Online Appendix H for

Discrimination and Employment Protection

Steinar Holden[1] and Åsa Rosén[2]

January 6, 2014

[1]Department of Economics, University of Oslo, Steinar.Holden@econ.uio.no.
[2]Stockholm University (SOFI) and University of Oslo (ESOP). Asa.Rosen@sofi.su.se.

H Online appendix

H.1 Taste for discrimination and neutral hiring quotas

In this section we explore the case where a proportion $m \in (0,1)$ of the vacancies for exogenous reasons always apply the discriminatory hiring rule: $\gamma_G^S = \underline{\gamma}$ and $\gamma_R^S = \overline{\gamma}$.[1] Thus, these firms are willing to hire Red workers if they observe a good signal, but not with a bad. Such behavior could arise from these employers receiving a certain disutility from hiring Red workers, as in Becker's model. However, as we want to focus on how the existence of the discriminatory firms affects the behaviour of the profit maximising firms, we do not include any such disutility explicitly in the model. This simplification implies that wages, given productivity, is the same in the two types of firms, implying that there is no reason for workers in a high productive match to search. For the profit maximising firms we still assume that the value of a vacancy is zero in equilibrium.

If the profit maximizing firms apply the neutral hiring rule $\gamma_G^S = \gamma_R^S = \underline{\gamma}$, the expected profits from hiring a Red worker with a bad signal $\underline{\gamma}$ is (using that $V = 0$ in equilibrium)

$$J(\underline{\gamma}, p_R) = \frac{\underline{\gamma}}{r+s}(1-\beta)y^H + \frac{1-\underline{\gamma}}{r+s+\phi p_R}(y^L - w^L), \tag{H.1}$$

where $p_R = 1 - m + m\eta$. We can show the following result (proof in Section H.4).

Proposition H.1 *Assume that a proportion $m \in (0,1)$ of the vacancies exogenously apply the discriminatory hiring rule $\gamma_G^S = \underline{\gamma}$, $\gamma_R^S = \overline{\gamma}$. Then there exists a critical value $\tilde{m} \in (0,1)$, given by $J_R(\underline{\gamma}, p_R) = 0$ in (H.1) and $p_R = 1 - m + m\eta$, such that for $m > \tilde{m}$*

[1] While it is simpler to work with proportions of discriminatory vacancies rather than numbers, one can show that the results also go through when assuming an exogenous number (measure) of discriminatory vacancies. (Proof available in Section H.9.)

an equilibrium where profit-maximizing firms apply the neutral hiring rule $\gamma_G^S = \gamma_R^S = \underline{\gamma}$ *does not exist, while the discriminatory equilibrium where they apply* $\gamma_G^S = \underline{\gamma}$, $\gamma_R^S = \overline{\gamma}$ *exists.*

Thus, we observe that the existence of firms with taste for discrimination has the opposite effect of what it has in Becker's model: neutral (non-discriminatory) behavior becomes less profitable. Furthermore, while discriminatory hiring is less profitable if there are only a few discriminatory firms, the opposite is true if the share of discriminatory firms is above the critical value $\tilde{m}$. In this case, it is neutral hiring that is less profitable, implying that it is the non-discriminatory hiring that is driven out of the market. In other words, in this case even the profit maximising firms without any taste for discrimination will employ the same discriminatory hiring rule. As both types of firms apply the same hiring rule, they also make the same profits.[2]

Next we consider how the outcome is affected by anti-discriminatory rules. We capture a hiring quota by assuming that a proportion a of vacancies always apply the neutral hiring rule: $\gamma_G^S = \gamma_R^S = \underline{\gamma}$. With this specification, the hiring quota case is the mirror image of the case with taste for discrimination above. Thus the next Corollary follows immediately.

Corollary H.1 *Assume that a proportion* $a \in (0, 1)$ *of the vacancies exogenously apply the neutral hiring rule* $\gamma_G^S = \gamma_R^S = \underline{\gamma}$. *If* $a > \tilde{a} = 1 - \tilde{m}$, *an equilibrium where profit-maximizing firms apply the discriminatory hiring rule* $\gamma_G^S = \underline{\gamma}$, $\gamma_R^S = \overline{\gamma}$ *does not exist, while the neutral equilibrium where they apply the neutral hiring rule* $\gamma_G^S = \gamma_R^S = \underline{\gamma}$ *does exist.*

[2] In Section H.2 below, we show that this result may also hold when wages depend on the outside option, so that discriminated workers obtain lower wages.

Thus, we see that in our model, affirmative action in the form of minimum hiring quotas works to counteract discrimination. If a sufficiently large share of the employers use a hiring quota, the discriminatory equilibrium vanishes.

H.2 Outside opportunities affect bargaining

In this section consider an alternative wage setting mechanism, where the wage is also affected by the players' outside options. The upshot is that the weaker outside alternatives of the Red workers lead them to have a lower wage. Specifically, we assume that the wage in a high productive match, when $y = y^H$, maximizes the Nash product of each player's asset values, i.e.

$$(W_i^H - U_i)^\beta (J^H - V)^{1-\beta}, \tag{H.2}$$

where the outside options U_i and V are still given by (3) and (10). In line with the non-cooperative interpretation of Binmore et al (1986), (H.2) is the appropriate specification if there is a certain probability that the wage bargaining breaks down, leading the parties to separate so that both receive their outside options. As the worker in this case is not laid off by the firm, no firing costs has to be paid, implying that the firing costs do not enter the wage setting.[3] The solution to (H.2) is

$$w_i^H = \beta y^H + (1-\beta)(r+s)U_i - \beta rV, \tag{H.3}$$

Except for the novel wage equation (H.3) which replaces (11), the model is as before, with equilibrium as explained in Section 3. We shall analyse the possible existence of the discriminatory equilibrium derived above, where all Greens are hired, while Reds are only hired if $\gamma = \overline{\gamma}$. Proof of existence of equilibrium for a given hiring strategy is analogous to the proof of Lemma 1, see Section H.11.

Conditional on the signal γ, the expected profits from hiring a Green worker in a discriminatory equilibrium is (note that $V = 0$ in equilibrium, that we have substituted out for the wage equation (H.3), and finally that we now need a subscript indicating worker type as the outside options U_G and U_R differ, implying that the wage under high productivity differs between the types).

$$J_G(\gamma, 1) = \frac{\gamma}{r+s}(1-\beta)(y^H - (r+s)U_G) + \frac{(1-\gamma)}{r+s+\phi}\left(y^L - w^L\right). \tag{H.4}$$

[3]One can show that exactly the same results can be derived with a formuluation where firing costs affect the Nash Product, in which case it also affects the wage outcome, see Section H.10. While this is the more common approach, it would in fact only be appropriate for costs which are incurred by the firm even in the case when the worker leaves voluntarily, so that they could be used as a threat by the worker to push up wages. However, most firing costs are not of this type. For example, for a professor with tenure, the firing costs could be very large, in the sense that it would be very costly for the university to lay off the professor. However, as is well known, these firing costs cannot be used by the professor to push up his or her wage. Note that a similar argument is also acknowledged by Pries and Rogerson (2005), who write that "In reality, dismissal costs are not incurred in the case of voluntary separations, ... In what follows, we shall abstract from this aspect and assume that all separations in the model lead the entrepreneur to incur the cost d. In this sense, we are really analyzing a separation tax levied on employers, rather than a dismissal cost per se."

while for a Red worker it is

$$J_R(\gamma,\eta) = \frac{\gamma}{r+s}(1-\beta)(y^H - (r+s)U_R) + \frac{(1-\gamma)}{r+s+\phi\eta}\left(y^L - w^L\right). \tag{H.5}$$

For a discriminatory equilibrium to exist it is necessary and sufficient that $J_G(\underline{\gamma},1) \geq 0 > J_R(\underline{\gamma},\eta)$ in equilibrium. There are two opposing mechanisms at work. On the one hand, Red workers have the disadvantage that they are less likely to find another job, implying that the expected duration of a low productive match is longer. On the other hand, the weaker outside option of Red workers implies that their wage is lower under high productivity. Using (H.4) and (H.5), the necessary condition that $J_G(\underline{\gamma},1) > J_R(\underline{\gamma},\eta)$ can be rewritten as

$$(1-\underline{\gamma})(\frac{w^L - y^L}{r+s+\phi\eta} - \frac{w^L - y^L}{r+s+\phi}) > \underline{\gamma}(1-\beta)(U_G - U_R). \tag{H.6}$$

The expressions for U_G and U_R are derived in Section H.5. The left hand side of (H.6) is the probability that the match is of low productivity, multiplied with the extra loss associated with a Red worker in this match, due to the expected duration of the unprofitable match is greater for Red workers. The right hand side is the probability of high productivity, multiplied with the extra cost associated with a Green worker in this match, arising from Green workers receiving higher wages, due to their better outside options. For a discriminatory equilibrium to exist, the expected additional loss for Red workers under low productivity must be greater than the expected additional wage cost for the Green workers under high productivity.

