Primitive Equations

\[
\frac{du}{dt} - fv = -\frac{1}{\rho} \frac{\partial p}{\partial x}
\]

\[
\frac{dv}{dt} + fu = -\frac{1}{\rho} \frac{\partial p}{\partial y}
\]

\[
\frac{dp}{dz} = -\rho g
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{d\rho}{dt}
\]

\[
c_p \frac{dT}{dt} - \alpha \frac{dp}{dt} = Q
\]

\[
p = \rho RT
\]
Primitive Equations

\[
\frac{du}{dt} - fv = -\frac{1}{\rho} \frac{\partial p}{\partial x}
\]

x-component momentum equation

\[
\frac{dv}{dt} + fu = -\frac{1}{\rho} \frac{\partial p}{\partial y}
\]

y-component momentum equation

\[
\frac{dp}{dz} = -\rho g
\]

hydrostatic equation

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{d\rho}{dt}
\]

continuity equation

\[
c_p \frac{dT}{dt} - \alpha \frac{dp}{dt} = Q
\]

thermodynamic energy equation

\[
p = \rho RT
\]

equation of state

6 equations with 6 dependent variables: \(u, v, w, p, \rho, T\)
Stratocumulus Clouds
Stratocumulus Clouds

- Altitude of these clouds?
- Approximate thickness (meters)?
- What are the processes that gives supersaturation and cloud formation?
- Why is there a cell-like structure of the clouds?
Why study stratocumulus? - radiative forcing

Annual ERBE Net Cloud Radiative Forcing

cloud forcing = cloudy TOA rad. flux - clear sky TOA rad. flux

Courtesy of Dennis Hartmann
Brewer-Dobson sirkulasjon i stratosfæren
Several massive wildfires were across southern California during October 2003. MODIS, on the NASA Terra satellite, captured smoke spreading across the region and westward over the Pacific Ocean on October 26, 2003. Credit: NASA.
Satelittbilder i IR

Brightness Temperature (K)
Figur 7.4: Et øyeblikksbilde av den enkleste formen for elektromagnetisk bølge, nærmig en plan bølge. En slik bølge kan oppnås langt fra kildene til bølgen og langt fra materialer som
# Maxwells Equations

<table>
<thead>
<tr>
<th>Name</th>
<th>Differential form</th>
<th>Integral form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauss's law</td>
<td>$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$</td>
<td>$\oiint_{\partial V} \mathbf{E} \cdot d\mathbf{A} = \frac{Q(V)}{\varepsilon_0}$</td>
</tr>
<tr>
<td>Gauss's law for magnetism</td>
<td>$\nabla \cdot \mathbf{B} = 0$</td>
<td>$\oiint_{\partial V} \mathbf{B} \cdot d\mathbf{A} = 0$</td>
</tr>
<tr>
<td>Maxwell–Faraday equation (Faraday's law of induction)</td>
<td>$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$</td>
<td>$\int_{\partial S} \mathbf{E} \cdot dl = -\int_{S} \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{A}$</td>
</tr>
<tr>
<td>Ampère's circuital law (with Maxwell's correction)</td>
<td>$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$</td>
<td>$\int_{\partial S} \mathbf{B} \cdot dl = \mu_0 I_S + \mu_0 \varepsilon_0 \int_{S} \frac{\partial \mathbf{E}}{\partial t} \cdot d\mathbf{A}$</td>
</tr>
</tbody>
</table>

E: Elektrisk felt, B: Magnetfelt.

What is the unit on the y-axis?

$I_\lambda$: Intensity (radians): Radiative energy per unit wavelength and Solid Angle

$I_\lambda : Wm^{-2} \text{ nm}^{-1} \text{ steradian}^{-1}$

Total radians (I):

$$I = \int_{\lambda=0}^{\lambda=\infty} I_\lambda d\lambda$$

Fig. 4.2 The curve represents a hypothetical spectrum of monochromatic intensity $I_\lambda$ or monochromatic flux density $F_\lambda$ as a function of wavelength $\lambda$. 
Solid Angle

\[ \omega = \frac{A}{r^2} \]

\[ \omega = 2\pi \]

