# Credit Rating and Debt Crises[☆]

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**Abstract**

We develop an equilibrium theory of credit rating in the presence of rollover risk. By influencing rational creditors, ratings affect sovereigns' probability of default, which in turn affects ratings. Our analysis reveals a pro-cyclical impact of credit rating: in equilibrium the presence of a rating agency increases default risk when it is high, and decreases default risk when it is low.

**Keywords:** credit rating agencies, sovereign debt, global games, coordination failure.

**JEL:** *G24, F34, D82, C72*

## 1. Introduction

When short term borrowing is used to finance long term needs, debtors are vulnerable to swings in market sentiment. This vulnerability is inherent in sovereign debt markets, where investors may suddenly refuse to roll over their short term claims on a country. Since the 1980s, such liquidity crises have recurred and created havoc in emerging market economies,

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such as Mexico, Russia and Argentina. More recently, similar crises occurred in Europe, where Greece, Ireland and Portugal were effectively shut out of the bond market in 2010 and 2011, and concerns spread that Spain and Italy could meet the same fate. It is widely believed that in these episodes, pessimistic investor expectations played a propagating role.

When investor expectations are decisive, any event that coordinates expectations might in principle be pivotal. In particular, credit ratings might serve such a role. Indeed, in crisis periods, politicians and observers often blame rating agencies. For example, in a joint letter dated 6 May 2010, Chancellor Angela Merkel and President Nicolas Sarkozy wrote that 'The decision of a rating agency to downgrade the rating of the Greek debt even before the authorities' programme and amount of the support package were known must make us ponder the rating agencies' role in propagating crises.'[2] A natural instinct is of course to discredit such statements as coming from politicians eager to cloud their own responsibility for an ongoing crisis. Yet, the question at hand is important, and should not be dismissed without a rigorous answer. This paper therefore scrutinizes the role of credit rating agencies in the presence of rollover risk.

We analyze the problem faced by a credit rating agency (CRA) about to rate sovereign debt in a situation where coordination failure among investors might cause default. As Manso (2013), we assume that the CRA optimally chooses ratings in order to preserve its track record of correct predictions.[3] The CRA realizes that its rating may affect whether or not

[2] See also Paul Krugman's New York Times article 'Berating the raters', April 26, 2010, or Ferri, Liu and Stiglitz (1999). Another example is Helmut Reisen (2010) Head of Research, OECD Development Centre: 'Unless sovereign ratings can be turned into proper early warning systems, they will continue to add to the instability of international capital flows, to make returns to investors more volatile than they need be, and to reduce the benefits of capital markets for recipient countries.'

[3] A quote of Thomas McGuire, former VP at Moody's, summarizes this motivation: 'What's driving us is primarily the issue of preserving our track record. That's our bread and butter' (Manso, 2013).

the sovereign defaults, and acts strategically by taking this effect into account in deciding what rating to give. Hence, there is potential feedback between rollover risk and ratings. Importantly, investors are rational and realize that the rating agency is strategic. When investors interpret a rating, they therefore take the CRA's incentives and constraints into account. Thus, a central contribution of our analysis is that ratings and investors' response to them are determined in equilibrium. This is important, as to address how CRAs affect coordination failure one must take investors and credit rating simultaneously into account. For instance, a policy forcing CRAs to issue more positive ratings in times of crisis is likely to fail as investors adjust their behavior accordingly.

To model the problem facing the rating agency, we use the framework in Morris and Shin (2006), where a sovereign needs to roll over short-term debt. We extend the model by adding a strategic rating agency, which observes a noisy signal of the fundamental (the sovereign's alternative means of short term financing) and reports a rating which is either good or bad. A mass of creditors decide whether or not to roll over their loan, based on the rating of the CRA as well as their own private signal of the fundamental. Withdrawing the loan involves a partial loss, but allows creditors to receive immediate payment and avoid the risk of a later default due to coordination failure. Investors discount delayed payments according to their immediate need for cash: when liquidity is tight, investors discount future payments more. Before making their decisions, investors update their beliefs based on the rating by the CRA. Investors' decisions and the realized value of the fundamental then determine whether the sovereign defaults. As highlighted above, the CRA takes these effects into account when choosing what rating to assign.

We first examine ratings' equilibrium impact. Tight liquidity makes investors more reluctant to roll their loan over, while it is of no *direct* concern to the CRA, which only wants to predict the correct outcome. Therefore, when liquidity is tight, an individual investor requires a better signal to roll over his loan than what the CRA requires to issue a positive rating. If the CRA in spite of its more lenient threshold issues a bad rating, it will therefore tend to influence investors' beliefs – and the incidence of default – more than a good rating would. Conversely, when investors have low liquidity needs, they are willing to accept a higher risk of default than what the CRA requires to announce a good rating. Consequently, when liquidity is easy, a good rating tends to influence investors' beliefs more than a bad rating would. Note that, in the absence of credit rating, default is already more likely when aggregate liquidity is tight. Hence, our analysis reveals a pro-cyclical impact of credit rating: CRAs increase default risk when it is high, and decrease default risk when it is low.

We then explore the implications of our main result for the equilibrium frequency of good and bad ratings. We show that when liquidity is easy, the frequency of good ratings is high compared to the 'private' assessment of an independent observer with the same information as the CRA. Symmetrically, when liquidity is tight, the frequency of bad ratings is high compared to the 'private' assessment of an independent observer. The basic intuition is straightforward: because the CRA aims to make correct predictions, it will exploit its ability to affect the outcome. Hence, the more strongly a rating affects default risk, the more attractive this rating becomes. Because tight liquidity makes a bad rating more influential than a good rating, it also makes a bad rating the more attractive choice. By the same token, when liquidity is easy, a good rating is attractive, relative to a bad one. Our analysis therefore suggests that in times of booms credit rating of sovereign debt may appear overly generous, whereas during crises credit rating can appear overly harsh. Note that these results do not invoke any exogenous bias in the payoff structure of CRAs. Instead, they highlight that rating standards might endogenously vary over time according to the state of the economy.

Theoretical research on credit rating agencies has burgeoned over the last years. Yet, to our knowledge Manso (2013) is the only previous paper to explore optimal CRA decisions and the feedback effects arising when ratings affect the performance of rated assets. While both papers explore the feedback effects of credit ratings, our paper and Manso's take substantively different approaches. In particular, Manso focuses on the strategic interaction between CRAs and issuers, while our focus is on the strategic interaction between CRAs and creditors, allowing us to derive new results regarding the pro-cyclical nature of credit rating.

Our analysis also relates to studies of how CRAs might coordinate investors. Boot et al. (2006) develop a model with moral hazard in which credit ratings provide a focal point for firms and investors, and help select the most efficient equilibrium. By contrast, Carlson and Hale (2006) show in a global games setting similar to ours how a non-strategic CRA, which simply passes its information on to the market, may induce multiple equilibria by publicly revealing its information. We extend their analysis by considering ratings as strategically chosen by the CRA, and all our interesting implications come from the endogenous choice of a rating.

In global games more generally, precise public information enhances the scope for multiplicity; see for instance Morris and Shin (2003) or Hellwig (2002). This effect naturally arises in our model, too. If the CRA has sufficiently precise information, its rating may coordinate investors completely. Thus, the CRA may essentially decide what the outcome will be. This relates to the literature on whether a large investor may coordinate other investors, making a currency crisis more likely (Corsetti, Dasgupta, Morris and Shin, 2004). Yet our focus is elsewhere. We want to explore a setting where the CRA may influence investors through their beliefs, but without having the superior position that prevails in a global games setting when there is an agent - the CRA in our case - which may provide a public signal with sufficiently high precision. Thus, we parameterize our model to obtain a unique equilibrium. Our main innovation lies in the fact that the rating agency behaves strategically, and takes the equilibrium effects of its rating into account. Relative to the global games literature overall, we thus contribute by studying a *strategic* sender of public information. In this respect, our paper is closer in spirit to Angeletos, Hellwig and Pavan (2006) who study the endogenous information generated by policy interventions. In a broader perspective, our paper relates to studies of self-fulfilling crises at large, such as the bank-run model of Diamond and Dybvig (1983).

