Equilibrium Rating of Sovereign Debt

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December 18, 2014

Abstract
We develop an equilibrium theory of credit rating in the presence of rollover risk. By influencing rational creditors, ratings affect sovereigns’ probability of default, which in turn affects ratings. In equilibrium, credit rating is pro-cyclical and magnifies underlying market conditions. Moreover, biased incentives of credit rating agencies are ultimately self-defeating – a bias toward issuers makes sovereign debt more risky.

Keywords: credit rating agencies, sovereign debt, global games, coordination failure.

JEL: G24, F34, D82, C72

1 Introduction

When short term borrowing is used to finance long term needs, debtors are vulnerable to swings in market sentiment. This vulnerability is inherent in sovereign debt markets, where

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We thank Patrick Bolton, Craig Furfine, Douglas Gale, Bård Harstad, Johannes Hörner, Espen Moen and Dagfinn Rime, as well as seminar participants at the European University Institute, the Norwegian School of Economics, the Norwegian Business School (BI), the University of Copenhagen, the Annual Conference on Bank Structure and Competition, and at various conferences for useful comments. A earlier version of this paper was previously circulated under the title ‘An Equilibrium Theory of Credit Rating’. The views expressed in this paper are those of the authors and not of Norges Bank.
investors may suddenly refuse to roll over their short term claims on a country. Since the 1980s, such liquidity crises have recurred and created havoc in emerging market economies, such as Mexico, Russia and Argentina. More recently, similar crises occurred in Europe, where Greece, Ireland and Portugal were effectively shut out of the bond market in 2010 and 2011, and later there was widespread concern that Spain and Italy might meet the same fate. It is widely believed that in these episodes, pessimistic investor expectations played a propagating role.

When investor expectations are decisive, any event that coordinates expectations might in principle be pivotal. In particular, credit ratings might serve such a role. Indeed, in crisis periods, politicians and observers often blame rating agencies. For example, in a joint letter 6 May 2010, Chancellor Angela Merkel and President Nicolas Sarkozy wrote that ‘The decision of a rating agency to downgrade the rating of the Greek debt even before the authorities’ programme and amount of the support package were known must make us ponder the rating agencies’ role in propagating crises.’ A natural instinct is of course to discredit such statements as coming from politicians eager to cloud their own responsibility for an ongoing crisis. Yet, the question at hand is important, and should not be dismissed without a rigorous answer. This paper therefore scrutinizes the role of credit rating agencies when there is rollover risk.

We analyze the problem faced by a rating agency about to rate sovereign debt in a situation where coordination failure among investors might cause default. Following Manso (2013), we assume that the CRA optimally chooses ratings in order to preserve its track record of correct predictions. The CRA realizes that its rating may affect whether or not the rated sovereign

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1See also Paul Krugman’s New York Times article ‘Berating the raters’, April 26, 2010, or Ferri, Liu and Stiglitz (1999). Another example is Helmut Reisen (2010) Head of Research, OECD Development Centre: ‘Unless sovereign ratings can be turned into proper early warning systems, they will continue to add to the instability of international capital flows, to make returns to investors more volatile than they need be, and to reduce the benefits of capital markets for recipient countries.’

2A quote of Thomas McGuire, former VP at Moody’s, summarizes this motivation: ‘What’s driving us is
defaults, and it acts strategically by taking this effect into account in its rating decision. Hence, there is potential feedback between rollover risk and ratings. Importantly, investors are rational and realize that the rating agency is strategic. They therefore take the CRA’s incentives and constraints into account when they interpret a rating. Thus, a central feature of our investigation is that ratings and investors’ response to them are determined in equilibrium. This is an important contribution of our analysis, as one cannot satisfactorily address how rating agencies affect coordination failure by studying either investors or ratings in isolation. For instance, a policy of simply regulating CRAs to issue more positive ratings in times of crisis is likely to fail as investors adjust their behavior accordingly.

To model the problem facing the rating agency, we use the framework in Morris and Shin (2004, 2006), where a sovereign needs to roll over short-term debt. We extend the model by adding a strategic rating agency, who observes a noisy signal of the fundamental (the sovereign’s alternative means of short term financing) and reports a rating, reflecting the likelihood of a default. The rating may be good or bad. A mass of creditors decide whether or not to roll over their loan, based on the rating of the CRA as well as their own private signal of the fundamental. Withdrawing the loan involves a partial loss, but allows creditors to receive immediate payment and avoid the risk of a later default due to coordination failure. Investors discount delayed payments according to their immediate need for cash: when liquidity is tight, investors discount future payments more.

Before making their decisions, investors update their beliefs based on the rating by the CRA. If the information content of the rating is entirely consistent with an investor’s private signal, it has no impact on his/her beliefs. In contrast, if the rating provides new information, the investor’s beliefs are updated accordingly. Investors’ decisions and the true value of the

3Carlson and Hale (2006) also consider the effect of a CRA in the Morris and Shin - model, but in their model, the CRA is non-strategic, implying an entirely different analysis.
fundamental then determine the probability of default. As highlighted before, the CRA takes these effects into account when assigning a rating.

We first examine ratings’ equilibrium impact. Tight liquidity makes investors more reluctant to roll their loan over, while it is of no direct concern to the CRA, which only wants to predict the correct outcome. Therefore, an individual investor requires a better signal to roll over the debt than what the CRA requires to issue a positive rating. If the CRA in spite of its more lenient threshold issues a bad rating, it will tend to influence investors’ beliefs – and thereby the incidence of default – while a good rating will not have this effect. Conversely, when investors have low liquidity needs, they are willing to accept a higher risk of default than what the CRA requires to announce a good rating. Consequently, good ratings tend to influence investors’ beliefs – and thereby the incidence of default – while bad ratings do not. Since, even in the absence of a CRA, default is more likely when aggregate liquidity is tight, our results imply a pro-cyclical impact of CRAs: they increase default risk when it is high, and decrease default risk when it is low.

We then explore our model’s implications for the frequency with which the CRA gives positive or negative ratings – how will the fact that a CRA can influence sovereign default affect rating standards? We define ‘positive bias’ as occurring when the frequency of good ratings is higher in equilibrium, under the influential CRA, than if they were privately assigned, and never released, by an observer with the same information as the CRA. This definition controls for the direct effect of liquidity on the incidence of default. When liquidity is tight, good ratings are uninfluential, while bad ratings make default more likely. Hence, because the CRA’s objective is to make correct predictions, bad ratings become attractive when liquidity

\footnote{A number of papers define ‘rating inflation’ as occurring if a CRA lies and gives too good ratings, while in other papers rating inflation occurs when the issuer picks the best among several unbiased ratings. Our definition of ‘positive bias’ is closely related, as it reflects the common feature of an excessive frequency of positive ratings, relative to the assessment by an independent observer. See also our discussion in Section 3.3.}
is tight: they allow the CRA to benefit from its own impact on the probability of default. This effect induces a ‘negative bias’ in the rating. By symmetry, easy liquidity induces a positive bias. Note that these results reflect an endogenous bias in equilibrium, without invoking any exogenous bias in the payoff structure of CRAs. Instead, they highlight that rating standards might endogenously vary over time depending on the state of the economy. In a boom, one might therefore observe a tendency of many positive ratings even in the sovereign debt market where ratings are unsolicited, and ‘rating shopping’ is not an issue.\(^5\)

Finally, we examine the effects of exogenous biases in CRAs’ incentives. A CRA might in principle be biased toward treating sovereigns favorably, for instance due to political pressure, or it might be biased toward prudent ratings, e.g. to avoid later responsibility for failed investments. We assume that the exogenous bias is common knowledge. In equilibrium, any bias is therefore taken into account when investors interpret ratings. If, for instance, a CRA favors issuers, investors will place little weight on good ratings, as it then is unclear if these are motivated by the outlook for the sovereign or simply reflect the bias of the CRA. By contrast, a bad rating will now be given more weight, since it indicates that the CRA has observed a sufficiently negative signal to overcome its bias. We show that a bias in the incentive structure of a CRA ultimately becomes self-defeating in equilibrium. When CRAs are biased in favor of issuers, ratings on average increase the incidence of default. When CRAs are biased in favor of bad ratings, ratings on average reduce the incidence of default.