Hemisphere
To integrate over solid angle we use spherical coordinates

$$d\omega = r^2 \sin \theta \, d\theta \, d\phi$$
Flux Density $F$ (irradians): Flux of radiation energy through a plane surface (up or down)

$$F = \int_{2\pi} I \cos \theta \, d\omega = \int_{\lambda=0}^{\lambda=\infty} \int_{2\pi} I(\lambda) \cos \theta \, d\omega$$
Exercise 4.2 The flux density $F_s$ of solar radiation incident upon a horizontal surface at the top of the earth's atmosphere at zero zenith angle is 1368 W m$^{-2}$. Estimate the intensity of solar radiation. Assume that solar radiation is isotropic (i.e., that every point on the "surface" of the sun emits radiation with the same intensity in all directions, as indicated in Fig. 4.4). For reference, the radius of the Sun $R_s$ is $7.00 \times 10^8$ m and the Earth-Sun distance $d$ is $1.50 \times 10^{11}$ m.
Exercise 4.2 The flux density $F_s$ of solar radiation incident upon a horizontal surface at the top of the earth’s atmosphere at zero zenith angle is 1368 W m$^{-2}$. Estimate the intensity of solar radiation. Assume that solar radiation is isotropic (i.e., that every point on the "surface" of the sun emits radiation with the same intensity in all directions, as indicated in Fig. 4.4). For reference, the radius of the Sun $R_s$ is $7.00 \times 10^8$ m and the Earth-Sun distance $d$ is $1.50 \times 10^{11}$ m.

Solution: Let $I_s$ be the intensity of solar radiation. If the solar radiation is isotropic and the sun is directly overhead, then from (4.5) the flux density of solar radiation at the top of the earth’s atmosphere is

$$F = \int_{\delta \omega} I_s \cos \phi d\omega$$

where $\delta \omega$ is the arc of solid angle subtended by the sun in the sky. Since $\delta \omega$ is
Exercise 4.2 The flux density $F_s$ of solar radiation incident upon a horizontal surface at the top of the earth’s atmosphere at zero zenith angle is $1368$ W m$^{-2}$. Estimate the intensity of solar radiation. Assume that solar radiation is isotropic (i.e., that every point on the "surface" of the sun emits radiation with the same intensity in all directions, as indicated in Fig. 4.4). For reference, the radius of the Sun $R_s$ is $7.00 \times 10^8$ m and the Earth-Sun distance $d$ is $1.50 \times 10^{11}$ m.

Solution: Let $I_s$ be the intensity of solar radiation. If the solar radiation is isotropic and the sun is directly overhead, then from (4.5) the flux density of solar radiation at the top of the earth’s atmosphere is

$$F = \int_{\delta\omega} I_s \cos \phi d\omega$$

where $\delta\omega$ is the arc of solid angle subtended by the sun in the sky. Since $\delta\omega$ is very small, we can ignore the variations in $\cos \phi$ in the integration. With this so called parallel beam approximation, the integral reduces to

$$F = I_s \times \cos \phi \times \delta\omega$$

and since the zenith angle, in this case, is zero,

$$F = I_s \times \delta\omega$$
radius of the Sun $R_s$ is $7.00 \times 10^8$ m and the Earth-Sun distance $d$ is $1.50 \times 10^{11}$ m.

The fraction of the hemisphere of solid angle (i.e., "the sky") that is occupied by the Sun is the same as the fraction of the area of the hemisphere of radius $d$, centered on the earth, that is occupied by the Sun that is,

$$\frac{\delta \omega}{2\pi} = \frac{\pi R_s^2}{2\pi d^2}$$
4.2 cont.

The fraction of the hemisphere of solid angle (i.e., "the sky") that is occupied by the Sun is the same as the fraction of the area of the hemisphere of radius \(d\), centered on the earth, that is occupied by the Sun that is,

\[
\frac{\delta \omega}{2\pi} = \frac{\pi R_s^2}{2\pi d^2}
\]

from which

\[
\delta \omega = \pi \left(\frac{R_s}{d}\right)^2 = \left(\frac{7.00 \times 10^8}{1.50 \times 10^{11}}\right)^2 = 6.84 \times 10^{-5} \text{ sr}
\]

and

\[
I_s = \frac{F}{\delta \omega} = \frac{1368 \text{ W m}^{-2}}{6.84 \times 10^{-5} \text{ sr}} = 2.00 \times 10^7 \text{ W m}^{-2} \text{sr}^{-1}
\]

If an alien living on Mars had done the same calculations – what would be the answer?
Black Body Radiation

A blackbody is a surface that

- completely absorbs all incident radiation, and

- emits radiation at the maximum possible monochromatic intensity in all directions and at all wavelengths.
Black Body Radiation

Teori: From cavity radiation

- Absorberer all innkommende stråling
- Utstråling gjennom hullet er bare avhengig av temperaturen til legemet
- Max Planck (1900): Hypotese basert på kvantifisering av stråling!
Black Body Radiation

A blackbody is a surface that

- completely absorbs all incident radiation, and

- emits radiation at the maximum possible monochromatic intensity in all directions and at all wavelengths.

