Chapter 1

Introduction

The wave equation is derived from what is often called governing equations. That covers equations for conservation of momentum and mass, and constitutive equations that relate e.g. pressure and density or stress and strain.

Considering these together may give the impression that there is little distinction between them and in some acoustics and elastic wave propagation texts, these equations are just lumped together and considered as a given.[1] But these equations don't have the same standing so therefore we make it a point here to distinguish between them.

1.1 Conservation principles

In acoustics and elastic wave propagation, the equation that expresses conservation of linear momentum is an expression for Newton's second law which relates force, mass and acceleration: $F = ma$. It can also be stated as $F = d(mv)/dt$ where the linear momentum, mv, is the product of mass and velocity. Conservation of linear momentum, which is closely related to the Euler equation in fluid dynamics, is usually stated as the equivalence between the rate of change of momentum per volume and the negative gradient of the pressure or the average force over a unit volume:

$$\rho \frac{\partial \mathbf{v}}{\partial t} = -\nabla p. \tag{1.1}$$

where ρ is density, $\mathbf{v}$ is the velocity vector and p is the pressure. Conservation of linear momentum is a fundamental principle of physics which derives from the principle of symmetry in space first expressed by Noether (see sidebar).

The second conservation principle is concerned with energy. In a system at rest (non-relativistic) it is the same as conservation of mass via $E = mc^2$ [Hecht, 2009], and local conservation of mass is expressed in the continuity equation. It states that the rate at which mass enters a closed system in steady

Emmy Noether (1882–1935) German and Jewish, who emigrated to USA. In theoretical physics she is known for Noether's theorem which connects symmetries and conservation laws:

- Conservation of linear momentum is a consequence of invariance to spatial translation
- Conservation of energy is a consequence of invariance under time translations
- Conservation of angular momentum is due to invariance with respect to rotation

See [Landau and Lifshitz, 1976] (ch. II) for detailed derivations. Image: Public domain, from Wikipedia Commons. (Wikipedia)

[1] Calling these equations 'governing' may give the impression that they actually are the cause of the phenomena they describe. This may be so for legislation in a country governed by the rule of law. But it shouldn't take much reflection to realize that it is not the case for physical laws as they are only descriptive.

state is equal to the volume expansion rate:

$$\frac{\partial \rho}{\partial t} + \rho \nabla \cdot \mathbf{v} = 0. \tag{1.2}$$

The underlying principle of conservation of energy is Noether's theorem of symmetry in time.

The conservation principles relate to the properties that an experiment may be performed in a different place with the same result – symmetry in space – or performed at a different time with the same outcome – symmetry in time [Gross, 1996]. These are properties which are at the foundation of physics and many would say that what they describe is so obvious that one rarely needs to think about it. Such statements, which are accepted without controversy, are usually called axioms, and as such belong to what one may call meta-science. Axioms are mostly known from mathematics, and not in so many other sciences. Physics may be an exception, as space- and time-invariance there may be considered to have an axiomatic standing.[2]

Albert Einstein (1879–1955) German. He developed the special and general theories of relativity marking that Newtonian mechanics was no longer adequate. He is also known for his influence on the philosophy of science. His Nobel Prize in Physics in 1921 was for the discovery of the law of the photoelectric effect. Einstein lived through turbulent times: First his "German theory" was hard to accept during the time of the first world war [Stanley, 2016], and later he found it best to stay behind in the US after a visit in 1933 because of his Jewish origins. Image: E. O. Hoppe, published on (LIFE)

1.2 Constitutive equations

A constitutive equation is different from a conservation law because of its empirical nature. That means that it is primarily based on measurements of material characteristics, although it can be justified in some underlying physical principle as well. But these principles are not as fundamental as the symmetry principles behind the conservation laws. This is easily overlooked and it is not uncommon to grant a constitutive equation a similar standing as the conservation laws. This is especially so when some of the descriptions and equations are attached to famous names like Newton, Navier, and Stokes. It may therefore be good to heed to Einstein who wrote the following in the obituary of Ernst Mach [Einstein, 1916](p. 102):

> *Concepts that have proven useful in ordering things easily achieve such authority over us that we forget their earthly origins and accept them as unalterable givens. Thus they come to be stamped as 'necessities of thought,' 'a priori givens,' etc. The path of scientific progress is often made impassable for a long time by such errors. Therefore it is by no means an idle game if we become practiced in analyzing longheld commonplace concepts and showing the circumstances on which their justification and usefulness depend, and how they have grown up, individually, out of the givens of experience. "* [Howard, 2005].

[2] Gödel's incompleteness theorem states that the proof of consistency of a set of mathematical axioms only can be found outside the set. The same limitation may therefore apply to physics [Jaki, 1966](pp. 128–130), [Barrow, 2011]. This may have implications regarding how much a "theory of everything" in physics really can encompass, as later rediscovered and made more widely known by [Hawking, 2002]. On first glance this may cause pessimism on behalf of science, but on second thought it should rather give reason for optimism as science will never come to an end, and there will always be new discoveries to explore. The open-endedness of physics due to Gödel's theorem is also discussed in [Dyson, 1996].