Delegation of Regulation*

Tapas Kundu†‡  Tore Nilssen§

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Abstract

We discuss a government’s incentives to delegate regulation to bureaucrats. The government faces a trade-off in its delegation decision: bureaucrats have knowledge of the firms in the industry that the government does not have, but at the same time, they have other preferences than the government, so-called bureaucratic drift. We study how the bureaucratic drift and the firm’s private information interact to affect the incentives to delegate regulation. Furthermore, we discuss how constrained delegation, i.e., delegation followed by laws and regulations that restrict bureaucratic discretion, increases the scope of delegation. We characterize the optimal delegation rule and show that, in equilibrium, three different regimes can arise that differ in the extent of bureaucratic discretion: no delegation, strict delegation, and weak delegation. We find that bureaucratic discretion reduces with bureaucratic drift.

Because of the nature of the regulation problem, the effect of increased uncertainty about the firm’s technology on the bureaucratic discretion depends on how that uncertainty changes.

Keywords: Bureaucracy; Delegation; Regulation.

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†Oslo Business School, Oslo Metropolitan University. Email: tapas.kundu@oslomet.no.
‡School of Business and Economics, UiT the Arctic University of Norway.
§Department of Economics, University of Oslo. Email: tore.nilssen@econ.uio.no.
1 Introduction

Governments often delegate to bureaucrats to deal with industry. Delegation has contrasting
effects, though. On one hand, society benefits from bureaucrats’ industry-specific knowledge. On
the other hand, society loses control over policy as non-elected officials will be making decisions.
So when and how should such regulatory decisions be delegated? Without delegation, an incom-
pletely informed government will have to resort to formulating a menu-based regulatory policy,
so that low-cost firms receive an information rent and high-cost firms’ production is distorted;
see, e.g., Baron [3]. With delegation, regulation is carried out by an informed bureaucrat, and
there is no longer a need to provide low-cost firms with an information rent. But the bureau-
crat, if she is biased, will distort production for both low-cost and high-cost firms, relative to
the government’s first best. In order to restrain these distortions, while still benefiting from the
bureaucrat’s knowledge, the government can introduce restrictions on the bureaucrat’s conduct,
which we call constrained delegation: various laws and rules to go with the bureaucrat’s license
to deal with industry.

In this paper, we set up a model to discuss which kind of such restrictions the government may
choose. Our model has two key components: a regulated firm with private information about its
production technology, and a bureaucrat who, if delegated the power to do so, will carry out the
regulation of the firm on behalf of the government. The firm provides a public project and has
private information about its production technology. Its production costs are a function of that
technology as well as of the effort that the firm puts into the production. While the government
observes the quality of the project as well as the firm’s costs, it cannot observe the firm’s efforts.
And it only knows the firm’s technology up to a probability distribution. In particular, the firm
is one of two types, the low-cost and the high-cost types. Without the presence of the bureaucrat,
this is a problem of regulation under asymmetric information originally studied by Laffont and
Tirole [18, 22]; like them, we assume there is a positive cost of public funds, so that transfers to
the firm are costly to society.

The bureaucrat, if she is hired, is not a strict adversary to the government. Rather, she differs
from it merely by putting a higher value on the project’s quality than the government does; this
aspect of our model catches what in the political-science literature is called bureaucratic drift.1
In addition, the bureaucrat has more information about the industry than the government does.
In order to make our point in a simple manner, we make the extreme assumption that the
bureaucrat has complete information about the firm’s technology. The implication is that the
bureaucrat can contract with the firm without offering it an information rent if it is low-cost or a
distorted contract if it is high-cost. Finally, we disregard any payment from the government to the
bureaucrat, in effect assuming that incentivizing her through a contract is not feasible; in this, we
are aligned with the literature on the delegation problem, originally studied by Holmström [12].

We find that the government will choose one of three options. One is not to delegate, because

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1See, e.g., Huber and Shipan [13] and Moe [26] for reviews.
the bureaucrat’s bias is too costly. A second option is what we call weak delegation: the government puts a cap on the bureaucrat’s choice set, so that undistorted production by a low-cost firm is ensured; this happens when the bureaucrat’s bias is less costly. But we also find scope for a third option: When it is likely that the firm is high-cost, and/or the distortion that an unrestrained bureaucrat would impose on this firm type is big, the government will choose a stricter cap, which is based on the firm’s expected technology; we call this strict delegation.

The need for capping the bureaucrat’s actions based on expected technology rather than on that of a low-cost firm through strict delegation stems from the government’s informational disadvantage, leading to cases where capping based on the firm being low-cost is ineffective. The occurrence of strict delegation has two implications that distinguish our model from most of the previous literature. First, strict delegation implies that there is distortion at the top, in the sense that the firm, when it is low-cost, does not offer the project at the first-best quality level. This is in contrast to most models of regulation under asymmetric information, where distortions are imposed only on firm types other than the most effective one.

Secondly, strict delegation occurs mainly when the probability of the firm having high cost is high. This way, the possibility of strict delegation impacts on how the government’s incentives to delegate respond to changes in uncertainty. In our model, it is not the case that the government necessarily responds to reduced uncertainty by allowing more delegation. Rather, we find that, if the government becomes more certain that the firm has high cost, it will likely go for strict delegation, while it is more interested in weak delegation if the firm most likely has low cost.