The intensity of radiation emitted by a blackbody is uniquely determined by the Planck function

\[
B_\lambda(T) = \frac{c_1 \lambda^{-5}}{\pi \left( e^{c_2/\lambda T} - 1 \right)}
\]

\(T\): Temperature (K), \(\lambda\): Wavelength

Constants: \(c_1 = 3.74 \cdot 10^{-16}\) Wm\(^{-2}\) \(c_2 = 1.45 \cdot 10^{-2}\) m K
Fig. 4.5 Blackbody emission (also referred to as the Planck function) for bodies with absolute temperatures as indicated, plotted as a function of wavelength on a linear scale.
 Radiation Transmitted by the Atmosphere

Dowgoing Solar Radiation
70-75% Transmitted

Upgoing Thermal Radiation
15-30% Transmitted

Spectral Intensity

5525K

UV Visible Infrared

Wavelength (µm)

Percent

Total Absorption and Scattering

Stefan Boltzmanns lov

II. STEFAN-BOLTZMANN LAW: \[ I = \int_0^\infty R(\lambda) \, d\lambda = \sigma T^4 \]

\[ I = \int_0^\infty R(\lambda) \, d\lambda = \int_0^\infty \frac{8\pi c}{4} \frac{hc}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1} \, d\lambda \]

Let \( x = \frac{hc}{\lambda k_B T} \) then \( \lambda = \frac{1}{x} \frac{hc}{k_B T} \) and \( d\lambda = -\frac{dx}{x^2} \frac{hc}{k_B T} \) \( (7) \)

Substituting for \( \lambda \) and \( d\lambda \) in terms of \( x \) and \( dx \), the integral in Eq. 6 becomes,

\[ I = \frac{8\pi c}{4} \frac{(k_B T)^4}{(hc)^3} \int_0^{\infty} \frac{x^3 \, dx}{e^x - 1} = \frac{8\pi c}{4} \frac{(k_B T)^4}{(hc)^3} \left[ \pi^4/15 \right] \]

\( (8) \)

So,

\[ I = \frac{2\pi^5}{15} \frac{(k_B T)^4}{c^2 h^3} = \sigma T^4 \]

\( (9) \)

where

\[ \sigma = \frac{2\pi^5}{15} \frac{k_B^4}{c^2 h^3} \]

\( (10) \)

\[ \sigma = \frac{2\pi^5}{15} \frac{(1.381 \times 10^{-23} J/K)^4}{(3.00 \times 10^8 m/s)^2 (6.626 \times 10^{-34} J - sec)^3} = 5.67 \times 10^{-8} \frac{W}{m^2 - K^4} \]

\( (11) \)
Exercise 4.5 Calculate the equivalent blackbody temperature of the solar photosphere (i.e., the outermost visible layer of the Sun) based on the following information. The flux density of solar radiation reaching the earth, \( F_{\text{earth}} \), is 1368 W m\(^{-2}\). The Earth-Sun distance \( d \) is 1.50 \( \times \) 10\(^{11} \) m and the radius of the solar photosphere \( R_s \) is 7.00 \( \times \) 10\(^{8} \) m.

**Solution:** We first calculate the flux density at the top of the layer, making use of the inverse square law (4.7).

\[
F_{\text{photosphere}} = F_{\text{earth}} \left( \frac{R_s}{d} \right)^{-2}
\]

\[
F_{\text{photosphere}} = 1,368 \times \left( \frac{1.50 \times 10^{11}}{7.00 \times 10^{8}} \right)^{2} = 6.28 \times 10^{7} \text{ W m}^{-2}.
\]

From the Stefan-Boltzmann law

\[
\sigma T_E^4 = 6.28 \times 10^{7} \text{ W m}^{-2}
\]

Therefore, the equivalent blackbody temperature is

\[
T_E = \left( \frac{6.28 \times 10^{7}}{5.67 \times 10^{-8}} \right)^{1/4}
\]

\[
= (1108 \times 10^{12})^{1/4}
\]

\[
= 5.77 \times 10^{3}
\]

\[
= 5770 \text{ K}
\]
Table 4.1 Equivalent blackbody temperatures of some of the planets, based on the assumption that they are in radiative equilibrium with the Sun. Astronomical units are multiples of Earth-Sun distance.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Distance from Sun (A.U.)</th>
<th>Albedo</th>
<th>$T_E$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>0.39</td>
<td>0.06</td>
<td>439</td>
</tr>
<tr>
<td>Venus</td>
<td>0.72</td>
<td>0.78</td>
<td>225</td>
</tr>
<tr>
<td>Earth</td>
<td>1.00</td>
<td>0.30</td>
<td>255</td>
</tr>
<tr>
<td>Mars</td>
<td>1.52</td>
<td>0.17</td>
<td>216</td>
</tr>
<tr>
<td>Jupiter</td>
<td>5.18</td>
<td>0.45</td>
<td>105</td>
</tr>
</tbody>
</table>

Hvorfor er $T_E$ lavere for Venus enn for Jorda?