Theoretical research on credit rating agencies has burgeoned over the last years. Yet, to our knowledge Manso (2013) is the only previous paper to explore optimal CRA decisions and the feedback effects arising when ratings affect the performance of rated assets. In Manso (2013) a firm issues debt with interest payment that decreases with the firm’s credit rating. Equity

\(^5\)Rating shopping refers to the ability of issuers to pay for and disclose ratings after they privately observe them, making it possible to choose the rating agency assigning the highest rating.
holders then choose the default time which maximizes equity value. The CRA’s objective is to set accurate ratings that inform investors about the probability of default over a given time horizon. By determining the interest rate, a rating affects the optimal default decision of the issuer, which in turn influences the rating itself. While both papers explore the feedback effects of credit ratings, our paper and Manso’s therefore take substantively different approaches. In particular, Manso focuses on the strategic interaction between CRAs and issuers, while our focus is on the strategic interaction between CRAs and creditors, allowing us to derive new results regarding the pro-cyclical nature of credit rating. Section 3.4 further highlights the starkly different normative implications of the two papers. While Manso’s results advocate for a CRA bias in favor of issuers so as to reduce the incidence of default, our results imply that such a bias might instead increase the incidence of default. The reason why our policy implication diverges, is that investors in our framework take CRA incentives into account before they act.

Our analysis also relates to studies of how CRAs might coordinate investors. Boot et al. (2006) develop a model with moral hazard in which credit ratings provide a focal point for firms and investors, and help select the most efficient equilibrium. By contrast, Carlson and Hale (2006) show in a global games setting similar to ours how a non-strategic CRA, which simply passes its information on to the market, may induce multiple equilibria by publicly revealing its information.

In global games more generally, precise public information enhances the scope for multiplicity; see for instance Morris and Shin (2003) or Hellwig (2002). While this effect naturally arises in our model too, our focus is elsewhere. Instead, we parameterize our model to obtain a unique equilibrium. Our main innovation lies in the fact that the rating agency behaves strategically, and takes the equilibrium effects of its rating into account. Relative to the global games literature overall, we thus contribute by studying a strategic sender of public
information. In this respect, our paper is closer in spirit to Angeletos, Hellwig and Pavan (2006) who study the endogenous information generated by policy interventions. In a broader perspective, our paper relates to studies of self-fulfilling crises at large, such as the bank-run model of Diamond and Dybvig (1983).

An important strand of the rating literature explores the link between rating shopping and rating inflation. This includes, among others, Skreta and Veldkamp (2009), Sangiorgi and Spatt (2011), Bolton, Freixas and Shapiro (2012), and Opp, Opp and Harris (2013). Compared to that literature, our paper highlights that the frequency of good ratings might be elevated even in the absence of issuers’ rating shopping, due instead to creditors’ changing liquidity needs. Mathis et al. (2009) is also, in spirit, related to that strand of literature: A rating agency is inclined to inflate ratings in order to secure higher fees. Their focus, however, is on the reputational cycles that result when CRAs first report truthfully to build up a reputation, and thereafter milk their reputation down by repeatedly inflating ratings.

Our paper naturally relates to the extensive literature on expectations-driven sovereign debt crises. Cornerstone contributions there are Calvo (1988) and Cole and Kehoe (2000). The model we base our analysis on has previously been used by Morris and Shin (2006) and Corsetti, Guimaraes and Roubini (2006) to study the role of IMF interventions in curbing sovereign debt crises. Importantly, in their studies the IMF decision to support a sovereign with short term assistance does not serve as a signal to investors, but affects outcomes by directly reducing the sovereign’s need for market financing. In contrast, the signalling effect

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6In Skreta and Veldkamp (2009) a series of non-strategic CRAs truthfully report a noisy signal of an asset’s return. Issuers then exploit investors’ failure to account for the fact that they strategically select the best of all ratings. Published ratings are thus biased signals of assets’ true values, and prices distorted. Sangiorgi and Spatt (2011) extend the analysis by showing that similar outcomes result when investors behave rationally but are uncertain about the number of ratings observed by issuers. In Bolton et al. (2012), with good ratings an issuer can sell more assets to investors who take ratings at face value. Good ratings thus allow issuers to raise profits, creating high willingness to pay for these ratings. This in turn motivates CRAs to produce good ratings, for which they receive higher fees. Opp et al. (2013) build on this approach and show that introducing rational investors dampens rating inflation, since inflated ratings are less informative – and therefore less valuable – for investors.
is exactly how a CRA affects outcomes in our study.

While our study focuses on sovereign debt, its basic insights might carry over to other settings where asset performance is sensitive to ratings. Our prediction of procyclical ratings is consistent with Griffin and Tang’s (2012) finding that as the coming recession became imminent in April 2007, rating standards at a top rating agency became markedly tougher. Amato and Furfine (2004) find that while average ratings are acyclical, new ratings and rating changes typically have a procyclical effect. Our results are also related to the widespread empirical finding that ratings tend to affect investor behavior asymmetrically, with negative rating events having stronger effects than positive events, or vice versa. The pattern was first highlighted by Holthausen and Leftwich (1986), and multiple studies have later shown that this pattern holds for a broad set of asset classes. Among these studies, some have documented the pattern for sovereign debt, such as Afonso, Furceri and Gomes (2012) and Ismailescu and Kazemi (2010). Our model implies asymmetric rating effects, and ties asymmetry to the state of the economy and biases in CRA incentives. For instance, if CRAs are biased in favor of issuers, or if liquidity is tight, then bad ratings will matter more than good ones.\footnote{Given that our model is static, we cannot strictly account for downgrades or upgrades. However, if a CRA has given a rating in the past, then as time elapses this rating looses relevance. By the time a new rating is announced, the situation may therefore to some extent be similar to one where no previous rating existed, as our model assumes.}

The paper is organized as follows. The model is presented in Section 2. Section 3 solves the model and presents our results. Section 4 concludes. All proofs are contained in the Appendix.
2 Model

Our framework builds on Morris and Shin’s (2004, 2006) model of the rollover problem encountered by a firm or a sovereign relying on short-term debt. We first lay out the basic model, which we later enhance by introducing a strategic credit rating agency.

