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WAGE BARGAINING, HOLDOUT, AND INFLATION

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In many countries it is customary that production continues under the terms of the old contract during wage negotiations (holdout), unless a work stoppage is initiated. This paper analyses a model where the workers deliberately work less efficiently during a holdout, while the firm reduces bonus payments. If a holdout is more costly to the firm than to the workers, the wage bargaining will result in a nominal wage increase. The model implies a Phillips curve that consists of two vertical parts; one with high inflation and low unemployment and one with low inflation and high unemployment.

1. Introduction

In recent years a number of countries have introduced formal targets for the rate on inflation. However, a possible problem in realizing the targets lies in the possible existence of an inflationary tendency in the system of wage formation. After all, in most countries the present system of wage determination has evolved over decades of higher inflation than what is now deemed acceptable, and this may have caused the system of wage determination itself to adapt to the high rate of inflation.

Although the possibility of an inflationary tendency in the system of wage determination also is acknowledged by keen advocates of inflationary targets (e.g. Svensson, 1994), this possibility has received little attention in the literature. A possible reason is that in the typical macroeconomic model with wage negotiations, there is no room for an inflationary tendency. In models of the type of Layard et al. (1991) or Dixon (1987), the classical dichotomy holds; the wage and price setting determine the real wage level and the equilibrium unemployment rate, whereas the money growth determines the rate of inflation.

The aim of the present paper is to show that there may exist a tendency to inflation in wage determination systems in many modern economies. The source of this inflationary tendency lies in the possibility of holdout in wage negotiations, i.e. that production can take place under the terms of the old contract while the parties are bargaining. This holdout possibility has received scant attention in the literature (some exceptions are Cramton and Tracy, 1992; Holden, 1994; and MacLeod and Malcolmson, 1992), yet holdouts appear to be more frequent than strikes in both the US (Cramton and Tracy, 1992) and the Netherlands (van Ours and van de Wijngaert, 1992).

As emphasised in the literature on incomplete contracts, a contract will rarely be so specific that it covers all aspects that determine the payoffs of the parties. Usually, there is some scope for the parties to inflict a cost at the opponent while still observing the contract (or at least not undertake any verifiable
violations of the contract). This may be done to affect the bargaining outcome during a holdout. An inflationary bias may result if a holdout is more costly to the firm than to the workers. Typically, the workers may follow the working rules meticulously in order to reduce profits (work-to-rule), yet the firm may have to pay the nominal wage of the old contract, as there are no verifiable violations of the contract on the part of the workers. In this situation the workers may use the old nominal wage as a leverage, and the firm will accept a nominal wage rise to avoid work-to-rule.

Inflation is, however, not inevitable in real life, nor is it in the present model. In the model, this is ensured by the assumption that the firms may break the holdout threats by threatening to close the plant, fire all workers, and open a new plant with a new workforce. However, this threat is only credible if the firm would actually benefit from doing so if the workers demanded a wage rise. In the present model, a threat of closing the plant is credible only if the unemployment rate is sufficiently high, in which case it will be relatively easy for the firm to hire new labour at lower wages.

Correspondingly, if there is a shortage of labour, workers can use threats of leaving the firm in order to obtain wage rises above what they can achieve with holdout threats. Thus, the inflationary tendency in the model does not imply a specific rate of inflation. As in standard macroeconomic models, in equilibrium the rate of inflation is determined by the rate of money growth. The inflationary tendency does, however, imply that the rate of money growth affects the level of unemployment. It turns out that the Phillips curve consists of two vertical parts; one with high inflation and low unemployment and one with low inflation and high unemployment. For the intermediate level of inflation, the Phillips curve is flat, with unemployment being in the range between low and high unemployment.

Intuitively, the basis for this particular shape of the Phillips curve is the following. Under holdout threats, the bargaining position of the workers depends on the rate of inflation, as the payoffs during a holdout depend on the old nominal contract. With low inflation, holdout threats are powerful, giving the workers a strong bargaining position. However, in equilibrium the real wage is given by the price setting, so there must be something that weakens the bargaining position of the workers. The rate of unemployment has this function; unemployment must be high, so that firms may break the holdout threats by credibly threatening to close the plant.

With high inflation, on the other hand, holdout threats are weak and not sufficient for the workers to obtain the equilibrium real wage. Thus, unemployment must be low, so that workers may use threats of leaving the firm to obtain the equilibrium real wage.

The virtues of the present paper are twofold. First, it draws attention to an important empirical phenomenon of wage negotiations, holdouts, as a possible source of inflation. Thus, the model points directly to those aspects of the wage setting which contribute to inflation. For instance, the introduction of bonus payments that are not paid during a holdout would weaken the bargaining
position of the workers and thus reduce the rate of money-wage growth resulting from wage negotiations under holdout threats (the rate of growth in money wages may even become negative).

Secondly, the paper shows how strategic bargaining theory of the Binmore et al. (1986) type can be used to analyse holdouts within a macroeconomic framework. A special problem in this respect is that under holdout threats the outcome of the wage bargaining in one year may affect the wage bargaining in subsequent years. To make the analysis tractable, I restrict attention to steady-state equilibria of the model.

In a related paper (Holden, 1994), I set up a wage bargaining model with endogenous type of conflict (strike, lock-out, or holdout), which is also integrated into an imperfect competition macroeconomic model. However, the present paper allows for a holdout being costly to the parties, which is the basis for the inflationary bias. Furthermore, in Holden (1994) attention is restricted to one contract period, while in the present paper there is an infinite horizon of wage contracts, allowing for an explicit analysis of the relationship between consecutive contract periods. On the other hand, the possibility of strikes and lock-outs is neglected in the present paper (cf. discussion in Section 6 below).

Section 2 below discusses the key assumptions of the bargaining model, and Section 3 sets out the basic macroeconomic model. Section 4 analyses the equilibrium of one year in isolation, while the full equilibrium of the model is analysed in Section 5. Some concluding remarks are given in Section 6. All proofs are in the Appendix.

2. A motivation of the basic assumptions

An essential assumption of the model is that the wage is given by the old contract during a holdout. In this section I argue that this is a good description of wage negotiations in many countries. In most European countries, union contracts are usually of finite duration. However, unless a work stoppage has been initiated, it is in most countries a well-established practice that production continues under the terms of the old contract until a new agreement is reached, even after the old contract has expired. In many European countries—at least Austria, France, Norway, Spain, and Sweden—the firm and the union are bound by the law to observe the old contract (cf. the country chapters in Blanpain, 1987).