Our paper relates to several strands of literature. Firstly, there is a large literature in political science on when and how to delegate decision power to bureaucrats; see Bendor, et al. [4], Huber and Shipan [13], Gailmard and Patty [8], and Moe [26] for summaries of this literature, and in particular Gilardi [9] on the delegation of regulatory tasks. This literature points to several factors influencing the decision to delegate, two of which are particularly pertinent to our analysis. One is the so-called ally principle, which states that there is more delegation, the more aligned the government and the bureaucrat are, or the lower the bureaucratic drift is. This principle shows up clearly in our analysis: the more weight the bureaucrat puts on the project’s quality, the less delegation there will be in equilibrium. The other is called the uncertainty principle: the more uncertain the government is about the effect of the decisions to be made, the more willing it is to delegate those decisions to an informed bureaucrat. In our setting, the government is incompletely informed about the regulated firm’s production technology. The uncertainty is the highest when, in our two-type case, it is equally likely that the firm has low and high costs, respectively. What we find is slightly in contrast to the uncertainty principle. The bureaucratic drift gives rise to an upward bias in quality levels: the bureaucrat prefers higher quality level than the government does. Weak delegation means putting a cap on the bias. When this is not enough, the government may want to resort to strict delegation. However, such strict delegation is based on an ex-ante expectation of the firm’s production technology and does not work well when the firm is likely to have low costs. The upshot is that, in discussions of delegation of
regulatory tasks, one cannot expect the uncertainty principle to hold unmodified.

Secondly, we are related to the literature on the delegation problem, which started out with the seminal work of Holmström [12]; see Alonso and Matouschek [1] and Amador and Bagwell [2] for more recent work. There, a relationship between a principal and an agent is modeled, where incentive contracts are not feasible and the agent is biased and privately informed; all these features are shared with our model, where the two are called government and bureaucrat. Of particular interest is work by Koessler and Martimort [15], Frankel [6, 7], and Letina, et al. [23], where the action space is multi-dimensional, which corresponds to our bureaucrat’s making decisions over a three-dimensional contract space; and work by Melumad and Shibano [25], whose action space is one-dimensional, but who distinguish between decision rules that are what they call communication dependent and those that are communication independent, a distinction that closely resembles the one we make here between weak and strict delegation.

A common theme between these papers and ours is the need to cap the bias. In our case, this means putting an upper bound on quality because the bureaucrat is more interested in quality than is the government. However, the focus is still quite different. The papers cited are mostly interested in finding out whether interval delegation is optimal, meaning that the set of actions that the principal optimally admits is an interval (or the equivalent in multi-dimensional versions). Our model differs from those discussed in this literature in that, without delegation, there is a regulation problem with asymmetric information. Our primary concern is to discuss how delegation of regulation to an informed and biased bureaucrat is best done. In order to do this in a transparent way, we introduce a model with two types of firms, so that the government’s first-best choice is one of two single points in the action (quality-cost-transfer) space. Moreover, we limit the government to put constraints in one dimension only, quality, and we impose on the government a requirement that the constraint is an interval.\(^2\) Of particular interest, relative to the literature on the delegation problem, is our finding of a scope for strict delegation. In particular, it may be optimal to delegate stricter than merely capping the bias: the government’s lack of information about the regulated firm, a feature which is novel to our delegation problem, may cause it to limit the bureaucrat to perform a uniform regulatory policy. Finally, in our analysis, the principal has a natural outside-option: to go about regulating the firm himself by way of a menu of self-selection contracts. This way, we are able not only to discuss what is the best way to delegate, but also whether this optimum delegation is better than no delegation at all.

Thirdly, there is a literature discussing bureaucrats regulating firms. In one strand of this literature, the focus is on how to avoid regulatory capture; see, e.g., Laffont and Tirole [21,22]. In these models, regulation is modeled as a three-tiered principal-agent problem with the bureaucrat in the middle tier, observing the firm’s true type with a certain probability. Regulatory capture is modeled as collusion between the bureaucrat and the firm, and the focus is on how to formulate

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\(^2\)But see our discussion in Section 6.2 of constrained delegation with a constraint on transfers rather than on quality.
contracts with the bureaucrat and the firm that are collusion-proof, thus avoiding regulatory capture. While we certainly believe that regulatory capture is a problem that should be taken seriously, we distract from it here in order to focus on the government’s use of various forms of constrained delegation in order to make delegation less harmful and therefore more useful.\(^3\) In this literature on regulatory capture, delegation is taken as a given, and there is little discussion of how one can limit bureaucrats’ discretion in order to avoid regulatory capture.\(^4\) Moreover, this literature assumes that incentive contracts between the government and bureaucrats are feasible, whereas we let bureaucrats be hired at an unmodeled fixed salary.

Other models of bureaucrats regulating firms include Khalil, et al. [14]. They model a bureaucrat who procures a good from a privately informed firm and who is given a fixed budget. The bureaucrat benefits in part from funds kept in the bureaucracy and not payed out to the firm. Although this is not a model of delegation and the bureaucrat is not informed, as ours is, there are some similarities in result. In their model, the government will keep the bureaucrat’s budget low, to which the bureaucrat may choose to respond by offering the firm a pooling contract. This resembles our strict delegation, where the bureaucrat is tied up and, while not strictly speaking offering a pooling contract, at least is restricted to offer both firm types the same level of quality. The work of Hiriart and Martimort [11] is quite complementary to ours. Like us, they discuss how much discretion the government should give when delegating a regulatory task to a bureaucracy. But that regulatory task is quite different from ours, since the incentive problem involved is one of moral hazard rather than of asymmetric information. Moreover, the bureaucrat’s bias in their model is pro-firm rather than pro-quality. Also, they do not discuss whether or not the government should delegate at all, as we do here. In our companion paper, Kundu and Nilssen [16], we discuss a setting where a privately informed firm needs pollution permits from the government in order to start production. We again discuss the government’s incentives to delegate the regulatory task to an informed and biased bureaucrat.\(^5\)

Finally, our finding that strict delegation may occur in equilibrium implies that there may be “distortion at the top”, i.e., that the quality produced by the low-cost firm may not be first best. This resonates well with several models of regulation under asymmetric information where, for various reasons, the outcome is an equilibrium that is pooling or semi-pooling and hence features distortion at the top. Reasons cited in the literature for pooling regulatory contracts are countervailing incentives among the regulated firms (Lewis and Sappington [24]), type-dependent externalities (Greenwood and McAfee [10]), scope for renegotiation of the regulatory contracts (Laffont and Tirole [20, 22]), and the dynamics of short-term regulatory contracts (Laffont and Tirole [19, 22]). While we hesitate to call strict delegation a pooling outcome, the effect is the

\(^3\)But see our discussion on extrinsically motivated bureaucrats in Section 6.1.