A large strand of the credit rating literature focuses on potential sources of rating inflation. One explanation is "rating shopping",- debtors' option to strategically shop among alternative agencies' assessments. Prominent examples here are Skreta and Veldkamp (2009) and Sangiorgi and Spatt (2011). Compared to these studies, our paper highlights that the frequency of good ratings might be elevated even in the absence of issuers' rating shopping, due instead to creditors' changing liquidity needs. Another branch of this literature focuses on how rating inflation may be driven by strategic behavior by CRAs motivated by rating fees. For example, Mathis et al. (2009) explores how CRAs might first build a reputation via truthful reporting, only to milk it down by repeatedly inflating ratings thereafter. Bolton, Freixas and Shapiro (2012) focus on how an agency might exploit naive investors who mechanically follow their advice. Opp, Opp and Harris (2013) study how a CRA might exploit regulatory rules that are based on ratings.

Our paper naturally relates to the extensive literature on expectations-driven sovereign debt crises. Cornerstone contributions there are Calvo (1988) and Cole and Kehoe (2000). The model we base our analysis on has previously been used by Morris and Shin (2006) and Corsetti, Guimaraes and Roubini (2006) to study the role of IMF interventions in curbing sovereign debt crises. Importantly, in their studies the IMF decision to support a sovereign with short term assistance does not serve as a signal to investors, but affects outcomes by directly reducing the sovereign's need for market financing. In contrast, the signalling effect is exactly how a CRA affects outcomes in our study.

While our study focuses on sovereign debt, its basic insights should carry over to other settings where asset performance is sensitive to ratings. Our prediction of procyclical ratings is consistent with Amato and Furfine (2004), who find that new ratings typically have a procyclical effect. Our results are also related to the widespread empirical finding that ratings tend to affect investor behavior asymmetrically, with negative rating events having stronger effects than positive events, or vice versa. The pattern was first highlighted by Holthausen and Leftwich (1986), and multiple studies have later shown that this pattern holds for a broad set of asset classes. Among these studies, some have documented the pattern for sovereign debt, such as Afonso, Furceri and Gomes (2012) and Ismailescu and Kazemi (2010). Our model implies asymmetric rating effects, and ties this phenomenon to the state of the economy. For instance, if liquidity is tight, in our model bad ratings matter more than good ones.[4]

The paper is organized as follows. The model is presented in Section 2. Section 3 solves the model and presents our results. Section 4 concludes. All proofs are contained in the Appendix.

## 2. Model

Our framework builds on Morris and Shin's (2006) model of the rollover problem encountered by a sovereign relying on short-term debt. We lay out that model and enhance it by introducing a *strategic* credit rating agency (CRA) communicating information to creditors about the fundamental status of the sovereign.

**Debt rollover.** There are two main periods, $t = 1, 2$. A unit mass of investors (or creditors), indexed by $i$, are financing a sovereign using a conventional debt contract. At

[4] Given that our model is static, we cannot strictly account for downgrades or upgrades. However, if a CRA has given a rating in the past, then as time elapses this rating looses relevance. By the time a new rating is announced, the situation may therefore to some extent be similar to one where no previous rating existed, as our model assumes.

$t = 1$ each investor faces the option to (a) liquidate his loan to the sovereign for a payment normalized to 1 or (b) rollover his loan to the sovereign. In the latter case the contract specifies a final period payment $V$, unless the sovereign defaults, in which case investors receive 0.

As our main goal is to examine the effect of the strategic CRA in the presence of rollover risk, we follow Morris and Shin (2006) and assume that the sovereign's ability to meet short term claims is the sole source of uncertainty; this is the decisive factor for whether the sovereign defaults or not. Let $l$ denote the mass of investors liquidating at time $t = 1$. The ability to meet short term claims, and thereby avoid default, is summarized by the random variable $\theta$. One may think of $\theta$ as the sovereign's stock of liquid reserves, including all assets it can liquidate in the short run, or access to alternative credit lines other than the debt market. If $\theta \geq l$, then the sovereign meets its short term claims, and investors who in the first period chose to roll over obtain payment $V$ at $t = 2$. If $\theta < l$, the sovereign defaults, and those who chose to roll over get nothing. Note that if $\theta \geq 1$, the sovereign never defaults, even if all creditors liquidate. By contrast, default occurs with certainty if $\theta < 0$. However, if $\theta \in [0, 1)$, the sovereign may or may not default, depending on the behavior of creditors. In that case, a coordination problem prevails: Each investor would gain if all were to roll over their loan to the sovereign, but no investor would gain from being the only one to do so. Let $I$ denote the indicator variable taking value 1 if the sovereign repays, and 0 if it defaults. Then $I = 1$ if and only if $\theta \geq l$.

**Information.** The fundamental, $\theta$, is uniformly distributed over $\Theta = [\underline{\Theta}, \overline{\Theta}]$. While $\theta$ is unobserved, its distribution is commonly known. In addition, investors are privately informed about $\theta$ through the signals $\{x_i\}$ uniformly distributed over $[\theta - \beta, \theta + \beta]$. The CRA for its part is privately informed about $\theta$ through the signal $y$, uniformly distributed over $[\theta - \alpha, \theta + \alpha]$. All signals are conditionally independent and $\underline{\Theta} < -\max\{\alpha, \beta\}$ while $\overline{\Theta} > 1 + \max\{\alpha, \beta\}$. With a slight abuse to save on notation we use $\Theta$ for the support of all random variables above.

**Ratings and strategies.** Before $t = 1$, all creditors observe the rating of the CRA. For tractability, we follow the bulk of the literature and assume that a credit rating, $r$, is a binary variable, where a rating is either 'good' ($r = 1$) or 'bad' ($r = 0$).[5] More specifically, a CRA strategy is a threshold $\tau \in \Theta$ specifying $r = 1$ if $y \geq \tau$ and $r = 0$ if $y < \tau$.[6] The strategy of investor $i$ is a map $\sigma_i : \Theta \times \{0, 1\} \to \{0, 1\}$, specifying whether to liquidate ($\sigma_i = 0$) or roll over ($\sigma_i = 1$) at time $t = 1$ as a function of his private signal $x_i$ and the credit rating $r$.

For any $z \in \Theta$, define the intervals $R_z^+$ and $R_z^-$ by: $R_z^+ := [z, \overline{\Theta}]$, $R_z^- := [\underline{\Theta}, z]$. Notice that if the CRA uses strategy $\tau$, then $r = 0 \Rightarrow \theta \in R_{\tau+\alpha}^-$ and similarly $r = 1 \Rightarrow \theta \in R_{\tau-\alpha}^+$. Hence for convenience, if the CRA uses strategy $\tau$ we will sometimes use the interval $R_{\tau+\alpha}^-$ to represent the rating $r = 0$ and $R_{\tau-\alpha}^+$ to represent $r = 1$. This is useful for the exposition as it identifies broadly speaking a credit rating, which is an announcement in $\{0, 1\}$, with the informational content of that announcement.[7]

Next, let $\mathcal{R}^- := \{R_z^-\}_{z\in\Theta}$, $\mathcal{R}^+ := \{R_z^+\}_{z\in\Theta}$, and $\mathcal{R} := \mathcal{R}^- \cup \mathcal{R}^+$. When we wish to identify

[5]There are two reasons why binary ratings simplify our analysis. First, our analysis starts from the premise that the CRA aims to be correct, which is straightforward to define with binary ratings but has no obvious definition with more ratings. Second, with a strategic CRA the number of ratings, as defined by their information content, would be an equilibrium object and hence an additional variable to solve for in itself. In an early version of our paper, we allowed for any finite number of ratings and found sufficient conditions under which the CRA effectively chooses only two ratings in equilibrium. However, tractability then required strong assumptions on CRA preferences (see Holden, Natvik, and Vigier (2012)).

