, to obtain

$$\Pi^D = \frac{bH(\tau, \delta)}{K(\tau, \delta)} = \frac{xe^x - 2e^x + 2 + x}{\delta x (e^x - 1)} \quad \text{(A.13)}$$

Now, $d \Pi^D / dx$ is proportional to $d \Pi^D / d\tau$. Differentiation of $\Pi^D$ with respect to $x$ gives the numerator (the denominator is squared and hence positive)

$$\delta (e^x + xe^x - 2e^x + 1) x (e^x - 1) - \delta (e^x - 1 + xe^x)(xe^x - 2e^x + 2 + x)$$

$$\quad \text{(A.14)}$$

which simplifies to

$$2(e^{2x} - (x^2 + 2)e^x + 1). \quad \text{(A.15)}$$

Let

$$g(x) = e^{2x} - (x^2 + 2)e^x + 1. \quad \text{(A.16)}$$

Now $g(0) = 0$ while

$$g'(x) = 2e^x(e^x - x - 1 - x^2/2). \quad \text{(A.17)}$$

Inspection of (A.17) reveals $g'(x) > 0 \forall x$, because

$$e^x = 1 + x + x^2/2 + x^3/3 + \ldots. \quad \text{(A.18)}$$

Thus $d \Pi^D / d\tau > 0$.
Proof of Proposition 3. (14) has two solutions:

\[
\tilde{\Pi} = \begin{cases} 
\frac{2bH(\tau, \delta)}{K(\tau, \delta)(1 + \beta)} - \Pi^D = \Pi^D \left( \frac{2}{1 + \beta} - 1 \right), \\
\Pi^D.
\end{cases}
\] (A.19)

It is clear that the former is the smaller, as \(2/(1 + \beta) - 1 < 1\). □

Proof of Proposition 4. \(\partial \tilde{\Pi}/\partial \tau > 0\) follows from \(\partial \Pi^D/\partial \tau > 0\) and \(\partial \beta/\partial \tau < 0\). □

Proof of Proposition 5. (i)–(ii) as in the text, (iii) because \(\partial H(\tau, \delta)/\partial \tau > 0\). □

Proof of Proposition 6. The socially optimal contract length is found by minimizing the expected costs of the private agents \(R(\tau)\). From the proofs of Proposition 1 and Proposition 3 we may write the problem faced by a social planner as

\[
\min_{\tau} R(\tau) = \min_{\tau} \left\{ S(\tau) + \int_{t=0}^{\infty} \frac{1}{2} \tilde{\Pi}(\tau, \delta)^2 dt \right\}
\] (A.20)

where

\[
S(\tau) = E \left\{ \sum_{j=0}^{\infty} \int_{t=j \tau}^{(j+1) \tau} e^{-rt} \left[ P(t) - P^E(t, j \tau) \right]^2 dt \right\} + \sum_{j=0}^{\infty} e^{-rt} c.
\] (A.21)

From Proposition 1, \(\tau^*\) is determined by the first-order condition \(S'(\tau^*) = 0\) (each agent treats the rate of inflation as exogenous). Moreover, (A.9) ensures that \(S'(\tau) > 0\) for all \(\tau > \tau^*\). From Proposition 4, \(\tilde{\Pi}\) is increasing in \(\tau\), so we also have that \(R'(\tau) > 0\) for all \(\tau \geq \tau^*\). Thus, contract length longer than \(\tau^*\) involves higher social costs than a contract length of \(\tau^*\).

Now, \(R(\tau)\) is a continuous function, so it must have a minimum over the closed interval \([0, \tau^*]\). Thus there exists a socially optimal contract length (which minimizes (A.20)) \(\tau^* < \tau^*\). Note that the last term in (6') approaches infinity when \(\tau\) approaches zero, so we must have \(\tau^* < 0\). (Note however that \(\tau^*\) is not necessarily unique). □

Proof of Proposition 7. Rewriting the first-order condition from the private agents minimization problem gives

\[
\sigma^2 \left[ \tau - \left( \frac{1 - e^{-r\tau}}{r} \right) \right] - rc(1 - k) = 0.
\] (A.22)

We know: (i) at \(k = 0\) \(\tau^* > \tau^* > 0\); (ii) at \(k = 1\) \(\tau^* = 0 < \tau^*\); (iii) the first-order condition is continuous in \(\tau\) and \(k\). Hence \(\exists k \in [0, 1]\) such that
\[ \tau^* = \tau^{**}, \text{ such a } k \text{ will be unique if } \tau \text{ is monotonically decreasing in } k. \]

Differentiating the FOC and rearranging gives

\[ \frac{d\tau}{dk} = \left[ \frac{r \epsilon}{\sigma^2 (1 - 1/e^{-r\tau})} \right] < 0 \quad \forall \tau \in [0, 1]. \]

### References