The employer can, however, unilaterally terminate the agreement, following specific legal procedures and after some time delay. But if the union contract is terminated, the terms of this contract are in many countries considered to be included in the individual employment contracts. Thus, even in this situation the employer cannot easily reduce employees' money wages without the employees' consent. In general the employer would have to give the employees conditional notice, to be effective if the employees do not accept the wage reduction. If the employees do not accept the wage reduction, they are entitled to keep their old wage during the period of notice. Moreover, in many countries
this type of conditional notice is only lawful if the wage reduction can be justified by the economic situation of the firm. Although the employer may in the end be able to unilaterally reduce money wages, the uncertainty of the outcome and the risk for a costly legal process may to some extent deter the employer from trying.¹

In the US, the employer is bound by the law to observe the old union contract until a bargaining impasse is reached. As an impasse can be broken (for example, if the union reduces its wage demand), it might be difficult for the employer to unilaterally reduce wages during the bargaining (Gold, 1989, p. 36).

The feature that a wage level prevails until it is changed by mutual consent is, however, not confined to the union sector. In some countries similar rules apply also in the non-union sector. In England, this was shown by Rigby v Ferodo Limited where the employer had unilaterally proposed a 5% wage reduction. The employee had continued to work, without accepting the wage reduction. The courts held that the wage reduction was in breach of contract, so that the employee could claim the arrears of pay wrongfully withheld from him under the contract (McCullen, 1992). The employer can circumvent this problem by terminating the contract with due notice, and then offering a new contract with lower pay. But here also there is a risk that the employee will claim unfair dismissal (McCullen, 1992).

To what extent is holdout important empirically? Cramton and Tracy (1992) present data indicating that holdouts occur more frequently than strikes in the US, and van Ours and van de Wijngaert (1992) show that this also holds in the Netherlands. Furthermore, it is well-known that unions often use other weapons than strikes, e.g. work-to-rule, go-slow, etc. (Blanpain, 1987), which may indicate that a holdout takes place. Evidence for Britain reported in Salamon (1987) indicates that other types of industrial action than strikes, like overtime ban, work-to-rule, and go-slow, account for 55% of the total number of industrial actions taken by manual workers, while for non-manual workers the corresponding number is 60–70%. Work-to-rules and go-slows are also well-known in local wage negotiations in Scandinavia.

The formal model is set up with the aim of capturing the most important aspects as simply as possible. I have chosen a setting where the workers (possibly organized in a local union) bargain collectively with the firm. I shall assume that holdout is the relevant type of dispute in the bargaining. The wage outcome will, however, also be restricted by the outside options of the parties: the firm may close the plant and open a new one with a new workforce, while if there is full employment, the workers may leave the firm and find a new job.

¹ Quoting Blanpain (1987, Vol. 2, p. 87) on Austria: 'An ex post facto reduction of the pay agreed in an individual agreement down to the level of the minimum wage specified in a collective agreement by one-sided directive by the employer is not permissible. An agreement is required. If the employee refuses, the employer can, as otherwise in the case of non-acceptance of desired contract alterations, give conditional notice (notice of termination pending a change of contract). That means that he gives the employee notice to be effective in the event that he, the employee, does not agree to the intended wage reduction. However, there is also protection from unwarranted dismissal with notice which is effective against this conditional notice.'
3. The model

We consider an economy consisting of $K$ symmetric firms, each producing a different good. There are $L$ workers associated with each firm, each supplying inelastically one unit of labour, so that the total labour supply in the economy is $KL$ (assumed to be constant over time). Between the end of one year and the beginning of the next, a fraction $\sigma$ of the workers retires and is replaced by the same number of young workers. As each firm is small, a single firm will have only a negligible influence on the aggregate price and output level. Thus I assume that the firms regard the aggregate variables as exogenous. All agents are assumed to have complete information.

The length of the wage contract period is exogenous, and set to be one year. The model considers an infinite horizon of years. Within each year, the sequence of moves is the following. First, the government distributes an increase (or deducts a decrease) $\Delta M_t$ in the nominal money stock to the private sector, so that total nominal money is $M_t = \Delta M_t + M_{t-1}$. Secondly, in each firm the workers and the firm (i.e. the management) bargain collectively over the real wage to prevail that year. Thirdly, each firm sets the price level of its output, and hires the number of workers that is necessary to supply the quantity of output that is demanded. (The assumption that the employment level is set unilaterally by the firm after the wage is set, is consistent with empirical findings for the US and the UK, cf. Oswald, 1984). The new workers are assumed to receive the same wage as the existing workforce. Moreover, the firm is not allowed to replace any of the existing workforce with new workers, so that the unemployed cannot underbid the existing workforce.

Each firm $i$ has a constant returns to scale production function $Y_{it} = N_{it}$, where $Y_{it}$ is output and $N_{it}$ is employment. The real profits of firm $i$ in year $t$ are

$$\pi_{it} = Y_{it} (P_{it} - W_{it}) / P_t$$  \hspace{1cm} (1)

where $P_{it}$ is the price of output, $W_{it}$ is the wage level in firm $i$, and $P_t = P(P_{1t}, \ldots, P_{Kt})$ is some price index for the whole economy, assumed to be symmetric and homogeneous of degree one in the $P_t$'s. The demand function facing each firm is

$$Y_{it} = Y(P_{it}/P_t, M_t/P_t) = (P_{it}/P_t)^{-\varepsilon} m M_t / P_t$$  \hspace{1cm} (2)

where $E > 1$ is the elasticity of demand and $m$ is a positive constant. (As in Weitzman, 1985, this demand function could be derived from a model where all wages and profits are distributed to the households, who set their consumption so as to maximize their CES utility functions.) Let $N_t = \Sigma_i N_{it}$ denote aggregate employment and $W_t = W(W_{1t}, \ldots, W_{Kt})$ be the aggregate nominal wage level (where $W()$ is homogeneous of degree one), so that $w_t = W_t / P_t$ is the aggregate real wage.

In the bargaining the workers are only concerned with the welfare of the existing workforce in the firm, that is, the workers that are left over from the firm from last year (the insiders, cf. Lindbeck and Snower, 1986; and Blanchard and Summers, 1986). The instantaneous utility function is of the
form \( u(N_t - (1 - \sigma)N_{t-1}, W_t/P_t) \), where the partial derivative \( u_1 = 0 \) for \( N_t \geq (1 - \sigma)N_{t-1} \) reflecting that the workers do not care about welfare of the possible new workers to the firm. Assuming linearity, the workers' utility can be set equal to the real wage level, \( w_t = W_t/P_t \) as long as \( N_t > (1 - \sigma)N_{t-1} \). As I shall focus on steady-state equilibria where employment is constant over time, it is not necessary to specify this utility function further.