\(^4\)One exception is the work of Laffont and Martimort [17], who discuss how the government can institute multiple regulators of the same firm, in order to reduce bureaucrats’ discretion and this way make regulatory capture more difficult.

\(^5\)See also Naso [27], who builds on our framework to discuss how bureaucratic discretion affects the prevalence of corruption in South Africa.
same: because of the regulated firm’s private information about its technology, the government is not willing to delegate to the bureaucrat unless at the same time being so restrictive that both the low-cost and high-cost contracts are distorted relative to the first-best.

The paper is organized as follows. In the next section, we present our model. While Section 3 contains some preliminary findings, Sections 4 and 5 analyze the full and constrained delegation problems, respectively. In Section 6, we discuss extensions of the analysis relating to bureaucrats’ motivation and to how constrained delegation is carried out. Section 7 concludes. Appendix A contains proofs of our results, while Appendix B contains additional results and proofs.

2 The model

We consider the problem of regulating a firm under asymmetric information. The firm provides a fixed-size public project. The firm’s cost function is given by $cq$, where $q \geq 0$ is the quality of the project. The marginal cost is given by

$$c = \theta - e,$$  \hspace{1cm} (1)

where $\theta > 0$ is an efficiency parameter, and $e \in [0, \theta]$ is the firm’s effort. For a given effort, a high $\theta$ implies a high marginal cost and therefore low cost efficiency. Following Laffont and Tirole [18,22], we assume that the marginal cost and quality of the project are observable and verifiable. However, effort is not observable and the cost parameter $\theta$ is the firm’s private information. For simplicity, we make the assumption that $\theta$ can take only two values, $\{\underline{\theta}, \overline{\theta}\}$, with $0 < \underline{\theta} < \overline{\theta}$. Let $\nu \in (0,1)$ be the probability that the firm is low-cost with type $\theta = \underline{\theta}$, so that the expected value of $\theta$ is $E_\theta \theta = \nu \underline{\theta} + (1 - \nu) \overline{\theta}$.

A regulatory contract is a combination $\alpha = (q, c, t) \in A = \mathbb{R}^3$, where $t$ is the monetary transfer from the government over and above the reimbursement of the project cost $cq$. Given a transfer $t$, the firm’s payoff is

$$U_P = t - \frac{k}{2} e^2,$$  \hspace{1cm} (2)

where $\frac{k}{2} e^2$ is the disutility from effort and $k > 0$ is a constant. We express the firm’s payoff, inserting for $e \in [0, \theta]$ from (1), as a function of the regulatory contract $\alpha$:  

$$U_P(\theta, \alpha) = t - \frac{k}{2} \left(\max\{\theta - c, 0\}\right)^2.$$  \hspace{1cm} (3)

The government raises public funds to compensate costs, and we let $\lambda > 0$ denote the distortionary costs of raising public funds. The project has value $S(q)$ for consumers. We make the following assumption on $S$.\footnote{In effect, we assume that a firm of type $\theta$ can obtain any marginal cost $c \geq \theta$ with zero effort. An alternative would be to consider the range of $e$ to be $(-\infty, \theta]$ with effort cost at zero for any $e \leq 0$.}

\footnote{The restriction on $S''$ ensures a unique solution of the full-information problem. The two restrictions on $S'$}
Assumption 1. (i) $S(0) = 0$; (ii) $S'(0) > (1 + \lambda)\overline{\theta}$; (iii) $S'(k\overline{\theta}) < 0$; and (iv) $S'' < -\frac{1+\lambda}{k}$.

The government’s payoff from a contract $\alpha$ is the social value of the project:

$$U_G(\theta, \alpha) = S(q) - (1 + \lambda)(cq + t) + U_P(\theta, \alpha)$$

$$= S(q) - (1 + \lambda)\left(cq + \frac{k}{2} \left(\max\{\theta - c, 0\}\right)^2\right) - \lambda U_P(\theta, \alpha). \quad (4)$$

The second equality follows from inserting for $t$ from (3).

We assume that the government can either regulate the firm using an incentive compatible regulatory contract menu under asymmetric information or delegate the regulatory decision-making to an independent regulator, a bureaucrat $B$. We assume that the bureaucrat is informed about the firm’s cost parameter. The bureaucrat can therefore implement a type-contingent regulatory policy. If the government delegates, then the bureaucrat has authority to choose a regulatory policy according to her own preferences. We assume that $B$ is intrinsically motivated by the project’s quality. Specifically, the bureaucrat’s payoff from a contract $\alpha$ is a weighted average of the consumers’ utility from quality, $S(q)$, and the government’s payoff:

$$U_B(\theta, \alpha) = \beta S(q) + U_G(\theta, \alpha)$$

$$= (1 + \beta)S(q) - (1 + \lambda)\left(cq + \frac{k}{2} \left(\max\{\theta - c, 0\}\right)^2\right) - \lambda U_P(\theta, \alpha), \quad (5)$$

where $\beta \geq 0$ measures the bureaucratic drift. The higher is $\beta$, the more the bureaucrat is concerned about the project quality.

The game proceeds as follows.

- Stage 1. The government decides whether or not to delegate the decision-making authority to an independent bureaucrat $B$. If it does not delegate, then the authority remains with the government.

- Stage 2. The firm learns its type $\theta$, which can be either $\underline{\theta}$ with probability $\nu$ or $\overline{\theta}$ with probability $1 - \nu$. $B$ also learns the firm’s type.

- Stage 3. The player with decision making authority — government or bureaucrat — determines the regulatory policy.

- Stage 4. Production takes place. Payoffs are realized. The game ends.