Debt rollover. There are two periods, $t = 1, 2$. A unit mass of investors (or creditors), indexed by $i$, are financing a sovereign, using a conventional debt contract. At $t = 1$ each investor faces the option to (i) liquidate his loan to the sovereign for a payment normalized to 1, or (ii) rollover his loan to the sovereign. In the latter case the contract specifies final-period payment $V$ if the sovereign does not default, while if the sovereign defaults, investors receive 0.\(^8\)

Our main goal is to examine the role played by CRAs when there is rollover risk. We thus assume that the sovereign’s ability to meet short-term claims is the sole source of uncertainty; this is the decisive factor for whether the sovereign defaults or not. Let $l$ denote the mass of investors liquidating at time $t = 1$. The ability to meet short-term claims, and thereby avoid default, is summarized by a variable $\theta$, unknown to creditors. One may think of $\theta$ as the sovereign’s stock of liquid reserves, including all assets it can liquidate in the short run, or access to alternative credit lines other than the debt market. If $\theta \geq l$, the sovereign meets its short-term claims, and investors who in the first period chose to roll over obtain payment $V$ at $t = 2$. If $\theta < l$, the sovereign defaults, and those who chose to roll over get nothing. Observe in particular that if $\theta \geq 1$, the sovereign survives even if all creditors were to liquidate. By contrast, default occurs with certainty if $\theta < 0$. However, if $\theta \in [0, 1)$, the sovereign may or may not default, depending on investors’ behavior. A coordination problem then prevails: Each investor would gain if all were to roll over their loan to the sovereign, but no investor

\(^8\)The loan may be backed on collateral for instance, whose liquidation value is $V_1$ in the first period and $V_2$ in the second period. Following Morris and Shin (2004) our baseline model sets $V_1 = 1$ and $V_2 = 0$. 

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would gain from being the only one to do so. Let $I$ denote the indicator variable taking value 1 if the sovereign repays, and 0 if it defaults. Thus

$$I = 1 \Leftrightarrow \theta \geq l. \quad (1)$$

**Information structure and CRA.** While $\theta$ is unknown to creditors, it is common knowledge that $\theta$ is uniformly distributed over $\Delta = [\underline{\theta}, \overline{\theta}]$, where $\underline{\theta} \ll 0$, and $\overline{\theta} \gg 1$. Before $t = 1$, each investor receives a private signal $x_i$ which conditional on $\theta$ is uniformly distributed over $[\theta - \beta, \theta + \beta]$. In addition investors observe, before $t = 1$, the rating $R$ of a credit rating agency. For tractability, and following much of the literature on the effect of rating agencies, e.g. Bolton et al (2012) and Mathis et al (2009), we restrict attention to a binary rating by the CRA. Thus, a rating is either good ($R = 1$), or bad ($R = 0$). We assume that the CRA receives information in the form of a private signal $y$, which conditional on $\theta$ is uniformly distributed over $[\theta - \alpha, \theta + \alpha]$. All signals are independent conditional on $\theta$.

**Strategies.** An investor’s strategy is a mapping $\sigma_i : \Delta \times \{0, 1\} \to \{\text{liquidate, rollover}\}$ specifying whether to liquidate or roll over at time $t = 1$ as a function of the private signal $x_i$ and the rating $R$.\(^\text{10}\)

A rating strategy for the CRA is a mapping $r : \Delta \to \{0, 1\}$, assigning a rating to each possible signal $y$ received by the CRA. The rating strategy $r$ partitions $\Delta$ into $r^{-1}(0)$ and $r^{-1}(1)$. **Threshold rating strategies**, in which the CRA announces rating 1 if and only if it observes a signal $y$ above a given threshold, play a prominent role in the analysis.\(^\text{11}\) We let $r_t$ denote the threshold rating strategy with threshold $t \in \Delta$. With a slight abuse of notation, we will interchangeably use $R$ to denote the actual rating given (0 or 1) or a subset

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\(^9\)This ensures that all relevant $\theta$ values arise with positive probability.

\(^{10}\)We are slightly abusing notation here. Strictly speaking, the set of possible signals is $[\underline{\theta} - \beta, \overline{\theta} + \beta]$. This is however innocuous, since we are assuming $\underline{\theta} \ll 0$ and $\overline{\theta} \gg 1$.

\(^{11}\)We later establish that any equilibrium must entail a threshold rating strategy.
of $\Delta$ (corresponding to $r^{-1}(0)$ or $r^{-1}(1)$). Doing so allows us to identify ratings with their actual informational content. For example, a rating from a CRA behaving according to rating strategy $r_t$ has informational content either $y \in \left[\delta, t\right]$, or $y \in \left[t, \delta\right]$.

**Payoffs.** Investors have preferences given by

$$u(w_1, w_2) = vw_1 + w_2$$

where $w_t$ indicates payments received in period $t$. The parameter $v$ captures investors’ valuation of immediate payoff relative to delayed payoff, and plays the same role as ‘patience’ plays in the tradition of Diamond and Dybvig (1983).\(^{12}\) We will interpret $v$ as a measure of investors’ liquidity needs. Thus $v$ is high when investors have strong immediate needs for cash, as for instance in a liquidity squeeze, or when investors face important margin calls on other positions. Importantly, $v$ is common to all investors, and therefore reflects the aggregate state of the economy. The ratio $v/V \equiv \lambda$ plays a key role. We will say that liquidity is *tight* when $\lambda$ is high, and that liquidity is *easy* when $\lambda$ is low.

Following Manso (2013), we assume that the CRA is driven by a desire to preserve its track record of correct predictions.\(^{13}\) There are two ways to be correct in this environment: the CRA may give a good rating to a sovereign that later repays, or it may give a bad rating to a sovereign that defaults. For the time being, we treat these two alternatives symmetrically, and assume that the CRA obtains a payoff normalized to 1 if the rating is $1$ and the sovereign later repays, or if the rating is $0$ and the sovereign later defaults. The CRA obtains zero payoff in the two other cases, where its rating turns out to be ‘incorrect’. We let $\Pi(R, I)$ denote the

\(^{12}\)See also Ennis and Keister (2000).

\(^{13}\)Our approach is also akin to the reduced-form models of reputation used in for example Morgan and Stocken (2003) or Bolton, Freixas, and Shapiro (2012).
payoff to the CRA which has given the rating $R$ to a sovereign with outcome $I$. Hence,

$$\Pi(R, I) = IR + (1 - I)(1 - R).$$

**Beliefs.** Before making a choice, each investor updates his beliefs regarding $\theta$ based on the information contained in his private signal $x_i$ and the public rating with informational content $R = [\underline{r}, \overline{r}]$. We shall say that an investor updates beliefs according to A1-A3 if he follows the following simple rules:

**A1** If $x_i - \beta > \overline{r} + \alpha$, he assigns probability 1 to the event $\theta = \overline{r} + \alpha$.

**A2** If $x_i + \beta < \underline{r} - \alpha$, he assigns probability 1 to the event $\theta = \underline{r} - \alpha$.

**A3** If $x_i - \beta \leq \overline{r} + \alpha$ and $x_i + \beta \geq \underline{r} - \alpha$, he assumes $\theta$ is uniformly distributed on $[x_i - \beta, x_i + \beta] \cap [\underline{r} - \alpha, \overline{r} + \alpha]$.