As there is perfect foresight and the aggregate price level is exogenous to a single firm, it does not matter whether the parties contract over nominal or real wages: the parties have preferences over the real wage, so this is clearly what is of importance in the bargaining. However, a crucial assumption is that there is no indexation of money wages after the contract has expired (which is in accordance with Norwegian law, cf. Labour Court Decision on 17 January, 1992). This ensures that the old contract only determines the nominal wage that prevails during a holdout, while the real wage during a holdout also depends on the aggregate price level.

4. One-year equilibrium

The equilibrium of the model clearly depends on how agents expect the outcome of the current year to affect the outcome of future years. In most macroeconomic models with imperfect competition, the outcome in one year has no effect on later years. However, under holdout threats, the bargaining outcome is influenced by the wage of the old contract, and the bargaining outcome for the current year will clearly affect the bargaining outcome in future years. In the present model, there are circumstances where this relationship is very simple: the real wage and employment outcomes of the bargaining in future years will be identical to the real wage and employment outcome of the present bargaining. In particular, if there is a deviation from the equilibrium path, e.g. that the real wage outcome in one firm is higher than it should have been in equilibrium, the new real wage outcome will remain constant in all future years. This property, that the model comes in a new steady-state equilibrium after a deviation from the initial equilibrium, is clearly extreme. However, it simplifies the analysis considerably, because it ensures that it is only necessary to solve the model in steady-state equilibrium.

To show these assertions, I first solve for the one-year equilibrium of the model under the assumption that all agents expect the real wage and employment outcomes of the present year to remain constant in all future years. In Section 5 below, I show under which circumstances these expectations are correct. Under these circumstances, the economy is in a steady-state equilibrium, that is, all real variables are constant over time. Clearly, there also exist circumstances (in particular, paths for the nominal money stock) for which no steady-state equilibrium exists, as the real wage and employment outcomes vary from year to year. I have not been able to solve the model under these circumstances.

In the analysis of the price setting and wage bargaining in a single firm, I
take the equilibrium values of the aggregate variables $M_t$ and $P_t$ as exogenous (in overall equilibrium, $P_t$ is clearly endogenous). The maximization problem of the firm is identical to the usual static one, as the price setting has no consequences for future years

$$
\max_{P_t} \left\{ Y_t(P_t/P_t, M_t/P_t)(P_t - W_t)/P_t \right\} \tag{3}
$$

As is well known, profit maximisation with constant elasticity of demand implies mark-up pricing, thus the price is given by the first order condition

$$
P_t = vW_t \tag{4}
$$

where $v = 1/(1 - 1/E) > 1$. As profits are concave in $P_t$, the first order condition of (3) is sufficient to ensure a unique maximum. Using (1), (2), and (4), the profits of the firm can be written as

$$
\pi \left( \frac{W_t}{P_t}, \frac{M_t}{P_t} \right) = v^{-E} m \frac{M_t}{P_t} (v - 1) \left( \frac{W_t}{P_t} \right)^{1 - E} \tag{5}
$$

4.1. The wage bargaining

The outcome of the wage bargaining is assumed to be given by the Nash bargaining solution which, as shown by Binmore et al. (1986), can be justified strategically as the subgame perfect equilibrium of a Rubinstein alternating offers bargaining game. However, I also allow for outside options for the two parties. The firm may close down the plant and open up a new one with a new workforce, and the workers may leave the firm for a job somewhere else. As shown by Binmore et al. (1989), the outside option of the firm will set an upper bound for the bargaining outcome, and the outside option of the workers set a lower bound. However, if the Nash bargaining solution is between these bounds, the outcome of the bargaining will not be affected by the existence of the outside options.\(^2\)

To keep the exposition simple, I first focus on the Nash bargain, and introduce the outside options afterwards. The Nash bargaining solution is

$$
W_t = \operatorname{argmax} \left\{ [\pi_{it} - \pi_{0it} + \beta(V_{it+1} - V_{0it+1})][w_{it} - q_{it} + \beta(U_{it+1} - U_{0it+1})] \right\} \tag{6}
$$

where $\beta$ is the common annual discount factor,\(^3\) $V_{it+1}$ and $U_{it+1}$ denote the expected total discounted future utility of the firm and the workers respectively, $\pi_{0it}$ and $q_{it}$ are the disagreement points, while $V_{0it+1}$ and $Q_{0it+1}$ are the respective expected future payoffs if no agreement is reached in the bargaining.

\(^2\) Technically, in an alternating offers framework this requires that a player can only choose the outside option after having rejected an offer from the opponent. However, this seems to be the reasonable assumption to make, cf. Shaked (1987).

\(^3\) Allowing for different discount factors for the workers and the firm (or different bargaining powers) would complicate the expressions, but not alter the qualitative results.
When all real variables are expected to remain constant in all future years, the expected total future payoff is

$$V_{it+1} = U\left(\frac{W_{it}}{P_t}\right) = \frac{1}{1 - \beta} \left(\frac{W_{it}}{P_t}\right)^{\nu - 1 - \epsilon}$$

for the workers, while for the firm, it is

$$V_{it+1} = V\left(\frac{W_{it}}{P_t} \frac{M_t}{P_t}\right) = \frac{1}{1 - \beta} \left(\frac{W_{it}}{P_t}\right)^{\nu - 1 - \epsilon}$$

It equilibrium, there will be an immediate agreement in the bargaining, and no holdout will ever be observed. However, to find the equilibrium outcome one must analyse what would happen if there were a delay in reaching an agreement in the bargaining; as emphasized by Binmore et al. (1986), the appropriate disagreement points in a Nash bargain are the parties’ payoffs in a delay. I assume that in a delay, production continues under the terms of the old contract (holdout), yet the workers withdraw cooperation, thus reducing production without actually breaking the working rules (work-to-rule). During a holdout, production is given by

$$Y_{0it} = \Theta N_{0it}$$

where \(\Theta \in (0, 1)\) is the output per worker during a holdout. In general the firm is not in a position to legally reduce the wage rates, cf. discussion above. However, I allow for the possibility of a reduction in the effective remuneration of the workers, for example owing to the withdrawal of bonus payments. The workers are assumed to have no direct utility or disutility from work-to-rule, so the workers’ payoff during a holdout is\(^4\)

$$q_{it} = \alpha W_{it-1}/P_t \quad \alpha \in (0, 1]$$

\(\alpha\) and \(\Theta\) are exogenously given by institutional and technological constraints, cf. discussion in Section 6 below. The payoff of the firm during a holdout is\(^5\)

$$\pi_{0it} = \max_{\rho_{0it}} \left[ Y\left(\frac{P_{0it}}{P_t}, \frac{M_t}{P_t}\right) \frac{P_{0it}}{P_t} - \alpha \left(\frac{W_{it-1}}{P_t}\right) Y\left(\frac{P_{0it}}{P_t}, \frac{M_t}{P_t}\right) \frac{1}{\Theta}\right]$$

\(^4\) In many countries it is common to backdate wage increases, so that if a wage increase is given after, say, a holdout of one month, then the workers will obtain the agreed wage increase also for this month. This is neglected in the present model. However, the outcome of the Nash bargain is a division of payoffs. Whether these payoffs are implemented by a certain wage for the remainder of a contract period (after a holdout) or a slightly lower wage for the whole contract period, is of minor importance; it would be straightforward (but messy) to modify the model to allow for backdating.