We study the Perfect Bayesian Nash equilibrium of the game. We solve the game by backward induction.

and the restriction on $S(0)$ ensure that that unique solution lies in $(0, k\overline{\theta})$. Parallel restrictions to these are found in Laffont and Tirole [18, Assumption 1].
3 Preliminary analysis

We first describe the regulatory contract that the government chooses if it has complete information about \( \theta \). The contract for type \( \theta \) solves the following problem:

\[
\max_\alpha U_G (\theta, \alpha) \quad (6)
\]

subject to \( U_P (\theta, \alpha) \geq 0 \).

We denote the solution with subscript \( GI \). Note that, at the solution, the firm’s participation constraint is binding, which gives \( U_P = 0 \). After replacing \( U_P \) in (6), the objective function is strictly decreasing in \( c \) for \( c > \theta \). Therefore, \( c \leq \theta \) at the optimum, and the first-order condition with respect to \( c \) is

\[
q - k (\theta - c) = 0. \quad (7)
\]

We rewrite the optimization problem (6), inserting for \( c \) from (7), as

\[
\max_q S(q) - (1 + \lambda) \left( \theta q - \frac{q^2}{2k} \right). \quad (8)
\]

By Assumption 1, (8) has a unique solution \( q_{GI}(\theta) \in (0, k\theta) \) to the first-order condition

\[
S'(q) = (1 + \lambda) \left( \theta - \frac{q}{k} \right). \quad (9)
\]

It follows from (9) and Assumption 1 that \( \frac{\partial}{\partial \theta} q_{GI}(\theta) < 0 \); a low-cost firm has higher quality in equilibrium. The marginal costs and the transfer are given by

\[
c_{GI}(\theta) = \theta - \frac{q_{GI}(\theta)}{k}, \quad (10)
\]

\[
t_{GI}(\theta) = \frac{k}{2} (\theta - c_{GI}(\theta))^2 = \frac{q^2_{GI}(\theta)}{2k}. \quad (11)
\]

The complete-information contract \( \alpha_{GI}(\theta) \) is given by \( \left( q_{GI}(\theta), \theta - \frac{q_{GI}(\theta)}{k}, \frac{q^2_{GI}(\theta)}{2k} \right) \). \( G \)'s ex-ante expected payoff under full information is

\[
U^{FI}_G = \nu U_G (\bar{\theta}, \alpha_{GI}(\bar{\theta})) + (1 - \nu) U_G (\bar{\theta}, \alpha_{GI}(\bar{\theta})). \quad (12)
\]

3.1 Regulation by the government under asymmetric information

With no delegation at stage 1, the uninformed government offers an incentive-compatible pair of contracts \( (\alpha, \bar{\alpha}) = \left( (q, c, \bar{q}), (\overline{q}, \overline{c}, \overline{T}) \right) \) to the firm at stage 3. The contract pair solves the following problem:
\[
\max_{\alpha, \bar{\alpha}} \nu U_G (\theta, \alpha) + (1 - \nu) U_G (\bar{\theta}, \bar{\alpha})
\]

subject to

\[
U_G (\theta, \alpha) \geq U_G (\bar{\theta}, \bar{\alpha}), \quad \text{(ICH)}
\]

\[
U_G (\theta, \alpha) \geq U_G (\bar{\theta}, \bar{\alpha}), \quad \text{(ICL)}
\]

\[
U_G (\theta, \alpha) \geq 0, \quad \text{(IRH)}
\]

\[
U_G (\theta, \alpha) \geq 0, \quad \text{(IRL)}
\]

where (ICH) and (ICL) are the two firm types’ incentive-compatibility constraints and (IRH) and (IRL) are their individual-rationality constraints. We denote the solution with subscript GN. We characterize the optimal contract menu in Lemma A.1 in Appendix A.

Define \( \triangle \theta := \theta - \bar{\theta} \) and \( \eta := \eta (\nu) = \frac{\nu \lambda}{1 - \nu \lambda}, \) and let \( \nu^* \in [0, 1) \) be the \( \nu \) that uniquely solves

\[
S' (k \triangle \theta (\eta (\nu) + 1)) = (1 + \lambda) \theta.
\]

(14)

The following proposition compares the optimal contracts under asymmetric information with the full-information ones.

**Proposition 1.** Assume that the government regulates the firm under asymmetric information. The contract pair \((\alpha_{GN} (\theta), \alpha_{GN} (\bar{\theta}))\) offered to the firm has the following properties:

\[
q_{GN} (\theta) = q_{GI} (\theta), \quad c_{GN} (\theta) = c_{GI} (\theta), \quad \text{and}
\]

\[
q_{GN} (\bar{\theta}) < q_{GI} (\bar{\theta}), \quad c_{GN} (\bar{\theta}) > c_{GI} (\bar{\theta}).
\]

Moreover, the government offers to the low-cost firm an information rent that is given by

\[
\begin{cases}
\frac{k \triangle \theta}{2} (\theta + \bar{\theta} - 2c_{GN} (\bar{\theta})), & \text{if } \nu \leq \nu^*; \\
\frac{k}{2} (\theta - c_{GN} (\bar{\theta}))^2, & \text{if } \nu > \nu^*.
\end{cases}
\]

In absence of complete information, there is a downward distortion in the quality set for the high-cost firm, and an information rent is offered to the low-cost firm. The information rent is decreasing in marginal costs. For \( \nu \geq \nu^* \), the low-cost firm can choose the contract set for the high-cost firm at zero effort cost, thereby making the government set an information rent identical to the transfer to the high-cost firm.

The government’s expected payoff under no delegation is

\[
U_{ND}^G = \nu U_G (\theta, \alpha_{GN} (\theta)) + (1 - \nu) U_G (\bar{\theta}, \alpha_{GN} (\bar{\theta})).
\]

(15)
3.2 Regulation by a bureaucrat

Consider a case where an informed bureaucrat regulates the firm without any restrictions. The contract for type $\theta$ solves the following problem:

$$\max_{\alpha} U_B (\theta, \alpha)$$

subject to $U_P (\theta, \alpha) \geq 0$.