Rules A1-A2 pin down investors’ out-of-equilibrium beliefs, i.e. whenever information conveyed by the rating is inconsistent with their own information.\(^{14}\) A3 approximates the Bayesian updating procedure by assuming uniform posterior distributions throughout, and plays no other role than ensuring tractability.\(^{15}\)

**Equilibrium.** We next formulate our equilibrium concept. A profile of strategies\(^{16}\) $(\sigma^*, r^*)$ for investors and CRA constitute a *rating equilibrium* if and only if it satisfies the following conditions:

\(^{14}\)The obvious alternative would be to set probability 1 to $\theta = x_i - \beta$ if $x_i - \beta > \sup R$, and probability 1 to $\theta = x_i + \beta$ if $x_i + \beta < \inf R$. However, that assumption opens the possibility of multiple equilibria in the investment game and thus defeats the purpose of using global games to ensure uniqueness of equilibrium. See Carlsson and Van Damme (1993) and Morris and Shin (2003) for thorough discussions of the issue.

\(^{15}\)Notice that rule A3 exactly coincides with Bayesian updating whenever $\underline{r} = \overline{r}$. In general however, the probability density of $\theta$ conditional on $y \in R$ tapers off at the edges of its domain. If $[\underline{r} + \alpha \leq \overline{r} - \alpha]$ for instance, then the conditional distribution is uniform on that interval and tapers off on $[\underline{r} - \alpha, \underline{r} + \alpha] \cup [\overline{r} - \alpha, \overline{r} + \alpha]$.

\(^{16}\)We do not explicitly consider mixed strategies. Mixed strategies complicate the exposition but do not affect the results of our paper.
1. Investors update beliefs according to A1-A3.

2. Given any signal $x_i \in \Delta$ and rating $R \in \{0, 1\}$, investors maximize expected utility:
   $$\mathbb{E}[u(\sigma_{i}^{*}, \sigma_{-,i}^{*}, r^{*})] \geq \mathbb{E}[u(\sigma_{i}', \sigma_{-,i}^{*}, r^{*})],$$
   for all $i$ and $\sigma_{i}' \neq \sigma_{i}^{*}$.

3. Given any signal $y \in \Delta$, the CRA maximizes expected payoff:
   $$\mathbb{E}[\Pi(\sigma^{*}, r^{*})] \geq \mathbb{E}[\Pi(\sigma^{*}, r')],$$
   for all rating strategies $r'^{*}$.

Thus, besides the computational approximation of rule A3, our definition of equilibrium is that of a Perfect Bayesian Equilibrium.

Finally, we impose a number of parameter restrictions. Observe that if $\lambda \geq 1$, then all investors always liquidate. If on the other hand $\lambda = 0$, then all investors always roll over. So $\lambda \in (0, 1)$ is the critical region of interest, which we henceforth restrict attention to. We assume throughout that $\alpha$ and $\beta$ are strictly greater than $1/2$. Furthermore, overly precise information on the part of the CRA introduces multiple equilibria as in the Diamond and Dybvig (1983) framework of self-fulfilling crises, which would counter the purpose of using global games to facilitate exploration of coordination risk by ensuring equilibrium uniqueness.\(^{17}\) In our model, this would be the case if $\alpha < \beta$. We thus assume $\alpha > \beta$, in order to preserve equilibrium uniqueness. Finally, to guarantee the existence of an equilibrium, we assume that $1/(2\beta + 1) < \lambda < 1 - 1/(2\beta + 1)$.

### 3 Analysis and Results

The equilibrium is solved backwards, i.e. by first investigating the impact of a given rating on the incidence of default. We define for that purpose the investment game given rating $R$ as the game played among investors observing $R$ and following rules A1-A3. Section 3.1 studies that game. While the investment game treats ratings as exogenous, section 3.2 endogenizes

\(^{17}\)See e.g. Morris and Shin (2003), or Proposition 2 in Carlson and Hale (2006).
ratings by incorporating strategic behavior on the part of the credit rating agency. This section addresses the central question of our paper: do credit ratings affect the incidence of default, and, if so, how? Section 3.3 examines how the frequency of good versus bad ratings might endogenously diverge from the judgment of an independent observer, and relates these findings to the classic question of rating inflation. Section 3.4 extends the basic model in order to explore the effect of a possible exogenous bias in the preferences of the credit rating agency.

3.1 The Investment Game

Equilibrium in the investment game is characterized by two threshold values.\textsuperscript{18} There is first a threshold value for investors’ signals, denoted $x^*(R)$, such that investors observing $x_i < x^*(R)$ liquidate while investors observing $x_i \geq x^*(R)$ roll over. There is then a threshold value for $\theta$, denoted $\theta^*(R)$, below which the sovereign defaults and above which it repays. To shorten notation, we omit the dependence of the thresholds on the rating $R$ where this is unlikely to create confusion.

We will say that an investor is a marginal investor if he receives the private signal $x_i = x^*(R)$. A marginal investor is thus indifferent between liquidating and rolling over. The indifference equation of the marginal investor can be written as

$$\mathbb{P}(\theta > \theta^* | x_i = x^*, R) = \frac{v}{V} = \lambda. \quad (2)$$

Since all investors are ex ante identical, then given $\theta$, the mass $l$ of investors liquidating equals the probability that an arbitrary investor receives signal $x_i < x^*$. Thus

$$l = \mathbb{P}(x_i < x^* | \theta). \quad (3)$$

\textsuperscript{18}The analysis of the investment game is standard in the global games literature. See e.g. Morris and Shin (1998) or Carlson and Hale (2006).
Combining equations (1) and (3), $\theta^*$ is given by

$$\theta^* = \mathbb{P}(x_i < x^* | \theta = \theta^*).$$

(4)

To set a benchmark, we will start by solving the investment game given rating $R = \Delta$. In this case, the rating provides no information, thus capturing the situation prevailing in the absence of a CRA. When $R = \Delta$, the posterior beliefs of an investor observing signal $x_i$ are uniform on $[x_i - \beta, x_i + \beta]$. Equation (2) thus yields

$$x^* = \theta^* - \beta + 2\beta\lambda$$

(5)

while equation (4) gives

$$\theta^* = \frac{x^* + \beta}{2\beta + 1}$$

(6)

Finally, (5) and (6) together yield

$$\theta^* = \lambda$$

(7)

$$x^* = \lambda(2\beta + 1) - \beta$$

(8)

Equation (7) establishes the key link existing in our model – independently of the CRA – between liquidity on the one hand and default risk on the other: the easier liquidity (i.e. the lower $\lambda$), the lower the chance of default (i.e. the lower $\theta^*$).

We next turn to the analysis of the investment game given a rating $R = [\underline{r}, \bar{r}] \neq \Delta$. While ratings have no impact on equation (4), they affect investors’ beliefs and the indifference equation of the marginal investor, yielding

$$\mathbb{P}(\theta > \theta^* | \theta \in [x^* - \beta, x^* + \beta] \cap [\underline{r} - \alpha, \bar{r} + \alpha]) = \lambda$$

(9)
If the interval $[\tau - \alpha, \tau + \alpha]$ covers the support of the marginal investor’s beliefs, which from (8) is given by $[\lambda(2\beta + 1) - 2\beta, \lambda(2\beta + 1)]$, then the marginal investor’s behavior is not influenced by the rating. The equilibrium is therefore unaffected. If, however, the rating $R$ induces the marginal investor to revise his beliefs concerning $\theta$, then the rating will affect his behavior and, thereby, both threshold equilibrium values. A ‘negative’ rating such that the upper bound of the rating plus noise lies below the upper bound of the marginal investor’s beliefs, i.e. such that $\tau + \alpha < \lambda(2\beta + 1)$, induces him to liquidate. The threshold $x^*$ thus rises, causing in turn $\theta^*$ to rise which makes default more likely. In a similar way, a rather ‘positive’ rating such that the lower bound of the rating minus noise lies above the lower bound of the marginal investor’s beliefs, i.e. such that $\tau - \alpha > \lambda(2\beta + 1) - 2\beta$, induces him to strictly prefer rolling over; $x^*$ decreases, which in turn causes $\theta^*$ to fall, making default less likely. Lastly, whenever the rating is sufficiently negative, $\tau + \alpha > 0$, to ensure that $\theta < 1$, the rating affects all investors and triggers default. Similarly, whenever the rating is sufficiently positive, $\tau - \alpha > 0$, to ensure that $\theta \geq 0$, the rating affects all investors and prevents default. The following Lemma summarizes these observations.