Hvordan er overflatetemperaturen på Venus i forhold til på Jorda?
Exercise 4.7 A small blackbody satellite is orbiting the earth at a distance far enough away so that the flux density of earth-radiation is negligible, compared to that of solar radiation. Suppose that the satellite suddenly passes into the earth’s shadow. At what rate will it initially cool? The satellite has a mass \( m = 10^3 \) kg and a specific heat \( c = 10^3 \) J (kg K\(^{-1}\))\(^{-1}\): it is spherical with a radius \( r = 1 \) m, and temperature is uniform over its surface.

Albedo of the satellite is \( a = 0 \)

\( \frac{dT}{dt} = 0 \) for the satellite before entering the shadow
Solution: Consideration of the energy balance, as in the previous problem, but with an albedo of zero yields an emission \( F = 342 \) W m\(^{-2}\) from the satellite, which implies an equivalent blackbody temperature \( T_E = 279 \) K. When the satellite passes into the earth’s shadow, it will no longer be in radiative equilibrium. The flux density of solar radiation abruptly drops to zero while the emitted radiation, which is determined by the temperature of the satellite, drops gradually as the satellite cools by emitting radiation. The instant that the satellite passes into the shadow, it’s temperature is still equal to \( T_E \), so we can write

\[
m c \frac{dT}{dt} = 4 \pi r^2 \sigma T_E^4
\]

Solving, we obtain \( dT/dt = 4.30 \times 10^{-3} \) s\(^{-1}\) or 15.5 K hr\(^{-1}\)

Print Error: Should of course be K \cdot s\(^{-1}\)
4.3.3 Kirchhoff’s Law

Absorbed flux density: \( F_{\text{abs}} = \alpha \cdot F_{\text{inn}} \)

Emitted flux density: \( F_{\text{ut}} = \varepsilon \cdot \sigma T^4 \)

Kirchhoff’s Law:

Emissivity (\( \varepsilon \)) = Absorpsivity (\( \alpha \))
At equilibrium: \( T_A = T_{BB} \)

If \( T_A > T_{BB} \) at equilibrium then there must be a state where \( T_A - dT > T_{BB} + dT \), and where heat is transported spontaneously from a colder body to a warm body. This is not allowed according to the second law of thermodynamics.

A absorbs \( \alpha \% \) of incoming radiation \( \Rightarrow F_A(\text{abs}) = \alpha \cdot \sigma T_{BB}^4 \)

At equilibrium (constant temperature) equal amounts of energy is emitted as is absorbed \( \Rightarrow F_A(\text{em}) = \varepsilon \cdot \sigma T_A^4 \)

\( \Rightarrow \varepsilon = \alpha \) Kirchoffs law,

NB!! Valid also at non-equilibrium AND \( \varepsilon_\lambda = \alpha_\lambda \)
4.3.3 Kirchhoff’s Law

Does Kirchhoff’s law imply that a body always absorbs and emits equal amounts of radiative energy?
The greenhouse Effect
Exercise 4.8 The Earth-like planet considered in Exercise 4.6 has an atmosphere consisting of multiple isothermal layers, each of which is transparent to shortwave radiation and completely opaque to longwave radiation. The layers and the surface of the planet are in radiative equilibrium. How is the surface temperature of the planet affected by the presence of this atmosphere?

Strålingslikevekt gjennom alle grenseflater mellom lagene

TOA: \( F_\downarrow = F_\uparrow \)

Mellom lag 1 og 2: \( F_\downarrow (SW) + F_\downarrow (LW) = 2F_\uparrow (LW) \)

Mellom lag 2 og 3: \( F_\downarrow + 2F_\downarrow = 3F_\uparrow \)

\[ T_s = \left( \frac{(N+1)F}{\sigma} \right)^{1/4} \]  

dvs. \( T_s \) øker med antall lag

Klimaskeptikere sier ofte at fordi det er allerede er nok CO\(_2\) i atm slik at all stråling absorberes vil ikke \( T_s \) kunne øke.

Hva mener du?
- Top layer is always at $T_E=255K$
- Increasing optical thickness $\Rightarrow$ Thinner (geometrical) layers $\Rightarrow$ Increasing lapse rate $\Rightarrow$ Convection

$$T_s = ((N+1) \times F / \sigma)^{1/4}$$