We denote the solution with subscript $BI$. We characterize the optimal contract in Lemma A.2 in Appendix A. The following proposition compares the optimal contract with the full-information contract.

**Proposition 2.** Assume that the government delegates the decision-making authority to a bureaucrat. Then, for $\theta \in \{\theta, \overline{\theta}\}$,

$$q_{BI}(\theta) > q_{GI}(\theta), \ c_{BI}(\theta) < c_{GI}(\theta).$$

The bureaucrat shares no information rent with the firm.

Because of her intrinsic motivation, the bureaucrat’s choice of quality level is above the government’s full-information choice, which, in turn, implies that $c_{BI}(\theta) < c_{GI}(\theta)$. Together, Propositions 1 and 2 imply the following:

$$q_{GN}(\theta) = q_{GI}(\theta) < q_{BI}(\theta),$$

(17)

$$q_{GN}(\overline{\theta}) < q_{GI}(\overline{\theta}) < q_{BI}(\overline{\theta}).$$

(18)

4 Full delegation

**Full delegation** refers to a case where the government delegates decision-making authority to a bureaucrat without imposing any constraint on the latter’s choice set. The government’s expected payoff under full delegation is

$$U_{G}^{FD} = \nu U_G (\theta, \alpha_{BI}(\theta)) + (1 - \nu) U_G (\overline{\theta}, \alpha_{BI}(\overline{\theta})).$$

(19)

The condition under which the government prefers no delegation to full delegation is

$$\triangle D := U_{G}^{ND} - U_{G}^{FD} > 0.$$ 

(20)

Below, we discuss how the sign of $\triangle D$ changes with respect to $\beta$ and $\nu$.

The effect of $\beta$ is straightforward. The government’s payoff under full delegation decreases with $\beta$ whereas $\beta$ has no impact on that payoff under no delegation. Therefore, the government prefers no delegation to full delegation if and only if $\beta$ is above a threshold. This finding is similar
to the *ally principle* (e.g., Huber and Shipan [13]), which suggests the government prefers to give more discretion to more aligned bureaucrats.

How does \( \nu \) affect delegation? In Lemma A.3 in Appendix A, we show that \( U_G^{ND} \) is strictly convex in \( \nu \), which in turn implies that \( \Delta D \) is strictly convex in \( \nu \). Furthermore, \( \Delta D \) takes positive values at \( \nu = 0 \) and \( \nu = 1 \). Therefore, it can only take negative values at an intermediate range of \( \nu \), i.e., when the uncertainty about the firm’s type is high. This is because the government’s benefit from the bureaucrat’s informational advantage is high in situations with high uncertainty. Our result is in line with the *uncertainty principle*, which suggests that the government prefers more bureaucratic discretion in situations with high uncertainty (Huber and Shipan [13]).

The following Proposition documents the above findings.

**Proposition 3.** Consider the game in which the government chooses between the alternatives of full delegation and no delegation. The equilibrium is characterized as follows:

(i) For a given \( \nu \), full delegation occurs if and only if \( \beta \leq \beta^{FD} \) for some \( \beta^{FD} > 0 \).

(ii) For a given \( \beta \), full delegation occurs if and only if \( \nu^{FD} \leq \nu \leq \nu^{FD} \) for some \( 0 < \nu^{FD} \leq \nu^{FD} < 1 \). The interval \([\nu^{FD}, \nu^{FD}]\) can be a null set.

Consider a numerical example depicted in Figure 1 with primitives satisfying Assumption 1. Panels a and b plot \( \Delta D \) against the bureaucratic drift \( \beta \) and the probability of low-cost type \( \nu \), respectively. Panel c plots the government’s preferences over full delegation and no delegation in \((\nu, \beta)\) space. Note that, in accordance with Proposition 3, there is in Panels b and c a range of \( \nu \) close to 1, in which \( \Delta D \) is positive and hence, no delegation is optimal. As this example indicates, the midrange of \( \nu \) for which full delegation is optimal may well be skewed, in this case towards high values of \( \nu \).
5 Constrained delegation

The government can improve its payoff from delegation by restricting the bureaucrat’s choice set. As the bureaucrat has an intrinsic interest in the project’s quality, her preferred quality level is always above that of the government. The government can therefore improve its payoff by imposing an upper bound on the bureaucrat’s choice of this level. But being uninformed, it cannot impose type-dependent bounds. In order to study the government’s interest in setting bounds on quality levels, we will be considering a bureaucrat choosing regulatory contracts $\alpha (\theta) = (q (\theta), c (\theta), t (\theta))$, $\theta \in \{\overline{\theta}, \underline{\theta}\}$ under the constraint that $q (\theta) \in [q_1, q_2] \subseteq [0, k\underline{\theta}]$; it is this constraint that we call constrained delegation.

This notion of constrained delegation resembles interval delegation (see, e.g., Alonso and Matouschek [1] and Amador and Bagwell [2]). Since the task to be delegated is one of regulation, we have on one hand a multi-dimensional action space and on the other hand a two-type information issue. Here, we do not consider whether interval regulation, or any multidimensional equivalent (such as, e.g., in Frankel [7]), is optimal and limit our attention to the above notion of constrained delegation.$^8$

Below we first look at how constrained delegation affects the bureaucrat’s choice of regulation contracts. Her optimal contract for type $\theta$ solves the following problem:

$$\max_{\alpha} U_B (\theta, \alpha)$$
$$\text{subject to } U_P (\theta, \alpha) \geq 0 \text{ and } q \in [q_1, q_2].$$