**Lemma 1** An equilibrium of the investment game given rating $R$ exists, and is unique. This equilibrium is characterized by the threshold $\theta^*(R) \in [0,1]$ such that: $I = 1 \iff \theta \geq \theta^*(R)$.

In particular, ratings affect $\theta^*$ in the following way:

1. $\theta^*(\Delta) = \lambda$
2. $\tau + \alpha < (2\beta + 1)\lambda \iff \theta^*(R) > \lambda$
3. $\tau - \alpha > (2\beta + 1)\lambda - 2\beta \iff \theta^*(R) < \lambda$
4. $\tau + \alpha < 1 \iff \theta^*(R) = 1$
5. $\tau - \alpha > 0 \iff \theta^*(R) = 0$
3.2 The Equilibrium Impact of Credit Ratings

This section endogenizes ratings by incorporating strategic behavior on the part of the credit rating agency. We show that rating equilibria exist, and explore their fundamental properties.

Our framework possesses a trivial equilibrium, henceforth denoted \((\sigma_0, r_0)\), where the CRA assigns ratings independently of which signal it receives.\(^{19}\) Since the CRA here provides no additional information to creditors, this babbling equilibrium captures the situation prevailing in the absence of a CRA. It therefore provides the natural benchmark to compare our results to. We begin this section with the necessary notation and definitions.

Let \(\mathbb{P}(I = 1|R, y)\) denote the probability of rollover, i.e. that the sovereign does not default, as seen from the perspective of a CRA observing signal \(y\) and announcing rating \(R\); thus \(\mathbb{P}(I = 1|R, y) = \mathbb{P}(\theta \geq \theta^*(R)|y)\). Given rating strategy \(r\), define the rollover probability as the function \(Q^r : \Delta \to [0, 1]\) such that

\[
Q^r(y) = \mathbb{P}(I = 1|r(y), y).
\]

The difference \(Q^r - Q^{r_0}\) measures the impact of rating strategy \(r\) on the incidence of rollover, as compared to a CRA which gives no information to the investors. Thus, a CRA following rating strategy \(r\) where \(Q^r = Q^{r_0}\) does not affect the sovereign’s probability of default. Typically, there will be many possible strategies that do not affect the default probability. We define a rating strategy \(r\) which differs from the empty rating, and yet does not affect the rollover probability (i.e. it satisfies both \(r \neq r_0\) and \(Q^r = Q^{r_0}\)) as reducible to \(r_0\). By extension, a rating equilibrium \((\sigma, r)\) will be called irreducible if and only if \(r\) cannot be reduced to \(r_0\).

Let \(Q^r > Q^{r_0}\) denote the case where \(Q^r(y) \geq Q^{r_0}(y)\) for all \(y \in \Delta\) and with strict inequality

\(^{19}\)Since for expositional purposes we are restricting attention to pure strategies, \(r_0\) here denotes any rating strategy which assigns one and the same rating to all signals received by the CRA.
for some \( y \in \Delta \). A rating strategy \( r \) such that \( Q^r > Q^{r_0} \) therefore facilitates rollover and reduces the incidence of default. By symmetry, a rating strategy with \( Q^r < Q^{r_0} \) impedes rollover and increases the incidence of default.

Note that default induced by coordination failure from investors is inefficient. Thus, a rating strategy which facilitates rollover increases welfare positively. Likewise, a strategy which impedes rollover decreases welfare.\(^{20}\)

Finally, observe that if a threshold rating strategy \( r_t \) facilitates rollover, so that \( Q^{r_t} > Q^{r_0} \), then \( Q^{r_t}(y) \) must equal \( Q^{r_0}(y) \) for all \( y < t \). This property follows from the fact that, as far as threshold rating strategies are concerned, a bad rating can never be of any help to the sovereign. In a similar way, a good rating cannot be negative for the sovereign. Hence, if \( Q^{r_t} < Q^{r_0} \) then \( Q^{r_t}(y) \) must equal \( Q^{r_0}(y) \) for all \( y > t \).

We begin the analysis with a simple observation concerning the CRA’s equilibrium behavior. For any rating \( R \), note that \( \mathbb{P}(I = 1|R, y) \) is non-decreasing in \( y \). Intuitively, given the rating, a better signal \( y \) leads to a higher rollover probability. Thus, if the CRA announces a good rating for \( y = y' \), it will also choose a good rating for all \( y > y' \). It follows that an equilibrium rating strategy must be a threshold rating strategy. We thus restrict attention to this class of strategies. For expository purposes, we use the notation \( R_t^- = [\delta, t] \), and \( R_t^+ = [t, \delta] \). A CRA using strategy \( r_t \) thus communicates that \( y \in R_t^+ \) if its rating is good, and that \( y \in R_t^- \) if its rating is bad.

A threshold rating strategy \( r_t \) is part of a rating equilibrium if and only if a CRA observing \( y = t \) is indifferent between announcing a good rating and announcing a bad rating. Thus, in equilibrium, we have

\[
\mathbb{P}(I = 1|R_t^+, t) = \mathbb{P}(I = 0|R_t^-, t). \tag{10}
\]

\(^{20}\)See Lemma 2 in the Appendix.
The left-hand side of this equation is the rollover probability, as seen from the perspective of a CRA observing signal $y = t$ and announcing $R = 1$. The right-hand side is the probability that a sovereign defaults, as seen from the perspective of a CRA observing $y = t$ and announcing $R = 0$.

In order to understand the equilibrium mechanics, it is useful to start from the special case where $\lambda = 1/2$. When $\lambda = 1/2$, each investor bases his individual decision on what he perceives as the most likely outcome. From Lemma 1, point 2. and 3., a CRA using threshold $t = 1/2$ never affects investor behavior, irrespective of whether it announces $R = 1$ or $R = 0$; in both cases $\theta^* = x^* = 1/2$. If the CRA receives a signal $y = 1/2$, it assigns 50% chance to a successful refinancing and 50% chance to the sovereign’s default, so that (10) holds. Thus, in this knife-edge case $t = x^*$, which means that the CRA and investors base their decisions on the same threshold.

In contrast, when $\lambda \neq 1/2$ there is a wedge between the CRA and investors: Investors then demand a different default probability in order to refinance than a CRA requires in order to announce $R = 1$. For instance, when $\lambda > 1/2$ investors require that the probability of repayment exceeds 50% in order to refinance the sovereign, while the CRA, as always, bases its rating only on what it sees as the most likely outcome. A marginal investor must therefore observe $x^* > t = \lambda$. In turn, as $x^*$ rises above $t$, bad ratings tend to become more informative to a marginal investor (while good ratings tend to become less informative). At $\lambda = \alpha/2\beta > 1/2$ (c.f. Lemma 1), a bad rating starts affecting outcomes, raising $\theta^*$ above $\lambda$. For $\lambda$ high, therefore, ratings on average increase the incidence of default. But high values of $\lambda$ are associated with high default risk (Lemma 1). Credit ratings thus exhibit a pro-cyclical impact: they increase default risk when it is high, and decrease default risk when it is low.