[6]Thus a CRA strategy can be viewed as a commitment to announce $r = 1$ if and only if $y \geq \tau$. It can be shown show that, if investors interpret credit ratings as resulting from a CRA strategy $\tau$, then the CRA's best response as a function of $y$ exhibits a threshold $\tau'$ such that $r = 1$ if and only if $y \geq \tau'$. In equilibrium, we will require $\tau'$ to be equal to $\tau$.

[7]If the CRA strategy is $\tau$, one should think of the credit rating $r = 0$ as a message conveying $y < \tau$, and of $r = 1$ as a message conveying $y \geq \tau$.

a rating with its informational content, we will refer to $R \in \mathcal{R}$ as a rating. In this sense $\mathcal{R}$ denotes the set of all possible ratings. Finally, for all $R \in \mathcal{R}$, define $\overline{R} := \max\{\theta : \theta \in R\}$ and $\underline{R} := \min\{\theta : \theta \in R\}$.

**Payoffs.** The preferences of an investor are summarized by the function $u(w_1, w_2) = w_1 + \delta w_2$, where $w_t$ indicates payments received in period $t$, and $\delta \in (0, 1)$ is a discount factor. In the context of our paper the discount factor is naturally interpreted in terms of creditors' liquidity needs: $\delta$ is low when investors have strong immediate needs for cash, as for instance in a liquidity squeeze, or when investors face important margin calls on other positions. The quantity $\lambda := (\delta V)^{-1}$ plays a key role in this paper. We will say that liquidity is *tight* when $\lambda$ is high, and that liquidity is *easy* when $\lambda$ is low. Observe in particular that if $\lambda \geq 1$, then all investors always liquidate. We thus focus on $\lambda \in (0, 1)$, as this is the critical region of interest.

As Manso (2013), we assume that the objective of the CRA is to preserve its track record of 'correct predictions'. There are two ways to be correct in this environment: the CRA may give a good rating to a sovereign that later repays, or give a bad rating to a sovereign that later defaults. The CRA obtains a payoff normalized to 1 if $r = 1$ and the sovereign later repays, or if $r = 0$ and the sovereign later defaults.[8] The CRA obtains zero payoff in the two other cases, where its rating turns out to be 'incorrect'. Let $\Pi(r, I)$ denote the payoff of the CRA as a function of the rating $r$ and outcome $I$: $\Pi(r, I) = Ir + (1 - I)(1 - r)$.

**Beliefs.** We say that investor $i$ updates beliefs using **(A1)-(A2)** if his posterior upon observing rating $R \in \mathcal{R}$ satisfies the following: **(A1)** the posterior attaches probability 1 to

[8]In an earlier version of this paper (Holden, Natvik, and Vigier (2012)), we examine asymmetric payoffs where the CRA prefers to successfully predict default rather than success, or vice versa. The main insight from this extension is that any such bias is self-defeating in equilibrium, since investors internalize how bias in CRA preferences affect ratings.

$\theta = \overline{R}$ if $\overline{R} < x_i - \beta$ and probability 1 to $\theta = \underline{R}$ if $x_i + \beta < \underline{R}$, **(A2)** the posterior is uniform on $[x_i - \beta, x_i + \beta] \cap R$ in all other cases. Let $\hat{\mathbb{P}}(.|x_i, R)$ denote the probability measure when $i$ updates beliefs using **(A1)-(A2)**, and $\hat{\mathbb{E}}[.|x_i, R]$ the corresponding expectation.

Assumption **(A1)** pins down investors' off-path beliefs, and guarantees the existence of a unique equilibrium in the investment game analyzed in Section 3.1 following deviations of the CRA.[9]

Assumption **(A2)** is enforced to keep the model analytically tractable.[10] We discuss in Section 3.2 why we expect our insights to carry through without this approximation.

**Timing.** The timing of the game is as follows. CRA and investors first observe their private signals. The CRA then announces its rating and the game moves into period 1, during which investors simultaneously decide whether to roll over their loans to the sovereign, or to liquidate. Investors who liquidate receive payment 1. In period 2, if $\theta < l$, investors who chose to roll over receive nothing; if $\theta \geq l$, investors who chose to roll over receive payment $V$.

**Approximate Equilibrium.** An *approximate rating equilibrium* is a pair $(\tau, \{\sigma_i\})$ consisting of a strategy $\tau$ for the CRA as well as a strategy $\sigma_i$ for each investor $i$, with the following

[9] Since our signals have bounded supports we need to specify an investor's beliefs following a credit rating inconsistent with his own private signal.

[10] With Bayesian updating, the posterior probability density of $\theta$ conditional on, say, $y < \tau$, tapers off at the edge of its domain and is uniform only on $[\underline{\Theta}, \tau - \alpha)$. This makes the Perfect Bayesian Equilibrium extremely complicated to characterize. To see why, start from $\mathbb{P}(\theta|r, x_i) = \mathbb{P}(r|\theta)\,\mathbb{P}(\theta|x_i) / \int_\theta \mathbb{P}(r|\theta)\,\mathbb{P}(\theta|x_i)\,d\theta$. Under, say, a bad rating, the denominator is given by $\int_{x_i-\beta}^{x_i+\beta} \mathbb{P}(r=0|\theta)\,\mathbb{P}(\theta|x_i)\,d\theta = \int_{x^*-\beta}^{\max(\tau-\alpha, x_i-\beta)} \mathbb{P}(r=0|\theta)\,\mathbb{P}(\theta|x_i)\,d\theta + \int_{\max(\tau-\alpha, x_i-\beta)}^{\min(x_i+\beta, \tau+\alpha)} \mathbb{P}(r=0|\theta)\,\mathbb{P}(\theta|x_i)\,d\theta$. The *min* and *max* operators imply that there is no single expression that characterizes posterior probabilities. Moreover, which expression is relevant to characterize an equilibrium will depend on the threshold $\tau$ and the signal of the marginal investor $x_i$, which both are to be determined in equilibrium.

properties:[11]

1. $\forall\, x_i \in \Theta, \forall\, (r, R) \in \{(0, R^-_{\tau+\alpha}), (1, R^+_{\tau-\alpha})\}$:

$$\hat{\mathbb{E}}[u(\sigma_i(x_i, r), \sigma_{-i}, \theta)|x_i, R] \geq \hat{\mathbb{E}}[u(1 - \sigma_i(x_i, r), \sigma_{-i}, \theta)|x_i, R],$$

2. $\forall\, y \geq \tau$: $\quad \mathbb{P}(I = 1|y, r = 1) \geq \mathbb{P}(I = 0|y, r = 0)$,
3. $\forall\, y < \tau$: $\quad \mathbb{P}(I = 0|y, r = 0) \geq \mathbb{P}(I = 1|y, r = 1)$.