In the model in Section 3, Assumption 1 is sufficient to ensure that it is always profitable to hire a worker with a high signal. When outside options affect the wage setting, this will not be true for all parameter values. For some parameter values, discriminatory hiring standards may imply that Green workers obtain so high wages that they are unprofitable to hire even with a high signal. This will prevent the existence of this type of discriminatory equilibrium. Thus, we must assume that firms will always hire a worker with a high signal. (In Section H.6 we show that the assumption $J_G(\overline{\gamma}, 1) > 0$ holds if either n_G, $\overline{\gamma}$ or y_H is large enough.)

Assumption H.1 We assume that the parameter values are such that $J_G(\overline{\gamma}, 1) > 0$.

As in Section 3, we want to show that there exists an interval for the low signal, $\left[\underline{\gamma}^0, \underline{\gamma}^1\right)$,[4] for which a discriminatory equilibrium exists (proof in Section H.14).

Lemma H.1 *Define the values* $\underline{\gamma}^0$ *and* θ^0 *by* $J_G(\underline{\gamma}^0, 1, \theta^0) = 0$ *and* $V(\theta^0; \underline{\gamma}^0) = 0$. *There exists an* $\underline{\gamma}^0 \in (0, \overline{\gamma})$ *such that in equilibrium* $J_G(\underline{\gamma}^0, 1, \theta^0) = 0$ *and* $J_G(\underline{\gamma}, 1, \theta^0) > 0$ *for* $\underline{\gamma} > \underline{\gamma}^0$.

Thus, $\underline{\gamma}^0$ is the critical value for hiring a Green worker with a low signal in a discriminatory equilibrium. Lemma H.1 ensures that it is profitable to hire a Green worker with a low signal if $\underline{\gamma} \in \left[\underline{\gamma}^0, \overline{\gamma}\right)$. We must then investigate if it can be optimal not to hire a Red worker with a low signal in a discriminatory equilibrium of this type. This is shown in the following lemma (proof in Section H.15).

Lemma H.2 $J_R(\underline{\gamma}^0, \eta) < 0$ *if*

$$\underline{\gamma}^0 (1 - \beta)(y^H - z) > (w^L - z). \tag{H.7}$$

[4]Note that we use the same symbols as in Section 3 to facilitate the reading, even if the values are different.

To provide intuition for condition (H.7), note that when $w^L - z$ is small, there is little gain in being in a bad match as opposed to being unemployed, implying that the difference in outside options between Green and Red workers mostly reflect that Green workers have a shorter expected duration in a bad match. As the benefit from this is shared by the parties, Green workers are more profitable for the firm than Red workers. However, if $w^L - z$ is large, the difference in outside options is likely to be much bigger, reflecting that Red workers have a higher unemployment rate than Green workers. This will push up the wage of Green workers in high productive matches, making them less attractive to hire relative to Red workers, possibly preventing the existence of a discriminatory equilibrium. Note also that (H.7) is more likely to hold when y^H is large. For marginal workers, a higher productivity in a good match goes together with a larger loss associated with a bad match. As Red workers are likely to stay longer in a bad match, they become less attractive the larger the associated loss.

We have the following result (proof in Section H.7):

Proposition H.2 *If Assumptions 1 and H.1, and condition (H.7) hold, then there exists an interval* $\underline{\gamma} \in \left[\underline{\gamma}^0, \underline{\gamma}^1\right)$ *for which a discriminatory equilibrium in pure hiring strategies exists.*

Note that Assumptions 1 and H.1, and (H.7) can all hold simultaneously if y^H is sufficiently high, and $w^L - z$ sufficiently close to zero. Thus, if this is the case, and $\underline{\gamma} \in \left[\underline{\gamma}^0, \underline{\gamma}^1\right)$, then a discriminatory equilibrium exists.

There are two main empirical predictions regarding the existence of this type of discrimination. First, discrimination is related to the loss of a bad match being relatively large, as captured by y^H being large relative to y^L. Second, it is related to the utility of the unemployed not being too small relative to the utility of workers in low productive jobs ($w^L - z$ not too large), as if $w^L - z$ is large, this would cause Red workers to have much lower outside options than Greens.

As the hiring strategies in the discriminatory equilibrium are the same as in Section 3, the equilibrium unemployment rates are also the same. In other words, Corollary 1 still holds, with Reds facing a higher unemployment rate than the Greens. However, in contrast to our previous results, Red workers now also have a lower wage for the same high productivity level, because their lower outside options affect the wage outcome. That $U_R < U_G$ follows formally directly from equations (H.15) and (H.16) in Section H.5. Intuitively $U_R < U_G$ as Red workers have a harder time to find a job both when unemployed and when in a bad match.

As in the model in Section 3, there exists neutral equilibria in the model. There may also exist equilibria in mixed strategies. For space considerations, we do not consider these other types of equilibria.

Most of the results from Section H.1 on taste for discrimination and hiring quotas also hold in the model in this section. If a sufficiently large share of the vacancies exogenously apply discriminatory hiring, then also profit-maximising firms will practice discriminatory hiring. Thus, a neutral equilibrium will not exist, while the discriminatory equilibrium will still exist. Correspondingly, if a sufficiently large share of the vacancies exogenously apply neutral hiring, then it is the discriminatory equilibrium which will not exist. Both results follow directly from the fact that the expected value of hiring an applicant is a continuous function of the share, for given hiring rules. Furthermore, the persistence of a prevailing equilibrium is also unchanged. However, in contrast to the result in Section

H.1, the critical values in the counterparts to Proposition H.1 and Corollary H.1 need not coincide, because the expected value of hiring an applicant is not necessarily monotone in the share of vacancies with exogenous hiring.

H.3 Numerical simulations

In this section we will present results from numerical simulations of the model in Section H.2, where the wage depends on the outside options. The aim is to show that a discriminatory equilibrium may exist under plausible circumstances, thus we base the parameter values on empirical estimates whenever possible. We have chosen the following parameter values, where the period length is assumed to be one quarter. Output with high and low productivity are set to $y^H = 1.0$ and $y^L = 0.3$. The interest rate $r = 0.012$, following Shimer (2005), corresponding to an annual rate of about five percent. The workers' bargaining power $\beta = 0.5$. The wage in a low productivity match $w^L = 0.6$, and the value of being unemployed (leisure and unemployment benefits) $z = 0.5$. In equilibrium, this will induce an average replacement rate, calculated as z divided by the average wage, of about 55%, which is about the level in many European countries. We use a Cobb-Douglas matching function, where the weight on vacancies $1 - \lambda = 0.4$, see Petrongolo and Pissarides (2001), who suggest that it should be between 0.3 and 0.5. Flow into unemployment in European countries is between 0.3% and 2.8% on monthly basis in European countries, see OECD (1995), page 27-28, and we choose $s = 0.03$ on a quarterly basis. The high and low probabilities that output takes a high value are $\overline{\gamma} = 0.7$ and $\underline{\gamma} = 0.3$, while Prob($\gamma = \overline{\gamma}$) $= \eta = 0.6$. The latter value ensures that in a discriminatory equilibrium, Red workers have a 40 percent lower unconditional probability of being hired than Green workers, in line with the difference often found in empirical studies with regard to the probability of being called for an interview. The cost of a vacancy $c = 0.1$ ensures that recruitment costs $c/q = c\theta/\phi = 0.28$, which is close to the value of 0.3 proposed by Mortensen and Pissarides (1999). The share of Green workers, $n_G = 0.9$, implying that the share of Red workers, $n_R = 0.1$. We set the parameter in the matching function, $A = 0.4$ which leads to equilibrium labour tightness $\theta = 1.16$, and the matching rate for job searchers $\phi = 0.42$. As this is on a quarterly basis, it is well within the range for the job finding rate for unemployed in European countries, which according to OECD (1995), page 27-28 is between 3% and 21% on monthly basis in European countries. The parameter values are summarized in Table H.1.