\(^5\) The firm is allowed to delay hirings of new workers until an agreement is reached in the bargaining, and I assume that the turnover is sufficiently large so that a possible reduction in employment that the firm might wish during a holdout can be undertaken without layoffs among the existing workforce.
The first order condition to (11) is

$$P_{t0} = v\alpha W_{t-1}/\Theta$$

(12)

Using (2), (11), and (12), the profits of the firm during a holdout can be written as

$$\pi_{0it} = \pi_0 \left( \frac{W_{t-1}}{P_t}, \frac{M_i}{P_t} \right) = v^{-E_m} \frac{M_i}{P_t} (v - 1) \left( \frac{\alpha W_{t-1}}{\Theta P_t} \right)^{1-E}$$

(13)

If no agreement is reached in the bargaining, the real wage outcome is $W_{t-1}/P_t$, and the expected total future payoffs are \(^6\)

$$U_{0it+1} = U \left( \frac{W_{t-1}}{P_t} \right) = \frac{1}{1 - \beta} \left( \frac{W_{t-1}}{P_t} \right)$$

(14)

$$V_{0it+1} = V \left( \frac{W_{t-1}}{P_t}, \frac{M_i}{P_t} \right) = \frac{1}{1 - \beta} \left( v^{-E_m} \frac{M_i}{P_t} (v - 1) \left( \frac{W_{t-1}}{P_t} \right)^{1-E} \right)$$

(15)

The following lemma characterizes the outcome of the Nash bargain in a steady-state equilibrium, where the outside options are neglected.

**Lemma 1** Assume that all agents expect the values of the real variables in year $t$ to remain constant in the future. For any wage distribution $(W_{t-1}, \ldots, W_{kt-1})$ in year $t - 1$, the Nash bargaining solution in firm $i$ in year $t$ is uniquely given by

$$W_{it} = s^* W_{it-1}$$

where $s^*$ is a function of $\Theta$, $\alpha$, $\beta$, and $E$ defined implicitly by

$$A(s^*)^E + (E - 2)s^* - (E - 1)B = 0$$

(16)

where $A = (1 - \beta)(\alpha/\Theta)^{1-E} + \beta$ and $B = (1 - \beta)\alpha + \beta$. Furthermore, $s^* > 1$ if and only if

$$\Theta < (1 - (E - 1)(1 - \alpha))^{1/(E - 1)}\alpha$$

(17)

$s^*$ is decreasing in $\Theta$ and $E$, increasing in $\alpha$, and decreasing in $\beta$ if and only if (17) holds.

The bargaining outcome is a rate of change $s^* - 1$ in nominal wages. Note that the rate of change is independent of the old real wage level, because the relative costs of a holdout are independent of the old wage level. This property ensures that in an equilibrium where holdout threats prevail, the relative nominal wage distribution among the firms will remain constant. In particular, if there is a deviation from the initial equilibrium in the wage bargaining in one firm, this new real wage level will remain constant in all future years.

\(^6\) Note that when the parties contemplate not reaching an agreement in the present bargaining, they still expect to reach an agreement in the bargaining in future years. This is in the spirit of the concept of subgame perfect equilibrium, where players expect equilibrium strategies to be followed even after a deviation from the equilibrium strategy.
The relationship between \( s^* \) and \( \alpha \) and \( \Theta \) shows how the institutional aspect \( s \) of the wage setting may affect the rate of inflation. \( s^* \) is decreasing in \( \Theta \) and increasing in \( \alpha \); wage growth is greater the more costly a holdout is to the firm, and the less costly it is to the workers. The intuition for \( s^* \) decreasing in \( E \) is that a high price elasticity of demand \( E \) makes the firm less vulnerable to a reduction in productivity and output. A low \( E \) can be associated with little competition in the product market, which according to Lemma 1 may lead to a high rate of wage inflation.

The intuition for \( s^* \) decreasing in \( \beta \) when (17) holds (so \( s^* > 1 \)) is somewhat more subtle: if a holdout is more costly to the firm, the firm is willing to raise wages so as to avoid a costly conflict. However, the fact that a wage rise now will raise wages in all future years makes the firm offer (and the workers accept) a lower wage increase than if there were no impact on future years. This effect is clearly increasing in the concern for the future, so that \( s^* \) is decreasing in \( \beta \). If (17) is not fulfilled, \( s^* < 1 \). In this case \( s^* \) is increasing in \( \beta \), because in this case it is the workers that will accept a larger wage reduction the less they are concerned about the future.

If (17) holds, \( s^* - 1 > 0 \), so that nominal wages increase over time. The intuition is that a holdout is more costly to the firm than to the workers, so that the firm is willing to raise wages to avoid a costly conflict. This seems to be the more plausible case: even with bonus schemes or piece rates, the workers usually have some choice in how to reduce productivity and they will choose methods where the firm faces the higher costs. It is also the common assumption; see Moene (1988), Holden (1988, 1989), and Cramton and Tracy (1992). Numerically, (17) is fulfilled for all \( \Theta < 1 \) when \( \alpha = 1 \). For \( \alpha = 0.9 \) and \( E = 3 \), (17) will be fulfilled for all \( \Theta < 0.8 \).