The structure of an approximate rating equilibrium is straightforward. The CRA announces $r = 1$ if $y \geq \tau$, and $r = 0$ if $y < \tau$. Accounting for this, each investor, upon observing the rating announced updates his beliefs using **(A1)-(A2)**, and decides optimally whether or not to roll over his loan given the behavior of other investors (part 1 in the definition). The final ingredient is as follows: given the behavior of investors described above, the announcement $r = 1$ must maximize chances of making a correct prediction for $y \geq \tau$ (part 2 in the definition), whereas $r = 0$ must maximize chances of a correct prediction for $y < \tau$ (part 3 in the definition). Note that our equilibrium concept is based on the standard definition of perfect Bayesian equilibrium, but is only an approximation due to **(A2)**.

## 3. Analysis

Equilibrium is solved backwards, by first investigating the impact of a given rating $R \in \mathcal{R}$ on the incidence of default. That problem is standard in the literature (see e.g. Morris and Shin (1998, 2006) or Carlson and Hale (2006)), and therefore summarized briefly in Section 3.1. Our contribution starts in Section 3.2, where we endogenize credit ratings by incorporating strategic behavior of the CRA. That section addresses the question: how do equilibrium credit ratings affect the incidence of default? Section 3.3 examines our model's

[11] Abusing notation slightly, $u(a_i, a_{-i}, \theta)$ denotes the payoff of creditor $i$ when the profile of actions of all creditors is $\{a_k\}$, and the fundamental is $\theta$.

predictions concerning the frequency of good and bad ratings as a function of the parameters of the model. Section 3.4 summarizes the paper's main insight, namely that the equilibrium impact of credit rating is procyclical: credit rating raises the probability of default when it is high, and reduces it when it is low.

*3.1. Preliminaries*

In this section, we focus on the impact of an exogenously given rating $R \in \mathcal{R}$ on the incidence of default. We define for that purpose the *investment game* given rating $R$ as the game played among investors observing $R$ and following rules **(A1)-(A2)** to update their beliefs. The following condition plays an important role:

**(A3)** $1/(2\beta + 1) < \lambda < 1 - 1/(2\beta + 1)$.

**Lemma 1.** *Assume **(A1)-(A2)**. There exists an equilibrium of the investment game with rating $R$, for all $R \in \mathcal{R}$, and equilibrium is unique if **(A3)** holds. In the latter case equilibrium is characterized by thresholds $x^*(R)$ and $\theta^*(R)$ such that $\sigma_i(x_i; R) = 1 \Leftrightarrow x_i \geq x^*(R)$ and $I = 1 \Leftrightarrow \theta \geq \theta^*(R)$.*

As **(A3)** guarantees equilibrium uniqueness in the investment game, in the rest of the analysis we will assume that **(A3)** holds. The $\theta$-threshold $\theta^*(R)$ above which the sovereign repays will thus be well-defined for all $R \in \mathcal{R}$, and will be the key variable of our analysis.

In the absence of credit rating, how does $\lambda$ affect the incidence of default? Extending previous notation, let $\theta^*(\Theta)$ denote the equilibrium $\theta$-threshold of the investment game in the absence of credit rating. As $\lambda$ is the value of liquidating relative to the discounted full repayment value, a creditor prefers rolling over to liquidating if and only if he assigns probability $\lambda$ or more to repayment occurring. As $\lambda$ rises, creditors become more reluctant to roll over their loan, all else equal. Unsurprisingly therefore, in the absence of credit rating, the higher $\lambda$, the higher too the incidence of default. In fact, Lemma 2 below shows that:

$$\theta^*(\Theta) = \lambda. \tag{1}$$

Equation (1) sets a natural benchmark against which to measure the impact of any given rating on the incidence of default. In particular, the rating $R \in \mathcal{R}$ reduces the incidence of default if and only if $\theta^*(R) < \lambda$, and increases the incidence of default if and only if $\theta^*(R) > \lambda$. The next lemma gathers the key properties of $\theta^*(R)$ for the analysis of Section 3.2.

**Lemma 2.** *Assume **(A1)-(A3)**. Then, in the absence of credit rating, (1) holds. With credit rating, the following hold:*

1. $\theta^*(R) \geq \lambda$ *if* $R \in \mathcal{R}^-$ *and* $\theta^*(R) \leq \lambda$ *if* $R \in \mathcal{R}^+$,
2. $\theta^*(R)$ *is non increasing in* $\overline{R}$ *if* $R \in \mathcal{R}^-$,
3. $\theta^*(R)$ *is non increasing in* $\underline{R}$ *if* $R \in \mathcal{R}^+$,
4. $\theta^*(R) > \lambda \Leftrightarrow R \in \mathcal{R}^-$ *and* $\overline{R} < (2\beta + 1)\lambda$,
5. $\theta^*(R) < \lambda \Leftrightarrow R \in \mathcal{R}^+$ *and* $\underline{R} > (2\beta + 1)\lambda - 2\beta$,
6. $\theta^*(R)$ *is continuous in* $\underline{R}$ *if* $R \in \mathcal{R}^+$ *and continuous in* $\overline{R}$ *if* $R \in \mathcal{R}^-$,
7. $\theta^*(R) = 1 \Leftrightarrow \overline{R} \leq 1$,
8. $\theta^*(R) = 0 \Leftrightarrow \underline{R} \geq 0$.

Lemma 2.1 says that a negative rating never reduces the incidence of default, while a positive rating never increases the incidence of default. Next, one may intuitively think of $\overline{R}$ as measuring the 'strength' of the information content in a negative rating $R \in \mathcal{R}^-$ and similarly think of $\underline{R}$ as measuring the 'strength' of a positive rating $R \in \mathcal{R}^+$. Part 2 of the lemma then says that the 'milder' the negative rating is, the smaller will its impact be. Symmetrically, Part 3 states that the 'stronger' the positive rating is, the greater will its impact be. Parts 4 and 5 embody the idea that ratings must have sufficient informational content to affect the incidence of default at all. Part 6 shows that the impact of a rating is continuous in the 'strength' of the rating. Recall that the maximum $\theta$-threshold possible is 1, and that the minimum $\theta$-threshold possible is 0. Hence the maximum impact of a negative rating is to push $\theta^*$ to 1, and the maximum impact of a positive rating is to push $\theta^*$ to 0. Parts 7 and 8 then state the requirements for ratings to have maximum impact, which will be referred to in the discussion of equilibrium multiplicity below.

*3.2. Equilibrium Impact of Ratings*

This section endogenizes credit ratings by incorporating strategic behavior of the CRA. We show that rating equilibria exist, and explore how good and bad ratings affect the incidence of default in equilibrium. Specifically, we show that when default risk is high (i.e. when $\lambda$ is large), then the positive impact of a good rating is greater than the negative impact of a bad rating, and vice versa when default risk is low.

In this section and the next we set a lower bound on the noise parameter $\alpha$ of the private signal $y$ of the CRA:

**(A4)** $\alpha \geq 2\beta \min\{\lambda, 1-\lambda\}$.

This assumption ensures equilibrium uniqueness. At the end of this section we discuss the consequences of relaxing it in some detail. Assumption **(A4)** implies that $\alpha \geq \beta$ if $\lambda = 1/2$, while for $\lambda \neq 1/2$, we may have $\alpha \gtreqless \beta$. Interestingly, even if $\alpha > \beta$, implying that the CRA cannot even evaluate creditworthiness more effectively than any individual investor, the CRA may still provide information that investors find valuable. In part, this is because the signal observed by the CRA is imperfectly correlated with the investor's private information. More interestingly, the ratings have extra value because they are *public* information and thus place bounds on what any individual investor might believe about other investors' beliefs. The latter is a possibly powerful source of influence since the investors' choices are strategic complements.