Table H.1: Parameter values

Parameter	Value	Parameter	Value
s	0.03	λ	0.6
r	0.012	c	0.1
y^H	1.0	η	0.6
y^L	0.35	$\overline{\gamma}$	0.7
β	0.5	$\underline{\gamma}$	0.3
w^L	0.6	n_G	0.9
z	0.5	n_R	0.1
A	0.4		

With these parameter values, we find that a discriminatory equilibrium exists, see Table H.2 for the simulation outcome.[5] Thus, the advantage for the employer from Green workers being more attractive on the job market, making them more likely to quit if badly matched, dominates the effect of Red workers being paid less. In other words, it is profitable to hire a Green with $\gamma = \underline{\gamma}$, while it is not profitable to hire a Red with $\gamma = \underline{\gamma}$.

[5] As noted in Section H.2, there also exist neutral equilibria in the model. However, the discriminatory equilibrium is unique, as can be verified by repeated simulation of the model for different values of A, starting out with a value that ensures a high θ, and then with stepwise reduction in θ.

The discriminatory behavior leads to a large difference in unemployment rates, which is 6.6% for Green workers, and 10.5% for Red workers. As Green workers are hired even with a bad signal, a slightly higher share of them are in low productivity matches, the respective shares are 5.3% for Green workers and 4.3% for Red workers. While workers in low productivity match receive a lower wage, the average wage for Green workers is still somewhat higher than that of Red workers, 0.92 versus 0.91, because the lower unemployment rate for Green workers improves their disagreement point in the wage bargaining, giving Green workers higher wages than Red workers, conditional on a high productivity match.

Table H.2: Simulation outcome

Variable	Value	Variable	Value
θ	1.16	u_G	0.066
ϕ	0.42	u_R	0.105
$J_G(\underline{\gamma}, 1)$	0.06	ε_G	0.053
$J_R(\underline{\gamma}, \eta)$	-0.07	ε_R	0.043
$\overline{w_G}$	0.92	w_G^H	0.94
$\overline{w_R}$	0.91	w_R^H	0.92

$\overline{w_G}$ and $\overline{w_R}$ are the average wages for Green and Red workers,
u_G and u_R are the respective unemployment rates.
ε_G and ε_R are the share of employed workers in low productivity matches.

In the model we have assumed that the firing costs are prohibitive. However, it is of interest to see how large the firing costs would have to be for the firms to be willing to retain workers even in a low productive match, as is assumed in the model. If the firm can lay off a worker at a cost $F > 0$, the firm will retain a worker of type i in a low productive match if (where we impose the equilibrium condition $V = 0$)

$$J_i^L = \frac{y^L - w^L}{r + s + \phi p_i} > -F$$

or

$$F - \frac{-y^L + w^L}{r + s + \phi p_i} > 0, \quad \text{where } p_G = 1 \text{ and } p_R = \eta$$

The firm has a stronger incentive to lay off a Red than a Green worker, so the critical values for the firing cost which are necessary for the firm to retain a worker in a low productive match are 0.54 and 0.84 for respectively Green and Red workers (with parameter values as in Table H.1). This corresponds to 0.56 and 0.87 as a share of one quarter with average productivity, equal to 0.97 in the numerical simulation. Thus, if the firing costs are above 0.87 of quarterly output, it would be unprofitable for the firm to lay off a Red worker in a low productivity match, as assumed in our model. This is well within the parameter range of firing costs explored by Mortenson and Pissarides (1999), which goes from zero to unity.

Finally, we want to explore the robustness of the discriminatory equilibrium to variation in the parameter values, to see whether the existence requires a rare combination of parameter values. To this end, we vary the value of one parameter at a time, while at the same time adjusting labor market tightness θ so that the value of a vacancy remains equal to zero. We observe that for most variables, the interval size is quite large as compared to the value of the lower bound, suggesting that the existence does not hinge on a very particular combination of parameter values.

Table H.3: Robustness of discriminatory equilibrium

Variable	$J_G(\underline{\gamma},1)=0$	$J_R(\underline{\gamma},\eta)=0$	Interval size in percent of lower bound
$\underline{\gamma}$	0.265	0.329	24%
$\overline{\gamma}$	0.976	0.526	86%
η	0.98	0.01	9700%
z	0.601	0.383	57%
w^L	0.632	0.575	10%
y^H	0.941	1.058	12%
y^L	0.309	0.381	23%
c	0.458	0.007	6443%
s	0.065	0.002	2625%

Bounds for the interval in which a discriminatory equilibrium exists. One parameter is changed in each row, and θ is adjusted to ensure that $V=0$, while other parameter values as in Table H.1. In the upper bound for z, we also increased w^L to $w^L=0.6005$, to ensure that $z\leq w^L$.

H.4 Proof of Proposition H.1:

First we derive an expression for α_G as a function of m if the profit-maximizing firms apply the neutral hiring rule: $\gamma_G^S=\gamma_R^S=\underline{\gamma}$. Denote the proportion of the color i workers that search by $\widetilde{u}_i$. We have that $\widetilde{u}_i=u_i+(1-u_i)\varepsilon_i$. The fraction of Green workers among the applicants is given by

$$\alpha_G=\frac{\widetilde{u}_G n_G}{\widetilde{u}_G n_G+\widetilde{u}_R n_R}. \tag{H.8}$$

Using (13) and (14) we have that

$$\widetilde{u}_G=\frac{s}{s+\phi}+(1-\frac{s}{s+\phi})\varepsilon_G=\frac{s}{s+\phi}+\frac{\phi}{s+\phi}\frac{s(1-\gamma^M)}{s+\gamma^M\phi}=\frac{s}{s+\gamma^M\phi}. \tag{H.9}$$

We now turn to the expression for $\widetilde{u}_R$. We have that the average value of γ for newly hired Red workers is

$$E\gamma^R(m)=\frac{\gamma^M(1-m)+\overline{\gamma}m\eta}{p_R(m)},$$

where

$$p_R(m) = 1 - m + m\eta.$$

Hence it follows that

$$E\gamma^R(m)p_R(m) = \gamma^M(1-m) + \overline{\gamma}m\eta. \tag{H.10}$$

Using (13), (14) and (H.10) gives

$$\begin{aligned}\widetilde{u}_R &= \frac{s}{s+\phi p_R(m)} + (1 - \frac{s}{s+\phi p_R})\varepsilon_R = \frac{s}{s+\phi p_R} + \frac{\phi p_R}{s+\phi p_R}\frac{s(1-E\gamma^R(m))}{s+E\gamma^R(m)\phi p_R(m)} \\ &= \frac{s}{s+(\gamma^M(1-m)+\overline{\gamma}m\eta)\phi}\end{aligned} \tag{H.11}$$

Whenever a discriminatory equilibrium exists with only profit maximizing firms, it is straightforward that it will also exist if a part of the firms exogenously apply a discriminatory hiring strategy. It remains to show under what circumstances a neutral equilibrium exists. If the profit-maximizing firms apply the neutral hiring rule: $\gamma_G^S = \gamma_R^S = \underline{\gamma}$, we know that $J_R(\underline{\gamma}, p_R) > 0$ for $m = 0$ and $J_R(\underline{\gamma}, p_R) < 0$ for $m = 1$ (where $J_R(\underline{\gamma}, p_R)$ is given by (H.1)). As $J_R(\underline{\gamma}, p_R)$ is continuous in m, it then suffices to show that $J_R(\underline{\gamma}, p_R)$ is decreasingly monotonically in m, as there then must exist a critical value for m above which a neutral equilibrium will not exist. The derivative is

$$\frac{dJ_R(\underline{\gamma}, p_R)}{dm} = \frac{\partial J_R(\underline{\gamma}, p_R)}{\partial m} + \frac{\partial J_R(\underline{\gamma}, p_R)}{\partial \phi}\frac{\partial \phi}{\partial m}.$$

We see from (H.1) that $\frac{\partial J_R(\underline{\gamma}, p_R)}{\partial m} < 0$ and $\frac{\partial J_R(\underline{\gamma}, p_R)}{\partial \phi} > 0$. Then consider $\frac{\partial \phi}{\partial m}$: Equilibrium θ and hence equilibrium ϕ is defined by $V = 0$. Using (10) we have that

$$-c + q(\theta)\left(\alpha_G J_G(\gamma^M, p_G) + (1-\alpha_G) J_R(\gamma^M, p_R)\right) = 0$$

where α_G is given by (H.8), (H.9), and (H.11).

For a given ϕ, which also implies a given θ, a higher m implies a lower p_R and hence a lower $J_R(\underline{\gamma}, p_R)$, leading to a lower V. Furthermore, a higher m leads to a higher $\widetilde{u}_R$, which reduces α_G, also implying a lower V (as $J_G > J_R$). A lower V reduces labor market tightness θ, implying that ϕ will also be lower. Hence, $\frac{\partial \phi}{\partial m} < 0$ and thus $\frac{dJ_R(\underline{\gamma}, p_R)}{dm} < 0$.