We now investigate the consequences of the outside options. If there is full employment, the workers are assumed to be able to costlessly obtain a job at the average real wage, so their reservation wage \( r_t = W_t/P_t \). However, if there is some unemployment in the economy their reservation wage \( r_t = 0 \), which ensures that the workers stay in the firm unless there is full employment.\(^7\)

Concerning the outside option of the firm, I assume that the firm can shut down the plant and open a new one with a new workforce. The firm offers the unemployed workers their reservation wage, which is equal to the unemployment benefits \( b_t \). For space considerations, I do not develop a formal model of this matching process, but just assume that the total costs of finding a new workforce are a continuous and strictly increasing function of the real money stock, \( g(M_t/P_t) \), where \( g(0) = 0 \) and \( g(M_t/P_t) \) approaches infinity when \( M_t/P_t \) approaches \( L/m \). The intuition here is that aggregate employment is increasing in the real money stock, so that the greater the real money stock, the fewer unemployed and the more difficult it is to find new labour. When the real money stock approaches \( L/m \), aggregate employment approaches the total labour force

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\(^7\) In effect, I assume that the workers are not eligible for unemployment benefits if they voluntarily leave the firm (the rationale for assuming \( r_t = 0 \) will be further discussed in footnote 8 below).
in a symmetric equilibrium (cf. (2)) and the firm is not able to find new workers. This is captured by the assumption that the costs of finding a new workforce approach infinity. Costs of actually building a new plant are neglected, but could easily be added without affecting the qualitative results. Using (1), (2), and (4), the total future payoff of the firm of closing the plant and opening a new one is

$$V^F\left( b_t, \frac{M_t}{P_t} \right) = \frac{1}{1 - \beta} \left( v^{-e} M_t \frac{M_t}{P_t} (v - 1) b_t^{1 - E} \right) - g\left( \frac{M_t}{P_t} \right)$$

(18)

Define $w^F(M_t/P_t)$ as the value of $W_{it}/P_t$ in (8) that makes (8) and (18) equal, that is, the firm is indifferent between continuing the existing plant at wage $W_{it}/P_t$ or opening a new plant. The wage bargaining cannot result in a wage greater than $w^F(M_t/P_t)$, because this would induce the firm to shut down the plant. $g$ being strictly increasing in $M_t/P_t$ ensures that $w^F$ is strictly increasing in $M_t/P_t$ too.

**Lemma 2** Assume that all agents expect the values of the real variables in year $t$ to remain constant in the future. For any wage distribution $(W_{1t-1}, \ldots, W_{kt-1})$ in year $t - 1$, the wage level in firm $i$ in year $t$ is uniquely given by

$$W_{it}^* = \frac{s^* W_{it-1}}{P_t} \frac{M_t}{P_t} \quad \text{for} \quad \frac{s^* W_{it-1}}{P_t} \in \left( r_t, w^F\left( \frac{M_t}{P_t} \right) \right)$$

$$W_{it}^* = r_t \quad \text{for} \quad \frac{s^* W_{it-1}}{P_t} \leq r_t$$

4.2. **One year equilibrium**

So far, we have considered the wage setting in a single firm. We now consider the whole economy. Again, the equilibrium is derived under the assumption that all agents expect the equilibrium values of the real variables to remain constant for all future years, expectations that will be shown to be correct in Section 5 below. I first derive the equilibrium under the assumption that the old nominal wage is the same in all firms (in this analysis I omit subscript indicating firm). Afterwards I consider the effects of one firm having a different value of the old wage than the other firms.

As firms are assumed to be symmetrical, Lemma 2 ensures that if the old wage level is the same in all firms, then the new wage level will also be the same in all firms, which again implies that all firms set the same price. As prices are set as constant markups on wages in all firms by (4), it follows that the aggregate real wage is $W_i/P_t = 1/v$. In overall equilibrium the real wage implied by the price setting must be consistent with the wage bargaining. In the wage bargaining, there are three regimes (cf. Lemma 2) so that one out of three
conditions must hold.

\[
\frac{1}{v} = w^F \left(\frac{M_t}{P_t}\right) \tag{19a}
\]
\[
\frac{1}{v} = \frac{s^* W_{t-1}}{P_t} \tag{19b}
\]
\[
\frac{1}{v} = r_t \tag{19c}
\]

As the right-hand side of (19a) is strictly increasing in the real money stock, there exists a unique value of the real money stock, which I denote \((M/P)^F\), that is consistent with (19a). The interpretation is that in an equilibrium where the outside option of the firm is an effective restriction in the wage bargaining, the real money stock must be equal to \((M/P)^F\). Correspondingly, (19c) can only hold when there is full employment so that the reservation wage of the workers is equal to the aggregate real wage. In a symmetric equilibrium, there is full employment if \(M_t/P_t = L/m\) (cf. (2)), so this is the only possible value for the real-money stock if the workers outside option is binding in the wage bargaining. When the real-money stock is between these values, \(M_t/P_t \in ((M/P)^F, L/m)\), then \(r_t < 1/v < w^F(M_t/P_t)\), so neither of the outside options can be binding in the wage bargaining, and the holdout solution must prevail.

**Proposition 1** Assume that all agents expect the values of the real variables of year \(t\) to remain constant in the future. There exists a unique (one year) equilibrium to the economy. The equilibrium is characterized as follows: for a given \(W_{t-1}\), there exist two critical values \(M_t^U\) and \(M_t^F\), given by \(M_t^U = (L/m)\) vs* \(W_{t-1}\) and \(M_t^F = (M/P)^F\) vs* \(W_{t-1}\), where \(M_t^U > M_t^F\), such that:

(i) if \(M_t \leq M_t^F\), the outside option of the firm is binding and the real money stock is equal to \((M/P)^F\);

(ii) if \(M_t \geq M_t^U\), the outside option of the workers is binding and the real money stock is equal to \(L/m\);

(iii) if \(M_t \in (M_t^F, M_t^U)\), the holdout solution prevails in the wage bargaining and the real money stock is equal to \(M_t/(\text{vs}^* W_{t-1})\).

In the analysis of the complete steady-state equilibrium, we can, however, not assume that the firms are always identical. To find the outcome of the Nash bargain in the holdout regime, we need to know the consequences of a marginal increase in the wage in one firm (the derivative of \(\pi, U\), and \(V\) with respect to \(W_t\)), and the consequences if no wage increase is given in one year in a single firm (that is, \((V - V_0)\) and \((U - U_0)\)) (in equilibrium neither of these events will occur). Above I have assumed that a new steady-state equilibrium exists for the new wage level, and we must check that this assumption is consistent with the model. Thus, assume that all firms but one have the same old nominal wage \(W_{t-1}\), while the old nominal wage of firm \(i\) is \(W_{t-1} \neq W_{t-1}\). As a single firm
is so small that it has no impact on the aggregate variables, the results in Proposition 1 above still holds. If \( W_{i,t-1} \) is marginally above \( W_{i-1} \), then \( s^* W_{i,t-1} < w^f(M_i/P_i) \) implies that \( s^* W_{i,t-1} \leq w^f(M_i/P_i) \) so that holdout will prevail in firm \( i \) also. (If the wage in firm \( i \) is more than marginally above the aggregate wage, this is not necessarily so. But we only need this to hold for an infinitesimal difference).