To start our analysis of endogenous ratings, observe that a strategy $\tau^*$ of the CRA is part of a rating equilibrium if and only if

$$\mathbb{P}(\theta < \theta^*(R^-_{\tau^*+\alpha})|y = \tau^*, r = 0) = \mathbb{P}(\theta \geq \theta^*(R^+_{\tau^*-\alpha})|y = \tau^*, r = 1). \tag{2}$$

This is an indifference condition, stating that a CRA which observes $y = \tau^*$ must perceive it as equally likely that a good rating is followed by repayment as it is that a bad rating is followed by default. To realize why this condition must hold, consider a situation where at $y = \tau^*$ a positive rating is more likely to be followed by repayment than a a bad rating is to be followed by default. Then, by continuity of the probabilities in $y$, the CRA would benefit from giving a good rating also for signals slightly above $\tau^*$, meaning that a higher threshold than $\tau^*$ would actually be optimal.[12] Equation (2) is the basis of this section's analysis.

We noted in Section 3.1 that a creditor is indifferent between rolling over and liquidating his loan if and only if he believes that the sovereign will repay with probability $\lambda$. In what follows we will say that creditor $i$ is a *marginal investor* if his private signal $x_i = x^*(R)$ is such that conditional on the rating $R$, he attaches exactly probability $\lambda$ to repayment occurring (see Lemma 1). Let also $x^*(\Theta)$ denote the equilibrium threshold private signal in the absence of credit rating, and denote by M the marginal investor in that benchmark. The private signal of M is thus $x_M := x^*(\Theta)$. With the uniform distribution, equation (2) then implies

$$\mathbb{P}(\theta \geq \theta^*(\Theta)|x_i = x_M) = \frac{(x_M + \beta) - \theta^*(\Theta)}{2\beta} = \lambda.$$

As $\theta^*(\Theta) = \lambda$ (see Lemma 2), this equation shows that without credit rating: (a) $\lambda > 1/2 \Rightarrow x_M > \lambda$, whereas (b) $\lambda < 1/2 \Rightarrow x_M < \lambda$. These remarks will be key in the next two paragraphs.

Suppose first that $\lambda = 1/2$. By Lemma 2 it then follows that $\theta^*(\Theta) = 1/2$, and by the remarks above $x_M = 1/2$. Consider now how a CRA using strategy $\tau = 1/2$ affects M's beliefs. As here $x_M = \tau$, the ratings $r = 0$ and $r = 1$ have symmetric effects on M's beliefs. Intuitively, their respective impact on the incidence of default must be symmetric too, i.e. $\theta^*(r = 0) - 1/2 = 1/2 - \theta^*(r = 1)$. Thus, in particular, (2) holds for $\tau^* = 1/2$. It follows that if $\lambda = 1/2$, then $\tau = 1/2$ is part of a rating equilibrium, and therefore that in equilibrium a good and a bad rating have symmetric impact on the incidence of default.

[12] A formal proof is in the appendix.

By contrast, suppose now that $\lambda > 1/2$. Then remark (a) above implies $x_M > \lambda$. In this scenario, consider how a CRA using strategy $\tau = \lambda$ affects M's beliefs. M now receives a signal that is greater than the CRA's threshold, that is, $x_M > \tau$. It follows that a positive rating will largely be consistent with M's private signal, and thus have little or no impact on M's beliefs. In contrast, a negative rating will be in conflict with M's relatively positive private signal, leading to a much greater revision of M's beliefs. Intuitively, the impact of $r = 0$ on the incidence of default will be larger too, i.e. $\theta^*(r = 0) - \lambda \geq \lambda - \theta^*(r = 1)$.

Naturally, the CRA strategy of $\tau = \lambda$ assumed above need not –and typically will not– be part of a rating equilibrium. Nevertheless, Proposition 1 below shows that the remarks made here about the effects of credit rating do actually hold in a rating equilibrium: if $\lambda > 1/2$, a bad rating increases default risk more than a good rating reduces it. Likewise, if $\lambda < 1/2$, a good rating reduces default risk more than a bad rating increases it.

**Proposition 1.** *Assume **(A1)-(A4)**. Then there exists a unique rating equilibrium. In equilibrium* $\theta^*(R = 1) \leq \lambda \leq \theta^*(R = 0)$ *and:*

1. *for* $\lambda \in [1 - \alpha/(2\beta), \alpha/(2\beta)]$*, ratings have no impact:* $\theta^*(R = 0) = \lambda = \theta^*(R = 1)$*;*
2. *for* $\lambda < \min\{1/2, 1 - \alpha/(2\beta)\}$*, the impact of a good rating is greater than that of a bad rating:* $\lambda - \theta^*(R = 1) > \theta^*(R = 0) - \lambda$*;*
3. *for* $\lambda > \max\{1/2, \alpha/(2\beta)\}$*, the impact of a bad rating is greater than that of a good rating:* $\theta^*(R = 0) - \lambda > \lambda - \theta^*(R = 1)$*.*

The basic intuition behind Proposition 1 is as follows. When $\lambda$ is high, the creditors rolling over their loans all have high private signals of the fundamental $\theta$ (to compensate for their immediate need for cash). Consequently, the effect of a bad rating on those creditors' beliefs –and therefore on the incidence of default– tends to be greater than the corresponding effect of a good rating. By the same token, when $\lambda$ is low, the creditors liquidating their loans all have low private signals of the fundamental $\theta$, and the effect of a good rating on those creditors' beliefs tends to be greater than the corresponding effect of a bad rating.

We next discuss the effect of the approximation made in **(A2)** concerning investors' updating of beliefs. This approximation vastly simplifies the technical analysis by ensuring that if the information provided by the rating is fully consistent with the investor's own signal, the rating has no effect on the beliefs of the investor. Thus, a rating only affects the beliefs of the investor when it restricts the possible interval for $\theta$. With Bayesian updating, i.e. without this approximation, any rating will in general affect the beliefs of the investor. Yet, it will still be the case that an investor's beliefs are more affected by ratings whose information content overlap less with the investor's prior beliefs. Thus, when $\lambda > 1/2$, which implies that $x_M > \lambda$, a negative rating based on a threshold $\tau = \lambda$ will have a larger impact on the marginal investor's beliefs than a positive rating will have. We thus expect the insights of this paper to remain valid also without the approximation implied by **(A2)**.

We conclude this section with a discussion of assumption **(A4)**. As noted above, the lower bound on $\alpha$ in **(A4)** guarantees a unique rating equilibrium. As $\alpha$ becomes small, the CRA's rating alone may determine whether or not the sovereign defaults. This is best illustrated by the extreme case where $\alpha = 0$. Suppose that the CRA with $\alpha = 0$ uses an arbitrary strategy $\tau \in (0, 1)$. Then, if the CRA issues a negative rating, investors will know for sure that $\theta < 1$, as otherwise default would be impossible, and the rating wrong with certainty. Likewise, if the CRA issues a positive rating, investors will know for sure that $\theta > 0$, as otherwise default would be certain. By Lemma 2.7 and 2.8, it then follows that $\theta^*(r = 1) = 0$ and $\theta^*(r = 0) = 1$, which implies that for all $\theta \in (0, 1)$ the rating determines the outcome. As $\tau \in (0, 1)$ and $\alpha = 0$, this implies that at $y = \tau$ the CRA is indifferent between announcing $r = 0$ and $r = 1$: In both cases the CRA is able to make a correct prediction with probability 1. Hence, any $\tau \in (0, 1)$ is in fact part of a rating equilibrium. Multiple rating equilibria thus exist. In this case, Proposition 1 may no longer hold for all possible equilibria. For example, if $\lambda = 0.9$, then the equilibrium impact of a good rating is $\lambda - \theta^*(r = 1) = 0.9$, whereas the equilibrium impact of a bad rating is $\theta^*(r = 0) - \lambda = 0.1$. However, in the appendix, Proposition 4, we show that the results of Proposition 1 will hold even if **(A4)** is not satisfied, as long as the $\theta^*$-threshold is within the interval $(0, 1)$.

