Waves with Power-Law Attenuation: Corrections

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8th February, 2020

These are corrections and additions for Holm (2019). Bold text needs to be added and replace stricken out text. Please send new suggestions to email: sverre (a) uio.no.

1 Introduction

- Page 12, Fig 1.4: New figure and additions to caption (no effect on main text)
- Page 12, misprint: replace \( d^{-t/\tau} \) by \( e^{-t/\tau} \) in:

\[
G(t) = E_e + E_e \left( \frac{\tau}{\tau_e} - 1 \right) e^{-t/\tau_e},
\]  

(1.14)

3 Models of linear viscoelasticity

- Page 79, Fig 3.7: \( \tau_\sigma \) in formula in upper figure should be \( \tau \)

5 Power-law wave equations from constitutive equations

- Page 126, Fig 5.4: New figure and additions to caption (no effect on main text)
- Page 133, Sect. 5.3.1: Missing minus after last equal sign:

\[
\Delta c_{ph} \approx \frac{c_0}{2} \tau^{-1} \sin \frac{\pi y}{\omega} = -c_0^2 \alpha_0 \tan \frac{\pi y}{2} \omega^{-1}
\]  

(5.35)

6 Phenomenological power-law wave equations

- Of the four references to Zhao & McGough (2016a), three of them should be to Zhao & McGough (2016b) instead (pages 163, 168, and 171)
- Page 166, line 1, Sect. 6.1.2.1: "... the phase velocity increases as a function of frequency, but then may start falling and eventually become negative zero."
Thus $c_{ph}$ will increase monotonically (except for $y = 1$) and this is consistent with the condition of (4.36). The lack of agreement with (4.35) for the attenuation for $y > 1$ seems to be consistent with the observation in [Zhao & McGough (2016b)] that this wave equation gives a non-causal, i.e. a non-passive, solution for $y = 1.139$, $y = 1.5$, and $y = 2$.

Thus $c_{ph}$ will decrease monotonically when $\tan(\pi y/2) > 0$, i.e. when $y < 1$. Since that is not consistent with the condition of (4.36), it can be concluded that this model does not satisfy the criterion for passivity of Sect. (4.3) regardless of whether $y < 1$ or $y > 1$. This is consistent with the observation in Zhao & McGough (2016a) that this wave equation gives a non-causal, i.e. a non-passive, solution for $y = 1.139$, $y = 1.5$, and $y = 2$.

7 Justification for power laws and fractional models

- Page 216, misprint in fourth line below Eq. (7.105): change (5.2.2) to (5.22): “just like the half-order fractional Newton model of (5.22)”

- Page 201, change text under Eq. (7.59) from “where the order may be in the range $0 \leq \alpha \leq 1$ to resulting in $\tilde{E}(\omega) \approx E_0^{1-\alpha} (i\omega\eta_0)^\alpha$ which extends (7.51) to the range $0 \leq \alpha \leq 1$.

- Page 218, change the equation to

$$T(y_0) = \frac{1}{\sqrt{2g}} \int_0^{y_0} \frac{1}{(y_0 - y)^{0.5}} \frac{ds}{dy} dy,$$

(7.108)

where $s(y)$ gives the shape of the curve and $g$ is the gravity of the Earth. It is proportional to the Caputo fractional derivative of order 0.5 of (A.44). Therefore this is considered to be the first physical problem that requires a fractional derivative.

Addendum

- Page 267, heading of ex. A.1: “Fractional derivative of order 0...1”, rather than “0.1”.

References


Fig. 1.4 Relaxation moduli of Zener model with an exponential time response, (1.13) (solid line) with $E = 1$, $\eta = 0.5$, and $\tau_\varepsilon = 1$, and for the fractional Zener model (dashed line) for $\alpha = 0.5$, $\tau_\varepsilon = 2$, which asymptotically approaches a power law function, (1.30). The asymptotic values are the glass modulus, $G_g = G(0^+)$ and the equilibrium modulus, $G_e$ for infinite time.

Fig. 5.4 Relaxation moduli of fractional Kelvin-Voigt model (upper) with $E = 1$, $\eta = 1$ and fractional Zener model (lower) with $\alpha = 0.5$, $\tau_\varepsilon = 4$