Let $\alpha^{P}_{BI} (\theta, q_1, q_2)$ denote the optimal contract. We characterize this contract in Lemma A.4 in Appendix A. We show there that the bureaucrat’s choice of contract under constrained delegation coincides with her choice under full delegation if $q_{BI} (\theta)$ lies in the bounded interval $[q_1, q_2]$; otherwise, the optimal choice lies at the boundaries. The government can therefore affect her choice by manipulating $q_1$ and $q_2$. The government’s choice of bounds $q_1$ and $q_2$ solves the following problem:

$$\max_{q_1, q_2} \nu U_G (\theta, \alpha^{P}_{BI} (\theta, q_1, q_2)) + (1 - \nu) U_G (\overline{\theta}, \alpha^{P}_{BI} (\overline{\theta}, q_1, q_2)) .$$

We characterize the government’s choice of the lower and the upper bound in Lemmas A.5 and A.6 in Appendix A. It follows from these lemmas that, for any given upper bound $q_2$, the government’s payoff is maximized at any $q_1 \leq \min \{q_2, q_{BI} (\overline{\theta})\}$. Disregarding the government’s indifference, we simply put its choice at $q_1 = q_{GI} (\overline{\theta})$.\(^9\) Recall that $q_{GI} (\overline{\theta})$ is the government’s

---

$^8$The alternative form of constrained delegation, putting constraints on transfers rather than on quality would not be substantially different, is discussed in Section 6.2 below. A discussion of doing both, that is, having constraints on both quality and transfers, would require enriching the present model to have more than two firm types and is left for future research.

$^9$We disregard the possibility $q_1 = q_2 < q_{GI} (\overline{\theta})$, since in this case, the government’s payoff is strictly increasing in $q_2$. 

preferred quality for the high-cost firm under full information, and that $q_{GI} (\overline{\theta}) < q_{BI} (\overline{\theta})$. The analysis of the optimal upper bound shows that, when the government delegates with a constraint, two possibilities may arise.

- **Weak Delegation** (WD): In this regime, the government chooses $q_1 = q_{GI} (\overline{\theta})$ and $q_2 = q_{GI} (\overline{\theta})$. In response, the bureaucrat sets $q^P_{BI} (\overline{\theta}, q_1, q_2) = q_{GI} (\overline{\theta})$ and $q^P_{BI} (\overline{\theta}, q_1, q_2) = q_{BI} (\overline{\theta})$. The government implements the full-information contract if the firm is low-cost. There is distortion at the contract offered to a high-cost firm, as $q_{BI} (\overline{\theta}) > q_{GI} (\overline{\theta})$. We refer to this regime as weak delegation. The government’s expected payoff under weak delegation is

$$U^{WD}_G = \nu U_G (\overline{\theta}, \alpha_{GI} (\overline{\theta})) + (1 - \nu) U_G (\overline{\theta}, \alpha_{BI} (\overline{\theta})).$$

\hfill (23)

- **Strict Delegation** (SD): In this regime, the government chooses $q_2 = q_{GI} (E_{\theta} \overline{\theta}) < q_{BI} (\overline{\theta})$. In response, the bureaucrat sets $q^P_{BI} (\overline{\theta}, q_1, q_2) = q^P_{BI} (\overline{\theta}, q_1, q_2) = q_{GI} (E_{\theta} \overline{\theta})$, resulting in a uniform quality for both types of firm. The government’s choice of $q_2$ is the optimal uniform quality level. Also note that the bureaucrat’s choice of quality does not depend on the lower bound $q_1$ for any $q_1 \leq q_{GI} (E_{\theta} \overline{\theta})$. In this case, we can therefore write the government’s optimal delegation strategy as setting $q_1 = q_2 = q_{GI} (E_{\theta} \overline{\theta})$, and the bureaucrat’s discretion is limited to setting the transfer. We refer to this regime as strict delegation. The government’s expected payoff under strict delegation is

$$U^{SD}_G = \nu U_G (\overline{\theta}, \alpha_{GI} (E_{\theta} \overline{\theta})) + (1 - \nu) U_G (\overline{\theta}, \alpha_{BI} (E_{\theta} \overline{\theta})) = U_G (E_{\theta} \overline{\theta}, \alpha_{GI} (E_{\theta} \overline{\theta})).$$

\hfill (24)

The following proposition documents the above findings.

**Proposition 4.** Suppose that the government delegates with the constraint that $q (\overline{\theta}), q (\overline{\theta}) \in [q_1, q_2] \subseteq [0, k_{\theta}]$. The government’s constrained-delegation rule takes one of the following two forms:

(i) **Weak Delegation:** $q_1 = q_{GI} (\overline{\theta})$ and $q_2 = q_{GI} (\overline{\theta})$, in which case the government implements the full-information contract to a low-cost firm and there is distortion to the contract offered to a high-cost firm;

(ii) **Strict Delegation:** $q_1 = q_2 = q_{GI} (E_{\theta} \overline{\theta})$, in which case the government implements a uniform quality.

Based on the government’s expected payoffs in the various cases, we observe three possible regimes in equilibrium — weak (WD), strict (SD), and no delegation (ND), with expected payoffs $U^{WD}_G, U^{SD}_G$, and $U^{ND}_G$, respectively. The following lemma compares the government’s payoff in the various regimes. Define

$$\overline{\beta} := \min \{ \beta > 0 : q_{BI} (\overline{\theta}) \geq q_{GI} (\overline{\theta}) \}. $$

\hfill (25)
Lemma 1. Consider the constrained delegation game.

(i) \( \exists \varphi^{SD}(\beta) \in (0, 1], \) increasing in \( \beta, \) such that \( U_G^{SD} \geq U_G^{WD} \) if and only if \( \nu \leq \varphi^{SD}(\beta). \)

Furthermore, \( \varphi^{SD}(\beta) < 1 \) for \( \beta < \overline{\beta}, \) and \( \varphi^{SD}(\beta) = 1 \) for \( \beta \geq \overline{\beta}. \)

(ii) \( \exists \varphi^{ND}(\beta) \in (0, 1], \) increasing in \( \beta, \) such that \( U_G^{ND} \geq U_G^{WD} \) if and only if \( \nu \leq \varphi^{ND}(\beta). \)

The following Proposition describes the equilibrium regimes.