Notably, the discussion above implies that the mere presence of a CRA with accurate information may cause equilibrium multiplicity. This insight is central in Carlson and Hale (2006), who study a CRA that non-strategically passes its private information on to the market. Our analysis shows that this insight will also hold when the CRA acts strategically. Notably, though, there is a subtle conceptual difference. When the CRA passes its information directly on, accuracy induces multiplicity in the game played between investors. In our setting with strategically chosen binary ratings, multiplicity arises as the CRA is indifferent between selecting alternative equilibria in the investment game. However, within the investment game, the equilibrium is unique.[13]

*3.3. Equilibrium Frequency of Good and Bad Ratings*

CRAs have been accused of delivering excessively many good ratings.[14] The theoretical literature has typically focused on the role of 'rating shopping' in explaining that phenomenom. 'Rating shopping' describes the ability of issuers to pay for and disclose ratings *after* they privately observe them. However, that explanation is less relevant for sovereign debt, as credit rating is then typically unsolicited. This Section examines the respective frequency of good and bad ratings arising in the equilibrium of our model and shows that, when $\lambda$ is low, then the frequency of a good rating is high compared to the 'private' assessment of an independent observer based on the same information as the ratings of the CRA. By symmetry, when $\lambda$ is

[13] Related here is also the study by Angeletos and Werning (2006), who show how an endogenous source of public information, an asset price in their model, enlarges the scope for equilibrium multiplicity to arise. Although ratings constitute an endogenous signal in our model, that phenomenon does not arise because ratings never are sufficiently precise to induce multiplicity in the investment game.

[14] See, e.g., Skreta and Veldkamp (2009) or Sangiorgi and Spatt (2011).

high, we show that the frequency of a bad rating is high compared to the 'private' assessment of an independent observer. Our model thus suggests that in times of booms, credit rating of sovereign debt may appear overly generous, whereas during crises, credit rating of sovereign debt can appear overly harsh.

Recall that in the absence of credit rating, the critical value for the fundamental $\theta^*(\Theta) = \lambda$. Thus, as the signal $y$ is distributed symmetrically around $\theta$, an independent observer aiming to maximize chances of making a correct prediction would (a) forecast repayment for $y \geq \lambda$, and (b) forecast default for $y < \lambda$. This implies that if in a rating equilibrium the CRA strategy is $\tau < \lambda$, then in that equilibrium the frequency of good ratings is higher than the frequency with which an independent observer would forecast repayment. Similarly, if in a rating equilibrium the CRA strategy is $\tau > \lambda$, then in that equilibrium bad ratings are more frequent than the default forecasts of an independent observer would be. These remarks motivate the following definition:

**Definition 1.** *A CRA strategy $\tau$ exhibits an excess of good ratings if $\tau < \lambda$, and exhibits an excess of bad ratings if $\tau > \lambda$.*

We next show that rating equilibria exhibit an excess of good ratings when $\lambda$ is small, and an excess of bad ratings when $\lambda$ is large. The intuition for that result is as follows. Suppose for the sake of illustration that $\lambda > \max\{1/2, \alpha/(2\beta)\}$. We argued in Section 3.2 that in this case a CRA using strategy $\tau = \lambda$ would have a greater impact with a bad rating than with a good rating. But note that then, if it observed the signal $y = \tau$, the CRA would prefer announcing $r = 0$ to announcing $r = 1$ because the bad rating would be more likely to come true. It follows that in this case equilibrium must entail $\tau > \lambda$, that is, an excess of bad ratings.

**Proposition 2.** *Assume **(A1)-(A4)**. Then:*

1. *for $\lambda \in [1 - \alpha/(2\beta), \alpha/(2\beta)]$, the rating equilibrium has $\tau = \lambda$.*
2. *for $\lambda < \min\{1/2, 1 - \alpha/(2\beta)\}$, the rating equilibrium exhibits an excess of good ratings,*

3. *for $\lambda > \max\{1/2, \alpha/(2\beta)\}$, the rating equilibrium exhibits an excess of bad ratings.*

The basic intuiton behind Proposition 2 is straightforward: when CRAs are motivated by the desire to make correct predictions, then a rating affecting the default outcome becomes attractive. Since with high $\lambda$ a bad rating has more effect on the outcome than a good one, this implies that bad ratings are attractive when $\lambda$ is high; by the same token, good ratings are attractive when $\lambda$ is low.

### *3.4. Main Result*

This section contains our main theorem, showing that the equilibrium impact of credit rating is procyclical.

Let $p_0$ denote the probability of default in the absence of credit rating. Then, by (1), $p_0 = \mathbb{P}(\theta < \theta^*(\Theta)) = \mathbb{P}(\theta < \lambda)$ and clearly $p_0$ is increasing in $\lambda$. So, even without credit rating, default risk is high when $\lambda$ is large. We now show that the equilibrium impact of credit rating is procyclical: credit rating raises the probability of default when it is high (i.e. when $\lambda$ is high), and reduces it when it is low (i.e. when $\lambda$ is low).

**Theorem 1.** *Assume **(A1)-(A4)**. For $\lambda \in [1 - \alpha/(2\beta), \alpha/(2\beta)]$, the probability of default is $p_0$, as it is without credit rating. Otherwise, the impact of credit rating is procyclical: in the unique rating equilibrium the probability of default is smaller than $p_0$ for $\lambda < \min\{1/2, 1 - \alpha/(2\beta)\}$, and greater than $p_0$ for $\lambda > \max\{1/2, \alpha/(2\beta)\}$.*

**Proof:** Let $L := \overline{\Theta} - \underline{\Theta}$. Then $p_0 = \mathbb{P}(\theta < \lambda) = (\lambda - \underline{\Theta})/L$. Consider next the probability of default $\mathbb{P}(I = 0)$ calculated in the rating equilibrium. Let $\tau^*$ denote the equilibrium strategy of the CRA. Then:

$$
\begin{aligned}
\mathbb{P}(I=0) &= \mathbb{P}\big(\theta < \theta^*(r=1)\big) \\
&\quad + \mathbb{P}\big(\theta^*(r=1) \le \theta < \theta^*(r=0)\big)\mathbb{P}\big(y < \tau^* | \theta^*(r=1) \le \theta < \theta^*(r=0)\big) \\
&= \frac{\theta^*(r=1) - \underline{\Theta}}{L} + \frac{1}{2}\left(\frac{\theta^*(r=0) - \theta^*(r=1)}{L}\right) \\
&= \frac{\frac{\theta^*(r=0)+\theta^*(r=1)}{2} - \underline{\Theta}}{L} \\
&= \frac{\tau^* - \underline{\Theta}}{L},
\end{aligned}
$$

where the second and last equality follow from noting that $\tau^* = (\theta^*(R=0) + \theta^*(R=1))/2$ (we show this formally in the appendix). From Proposition 2, $\tau^* > \lambda$ if $\lambda > \max\{1/2, \alpha/(2\beta)\}$, implying that $\mathbb{P}(I=0) > p_0$. Likewise, if $\lambda < \min\{1/2, 1-\alpha/(2\beta)\}$, then $\tau^* < \lambda$, implying that $\mathbb{P}(I=0) < p_0$. ■

## 4. Conclusion

Credit rating agencies have recently been criticized on different and often opposing grounds, in particular for being too lenient before the 2007-08 financial crisis, and for propagating refinancing problems thereafter. Our analysis shows that in markets where coordination failure is possible, specifically the sovereign debt market, there is a channel that generates such effects. We find that when aggregate liquidity is easy, sovereign ratings will be inflated and on average decrease coordination risk. Hence, credit ratings will tend to ease sovereign refinancing when the market initially is in a good state. By contrast, when liquidity is tight, ratings are deflated and make it even harder to roll sovereign debt over. While existing studies focus on how an issuer bias in CRA payoffs generates rating inflation, our study shows how an excessive frequency of positive ratings might arise due to booming market conditions. This is a novel prediction which may be tested empirically. To the best of our knowledge, our paper is the first to simultaneously: (i) account for strategic behavior on the part of CRAs, (ii) allow for the possibility that ratings affect the performance of the rated objects, and (iii) endogenize investors' response to credit ratings.