The next proposition summarizes our findings.

**Proposition 1** A rating equilibrium always exists. If $1 - \frac{\alpha}{2\beta} < \lambda < \frac{\alpha}{2\beta}$, then the babbling
equilibrium is the only irreducible rating equilibrium. There exists otherwise exactly one other irreducible rating equilibrium, \((\sigma^*, r_*)\). In this equilibrium, credit ratings have a pro-cyclical impact:

1. If \(\lambda < 1 - \frac{\alpha}{2\beta}\) then \(Q^{r*} > Q^0\).

2. If \(\lambda > \frac{\alpha}{2\beta}\) then \(Q^{r*} < Q^0\).

To sum up, tight liquidity makes investors require a lower risk of default (i.e. a more positive signal) to roll over their loans than the CRA requires to assign a good rating. Consequently, bad ratings tend to influence investors’ beliefs – and thereby the incidence of default – while good ratings do not. Conversely, when investors have low liquidity needs, they accept a higher risk of default than the CRA requires to assign a good rating. Consequently, good ratings tend to influence investors’ beliefs – and thereby the incidence of default – while bad ratings do not.\(^{21}\)

Figure 1 illustrates the effect of credit ratings. In the upper panel, \(\lambda > \alpha/2\beta\) and the threshold value for investors’ private signal \(x^*\) lies above the equilibrium rating threshold \(t^*\). Thus, good ratings, corresponding to \(y > t^*\), will not affect the beliefs of the marginal investor and therefore have no impact. Bad ratings on the other hand, corresponding to \(y < t^*\), move \(\theta^*\) up – potentially affecting the marginal investor and thereby sovereign rollover, thus lowering \(Q^{r*}\) below \(Q^0\). At \(y = \lambda - \alpha\) default occurs with probability 1 irrespective of the rating, implying that \(Q^{r*}\) and \(Q^0\) coincide for \(y \leq \lambda - \alpha\), despite the rating’s adverse effect on \(\theta^*\). The lower panel displays the effect of credit rating when \(\lambda < 1 - \frac{\alpha}{2\beta}\). Now, bad ratings have no effect while good ratings move \(\theta^*\) down, increasing the rollover probability, unless the signal is sufficiently good that rollover is certain irrespective of the rating.

\(^{21}\)With uniform distributions the asymmetric effect is taken to the extreme in the sense that a rating which overlaps with the beliefs of the marginal investor will have no effect on his posterior beliefs. With more general distribution functions there might be some, moderate, effect in this case too.
Figure 1: Liquidity and the Impact of Credit Rating.
3.3 Endogenous Bias and Cyclical Rating Levels

Rating inflation has recently attracted much attention. The theoretical literature has typically focused on the role of rating shopping – the ability of issuers to pay for and disclose ratings after they privately observe them – in motivating CRAs to assign excessively positive ratings. In contrast, sovereigns cannot shop ratings in the same way, as CRAs typically issue unsolicited ratings for sovereign debt. Yet, our model shows that the frequency of good sovereign ratings might still become elevated, relative to the assessment by an independent observer who has the same information as the CRA, but who cannot publish it as the CRA does.

By Lemma 1, in the absence of a CRA, \( \theta^* = \lambda \). Hence, an independent observer interprets his signal as ‘good news’ if and only if the signal is greater than \( \lambda \). A CRA using strategy \( r_t \) on the other hand announces a good rating for any signal above \( t \) and a bad rating for any signal below \( t \). Thus, if \( t < \lambda \), the frequency of good ratings is higher than the frequency with which an independent observer receives signals he interprets as ‘good news’. Similarly, if \( t > \lambda \), the frequency of bad ratings is higher than the frequency with which an independent observer receives signals he interprets as ‘bad news’. These observations motivate the following definitions:

**Definition 1** A threshold rating strategy \( r_t \) exhibits a positive bias if \( t < \lambda \), and exhibits a negative bias if \( t > \lambda \).

Given rating equilibrium \((\sigma^*, r_t^*)\), we also define the probability that the rating is correct as

\[
\pi(\sigma^*, r_t^*) = \mathbb{E}(IR + (1-I)(1-R)|\sigma^*, r_t^*),
\]

which records the sum of probabilities that (i) the CRA announces \( R = 1 \) and the sovereign avoids default, and (ii) the CRA announces \( R = 0 \) and the sovereign defaults. Thus \( \pi \)

---

22 Prominent examples are Skreta and Veldkamp (2009), Sangiorgi and Spatt (2011), Bolton et al. (2012), and Opp et al. (2013)
is a measure of ratings’ predictive power within the equilibrium \((\sigma^*, r^*)\). For comparison, 
\[
\pi_0 = \mathbb{P}(\theta > \theta^*(\Delta) | y > \theta^*(\Delta)) + \mathbb{P}(\theta < \theta^*(\Delta) | y < \theta^*(\Delta))
\]
measures the predictive power of an independent observer receiving an unbiased signal of \(\theta\) in the absence of a CRA.

We next show that in equilibrium our model exhibits positive bias when \(\lambda\) is low and negative bias when it is high. More specifically, when \(\lambda > \alpha/2\beta\), a bad rating induces an upward – and thus adverse – shift of \(\theta^*\) (Proposition 1). Thus, in that region, the indifference equation (10) no longer holds for \(t = \lambda\), and a CRA receiving signal \(y = \lambda\) finds it in its best interest to announce \(R = 0\): doing so allows it to benefit from its own impact on the probability of default. This implies that in equilibrium \(t > \lambda\): ratings exhibit a negative bias.

Note furthermore that in this case, both sides of equation (10) exhibit probabilities greater than 50% in equilibrium. Thus, the bias in the rating in fact increases its predictive power.

Proposition 2 The unique irreducible rating equilibrium, \((\sigma^*, r^*)\), exhibits a positive bias if \(\lambda < 1 - \frac{\alpha}{2\beta}\) and a negative bias if \(\lambda > \frac{\alpha}{2\beta}\). Moreover, the bias increases the ratings’ predictive power: \(\pi((\sigma^*, r^*)) > \pi_0\) whenever \(\lambda < 1 - \frac{\alpha}{2\beta}\) or \(\lambda > \frac{\alpha}{2\beta}\).

Our paper in no way contends the role of rating shopping or distorted incentives in generating rating inflation (see also Section 3.4 below concerning CRAs with an exogenous bias). Instead, we highlight that the frequency of good ratings may be elevated in markets where there are unsolicited ratings, as typically is the case for sovereign debt. Because a CRA might affect sovereigns’ refinancing prospects, it can deliberately choose to give a different rating than an uninfluential independent observer would do, in order to increase the probability of a correct prediction. In this sense, ratings become inflated even though CRAs only aim to be correct and issuers do not select the best among alternative ratings. Furthermore, in our model excessive rating levels depend on creditors’ changing liquidity needs. When creditors have high liquidity needs, good ratings do not reduce the probability of sovereign default, but bad ratings increase it. Thus, the CRA favors bad ratings over good ones, inducing a negative
bias as the CRA seeks to increase the probability of a correct prediction. Similarly, when liquidity is abundant and investors hungry for yield, bad ratings do not worsen a sovereign’s prospects, while good ratings do improve the chances that default is avoided. CRAs then favor good ratings over bad ones, inducing a positive bias. To the best of our knowledge, these insights are novel.