Proposition 5. Consider the game in which the government chooses between constrained delegation and no delegation.

(i) For a given \( \beta, \) weak delegation occurs in equilibrium if and only if \( \nu \geq \underline{\nu}^{WD}(\beta) := \max\{\varphi^{SD}(\beta), \varphi^{ND}(\beta)\}, \) and strict or no delegation otherwise, with \( \varphi^{SD}(\beta) \) and \( \varphi^{ND}(\beta) \) defined in Lemma 1.

(ii) For a given \( \nu, \) if ND (SD) occurs in equilibrium for some \( \beta, \) then SD (ND) cannot occur in equilibrium for any \( \beta. \)

Note that \( \underline{\nu}^{WD}(\beta), \) defined in part (i) of Proposition 5, is increasing in \( \beta, \) since both \( \varphi^{SD}(\beta) \) and \( \varphi^{ND}(\beta) \) are. Thus, \( \underline{\nu}^{WD}(\beta) \) delineates, in \((\nu, \beta)\) space, a region of high \( \nu \) and low \( \beta \) for which weak delegation is chosen by the government and a region of low \( \nu \) and high \( \beta \) for which either no or strict delegation is chosen. From part (ii) of Proposition 5, we note that, in the region where there is either no or strict delegation, a delineation between the two regimes, if there is one, is a vertical line; this is essentially because both \( U_G^{ND} \) and \( U_G^{SD} \) are independent of \( \beta. \)

Figure 2 illustrates this with the same numerical example as in Section 4. Panel a of the figure plots the payoff differences between regimes, \( U_G^{SD} - U_G^{WD}, U_G^{ND} - U_G^{WD}, U_G^{ND} - U_G^{SD} \), against the bureaucratic drift \( \beta, \) for the case when \( \nu = 0.5. \) In this example, we find \( \overline{\beta} = 1.46. \) Since \( q_{BI}(\overline{\beta}) \geq q_{GI}(\overline{\beta}) \) for \( \beta \geq \overline{\beta}, \) the bureaucrat implements a uniform quality \( q_{GI}(\overline{\beta}) \) for both types under WD. Consequently, the government’s payoff from WD becomes independent of \( \beta, \) which is reflected in this figure, where the payoff difference functions are parallel to the horizontal axis for \( \beta \geq 1.46. \) Panel b plots the payoff differences between regimes, \( U_G^{SD} - U_G^{WD}, U_G^{ND} - U_G^{WD}, U_G^{ND} - U_G^{SD} \), against the probability \( \nu \) of low-cost type, for the case when \( \beta = 0.5. \)

The analysis of the example continues in Figure 3. Panel a there illustrates Lemma 1 and shows the parameter values at which the government is indifferent between various regimes in \((\nu, \beta)\) space. Specifically, the dotted orange curve represents \( \varphi^{SD}(\beta) \), the parameter values at which the government receives the same payoff under WD and SD, while the dashed blue curve represents \( \varphi^{ND}(\beta) \), the parameter values at which the government receives the same payoff under WD and ND. Note that, at \( \beta \geq \overline{\beta}, \varphi^{SD}(\beta) = 1. \) In our example, \( U_G^{ND} - U_G^{SD} \), the payoff difference between the two regimes ND and SD, changes sign only once, and is positive for low values of \( \nu \) and negative for high values of \( \nu. \) This can be seen from the dotted green curve in Panel b of Figure 2 and the vertical line in Panel a of Figure 3.

Panel b of Figure 3 illustrates Proposition 5 and plots the equilibrium regimes in \((\nu, \beta)\) space. From the construction of \( \varphi^{SD}(\beta) \) and \( \varphi^{ND}(\beta) \) in Lemma 1, we have that ND is preferred to SD by the government for a given \( \beta \) if and only if, for that \( \beta, \varphi^{SD}(\beta) \geq \varphi^{ND}(\beta). \) It follows
that vertical lines in \((\nu, \beta)\) space delineating ND and SD regions, such as the one in this panel, can only occur for values of \(\beta\) such that \(\nu^{SD}(\beta) = \nu^{ND}(\beta)\). In the panel, we have that strict delegation is preferred to no delegation when \(\nu\) is sufficiently high. We are not able to show that this holds in general. What we do know, however, is that, if, for some value \(\lambda'\) of the marginal cost of public funds, SD is preferred to ND as \(\nu\) approaches 1, then this is the case also for any \(\lambda \geq \lambda'\); this follows from Lemma B.2 in Appendix B. Since we, in this example, put \(\lambda = 0.04\), this result implies that the property holds for all \(\lambda \geq 0.04\), holding fixed the other parameter values specified in the example.

![Panel a. Payoff-differences against \(\beta\), for \(\nu = 0.5\)](image1)

![Panel b. Payoff-differences against \(\nu\), for \(\beta = 0.5\)](image2)

**Figure 2:** Payoff-differences

*Notes.* Specification: \(S(q) = 180 \ln (1 + q) - q^2, k = 1, \bar{\theta} = 12, \bar{\theta} = 10, \) and \(\lambda = 0.04\).

To summarize our findings, bureaucratic discretion is higher under weak delegation than under strict delegation, and there is, of course, zero bureaucratic discretion when there is no delegation. Taken together, Propositions 4 and 5 detail the extent of bureaucratic discretion that the government chooses. The government’s benefit from weak delegation is the ability to implement the first-best contract if the firm is low-cost without spending any information rent on that type. However, with weak delegation, there is a distortion in the contract offered to a high-cost firm. This distortion depends on the bureaucratic drift, measured by \(\beta\). It follows that, when the high-cost type is likely and/or the distortions are high, the government is better off giving up weak delegation and instead opt for strict delegation, where quality levels for both firm types are distorted, or no delegation, where the first-best quality level for the low-cost type comes at the cost of an information rent.