## 5. Appendix

**Proof of Lemma 1:** To show existence, we consider 3 cases in turn:

*Case 1:* $\underline{R} \geq 0$. By **(A1)** each investor's posterior beliefs attach probability 1 to $\theta \geq 0$. Hence, an equilibrium of the investment game exists in which $\sigma_i(x_i, R) = 1$, for all $i$ and $x_i$.

*Case 2:* $\overline{R} < 1$. By **(A1)** each investor's posterior beliefs attach probability 1 to $\theta < 1$. Hence, an equilibrium of the investment game exists in which $\sigma_i(x_i, R) = 0$, for all $i$ and $x_i$.

*Case 3:* $\underline{R} < 0$ *and* $\overline{R} \geq 1$. Suppose $\underline{R} = \underline{\Theta}$. We will show that an equilibrium exists such that $\sigma_i(x_i; R) = 1 \Leftrightarrow x_i \geq x^*(R)$. Clearly, if it exists then that equilibrium has $\theta^*(R)$ such that $I = 1 \Leftrightarrow \theta \geq \theta^*(R)$. The indifference equation of the marginal investor rolling over his loan is $\hat{\mathbb{P}}(\theta > \theta^*(R)|x^*(R), R) = \lambda$, and $\theta^*(R)$ satisfies $\theta^*(R) = \mathbb{P}(x < x^*(R)|\theta = \theta^*(R))$. Let

$$H_1 := \{(x^*, \theta^*) : \theta^* = \mathbb{P}(x < x^*|\theta = \theta^*)\},$$

$$H_2 := \{(x^*, \theta^*) : \hat{\mathbb{P}}(\theta \geq \theta^*|x^*, R) = \lambda\}.$$

$H_1$ and $H_2$ constitute continuous curves in the $(x^*, \theta^*)$-space. $H_1$ is flat in the half spaces $x^* < -\beta$ and $x^* > 1 + \beta$, and coincides in between with the graph of $\theta^* = x^* + \beta / (2\beta + 1)$. $H_2$ coincides with the graph of $\theta^* = x^* + [2\beta(1-\lambda) - \beta]$ for $x^* < \overline{R} - \beta$, and with the graph of $\theta^* = \lambda x^* + [\overline{R}(1 - \lambda) - \lambda\beta]$ for $x^* \in [\overline{R} - \beta, \overline{R} + \beta]$. $H_1$ and $H_2$ thus cross at least once. They cross exactly once if $1/(2\beta + 1) < \lambda$.

The case $\overline{R} = \overline{\Theta}$ is similar. $H_1$ is unchanged. $H_2$ now coincides with the graph of $\theta^* = x^* + [2\beta(1-\lambda) - \beta]$ for $x^* > \underline{R} + \beta$, and with the graph of $\theta^* = (1-\lambda)x^* + [(\beta - \underline{R})(1-\lambda) + \underline{R}]$

for $x^* \in [\underline{R}-\beta, \underline{R}+\beta]$. $H_1$ and $H_2$ thus cross at least once. They cross exactly once if $1/(2\beta+1) < 1-\lambda$.

Uniqueness follows from standard arguments in global games (see e.g. Morris and Shin (2006)) together with the remarks that (a) $H_1$ and $H_2$ never intersect in Cases 1 and 2, (b) $H_1$ and $H_2$ cross exactly once in Case 3 if $1/(2\beta+1) < \lambda < 1-1/(2\beta+1)$, and (c) in Case 3, investors with signals below $-\beta$ must liquidate whereas investors with signals $1+\beta$ or above must roll over. Hence, 'corner' equilbria are precluded in Case 3, just as 'interior' equilibria are precluded in Cases 1 and 2.

For the last part of the lemma, set (a) $\theta^*(R) = 0$ and $x^*(R) = \underline{\Theta}$ in Case 1 and (b) $\theta^*(R) = 1$ and $x^*(R) = \overline{\Theta}$ in Case 2. ■

**Proof of Lemma 2:** Following the arguments of the proof of Lemma 1, the unique equilibrium without credit rating is obtained by solving

$$\begin{cases} \theta^* = \frac{x^*+\beta}{2\beta+1} \\ \theta^* = x^* + [2\beta(1-\lambda)-\beta]. \end{cases}$$

This yields

$$\begin{cases} \theta^* = \lambda \\ x^* = \lambda(2\beta+1)-\beta. \end{cases}$$

Properties 1-8 are now immediate from inspection of $H_1$ and $H_2$ defined in the proof of Lemma 1. ■

**Proof of Propositions 1 and 2** Fix $\tau$, and consider the CRA with signal $y$. By Lemma 1 the probability $\mathbb{P}(I=0|y,R=0)$ of a correct prediction following rating $R=0$ is $\theta^*(R^-_{\tau+\alpha}) - (y-\alpha)/(2\alpha)$ if $y \in [\theta^*(R^-_{\tau+\alpha})-\alpha, \theta^*(R^-_{\tau+\alpha})+\alpha]$, 0 above $\theta^*(R^-_{\tau+\alpha})+\alpha$, and 1 below $\theta^*(R^-_{\tau+\alpha})-\alpha$. Similarly, the probability $\mathbb{P}(I=1|y,R=1)$ of a correct prediction following rating $R=1$ is $(y+\alpha)-\theta^*(R^+_{\tau-\alpha})/(2\alpha)$ if $y \in [\theta^*(R^+_{\tau-\alpha})-\alpha, \theta^*(R^+_{\tau-\alpha})+\alpha]$, 1 above $\theta^*(R^+_{\tau-\alpha})+\alpha$, and 0 below $\theta^*(R^+_{\tau-\alpha})-\alpha$. In particular (a) $\mathbb{P}(I=0|y,R=0)$ and

$\mathbb{P}(I = 1|y, R = 1)$ are continuous in $y$, (b) $\mathbb{P}(I = 0|y, R = 0)$ is non-increasing in $y$, and (c) $\mathbb{P}(I = 1|y, R = 1)$ is non-decreasing in $y$. Hence, the indifference condition

$$\mathbb{P}(I = 0|\tau^*, R^-_{\tau^*+\alpha}) = \mathbb{P}(I = 1|\tau^*, R^+_{\tau^*-\alpha}) \tag{3}$$

is necessary and sufficient for a rating equilibrium.