3.4 Exogenously Biased Credit Rating Agencies

In this section, we examine the consequences of exogenous bias in CRA incentives.

While rating shopping biases CRAs in favor of issuers, the bias could in principal go either way when it comes to sovereign debt. On the one hand, the CRA might find it beneficial to issue positive ratings due to political pressure. On the other hand, when a rating is unsolicited, the CRA might be more afraid of guiding investors into purchasing debt that later defaults, than of warning against debt that eventually is honored. Moreover, the design of appropriate incentive structures in the ratings industry is a potentially important policy instrument for regulators. Together, these considerations motivate a general examination of the effect of biased incentives of the CRA.

We modify our specification of CRA payoffs to:

\[ \Pi(R, I) = \rho_1 IR + \rho_0 (1 - I)(1 - R) \]  

A CRA with \( \rho_1 > \rho_0 \) is biased in favor of issuers, while \( \rho_1 < \rho_0 \) implies a bias toward prudency (‘prudent bias‘). This is intended as a parsimonious way to represent biases in any direction, without taking a stance on the fundamental cause of them.

Biased incentives have the expected effect on rating standards, in the sense that a bias in favor of issuers leads to rating inflation, while a prudent bias leads to rating deflation. The result that an issuer bias leads to rating inflation is in line with the rating-shopping literature.
discussed in the previous section. The next proposition shows, however, that an issuer bias need not reduce the incidence of default; quite on the contrary.

**Proposition 3** Consider CRA payoffs given in (11). A rating equilibrium always exists. If
\[
\frac{\beta \lambda}{1-\frac{a}{a}} < \frac{\rho_0}{\rho_1} < \frac{1-\frac{a}{a}(1-\lambda)}{\rho_1},
\]
the babbling equilibrium is the only irreducible rating equilibrium. There exists otherwise exactly one other irreducible rating equilibrium, \((\sigma^*, r^*_t)\). In this equilibrium, any exogenous bias on the part of the CRA is 'self-defeating':

1. if \(\frac{\rho_0}{\rho_1} > \frac{1-\frac{a}{a}(1-\lambda)}{\rho_1}\) (prudent bias), then \(t^* > \lambda\), yet \(Q^{r^*_t} > Q^{r_0}\).

2. if \(\frac{\rho_0}{\rho_1} < \frac{\beta \lambda}{1-\frac{a}{a}}\) (issuer bias), then \(t^* < \lambda\), yet \(Q^{r^*_t} < Q^{r_0}\).

It follows directly from the proposition that any exogenous bias in CRA incentives reduces the predictive power of the ratings. Intuitively, a biased CRA may be willing to assign a rating with a probability of being correct that is less than 50%, if this increases the likelihood of the favored outcome. This contrasts sharply to the endogenous bias explored in Section 3.3, which increases the predictive power of the rating.

As the proposition states, any bias in the incentive structure of a CRA ultimately becomes self-defeating in equilibrium. When the fact that CRAs are biased in favor of issuers is common knowledge, ratings on average increase the incidence of default. When CRAs are biased toward bad ratings, ratings on average reduce the incidence of default.

The underlying mechanism is the following. Investors know CRAs’ exogenous bias, and take this into account when interpreting ratings. Hence, if CRAs favor issuers, investors place little weight on good ratings, as it is then unclear if these are motivated by the outlook of the rated country or simply reflect CRA bias. By contrast, a bad rating is now given more weight, since it indicates that a CRA has observed a sufficiently negative signal to overcome its bias. In short, rational investors discount ratings according to the known biases of CRAs.
The previous insight has important practical implications. An issuer bias dilutes the impact of good ratings. Thus, the objective of preventing inefficient sovereign debt crises is best achieved by means of a cautious bias on the part of CRAs.

4 Conclusion

Credit rating agencies have recently been criticized on different and often opposing grounds, in particular for being too lenient before the 2007-08 financial crisis, and for propagating refinancing problems thereafter. Our analysis shows that in markets where coordination failure is possible, specifically the sovereign debt market, such effects are likely to be present. We find that when aggregate liquidity is easy, sovereign ratings will be inflated and on average decrease coordination risk. Hence, credit ratings will tend to ease sovereign refinancing when the market initially is in a good state. By contrast, when liquidity is tight, ratings are deflated and make it even harder to roll sovereign debt over. While existing studies focus on how an issuer bias in CRA payoffs generates rating inflation, our study shows how an excessive frequency of positive ratings might arise due to booming market conditions. This is a novel prediction which may be tested empirically. Moreover, we show that biased incentives on the part of CRAs are ultimately self-defeating. To the best of our knowledge, our paper is the first to simultaneously: (i) account for strategic behavior on the part of CRAs, (ii) allow for the possibility that ratings affect the performance of the rated objects, and (iii) endogenize investors’ response to credit ratings.

Our paper contributes to the ongoing debate on how CRAs should be paid. A widely held concern is that existing remuneration schemes, in which CRAs typically are paid by issuers, cause too generous ratings.\textsuperscript{23} To resolve this issue, one suggestion has been that investors

\textsuperscript{23}For an overview of the debate, see e.g. Mathis, McAndrews and Rochet (2009) and Pagano and Volpin (2010).
– not issuers – should finance CRAs. Our analysis provides support for this view, but from a different angle than the arguments that have been used so far: Credit ratings can serve as a welfare improving coordination device, but only if agencies have a sufficiently negatively biased incentive structure. If rating agencies are known to side with issuers, good ratings will be discounted by investors while only bad ratings may affect investors’ beliefs. This view contrasts sharply to the arguments developed by, for example, Manso (2013), in support of the issuer-pay model – on the grounds that good ratings potentially limit the incidence of default. Fundamentally, the reason why our conclusion differs, is that Manso treats investors’ responses to credit ratings as exogenous, while we study investor behavior as an equilibrium outcome in which CRA biases are rationally taken into account. The role played by institutionally constrained investors – who are forced to sell once assets drop below investment grade – is thus crucial to this issue. Recent policy initiatives and reforms in the U.S. and E.U. explicitly aim to reduce rating reliance in law and regulation.\textsuperscript{24} These developments will give added weight to our conclusions.

Appendix

**Proof of Lemma 1:** Say that an equilibrium of the investment game is interior if in this equilibrium an investor’s behavior is contingent on his private signal $x$. Otherwise say that the equilibrium is a corner equilibrium.