If \( W_{i,t-1} \) is below \( W_{i-1} \), but there is not full employment, then \( s^* W_{i,t-1}/P_i > r_i = 0 \), so that a holdout will prevail in firm \( i \) also. When the holdout regime prevails, we will have \( W_i = s^* W_{i,t-1} \) even if \( W_{i,t-1} \neq W_{i-1} \). Thus, in the holdout regime, if the workers in one firm obtain a slightly higher real wage than in other firms one year, then they will keep the higher real wage for all future years. Correspondingly, if one year there is no wage increase in one single firm, so that the real wage in this firm is lower than in other firms, then the lower real wage will remain constant for all future years.\(^8\)

5. Full equilibrium

I now investigate the full equilibrium of the model. Proposition 2 shows that for some possible paths of the nominal money stock there exists a unique steady-state equilibrium with the properties described above.

**Proposition 2** There exist two critical values \( M^U_1 \) and \( M^F_1 \), given by \( M^U_1 = (L/m) vs^* W_0 \) and \( M^F_1 = (M/P)^F vs^* W_0 \), where \( M^U_1 > M^F_1 \). If the exogenous path of the nominal money stock satisfy:

(i) \( M_1 \leq M^F_1 \) and \( M_i/M_{i-1} \leq s^* \) for all \( t > 1 \), there exists a unique steady-state equilibrium to the economy, where the outside option of the firm is binding in the wage bargaining and the real money stock is constant and equal to \((M/P)^F\);

(ii) \( M_1 \geq M^U_1 \) and \( M_i/M_{i-1} \geq s^* \) for all \( t > 1 \), there exists a unique steady-state equilibrium to the economy, where the outside option of the workers is binding in the wage bargaining and the real money stock is constant and equal to \( L/m \);

(iii) \( M_1 \in (M^F_1, M^U_1) \), and \( M_i/M_{i-1} = s^* \) for all \( t > 1 \), there exists a unique steady-state equilibrium to the economy, where the holdout solution prevails in the wage bargaining and the real money stock is constant and equal to \( M_i/(vs^* W_0) \in ((M/P)^F, L/m). \)

For other possible paths of the nominal money stock than those described in Proposition 2, no steady-state equilibrium will exist. For example, if the nominal

\(^8\) It is necessary to have \( r_i = 0 \) if there is any unemployment in the economy for the following reason. No matter how many times there is a deviation from the equilibrium path and no agreement in the wage bargaining in one firm (so that there is no wage increase), it must always be better to stay in the firm than to accept the outside option. If not, the workers could threaten to accept the outside option unless they obtain a wage rise, and that would increase their bargaining power so that they would obtain a higher wage rise.
money stock initially has a low value compared to the existing nominal wage level, but the nominal money stock increases at a high rate, there would be a switch from a regime where the outside option of the firm is binding, via a holdout regime, to a regime where the outside option of the workers is binding. Clearly, no steady-state equilibrium would exist.

Note that a steady-state equilibrium of the infinite horizon bargaining game should not be confused with the equilibrium of a single period model. The present model does take into account that the wage outcome in one year may influence the wage outcome in future years. The analysis also takes into account that if there is a deviation from the equilibrium path in one firm, the real wages in this firm will not be constant over time. The point is that under certain circumstances, a steady-state equilibrium will exist in the infinite horizon bargaining model.

Proposition 2 can be seen in light of the relationship between wage inflation and the real money stock. As aggregate employment is monotonically increasing in the real money stock, this relationship is essentially a Phillips curve. If wage inflation is to be below \( s^* - 1 \) in a steady-state equilibrium, the outside option of the firm must be binding, and the real money stock must be low, equal to \( (M/P)^\phi \). With a high value of the real money stock, \( (L/m) \), the outside option of the workers is binding, and money wages must grow at least at the rate \( s^* - 1 \). For intermediate values of the real money stock, the holdout solution must prevail and wage inflation is \( s^* - 1 \). The implied relationship is thus as the Phillips curve consists of two vertical parts, one with high employment and wage inflation above \( s^* - 1 \), and one with low employment and wage inflation below \( s^* - 1 \). There will also be a horizontal part, with intermediate values of aggregate employment and wage inflation equal to \( s^* - 1 \) (Fig. 1).

6. Concluding remarks

In many countries it is customary that production continues under the terms of the old contract during wage negotiations (holdout), unless a work stoppage is initiated. This paper analyses a model where the workers deliberately work less efficiently during a holdout, while the firm may reduce bonus payments. If a holdout is more costly to the firm than to the workers, the wage bargaining will result in a nominal wage increase. Wage inflation can only be prevented if the firm can credibly threaten to close the plant. Threats to close the plant are only credible when unemployment is high; thus is not possible to combine low unemployment with zero wage inflation. The Phillips curve consists of two vertical parts, one with high employment and wage inflation above a certain level \( (s^* - 1) \), and one with low employment and wage inflation below the same level (Fig. 1).\(^9\)

In the model there is no productivity growth, so price inflation is equal to

\(^9\) Similar Phillips curves can be derived from models with downward rigidity of money wages and asymmetric demand shocks (Holden, 1990; Akerlof et al., 1996).
wage inflation. With a positive productivity growth, the price inflation would be equal to the wage inflation less productivity growth. In this case, zero price inflation could be combined with low unemployment and a positive wage inflation, as long as the productivity growth is at least equal to rate of wage growth in the holdout regime $s^* - 1$.

Throughout the paper, the system of wage determination, including the parameters that determine the costs of a holdout to the parties in the wage negotiations ($\alpha$ and $\Theta$), is treated as exogenous. The aim has been to analyse the consequences of a given system. It is shown that the rate of money wage growth under holdout threats is higher, the greater the costs the workers may induce on the firm during a holdout (the lower $\Theta$ is), and the smaller the costs to the workers during a holdout (the higher $\alpha$ is). The value of these parameters ($\alpha$ and $\Theta$) are determined by various technological and institutional aspects. The costs to the firm of a holdout depend among other things on to what extent it is possible to make detailed labour contracts that prevent work-to-rule. The costs to the workers during a holdout may for instance reflect the degree of flexibility of the pay system, e.g. if bonus payments may be suspended under a holdout.

This paper does not address the issue of what determines the system of wage setting, including the values of these parameters. Some institutional aspects are probably set by the management, while others are determined in negotiations between management and workers. In either case, a satisfactory analysis presumably requires an analysis of the relationship between these parameters and the payoffs of the parties (i.e. the wage), which has been the topic of the present paper. Several questions of considerable policy interest are immediate. For example, to what extent will the system of wage setting adapt to the rate of inflation? Will the employers be able to enforce the necessary changes in the wage setting system to reduce $s^*$ if the government makes a credible commitment.
to low inflation? These are clearly important issues that should be addressed in future work.