H.5 Expressions for U_G and U_R

The asset value of a Green, unemployed worker is (from (3))

$$(r+s+\phi)U_G = z + \phi E W_G(\gamma^M). \tag{H.12}$$

and the asset value to a Green worker employed in a low productivity match is (from (6))

$$(r+s+\phi)W_i^L = w_i^L + \phi E W_G(\gamma^M) \tag{H.13}$$

Using (H.12), (H.13), (5), (2) and (H.3) gives

$$EW_G(\gamma^M) = \frac{\gamma^M \frac{(\beta y^H + (1-\beta)(r+s)U_G - \beta r V)(r+s+\phi)}{r+s} + (1-\gamma^M) w^L}{r+s+\gamma^M \phi}. \tag{H.14}$$

Using (H.12) and that $V = 0$ in equilibrium gives (after some tedious but straightforward manipulations shown in H.12)

$$U_G = \frac{1}{r+s} \frac{z(r+s) + \phi\gamma^M \beta y^H + \phi(1-\gamma^M)(w^L - z)\frac{r+s}{r+s+\phi}}{r+s+\beta\gamma^M\phi}. \tag{H.15}$$

and analogously for Red workers

$$U_R = \frac{1}{r+s} \frac{z(r+s) + \phi\eta\overline{\gamma}\beta y^H + \phi(1-\overline{\gamma})(w^L - z)\frac{r+s}{(r+s+\phi\eta)}}{r+s+\beta\overline{\gamma}\phi\eta}. \tag{H.16}$$

H.6 Alternative sufficient conditions for Assumption H.1 to hold.

Lemma H.3 *Given the hiring strategies $\gamma_G^S = \underline{\gamma}$, and $\gamma_R^S = \overline{\gamma}$ and $V = 0$, we have that $J_G(\overline{\gamma}, 1) > 0$ if either n_G, $\overline{\gamma}$ or y_H is large enough.*

Proof. $V = 0$ implies that

$$\alpha_G(\eta J_G(\overline{\gamma}, 1) + (1-\eta)J_G(\underline{\gamma}, 1) + \alpha_R \eta J_R(\overline{\gamma}, \eta) = c/q, \tag{H.17}$$

In principle $J_G(\overline{\gamma}, 1)$ can be negative as long as $J_R(\overline{\gamma}, \eta) > 0$. However, in certain cases we know that $J_G(\overline{\gamma}, 1) > 0$. Consider the limit case when n_G is 1 (and thus $\alpha_G = 1$). Then we know that $\eta J_G(\overline{\gamma}, 1) + (1-\eta)J_G(\underline{\gamma}, 1) > 0$, and as $J_G(\overline{\gamma}, 1) > J_G(\underline{\gamma}, 1)$, that $J_G(\overline{\gamma}, 1) > 0$. If $\overline{\gamma} = 1$, it follows directly that $J_G(\overline{\gamma}, 1) > 0$. For the case with $n_G < 1$ and $\overline{\gamma} < 1$, it can be shown that $J_G(\overline{\gamma}, 1) > 0$ for y_H large enough (proof in Section H.13). ■

H.7 Proof of Proposition H.2

From Lemma H.1, we know that $J_G(\underline{\gamma}^0, 1) = 0$, and $J_G(\underline{\gamma}, 1) > 0$ for $\underline{\gamma} > \underline{\gamma}^0$. From Lemma H.2, we have that $J_R(\underline{\gamma}^0, \eta) < 0$; thus a discriminatory equilibrium exists for $\underline{\gamma}^0$. Furthermore, we have that $J_R(\underline{\gamma}, \eta)$ is continuous in $\underline{\gamma}$ and ϕ, and that ϕ continuous in $\underline{\gamma}$. This can be seen from (H.5), observing from (H.16) that U_R is continuous in $\underline{\gamma}$ and ϕ, and from (H.25) that ϕ is continuous in $\underline{\gamma}$. As $J_R(\underline{\gamma}, \eta)$ is continuous in $\underline{\gamma}$, (taking into account that ϕ changes as $\underline{\gamma}$ changes) it follows from the fact that $J_R(\underline{\gamma}^0, \eta) < 0$ that there exists an interval $\underline{\gamma} \in [\underline{\gamma}^0, \underline{\gamma}^1)$ with $\underline{\gamma}^1 > \underline{\gamma}^0$, for which $J_G(\underline{\gamma}, 1) > 0 > J_R(\underline{\gamma}, \eta)$ for all $\underline{\gamma} \in [\underline{\gamma}^0, \underline{\gamma}^1)$. Thus, a discriminatory equilibrium exists for all $\underline{\gamma}$ in this interval.

H.8 Tightness θ is continuous and increasing in $\underline{\gamma}$:

Lemma H.4 *For a given hiring strategy $\gamma_G^S \in \{\underline{\gamma}, \overline{\gamma}\}$ and $\gamma_R^S \in \{\underline{\gamma}, \overline{\gamma}\}$ we have that θ^* is continuous and increasing in $\underline{\gamma}$.*

Proof. First, consider the hiring rule $\gamma_G^S = \underline{\gamma}$, and $\gamma_R^S = \overline{\gamma}$. Using (10) we have that

$$V(\theta^*; \underline{\gamma}) = \frac{-c + q(\theta)\left(\alpha_G(\eta J_G(\overline{\gamma}, 1) + (1-\eta) J_G(\underline{\gamma}, 1)) + \alpha_R \eta J_R(\overline{\gamma}, \eta)\right)}{r + q(\theta)\left(\alpha_G + \alpha_R \eta\right)} = 0. \tag{H.18}$$

with J_i given by (19). Implicit differentiation of (H.18) with respect to $\underline{\gamma}$ gives us

$$\frac{\partial V(\theta^*; \underline{\gamma})}{\partial \theta^*} \frac{d\theta^*}{d\underline{\gamma}} + \frac{\partial V(\theta^*; \underline{\gamma})}{\partial \underline{\gamma}} = 0$$

or

$$\frac{d\theta^*}{d\underline{\gamma}} = -\frac{\frac{\partial V(\theta^*; \underline{\gamma})}{\partial \underline{\gamma}}}{\frac{\partial V(\theta^*; \underline{\gamma})}{\partial \theta^*}}$$

From (19) it follows that $J_G(\overline{\gamma}, 1)$ and $J_R(\overline{\gamma}, \eta)$ are independent of $\underline{\gamma}$ and $J_G(\underline{\gamma}, 1)$ increasing in $\underline{\gamma}$ (given θ). Thus, $\frac{\partial V(\theta^*;\underline{\gamma})}{\partial \underline{\gamma}} > 0$. Since $\frac{\partial V(\theta^*;\underline{\gamma})}{\partial \theta^*} < 0$, we have that $\frac{d\theta^*}{d\underline{\gamma}} > 0$. Continuity follows directly from (H.18) and (19).

The proof for the other three possible hiring rules ($\gamma_G^S = \gamma_R^S = \underline{\gamma}$, $\gamma_G^S = \gamma_R^S = \overline{\gamma}$ and $\gamma_G^S = \overline{\gamma}$, and $\gamma_R^S = \underline{\gamma}$) is analogous and therefore omitted. ■

H.9 Proof of Proposition H.1 with exogenous number (measure) of discriminatory vacancies:

Denote the measure of discriminatory vacancies by v_d and the measure of non-discriminatory by v_n. The fraction of discriminatory vacancies is $m = v_d/(v_d + v_n)$. We will here show that m is increasing in v_d. As in Section H.4 we denote the proportion of the color i workers that search by $\widetilde{u}_i$. From Section H.4 we know that

$$\widetilde{u}_G = \frac{s}{s + \gamma^M \theta q} \tag{H.19}$$

and

$$\widetilde{u}_R = \frac{s}{s + h(m)\theta q}, \tag{H.20}$$

where $h(m) = \gamma^M - m(1-\eta)(1-\underline{\gamma})$.