As indicated in the body of the paper, any interior equilibrium is characterized by a pair $(x^*, \theta^*)$ satisfying equations (4) and (9), and repeated here:

\textsuperscript{24}In the U.S., the 2010 Dodd Frank Act removes statutory references to credit rating agencies, and calls for federal regulators to review and modify existing regulations to avoid relying on credit ratings as the sole assessment of creditworthiness (U.S. Securities and Exchange Commission, 2014). Similar initiatives have been taken in the EU (European Commission, 2013).
\[
\begin{aligned}
\theta^* &= \mathbb{P}(x < x^* | \theta = \theta^*) \\
\mathbb{P}(\theta > \theta^* | \theta \in [x^* - \beta, x^* + \beta] \cap [r - \alpha, r + \alpha]) &= \lambda
\end{aligned}
\] (12)

Note in particular that the second equation of the system defines a broken line in the \((x^*, \theta^*)\)-plane joining \(A, B, C, D\) where

\[
A = (r - \alpha - \beta, r - \alpha)
\]

\[
B = (r - \alpha + \beta, r - \alpha + 2\beta(1 - \lambda))
\]

\[
C = (r + \alpha - \beta, r + \alpha - 2\beta\lambda)
\]

\[
D = (r + \alpha + \beta, r + \alpha)
\]

By (12) an interior equilibrium thus exists if and only if (i) \(r + \alpha > 1\), and (ii) \(r - \alpha < 0\). If \(r + \alpha \leq 1\), a corner equilibrium exists in which all investors liquidate, irrespective of private signals. Similarly, if \(r - \alpha \geq 0\) then a corner equilibrium exists in which all investors roll over, irrespective of private signals. Hence an equilibrium always exists. Furthermore the equilibrium is unique since any time (i) and (ii) hold corner equilibria are precluded (investors receiving private signal above 1 must roll over while investors receiving private signal below 0 must liquidate).

Properties 1-5 follow from simple computations using (12).

Lemma 2 A rating equilibrium induces higher (lower) welfare than the babbling equilibrium if it induces lower (higher) probability of default than the latter. Let \((\sigma^*, r^*)\) denote a rating equilibrium; formally:

1. \(Q^{r^*} > Q^0 \Rightarrow \mathbb{E}[u(\sigma^*, r^*)] > \mathbb{E}[u(\sigma_0, r_0)]\)
2. \( Q^{r^*} < Q^{r_0} \Rightarrow E[u(\sigma^*, r^*_{t^*})] < E[u(\sigma_0, r_0)] \)

**Proof:** We show the proof of the second part. The proof of the first part is similar. Observe that \( Q^{r^*} < Q^{r_0} \) implies \( x^*(R_{t^*}^-) > x^*(\Delta) = x^*(R_{t^*}^+) \). By definition of \( x^* \), in equilibrium an investor observing signal \( x_i < x^* \) has utility \( v \) while an investor observing signal \( x_i > x^* \) has (expected) utility strictly more than \( v \). Averaging ex ante (i.e. before the realization of \( \theta \)) thus gives the desired result.

\[ \blacksquare \]

**Proof of Proposition 1:** Rewriting (10), the threshold rating strategy \( r_i \) is part of a rating equilibrium if and only if \( t \) satisfies

\[ \mathbb{P}(I = 1|R_i^+, t) = 1 - \mathbb{P}(I = 1|R_i^-, t). \]  \hfill (13)

By Lemma 1 the LHS of the equation is 1 for \( t - \alpha > 0 \) and 0 for \( t + \alpha < \lambda \), while the RHS of the equation is 1 for \( t + \alpha < 1 \) and 0 for \( t - \alpha > \lambda \). Moreover, the LHS of the equation is strictly increasing for \( t \in [\lambda - \alpha, \alpha] \) and the RHS of the equation strictly decreasing for \( t \in [1 - \alpha, \lambda + \alpha] \). The two curves thus cross exactly once. Let \( t^* \) denote this unique \( t \).

We next explore the conditions under which \( r_{t^*} \) is irreducible. By Lemma 1, a necessary condition is that one of the following two condition holds (note that at most one of the two conditions holds at once):

- **Condition 1:** \( t^* + \alpha < (2\beta + 1)\lambda \) (in which case \( \theta^*(R_{t^*}^-) > \lambda \)).
- **Condition 2:** \( t^* - \alpha > (2\beta + 1)\lambda - 2\beta \) (in which case \( \theta^*(R_{t^*}^+) < \lambda \)).

We next show that if either condition holds then \( Q^{r_{t^*}} \neq Q^{r_0} \), i.e. \( r_{t^*} \) is irreducible. By (13), note that
\[ t^* = \frac{\theta^*(R_t^-) + \theta^*(R_t^+)}{2}. \tag{14} \]

Thus, in particular if (say) condition 1 holds then \( t^* > \lambda \) (recall that in that case \( \theta^*(R_t^-) > \lambda = \theta^*(R_t^+) \)). Suppose now for a contradiction that \( Q^{r_t^*} = Q^{r_0} \). Since \( \theta^*(R_t^-) > \lambda \) it must then be that \( t^* + \alpha < \lambda \), i.e. default occurs with certainty from the point of view of the CRA observing \( t^* \). Be we then have \( t^* + \alpha < \lambda \) and \( t^* > \lambda \), which is the desired contradiction. This finishes to show that if either of conditions 1-2 holds then \( r_t^* \) is irreducible.

The proof is concluded by noting that condition 1 is equivalent to

\[ P(I = 1|R_t^+, y = t = (2\beta + 1)\lambda - \alpha) > (1 - P(I = 1|R_t^-, y = t = (2\beta + 1)\lambda - \alpha)) \tag{15} \]

while condition 2 is equivalent to

\[ P(I = 1|R_t^+, y = t = (2\beta + 1)\lambda - 2\beta + \alpha) < (1 - P(I = 1|R_t^-, y = t = (2\beta + 1)\lambda - 2\beta + \alpha)). \tag{16} \]

Substituting in (15) using Lemma 1 gives

\[ \frac{[(2\beta + 1)\lambda - \alpha] + \alpha - \lambda}{2\alpha} > 1 - \frac{[(2\beta + 1)\lambda - \alpha] + \alpha - \lambda}{2\alpha} \]

which simplifies to \( 2\beta \lambda > \alpha \).

Substituting in (16) using Lemma 1 gives

\[ \frac{[(2\beta + 1)\lambda - 2\beta + \alpha] + \alpha - \lambda}{2\alpha} < 1 - \frac{[(2\beta + 1)\lambda - 2\beta + \alpha] + \alpha - \lambda}{2\alpha} \]

which simplifies to \( 2\beta \lambda < 2\beta - \alpha \).
Proof of Proposition 2: The first part follows from (14) and the observation that $\theta^*(R_{t^-}) > \lambda = \theta^*(R_{t^+})$ if condition 1 holds, while $\theta^*(R_{t^-}) = \lambda > \theta^*(R_{t^+})$ if condition 2 holds.

That ratings’ predictive power increases in both cases follows from the facts that (i) $\mathbb{P}(I = 1|R_{t^+}, t^*) = \mathbb{P}(I = 0|R_{t^-}, t^*)$, (ii) $\mathbb{P}(I = 1|R_{t^+}, t^*) > \mathbb{P}(I = 1|R_{t^-}, t^*)$, and (iii) $\mathbb{P}(I = 1|R_{t^+}, t^*) + \mathbb{P}(I = 0|R_{t^-}, t^*) = 1$. These together imply $\mathbb{P}(I = 1|R_{t^+}, t^*) = \mathbb{P}(I = 0|R_{t^-}, t^*) > 1/2$.

Proof of Proposition 3: Follows the steps of the proof of Proposition 1, substituting (13) with

$$\rho_1 \mathbb{P}(I = 1|R_{t^+}, t) = \rho_0 (1 - \mathbb{P}(I = 1|R_{t^-}, t)).$$

References


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