The analysis also shows that the rate of money wage growth under holdout threats is decreasing in the elasticity of demand. A possible interpretation is that low product market competition, which can be associated with a low elasticity of demand, may contribute to a high rate of inflation. Such a relationship is sometimes claimed by labour market observers, but is difficult to explain within standard models of wage determination without holdout threats, where product market competition affects the real wage level but not the rate of inflation.

In the model I neglect the possibility of strikes and lock-outs entirely. This simplifies the model considerably, because a bargaining model with endogenous type of conflict requires a strategic bargaining model, with a proper modelling of the determination of type of conflict (cf. e.g. Haller and Holden, 1990). This would be very complicated in a multi-year setting, where the link between the annual wage negotiations should be explicitly modelled (see Holden, 1994, for a similar model which allows for holdout, strike, and lock-out, but where the multi-year aspect is neglected). Furthermore, neglecting strikes and lock-outs should not be too serious a problem; in Holden (1994) strikes and lock-outs have the same effects as the outside options have in the present model, as they set lower and upper bounds to the bargaining outcome. The only important difference is due to the fact that allowing for strikes would improve the bargaining position of the workers considerably compared to the outside option possibility. This prevents the possibility of full employment, as the lower bound to the bargaining outcome will be binding before full employment is reached (but there will still be a range of unemployment levels for which the holdout regime prevails). Focussing on outside options rather than work stoppages also makes the model more relevant for the non-union sector.

The assumption that the nominal wage is unaltered until a new agreement is reached is crucial for the results of the paper. This assumption is in accordance with law and established practice in many modern economies. But why do such laws or established practice exist? The main justification behind the law is the desire to protect the worker, who is often the weaker part of an employment relationship. If the employer is allowed to unilaterally reduce the nominal wage, employment protection laws would have less meaning, as the employer could reduce wages so much that the employees quit voluntarily (in countries with minimum wages given by law this would be the lower bound for the wage). Moreover, MacLeod and Malcomson (1992) and Holden (1996) show that under certain conditions the holdout possibility is essential to induce efficient investments.

In the model the contract length is taken as exogenous. However, the fact that it may be difficult to verify that a work-to-rule takes place makes it natural to ask whether the workers could use work-to-rule threats to demand another wage rise immediately after a wage agreement is reached. But this argument neglects the fact that the firm is only willing to raise wages because this prevents
a new work-to-rule during the contract period. If the contract period is shorter, so that the time until a new work-to-rule may take place is also shorter, then the wage rise which the firm will accept will be lower. (This can be seen from $s^*$ being decreasing in $\beta$, remembering that $\beta$ would be decreasing in the length of the contract period.)

The present model has many similarities to the literature within the conflict/structuralist approach to inflation, which was popular 20 years ago (cf. e.g. Hines, 1969; Seitzovsky, 1978). However, in contrast to most of this literature the present paper has an explicit microfoundation based on modern bargaining theory and rational agents. Furthermore, in the present paper the source to inflation is identified in the nominal rigidities and asymmetries of workers and firm that arise due to holdout threats.

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REFERENCES


**APPENDIX**

1. **Proof of Lemma 1**

Note first that the Nash maximand (6) is zero for $W_0 = [1 - \beta(\alpha/\Theta)]^{-\delta} + \beta [1 - \delta(W_{t-1})$ and $W_0 = [(1 - \beta)\alpha + \beta]W_{t-1}$ (when one of the terms in (6) is equal to zero), while it is positive for all $W_0$ in the interval between these values. Moreover, the Nash maximand is continuous and differentiable in this interval, so a maximum will exist and be given by the first order condition

$$\frac{\partial \pi}{\partial W_t} + \beta \frac{\partial V}{\partial W_t} + \frac{1 + \beta}{U(W_t - q_t + \beta(U_{t+1} - U_{t+1})} = 0$$

which, using (5), (7), (8), (10), (11), (13), (14), and (15) and straightforward differentiation can be rewritten as

$$\frac{(1 - \delta)W_t^\delta}{W_t^\delta - [(1 - \beta)(\alpha/\Theta)]^{-\delta} + \beta [1 - \delta(W_{t-1})]W_{t-1}} + \frac{1}{W_t - [(1 - \delta)\alpha + \beta]W_{t-1}} = 0$$

(A2)

I want to show that there is a unique $W_0$ that solves (A2), and that $W_0 \geq W_{t-1}$ when (17) holds.

(A2) can be rewritten as

$$\frac{(1 - \delta)}{W_t - AW_{t-1}^\delta} + \frac{1}{W_t - BW_{t-1}} = 0$$

(A3)
where $A = (1 - \beta)(\alpha/\Theta)^{1-E} + \beta$, $B = (1 - \beta)x + \beta$ and $s = W_u/W_{u-1}$. By multiplying (A3) with $W_{u-1}$ and rearranging, we obtain

$$f(s) = As^E + (E-2)s - (E-1)B = 0$$

(A4)

Now, as $f''(s) = AE(E-1)s^{E-2} > 0$ for all $s > 0$, $f(s)$ is a strictly convex function in $[0, \infty)$. Furthermore, $f(0) = -(E-1)B > 0$ and $f(s) \to \infty$ as $s \to \infty$, so there is only one solution to (A4) in $[0, \infty)$. Let $s_0$ be this solution. $s_0 = W_u/W_{u-1} > 1$ is equivalent to $f(1) < 0$, which is again equivalent to $E < 1 + (1-A)/(1-B)$. By rearranging this latter condition and inserting for $A$ and $B$, we find that it is equivalent to (17). Thus, $W_0/W_{u-1} > 1$ if and only if (17) holds.

For (A3) we see that if $W_u = u > 0$ is a solution to (A3) for $W_{u-1} = x > 0$, then $W_u = tu > 0$ is a solution to (A3) for $W_{u-1} = t > 0$, for all $t > 0$. Thus, $W_u$ is homogeneous of degree one in $W_{u-1}$, so $W_u$ can be written on the form $W_u = s^*W_{u-1}$, where $s^* > 1$ defined implicitly by (16).