Using (H.19) we have that

$$\frac{\partial \widetilde{u}_G}{\partial \theta} = \frac{-s\gamma^M(q + \theta\frac{\partial q}{\partial \theta})}{(s + \gamma^M \theta q)^2} = \frac{-\widetilde{u}_G \gamma^M q(1-\mu)}{s + \gamma^M \theta q},$$

where μ is the absolute value of the elasticity of q with respect to θ; $\mu = -\frac{\partial q}{\partial \theta}\frac{\theta}{q}$. It follows from the properties of the matching function that $-1 < \mu < 0$. Similarly we have that.

$$\frac{\partial \widetilde{u}_R}{\partial \theta} = \frac{-sh(m)(q + \theta \frac{\partial q}{\partial \theta})}{(s + h(m)\theta q)^2} = \frac{-\widetilde{u}_R h(m) q(1-\mu)}{s + h(m)\theta q}$$

$$\frac{\partial \widetilde{u}_R}{\partial m} = \frac{-s \frac{\partial h}{\partial m} \theta q}{(s + h(m)\theta q)^2} = \frac{-\widetilde{u}_R \frac{\partial h}{\partial m} \theta q}{s + h(m)\theta q}.$$

Denote the value of a profit-maximizing vacancy that applies the neutral hiring rule: $\gamma_G^S = \gamma_R^S = \underline{\gamma}$ by V_n. We have that

$$rV_n = -c + q(\theta) \left(\alpha_G (E(J_G) - V_n) + (1 - \alpha_G)(E(J_R) - V_n) \right),$$

$$E(J_G) = \frac{\gamma^M}{r+s}((1-\beta)y^H + sV_n) + \frac{(1-\gamma^M)}{r+s+\phi} \left(y^L - w^L + (s+\phi)V_n \right),$$

$$E(J_R) = \frac{\gamma^M}{r+s}((1-\beta)y^H + sV_n) + \frac{(1-\gamma^M)}{r+s+\phi p_R} \left(y^L - w^L + (s+p_R\phi)V_n \right),$$

where

$$p_R(m) = m\eta + 1 - m,$$

and the proportion of the applicants that are Green is

$$\alpha_G = \frac{\widetilde{u}_G n_G}{\widetilde{u}_G n_G + \widetilde{u}_R n_R}. \tag{H.21}$$

After some straightforward manipulations we get

$$V_n = \frac{d_1}{d_0},$$

where

$$d_1 = -c + q(\theta)\left(\frac{\gamma^M}{r+s}(1-\beta)y^H + (\alpha_G \frac{(1-\gamma^M)}{r+s+\phi} + (1-\alpha_G)\frac{(1-\gamma^M)}{r+s+\phi p_R})\left(y^L - w^L\right)\right),$$

$$d_0 = r + q(\theta)(1 - \frac{\gamma^M s}{r+s} - \alpha_G \frac{(1-\gamma^M)(s+\phi)}{r+s+\phi} - (1-\alpha_G)\frac{(1-\gamma^M)(s+\phi p_R)}{r+s+\phi p_R}) > 0.$$

We now proceed to the proof of that $\frac{dm}{dv_d} > 0$. For a given v_d define the equilibrium θ and v_n by the largest v_n where $dV_n/dv_n < 0$, and $V_n = 0$ (analogous to before). Total differentiating V_n w.r.t v_n and v_d gives

$$\frac{\partial r V_n}{\partial v_n} dv_n + \frac{\partial r V_n}{\partial v_d} dv_d = 0,$$

$$\frac{dv_n}{dv_d} = -\frac{\frac{\partial r V_n}{\partial v_d}}{\frac{\partial r V_n}{\partial v_n}}.$$

We know from the definition of equilibrium that in equilibrium $\frac{\partial r V_n}{\partial v_n} < 0$ and $V_n = 0$. Hence if $\frac{\partial r V_n}{\partial v_d} < 0$ when $\frac{\partial r V_n}{\partial v_n} < 0$ and $V_n = 0$ we have that $\frac{dv_n}{dv_d} < 0$ and then surely that $\frac{dm}{v_d} > 0$.

Proof of $\frac{\partial r V_n}{\partial v_d} < 0$ *when* $\frac{\partial r V_n}{\partial v_n} < 0$ *and* $V_n = 0$

Define V_n as a function of θ, α_G, and m

$$V_n = V_n(\theta, \alpha_G(\theta, m), p_R(m))$$

with

$$\theta = \frac{v}{\widetilde{u}} = \frac{v_d + v_n}{\widetilde{u}_G(\theta) + \widetilde{u}_R(\theta, m(v_d, v_n))}$$

Differentiating with respect to v_d

$$\begin{aligned}\frac{\partial rV_n}{\partial v_d} &= \left(\frac{\partial rV_n}{\partial \theta}+\frac{\partial rV_n}{\partial \alpha_G}\frac{\partial \alpha_G}{\partial \theta}\right)\frac{d\theta}{dv_d}+\left(\frac{\partial rV_n}{\partial \alpha_G}\frac{\partial \alpha_G}{\partial m}+\frac{\partial rV_n}{\partial p_R}\frac{\partial p_R}{\partial m}\right)\frac{dm}{dv_d}\\ &= A\frac{d\theta}{dv_n}+B\frac{dm}{dv_d}\end{aligned}$$

As $\frac{\partial rV_n}{\partial \alpha_G}>0$, $\frac{\partial \alpha_G}{\partial m}<0$, $\frac{\partial rV_n}{\partial p_R}>0$, and $\frac{\partial p_R}{\partial m}<0$ we know that $B<0$. Furthermore, we know that $\frac{dm}{dv_d}>0$. Hence if $A<0$ and $\frac{d\theta}{dv_d}>0$ we know that $\frac{\partial rV_n}{\partial v_d}<0$. We start by proving that $A<0$.

In equilibrium we know that

$$\frac{\partial rV_n}{\partial v_n}=A\frac{d\theta}{dv_n}+B\frac{dm}{dv_n}<0.$$

We know that $B<0$ and that $\frac{dm}{dv_n}<0$ and hence we know that $A\frac{d\theta}{dv_n}<0$. Thus if $\frac{d\theta}{dv_n}>0$ we know that $A<0$.

Proof of $\frac{d\theta}{dv_n}>0$ *and hence that* $A<0$.

Tightness is given by

$$\theta=\frac{v}{\widetilde{u}}=\frac{v_d+v_n}{\widetilde{u}_G(\theta)+\widetilde{u}_R(\theta,m(v_d,v_n))}$$

Total differentiation gives (using that $\frac{\partial v}{\partial v_n}=\frac{\partial \widetilde{u}}{\partial \widetilde{u}_G}=\frac{\partial \widetilde{u}}{\partial \widetilde{u}_R}=1$)

$$\begin{aligned}\frac{d\theta}{dv_n} &= \frac{\frac{\partial v}{\partial v_n}\widetilde{u}-v(\frac{\partial \widetilde{u}}{\partial \widetilde{u}_G}\frac{\partial \widetilde{u}_G}{\partial \theta}+\frac{\partial \widetilde{u}}{\partial \widetilde{u}_R}\frac{\partial \widetilde{u}_R}{\partial \theta})\frac{d\theta}{dv_n}-v\frac{\partial \widetilde{u}_R}{\partial m}\frac{\partial m}{v_n}}{\widetilde{u}^2}\\ &= \frac{\widetilde{u}-v(\frac{\partial \widetilde{u}_G}{\partial \theta}+\frac{\partial \widetilde{u}_R}{\partial \theta})\frac{d\theta}{dv_n}}{\widetilde{u}^2}-\frac{v\frac{\partial \widetilde{u}_R}{\partial m}\frac{\partial m}{v_n}}{\widetilde{u}^2}.\end{aligned}$$

Using (H.19) and (H.20) we have that

$$v(\frac{\partial \widetilde{u}_G}{\partial \theta} + \frac{\partial \widetilde{u}_R}{\partial \theta}) = -v\left(\frac{\widetilde{u}_G \gamma^M q(1-\mu)}{s+\gamma^M \phi} + \frac{\widetilde{u}_R h(m) q(1-\mu)}{s+h(m)\phi}\right)$$
$$= -vq(1-\mu)\left(\widetilde{u}_G \frac{\gamma^M}{s+\gamma^M\phi} + \widetilde{u}_R \frac{h(m)}{s+h(m)\phi}\right).$$

Thus,

$$\frac{d\theta}{dv_n} = \frac{\widetilde{u} + vq(1-\mu)\left(\widetilde{u}_G \frac{\gamma^M}{s+\gamma^M\phi} + \widetilde{u}_R \frac{h(m)}{s+h(m)\phi}\right)\frac{d\theta}{dv_n}}{\widetilde{u}^2} - \frac{v\frac{\partial \widetilde{u}_R}{\partial m}\frac{\partial m}{v_n}}{\widetilde{u}^2}$$