By Lemma 2.6, $\theta^*(R^-_{\tau^*+\alpha})$ and $\theta^*(R^+_{\tau^*-\alpha})$ are continuous in $\tau^*$. Hence both sides of (3) are continuous in $\tau^*$. By Lemma 2.7, the left hand side of (3) is 1 for $\tau^* + \alpha < 1$ while the right hand side is 1 for $\tau^* - \alpha > 0$ . Note too that the left hand side of (3) is 0 for $\tau^* - \alpha > 1$ while the right hand side is 0 for $\tau^* + \alpha < 0$ . These observations show that a rating equilibrium exists.

To show uniqueness, note first that if one of the two sides in (3) is 0 then the other side of the equation must be 1. For example, if the right hand side is 0 then $\tau^* + \alpha \leq \theta^*(R^+_{\tau^*-\alpha}) \leq \lambda$, where the last inequality follows from Lemma 2.1. But then as (using Lemma 2.1 again) $\theta^*(R^-_{\tau^*+\alpha}) \geq \lambda$, we find that the left hand side of (3) is 1. Next, Lemma 2.2 shows that wherever the left hand side is in $(0, 1)$, it is strictly decreasing in $\tau^*$, and 2.3 shows that wherever the right hand side is in $(0, 1)$, it is strictly increasing in $\tau^*$. Hence the two sides in (3) cross at most once provided $\alpha > 1/2$. We now show that the latter condition holds. We consider two cases. *Case 1:* $\lambda \geq 1/2$. Then $\min\{\lambda, 1 - \lambda\} = 1 - \lambda$, and by assumption $\alpha \geq 2\beta(1-\lambda)$, so it is enough to show $2\beta(1-\lambda) > 1/2$. However by **(A3)**: $1-1/(2\beta + 1) > \lambda$. This yields $2\beta(1 - \lambda) > \lambda$ and ultimately $2\beta(1 - \lambda) > 1/2$. *Case 2:* $\lambda < 1/2$. Then $\min\{\lambda, 1 - \lambda\} = \lambda$, and by assumption $\alpha \geq 2\beta\lambda$, so it is enough to show $2\beta\lambda > 1/2$. However by **(A3)**: $\lambda > 1/(2\beta + 1)$. This yields $2\beta\lambda > 1 - \lambda$ and ultimately $2\beta\lambda > 1/2$.

Finally, we prove properties 1-3 of Proposition 1. We consider two cases separately.

*Case 1:* $\alpha \geq \beta$. By inspection of $H_1$ and $H_2$ defined in the proof of Lemma 1, $\alpha \geq \beta$ implies that for any $\tau$ at most one rating impacts default outcome: either $\theta^*(R^+_{\tau-\alpha}) = \lambda$ or

$\theta^*(R^-_{\tau+\alpha}) = \lambda$. So we only need to check the conditions for which $\theta^*(R^+_{\lambda-\alpha}) < \lambda$ and $\theta^*(R^-_{\lambda+\alpha}) > \lambda$, respectively. By Lemma 2.5, $\theta^*(R^+_{\lambda-\alpha}) < \lambda \Leftrightarrow \lambda - \alpha > (2\beta+1)\lambda - 2\beta$, i.e. if and only if $\lambda < 1 - \alpha/(2\beta)$. When the condition holds then at $\tau^* = \lambda$ the right hand side of (3) is strictly larger than the left hand side of (3). Hence, using arguments from the first part of the proof, in a rating equilbrium: $\tau^* < \lambda$. Since in a rating equilibrium we also have $\tau^* = \theta^*(R = 0) + \theta^*(R = 1)/2$ (formally, this is a consequence of the fact shown above that $\alpha > 1/2$), we obtain $\lambda - \theta^*(R = 1) > \theta^*(R = 0) - \lambda$. Similarly, one shows that $\theta^*(R^-_{\lambda+\alpha}) > \lambda \Leftrightarrow \lambda > \alpha/(2\beta)$.

*Case 2:* $\alpha < \beta$. Assume first $\lambda > 1/2$. We claim that $\theta^*(R^-_{\lambda+\alpha}) > \lambda$, whereas $\theta^*(R^+_{\lambda-\alpha}) = \lambda$. For the first part of the claim, by Lemma 2.4 we need to show that $\lambda + \alpha < (2\beta + 1)\lambda$, i.e. $\alpha < 2\beta\lambda$. But by assumption we have $\lambda > 1/2$ and $\alpha < \beta$, so we are done with the first part of the claim. For the second part, inspection of $H_1$ and $H_2$ defined in the proof of Lemma 1 shows that for the claim to hold it is enough that $\lambda-\alpha+\beta \leq \lambda(2\beta+1)-\beta$ (the left hand side is the horizontal coordinate of the kink in $H_2$, and the right hand side is the $x$-threshold of the equilibrium without credit rating uncovered in the proof of Lemma 2). This is the same as $\alpha \geq 2\beta(1-\lambda)$, which holds by assumption since $\alpha \geq 2\beta \min\{\lambda, 1-\lambda\}$ and $\lambda > 1/2$. Now one implication of the claim is that at $\tau^* = \lambda$ the left hand side of (3) is strictly larger than the right hand side of (3). Hence, using arguments from the first part of the proof, in a rating equilbrium: $\tau^* > \lambda$. Since in a rating equilibrium we also have $\tau^* = \theta^*(R = 0) + \theta^*(R = 1)/2$, we obtain $\theta^*(R = 0) - \lambda > \lambda - \theta^*(R = 1)$. The case where $\lambda < 1/2$ is symmetric. The case where $\lambda = 1/2$ does not apply here since $\lambda = 1/2 \Rightarrow 2\beta \min\{\lambda, 1-\lambda\} = \beta$, and by assumption $\alpha \geq 2\beta \min\{\lambda, 1-\lambda\}$.

■

**Proposition 3.** *Assume **(A1)-(A3)**. There exists at least one rating equilibrium. In any equilibrium $\theta^*(R = 1) \leq \lambda \leq \theta^*(R = 0)$ and one (or more) of the following holds:*

1. $\theta^*(r=0)=1$,
2. $\theta^*(r=1)=0$,
3. (a) *if* $\lambda = 1/2$*:* $\theta^*(r=0)-\lambda = \lambda - \theta^*(r=1)$,
   (b) *if* $\lambda > 1/2$*:* $\theta^*(r=0)-\lambda > \lambda - \theta^*(r=1) > 0$,
   (c) *if* $\lambda < 1/2$*:* $\lambda - \theta^*(r=1) > \theta^*(r=0)-\lambda > 0$.

**Proof of Proposition 3:** For existence, the proof of Proposition 1 applies. We next show the second part of the propositon.

Assume first $\lambda > 1/2$. Inspection of $H_1$ and $H_2$ defined in the proof of Lemma 1 shows that either (i) $\theta^*(R^-_{\lambda+\alpha}) - \lambda > \lambda - \theta^*(R^+_{\lambda-\alpha})$ or (ii) $\theta^*(R^-_{\lambda+\alpha}) = 1$. In Case (i), at $\tau^* = \lambda$ the left hand side of (3) is strictly larger than the right hand side of (3). Hence, in a rating equilbrium: $\tau^* > \lambda$. Since in a rating equilibrium we also have $\tau^* = \theta^*(R=0) + \theta^*(R=1)/2$, Case 3.b of the proposition holds. In Case (ii), at $\tau^* = \lambda$ either the left hand side of (3) is strictly larger than the right hand side of (3), and then the previous argument applies, or the left hand side is weakly smaller than the right hand side, in which case $\tau^* \leq \lambda$ in a rating equilibrium and, using Lemma 2.2, Case 1 of the proposition holds

The case where $\lambda < 1/2$ is symmetric. The case $\lambda = 1/2$ is trivial. ■

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