Turning now to the relationship between $s^*$ and $\Theta$, $x$, $\beta$, and $E$: implicit derivation of (16) shows that

$$\frac{\partial s^*}{\partial A} = \frac{-(s^*)^E}{AE(s^*)^{E-1} + E - 2}$$

(A5) and

$$\frac{\partial s^*}{\partial B} = \frac{E - 1}{AE(s^*)^{E-1} + E - 2}$$

(A6)

The denominator in (A5) and (A6) is equal to $f'(s^*)$ from (A4), and we know that $f''(s^*) > 0$ from the following observations (cf. above): $f(s)$ is strictly convex in $[0, \infty)$, $f(0) < 0$ and $f(s^*) = 0$. Thus, $\partial s^*/\partial A < 0$ and $\partial s^*/\partial B > 0$. It is straightforward to show that $\partial A/\partial \Theta > 0$, $\partial A/\partial x < 0$ and $\partial B/\partial x > 0$, which imply that $\partial s^*/\partial \Theta < 0$ and $\partial s^*/\partial x > 0$. To find the effect of $\beta$, observe that

$$\frac{\partial s^*}{\partial \beta} = \frac{\partial s^*}{\partial A} \frac{\partial A}{\partial \beta} + \frac{\partial s^*}{\partial B} \frac{\partial B}{\partial \beta}$$

(A7)

By utilizing (A5) and (A6), and the fact that $\partial A/\partial \beta = 1 - (\alpha/\Theta)^{1-E}$ and $\partial B/\partial \beta = 1 - \alpha$, we find that the numerator is strictly positive and that (A7) > 0 if and only if (17) holds.

Finally, implicit derivation of (16) gives us

$$\frac{\partial s^*}{\partial E} = \frac{-A(s^*)^E \ln E - s^* + B}{AE(s^*)^{E-1} + E - 2} < 0$$

(A8)

The denominator is strictly positive (as observed above), and the numerator is clearly negative, as $s^* > 1 > B$. □

2. Proof of Lemma 2

This is a straightforward application of the outside option principle of Binmore et al. (1989). When the Nash bargaining solutions $s^*W_{u-1}/P_i$ is in the interval between the wage associated with the outside options, the outside options do not affect the bargaining outcome. However, if the Nash bargaining solution is outside this interval, the bargaining outcome is equal to the outside option which is closer to the Nash solution.

3. Proof of Proposition 1

Consider first the equilibrium where the outside option of the firm is binding. From (19a) it is clear that the real money stock must be equal to $(M/P)^F$ for the price and wage setting to be consistent, implying that the price level is

$$P_t = M_t/(M/P)^F$$

(A9)

Lemma 2 requires that $s^*W_{t-1}/P_t \geq w^F$, which, using (19a), is equivalent to $s^*W_{t-1} \geq P_t$. By
inserting for \( P_t \) from (A9), we obtain

\[ M_t \leq s^* v W_{t-1} \frac{(M/P)^P}{L/m} \equiv M_t^P \]  

(A10)

Correspondingly, in the equilibrium where the outside option of the workers is binding, (19c) shows that the real money stock must be equal to \( L/m \) for the price and wage setting to be consistent. This implies that the price level is

\[ P_t = M_t/(L/m) \]  

(A11)

Lemma 2 requires that \( s^* v W_{t-1} / P_t \leq r_t \), which, using (19c), is equivalent to \( s^* v W_{t-1} \leq P_t \). By inserting for \( P_t \) from (A11), we obtain

\[ M_t \geq s^* v W_{t-1} \frac{L}{n} \equiv M_t^U \]  

(A12)

Finally, in the equilibrium where holdout threats prevail, the price level must be (using (19b))

\[ P_t = s^* v W_{t-1} \]  

(A13)

so that the real money stock is \( M_t / [(s^* v W_{t-1})] \). If \( M_t^C > M_t > M_t^P \), a comparison of (A10), (A12), and (A13) shows that the real money stock is in the interval \( ((M/P)^P, L/m) \), which is consistent with holdout threats being effective in the wage bargaining.

Uniqueness follows from the uniqueness of the wage bargaining and price setting, and the unique solutions to (19a) and (19c). \( \square \)

4. Proof of Proposition 2

Consider first the steady state equilibria where the outside option of the firm is binding. If the nominal money stock is sufficiently small compared to the old nominal wage, that is

\[ M_t \leq M_t^P = (M/P)^P \frac{v s^* W_{t-1}}{s^* v} \]  

(A14)

Proposition 1 ensures that the outside option of the firm is binding, on the condition that there is a steady-state equilibrium. The critical issue is thus whether the same condition will hold in all future years, so that Proposition 1 can be applied to all future years. In an equilibrium where the outside option of the firm is binding, the equilibrium nominal wage is \( W_t^P = M_t^P / [(M/P)^P v] \) using (A9) and (4). Substituting the expression for \( W_t^P \) into (A14) gives us

\[ M_{t+1} \leq M_{t+1}^P = (M/P)^P \frac{v s^* W_t^P}{s^* v} = (M/P)^P \frac{v s^* M_t}{[(M/P)^P v]} = s^* M_t. \]

Thus, if (A14) holds in year \( t \) and the rate of growth in nominal money is not greater than \( s^* \), then (A14) will hold in year \( t + 1 \) also, and the outside option of the firm will be binding in year \( t + 1 \) too. This argument can be used iteratively for all future years.

Then consider the steady-state equilibria where the outside option of the workers is binding. If the nominal money stock is sufficiently large compared to the old nominal wage, that is

\[ M_t \geq M_t^U = (L/m) s^* v W_{t-1} \]  

(A15)

Proposition 1 ensures that the outside option of the workers is binding, on the condition that there is a steady-state equilibrium. Again, the critical issue is whether the same condition will hold in all future years, so that Proposition 1 can be applied to all future years. In an equilibrium where the outside option of the workers is binding, the equilibrium nominal wage is \( W_t^U = M_t^U / [(L/m) v] \) using (A11) and (4). Substituting the expression for \( W_t^U \) into (A15) gives us

\[ M_{t+1} \geq M_{t+1}^U = (L/m) s^* v W_t^U = (L/m) s^* v M_t / [(L/m) v] = s^* M_t. \]

Thus, if (A15) holds in year \( t \) and the rate of growth in nominal money is not less than \( s^* \), then (A15) will hold in year \( t + 1 \) also, and the outside option of the workers will be binding in year \( t + 1 \) too.
Finally, consider the possible steady-state equilibria with a holdout regime in the bargaining. As nominal wages grow at a rate $s^*$, the nominal money stock must also grow at this rate. Moreover, the level of the money stock must be within the range where the holdout solution prevails, that is, $M_t \in (M_t^L, M_t^U) = ((M/P)^0_t vs^* W_{t-1}, (L/m)_t vs^* W_{t-1})$, cf. Proposition 1.

Uniqueness follows directly from the uniqueness in Proposition 1. $\square$