$\Longleftrightarrow$

$$\frac{d\theta}{dv_n} = \frac{1 + (1-\mu)\left(\widetilde{u}_G \frac{\gamma^M\phi}{s+\gamma^M\phi} + \widetilde{u}_R \frac{h(m)\phi}{s+h(m)\phi}\right)\frac{d\theta}{dv_n}}{\widetilde{u}} - \frac{\theta\frac{\partial \widetilde{u}_R}{\partial m}\frac{\partial m}{v_n}}{\widetilde{u}}$$

$\Longleftrightarrow$

$$\widetilde{u}\frac{d\theta}{dv_n} = 1 + (1-\mu)\left(\widetilde{u}_G \frac{\gamma^M\phi}{s+\gamma^M\phi} + \widetilde{u}_R \frac{h(m)\phi}{s+h(m)\phi}\right)\frac{d\theta}{dv_n} - \theta\frac{\partial \widetilde{u}_R}{\partial m}\frac{\partial m}{v_n}$$

$\Longleftrightarrow$

$$\widetilde{u}(1 - (1-\mu)\frac{1}{\widetilde{u}}\left(\widetilde{u}_G \frac{\gamma^M\phi}{s+\gamma^M\phi} + \widetilde{u}_R \frac{h(m)\phi}{s+h(m)\phi}\right))\frac{d\theta}{dv_n} = 1 - \theta\frac{\partial \widetilde{u}_R}{\partial m}\frac{\partial m}{v_n}$$

As $\widetilde{u} = \widetilde{u}_G + \widetilde{u}_R > \widetilde{u}_G \frac{\gamma^M\phi}{s+\gamma^M\phi} + \widetilde{u}_R \frac{h(m)\phi}{s+h(m)\phi}$ we know that $1 - (1-\mu)\frac{1}{\widetilde{u}}\left(\widetilde{u}_G \frac{\gamma^M\phi}{s+\gamma^M\phi} + \widetilde{u}_R \frac{h(m)\phi}{s+h(m)\phi}\right) > 0$. Thus if $1 - \theta\frac{\partial \widetilde{u}_R}{\partial m}\frac{\partial m}{v_n} > 0$ we know that $\frac{d\theta}{dv_n} > 0$. Now, (using (H.20), the definitions of $h(m)$ and m)

$$\frac{\partial \widetilde{u}_R}{\partial m} = \frac{-\widetilde{u}_R \frac{\partial h}{\partial m}\theta q}{s + h(m)\theta q}$$

$$\frac{\partial h}{\partial m} = -(1-\eta)\underline{\gamma}$$

$$\frac{\partial m}{v_n} = \frac{-v_d}{(v_d+v_n)^2} = -\frac{m}{v_d+v_n}$$

Hence,

$$\begin{aligned} 1-\theta\frac{\partial \widetilde{u}_R}{\partial m}\frac{\partial m}{v_n} &= 1+\theta\frac{\widetilde{u}_R(1-\eta)\underline{\gamma}\theta q}{s+h(m)\theta q}\frac{m}{v_d+v_n} \\ &= 1+\frac{\widetilde{u}_R}{\widetilde{u}}\frac{(1-\eta)\underline{\gamma}\theta q}{s+h(m)\theta q}m > 0 \end{aligned}$$

Thus, we have shown that $A<0$ and that the expression for $\frac{\partial\theta}{\partial v_n}$ is

$$\frac{\partial\theta}{\partial v_n} = \frac{1-\theta\frac{\partial\widetilde{u}_R}{\partial m}\frac{\partial m}{v_n}}{\rho} = \frac{1-v\frac{\widetilde{u}_R}{\widetilde{u}}\frac{(1-\eta)\underline{\gamma}\theta q}{s+h(m)\theta q}\frac{\partial m}{v_n}}{\rho}. \tag{H.22}$$

where $\rho = (1-(1-\mu)\frac{1}{\widetilde{u}}\left(\widetilde{u}_G\frac{\gamma^M\phi}{s+\gamma^M\phi}+\widetilde{u}_R\frac{h(m)\phi}{s+h(m)\phi}\right))>0$.

The next step in the proof is to show that $\frac{\partial\theta}{\partial v_d}>0$. Analogously to (H.22) we have that:

$$\begin{aligned} \frac{\partial\theta}{\partial v_d} &= \frac{1-\theta\frac{\partial\widetilde{u}_R}{\partial m}\frac{\partial m}{v_d}}{\rho} = \frac{1-v\frac{\widetilde{u}_R}{\widetilde{u}}\frac{(1-\eta)\underline{\gamma}\theta q}{s+h(m)\theta q}\frac{\partial m}{v_d}}{\rho} \\ &= \frac{1-v\frac{\widetilde{u}_R}{\widetilde{u}}\frac{(1-\eta)\underline{\gamma}\theta q}{s+h(m)\theta q}\frac{v_n}{v^2}}{\rho} \end{aligned}$$

We will now show that the numerator $N_d = 1-v\frac{\widetilde{u}_R}{\widetilde{u}}\frac{(1-\eta)\underline{\gamma}\theta q}{s+h(m)\theta q}\frac{v_n}{v^2}>0$. As $\frac{\widetilde{u}_R}{\widetilde{u}}<1$ we have that $N_d>0$ if $\frac{(1-\eta)\underline{\gamma}\theta q\frac{v_n}{v}}{s+h(m)\theta q} = \frac{(1-\eta)\underline{\gamma}\theta q(1-m)}{s+h(m)\theta q}<1$.

Thus, a sufficient condition for $N_d > 0$ is that

$$(1-\eta)\underline{\gamma}(1-m) \quad < \quad h(m) = \gamma^M - m(1-\eta)\underline{\gamma}$$

$$\Leftrightarrow$$

$$(1-\eta)\underline{\gamma} \quad < \quad \eta\overline{\gamma} + (1-\eta)\underline{\gamma}$$

Thus we have shown that $N_d > 0$ and hence that $\frac{\partial\theta}{\partial v_d} > 0$.

H.10 Including firing cost in the threat-point:

Consider the case where the wage is determined by

$$(W_i^H - U_i)^{\beta^F}(J^H - V + F)^{1-\beta^F}, \tag{H.23}$$

where $F > 0$ is the firing cost. The solution to (H.23) is (with $V = 0$)

$$w_i^{HF} = \beta^F y^H + (1-\beta^F)(r+s)U_i^F + \beta(r+s)F, \tag{H.24}$$

where β^F is workers' bargaining power and U_i^F the associated value to an unemployed. Note first that (12) holds if F is sufficiently large, as claimed in footnote 9 in the main paper.

Then we want to show that for every solution to (H.2) (i.e. without F in the threat point) with the bargaining power β there exists a $\beta^F < 1$ that gives exactly the same solution to (H.23) where F enters the threat point. As U_i^F only depends on β^F and F through the wage w_i^{HF}, if follows that if $w_i^{HF} = w_i^H$, we also have that $U_i^F = U_i$.

Using (H.3) and (H.24), $w_i^{HF} = w_i^H$ holds if

$$\beta^F y^H + (1-\beta^F)(r+s)U_i - \beta r V + \beta(r+s)F = \beta y^H + (1-\beta)(r+s)U_i - \beta r V$$

$\Leftrightarrow$

$$\beta^F(y^H - (r+s)U_i + (r+s)F) = \beta(y^H - (r+s)U_i)$$

$\Leftrightarrow$

$$\beta^F = \beta \frac{y^H - (r+s)U_i}{y^H - (r+s)U_i + (r+s)F}$$

H.11 Proof of existence of equilibrium for the specification in Section H.2:

Lemma H.5 : *(Existence of equilibrium for given hiring rule with wages set by (H.3)).* *Given $\gamma_G^S = \underline{\gamma}$, and $\gamma_R^S = \overline{\gamma}$ there exists a unique $\theta^* > 0$, where $V(\theta^*) = 0$, $V(\theta^*) < 0$ for $\theta > \theta^*$, and $\frac{\partial V(\theta^*)}{\partial \theta} < 0$.*

Proof. Given $\gamma_G^S = \underline{\gamma}$, and $\gamma_R^S = \overline{\gamma}$ we have from (10) that

$$V(\theta) = \frac{-c/q(\theta) + \big(\alpha_G J_G(\gamma^M, 1) + \alpha_R \eta J_R(\overline{\gamma}, \eta)\big)}{r/q(\theta) + \alpha_G + \alpha_R \eta} \tag{H.25}$$

Using (H.4) and (H.5) and that $\phi = 0$ and $\alpha_i = n_i$, and $U = z/(r+s)$ when $\theta = 0$, gives

$$\begin{aligned} V(0) \quad = \quad & n_G \frac{\gamma^M((1-\beta)(y^H - z) + (\beta r + s)V) + (1-\gamma^M)\big((y^L - w^L) + sV\big)}{(r+s)(n_G + n_R\eta)} \\ & + n_R\eta \frac{\overline{\gamma}((1-\beta)(y^H - z) + (\beta r + s)V) + (1-\overline{\gamma})\big((y^L - w^L) + sV\big)}{(r+s)(n_G + n_R\eta)}, \end{aligned}$$

Thus $V(0) > 0$ at $\theta = 0 \Leftrightarrow$

$$n_G(\gamma^M(1-\beta)(y^H-z)+(1-\gamma^M)(y^L-w^L))+n_R\eta(\overline{\gamma}(1-\beta)(y^H-z)+(1-\overline{\gamma})(y^L-w^L)) > 0. \tag{H.26}$$

$\Leftrightarrow$

$$(1-\beta)(y^H - z) > (y^L - w^L)\frac{n_G(1-\gamma^M) + n_R\eta(1-\overline{\gamma})}{n_G\gamma^M + n_R\eta\overline{\gamma}}$$

Hence, by Assumption 1 we have that $V(0) > 0$.

In the limit, we have $\lim_{\theta\to\infty} V = -c/r < 0$. As V is continuous in θ for given hiring strategies, positive for low values of θ and negative for high values, it follows that there exist an equilibrium θ^* (conditional on the discriminatory hiring strategy) where $V(\theta^*) = 0$, $\frac{\partial V(\theta^*)}{\partial\theta} < 0$ and $V(\theta) < 0$ for all $\theta > \theta^*$. ■

H.12 Deriving equation (H.15)

Using (H.12) and (H.14) gives

$$(r+s+\phi)U_G = z + \phi\frac{\gamma^M\frac{(\beta y^H + (1-\beta)(r+s)U_G - \beta rV)(r+s+\phi)}{r+s} + (1-\gamma^M)w^L}{r+s+\gamma^M\phi}$$

or

$$\begin{aligned} & (r+s+\gamma^M\phi)(r+s+\phi)U_G \\ = \quad & z(r+s+\gamma^M\phi) + \phi(\gamma^M\frac{(\beta y^H + (1-\beta)(r+s)U_G - \beta rV)(r+s+\phi)}{r+s} + (1-\gamma^M)w^L) \end{aligned}$$

or

$$
\begin{aligned}
& (r+s+\gamma^M\phi)(r+s+\phi)U_G - \phi(1-\beta)\gamma^M(r+s+\phi)U_G \\
= \; & z(r+s+\gamma^M\phi) + \phi(\gamma^M \frac{(\beta y^H - \beta r V)(r+s+\phi)}{r+s} + (1-\gamma^M)w^L)
\end{aligned}
$$

or

$$
U_G = \frac{z(r+s+\gamma^M\phi) + \phi(\gamma^M \frac{(\beta y^H - \beta r V)(r+s+\phi)}{r+s} + (1-\gamma^M)w^L)}{(r+s+\beta\gamma^M\phi)(r+s+\phi)}
$$

H.13 $J_G(\overline{\gamma}, 1) > 0$ for y_H large enough

The expected profits from hiring a Green worker with signal $\overline{\gamma}$ is (with $V = 0$)

$$
J_G(\overline{\gamma}, 1, \theta) = \frac{\overline{\gamma}}{r+s}(1-\beta)(y^H - (r+s)U_G) + \frac{(1-\overline{\gamma})}{r+s+\phi}\left(y^L - w_i^L\right). \tag{H.27}
$$

Using the expression for U_G (equation H.15) we have

$$
\begin{aligned}
& y^H - (r+s)U_G \\
= \; & y^H - \frac{z(r+s) + \phi\gamma^M\beta y^H + \phi(1-\gamma^M)(w^L - z)\frac{r+s}{r+s+\phi}}{r+s+\beta\gamma^M\phi} \\
= \; & (r+s)\frac{(y^H - z) - \phi(1-\gamma^M)(w^L - z)\frac{1}{r+s+\phi}}{r+s+\beta\gamma^M\phi}.
\end{aligned}
$$

Substituting out in (H.27) gives us

$$
J_G(\overline{\gamma}, 1, \theta) = \overline{\gamma}(1-\beta)\frac{(y^H - z) - (1-\gamma^M)(w^L - z)\frac{\phi}{r+s+\phi}}{r+s+\beta\gamma^M\phi} + \frac{(1-\overline{\gamma})}{r+s+\phi}\left(y^L - w^L\right). \tag{H.28}
$$

$J_G(\overline{\gamma}, 1) \geq 0 \Longleftrightarrow$

$$\overline{\gamma}(1-\beta)(y^H-z)-(1-\gamma^M)(w^L-z)\frac{\phi}{r+s+\phi}-(1-\overline{\gamma})\frac{r+s+\beta\gamma^M\phi}{r+s+\phi}\left(w^L-y^L\right) \geq 0 \quad \text{(H.29)}$$

As $0 < \frac{\phi}{r+s+\phi} < 1$ and $0 < \frac{r+s+\beta\gamma^M\phi}{r+s+\phi} < 1$ it is sufficient that

$$\overline{\gamma}(1-\beta)(y^H-z)-(1-\gamma^M)(w^L-z)-(1-\overline{\gamma})\left(w^L-y^L\right) \geq 0 \quad \text{(H.30)}$$

which is always satisfied for y^H high enough.

H.14 Proof of Lemma H.1

Analogous to (H.30) we have that a sufficient condition for $J_G(\underline{\gamma}, 1) \geq 0$ is that

$$\underline{\gamma}(1-\beta)(y^H-z)-(1-\gamma^M)(w^L-z)-(1-\underline{\gamma})\left(w^L-y^L\right) \geq 0 \quad \text{(H.31)}$$

The LHS is continuous and strictly increasing in $\underline{\gamma}$. Thus, since (H.31) holds for the limit case $\underline{\gamma} = \overline{\gamma}$, (from Assumption H.1) then by continuity of (H.31) in $\underline{\gamma}$, there will also exist a $\underline{\gamma} < \overline{\gamma}$, such that it holds. On the other hand, if $\underline{\gamma} = 0$, then $J_G(\underline{\gamma}, 1) \geq 0$ does not hold. By the continuity of $J_G(\underline{\gamma}, 1)$ in $\underline{\gamma}$, it then follows that there exists a $\underline{\gamma}^0 > 0$ such that $J_G(\underline{\gamma}^0, 1) = 0$ and $J_G(\underline{\gamma}, 1) > 0$ for $\underline{\gamma} > \underline{\gamma}^0$. To find this critical value $\underline{\gamma}^0$, one can start with $\underline{\gamma} = \overline{\gamma}$, and then gradually decrease $\underline{\gamma}$ until $J_G(\underline{\gamma}, 1) = 0$, in which case one has found the critical value $\underline{\gamma}^0$.

H.15 Proof of Lemma H.2

We want to show that for $\underline{\gamma} = \underline{\gamma}^0$, implying that $J_G(\underline{\gamma}^0, 1) = 0$, then $J_R(\underline{\gamma}^0, \eta) < 0$, if (H.7) holds. To this end, we show that if (H.7) holds, then $J_R(\underline{\gamma}^0, \eta) - J_G(\underline{\gamma}^0, 1) < 0$. Corresponding to (H.28), $J_G(\underline{\gamma}, 1) = 0 \Leftrightarrow$

$$(y^H - z)\frac{r+s+\phi}{r+s+\beta\gamma^M\phi} - (1-\gamma^M)\frac{\phi\frac{w^L-z}{\underline{\gamma}(1-\beta)}}{r+s+\beta\gamma^M\phi} - \frac{(1-\underline{\gamma})}{\underline{\gamma}}\frac{w^L-y^L}{1-\beta} = 0.$$

$\Leftrightarrow$

$$(y^H - z)\frac{r+s+\phi}{r+s+\beta\gamma^M\phi} - (1-\gamma^M)\frac{(r+s+\phi)\frac{w^L-z}{\underline{\gamma}(1-\beta)}}{r+s+\beta\gamma^M\phi} + (1-\gamma^M)\frac{(r+s)\frac{w^L-z}{\underline{\gamma}(1-\beta)}}{r+s+\beta\gamma^M\phi} - \frac{(1-\underline{\gamma})}{\underline{\gamma}}\frac{w^L-y^L}{1-\beta} = 0.$$

$\Leftrightarrow$

$$\frac{r+s+\phi}{r+s+\beta\gamma^M\phi}((y^H-z)-(1-\gamma^M)\frac{w^L-z}{\underline{\gamma}(1-\beta)}) + (1-\gamma^M)\frac{(r+s)\frac{w^L-z}{\underline{\gamma}(1-\beta)}}{r+s+\beta\gamma^M\phi} - \frac{(1-\underline{\gamma})}{\underline{\gamma}}\frac{w^L-y^L}{1-\beta} = 0. \tag{H.32}$$

The expected profits of hiring a Red worker with signal $\underline{\gamma}$ is

$$J_R(\underline{\gamma}, \eta) = \underline{\gamma}(1-\beta)\frac{(y^H - z) - (1-\eta\overline{\gamma})(w^L - z)\frac{\phi\eta}{r+s+\phi\eta})}{r+s+\beta\overline{\gamma}\phi\eta} + \frac{(1-\underline{\gamma})}{r+s+\phi\eta}\left(y^L - w^L\right). \tag{H.33}$$

Using (H.33), it is unprofitable to hire a Red worker $J_R(\underline{\gamma}, \eta) < 0 \Longleftrightarrow$

$$(y^H - z)\frac{r+s+\phi\eta}{r+s+\beta\overline{\gamma}\phi\eta} - (1-\overline{\gamma})\frac{\phi\eta\frac{(w^L-z)}{\underline{\gamma}(1-\beta)}}{r+s+\beta\overline{\gamma}\phi\eta} - \frac{(1-\underline{\gamma})}{\underline{\gamma}}\frac{w^L-y^L}{1-\beta} < 0. \tag{H.34}$$

Thus, $J_R(\underline{\gamma}^0,\eta) - J_G(\underline{\gamma}^0,1) < 0$ if (using (H.32) and (H.34))

$$(y^H - z)\frac{r+s+\phi\eta}{r+s+\beta\overline{\gamma}\phi\eta} - (1-\overline{\gamma})\frac{\phi\eta\frac{(w^L-z)}{\underline{\gamma}(1-\beta)}}{r+s+\beta\overline{\gamma}\phi\eta} - (y^H-z)\frac{r+s+\phi}{r+s+\beta\gamma^M\phi} + (1-\gamma^M)\frac{\phi\frac{(w^L-z)}{\underline{\gamma}(1-\beta)}}{r+s+\beta\gamma^M\phi} < 0. \tag{H.35}$$

After tedious but straightforward calculations (see below) we find that (H.35) is equivalent to

$$\frac{\underline{\gamma}^0(1-\beta)(y^H-z)}{(w^L-z)} > \frac{(r+s)(1-\underline{\gamma}^0) + \phi\eta\beta(\overline{\gamma}-\underline{\gamma}^0)}{(r+s)(1-\beta\underline{\gamma}^0) + \phi\eta\beta(\overline{\gamma}-\underline{\gamma}^0)}. \tag{H.36}$$

Observe that the RHS < 1, which completes the proof.

Deriving equation (H.36)

We simplify the notation in (H.35) by defining

$a = r + s, \quad d = \frac{(w^L-z)}{\underline{\gamma}(1-\beta)(y^H-z)}$

to obtain

$$\frac{a+\phi\eta}{a+\beta\overline{\gamma}\phi\eta} - (1-\overline{\gamma})\frac{d\eta\phi}{a+\beta\eta\overline{\gamma}\phi} - \frac{a+\phi}{a+\beta\gamma^M\phi} + (1-\gamma^M)\frac{d\phi}{a+\beta\gamma^M\phi} < 0$$

$\Leftrightarrow$

$$(a+\phi\eta)(a+\beta\gamma^M\phi) - (1-\overline{\gamma})d\eta\phi(a+\beta\gamma^M\phi)$$

$$-(a+\phi)(a+\beta\overline{\gamma}\phi\eta) + (1-\gamma^M)d\phi(a+\beta\overline{\gamma}\phi\eta) < 0$$

$\Leftrightarrow$

$$a^2 + a\phi\eta + a\beta\gamma^M\phi + \phi\eta\beta\gamma^M\phi - d\eta\phi(a + \beta\gamma^M\phi - \overline{\gamma}a - \overline{\gamma}\beta\gamma^M\phi)$$

$$-a^2 - a\phi - a\beta\overline{\gamma}\phi\eta - \beta\overline{\gamma}\phi\eta\phi + d\phi(a + \beta\overline{\gamma}\phi\eta - \gamma^M a - \gamma^M\beta\overline{\gamma}\phi\eta) < 0$$

$\Leftrightarrow$

$$\phi(a\eta + a\beta\gamma^M + \phi\eta\beta\gamma^M - a - a\beta\overline{\gamma}\eta - \beta\overline{\gamma}\phi\eta)$$

$$+d\phi(-\eta a - \beta\eta\gamma^M\phi + \eta\overline{\gamma}a + \eta\overline{\gamma}\beta\gamma^M\phi + a + \beta\overline{\gamma}\phi\eta - \gamma^M a - \gamma^M\beta\overline{\gamma}\phi\eta) < 0$$

$\Leftrightarrow$

$$a(\eta - 1) + a\beta(\gamma^M - \overline{\gamma}\eta) + \phi\eta\beta(\gamma^M - \overline{\gamma})$$

$$+d(a(1-\eta) + \beta\phi\eta(-\gamma^M + \overline{\gamma}\gamma^M + \overline{\gamma} - \gamma^M\overline{\gamma}) + a(\eta\overline{\gamma} - \gamma^M)) < 0$$

$\Leftrightarrow$

$$a(\eta - 1) + a\beta(1-\eta)\underline{\gamma}^0 + \phi\eta\beta((\eta-1)\overline{\gamma} + (1-\eta)\underline{\gamma}^0)$$

$$+d(a(1-\eta) + \beta\phi\eta(-\gamma^M + \overline{\gamma}) - a(1-\eta)\underline{\gamma}^0) < 0$$

$\Leftrightarrow$

$$(1-\eta)(-a + a\beta\underline{\gamma}^0 + \phi\eta\beta(\underline{\gamma}^0 - \overline{\gamma}))$$

$$+d(a(1-\eta)(1-\underline{\gamma}^0) + \beta\phi\eta(-\gamma^M + \overline{\gamma})) < 0$$

$\Leftrightarrow$

$$(1-\eta)(-a + a\beta\underline{\gamma}^0 + \phi\eta\beta(\underline{\gamma}^0 - \overline{\gamma}))$$

$$+d(a(1-\eta)(1-\underline{\gamma}^0) + \beta\phi\eta(1-\eta)(\overline{\gamma} - \underline{\gamma}^0)) < 0$$

$\Leftrightarrow$

$$-a + a\beta\underline{\gamma}^0 + \phi\eta\beta(\underline{\gamma}^0 - \overline{\gamma}) + d(a(1-\underline{\gamma}^0) + \beta\phi\eta(\overline{\gamma} - \underline{\gamma}^0)) < 0$$

$\Leftrightarrow$

$$a(\beta\underline{\gamma}^0 - 1) + \phi\eta\beta(\underline{\gamma}^0 - \overline{\gamma}) + d(a(1-\underline{\gamma}^0) + \beta\phi\eta(\overline{\gamma} - \underline{\gamma}^0)) < 0$$

$\Leftrightarrow$

$$d < \frac{a(1-\beta\underline{\gamma}^0) + \phi\eta\beta(\overline{\gamma} - \underline{\gamma}^0)}{a(1-\underline{\gamma}^0) + \phi\eta\beta(\overline{\gamma} - \underline{\gamma}^0)}$$

Substituting out for a and d to obtain the original notation, we obtain

$$\frac{(w^L - z)}{\underline{\gamma}^0(1-\beta)(y^H - z)} < \frac{(r+s)(1-\beta\underline{\gamma}^0) + \phi\eta\beta(\overline{\gamma} - \underline{\gamma}^0)}{(r+s)(1-\underline{\gamma}^0) + \phi\eta\beta(\overline{\gamma} - \underline{\gamma}^0)} > 1$$

or, equivalently,

$$\frac{\underline{\gamma}^0(1-\beta)(y^H - z)}{(w^L - z)} > \frac{(r+s)(1-\underline{\gamma}^0) + \phi\eta\beta(\overline{\gamma} - \underline{\gamma}^0)}{(r+s)(1-\beta\underline{\gamma}^0) + \phi\eta\beta(\overline{\gamma} - \underline{\gamma}^0)} < 1$$