We see that bureaucratic discretion reduces with bureaucratic drift \(\beta\); this is in line with the ally principle (Epstein and O’Halloran [5]; Huber and Shiplan [13]). With either strict or no delegation, the government’s payoff is independent of \(\beta\); thus, for a given \(\nu\), if weak delegation is dominated, then the government chooses either always no delegation or always strict delegation.
Bureaucratic discretion is also affected by uncertainty. However, when constrained delegation is feasible, the effect on the equilibrium delegation regime of a change in that uncertainty depends on how uncertainty changes — whether it happens by the firm becoming more likely to be low-cost or more likely to be high-cost. In particular, if the firm is more likely to be low-cost, so that $\nu > \nu^{WD}(\beta)$, then the government chooses weak delegation. This is because weak delegation, without an information rent, implements the full-information contract for the low-cost firm, which is the likely firm type when $\nu$ is high. In contrast, if a firm is more likely to be high-cost, so that $\nu \leq \nu^{WD}(\beta)$, then the government chooses to give the bureaucrat less discretion, through either strict or no delegation. It follows that the uncertainty principle of Bendor, et al. [4] and Huber and Shipan [13] does not carry over to a setting where the task to be delegated is that of regulation under asymmetric information. This is in contrast to our analysis in Section 4, where the only feasible delegation was full delegation, and where the outcome is quite close to the uncertainty principle.

Interestingly, when strict delegation is chosen by the government, the classic result of regulation theory, that the optimum contract features no distortion at the top, no longer holds: with high bureaucratic drift and a high probability of the firm being high-cost, the government prefers putting such a strict cap on the bureaucrat’s activities that contracts are distorted for both firm types. In this way, allowing for constrained delegation opens up for interesting theoretical predictions on how the government tackles the challenge of regulating industry.

Figure 3: Delegation regimes

Notes. Specification: $S(q) = 180 \ln (1 + q) - q^2$, $k = 1$, $\theta = 12$, $\bar{\theta} = 10$, and $\lambda = 0.04$. 

Panel a. Preferences over delegation regimes in $(\nu, \beta)$ space

Panel b. Equilibrium delegation in $(\nu, \beta)$ space
6 Discussion

Here, we extend the analysis in two directions. First, in Section 6.1, we discuss another motivation for bureaucratic drift than the bureaucrat’s intrinsic preference for quality. Next, in Section 6.2, we discuss the consequences of having constrained delegation by putting constraints on transfers rather than on quality.

6.1 Bureaucratic motivation

So far we have assumed that the bureaucrat is intrinsically motivated to have projects of high quality. Our basic model lends itself easily to cases where the bureaucrat is both intrinsically and extrinsically motivated. Consider, for example, a bureaucrat who, in addition to her intrinsic motivation for high quality, receives a bribe from the producer. We show that this can lead to a downward policy bias. In order to motivate this variant of our model, we assume that the producer likes to have high marginal cost as this would keep his efforts down. For simplicity, suppose the producer pays to the bureaucrat an exogenously given cost-contingent bribe

\[ b(c) = \begin{cases} 
bc & \text{if } c \leq \theta \\
 b\theta & \text{if } c > \theta 
\end{cases} \]

with \( b > 0 \). Then, the bureaucrat’s choice of contract solves the following problem:

\[
\max_a U_B(\theta, a) + b(c) \quad (26)
\]

subject to \( U_P(\theta, a) \geq b(c) \).

We denote the solution with subscript \( BI \) and a superscript \( B \). We characterize the optimal contract in Lemma A.7 in Appendix A. The following proposition compares this contract with the one set by a bureaucrat with no extrinsic motivation.

**Proposition 6.** Assume that the government delegates the decision-making authority to an extrinsically-motivated bureaucrat. Then, for \( \theta \in \{\theta, \overline{\theta}\} \),

\[ q_{BI}^B(\theta) < q_{BI}(\theta). \]

Furthermore, \( q_{BI}^B(\theta) \) decreases with \( b \).

Note that \( b \) is an indirect measure of the intensity of extrinsic motivation. Thus, compared to the basic model, an extrinsic motivation in the form of a bribe makes the bureaucrat less interested in quality.

The analysis of the basic model can give us insight into the optimal constrained-delegation problem also in this case. To see this, note that the bureaucrat has discretionary power to choose quality only in the case of weak delegation. If \( b \) is sufficiently low such that \( q_{GI}(\theta) \leq q_{BI}^B(\theta) \) for
both \( \theta \), the following is true: There is an upward bureaucratic policy bias, the cap on quality will be binding under weak delegation, and the government’s incentive for weak delegation will increase compared to the basic model. On the other hand, if \( b \) is sufficiently high such that \( q_{BI}^B (\theta) \leq q_{GI} (\theta) \) for both \( \theta \), then the direction of bureaucratic policy bias is downward, and a floor on quality will be binding under weak delegation. In this case, the government’s preference for weak delegation over other regimes critically depends on \( b \). For high values of \( b \), the downward policy bias can be costly, and the government would either do no delegation or introduce a stringent floor through strict delegation. For intermediate values of \( b \), the direction of the policy bias differs for the two types. If, for example, \( q_{BI}^B (\bar{\theta}) < q_{GI} (\bar{\theta}) < q_{BI}^B (\theta) \), then weak delegation implements the full-information contract. On the other hand, if \( q_{GI} (\bar{\theta}) < q_{BI}^B (\bar{\theta}) < q_{BI}^B (\theta) < q_{GI} (\bar{\theta}) \), then weak delegation results in distortion both at the top and at the bottom, just like strict delegation does.

Here, we introduce a source of bureaucrat’s extrinsic motivation in a very simple way. Interestingly, our findings show that intrinsic and extrinsic motivations can lead to policy biases in different directions. This has implications for how exactly the government sets limits to bureaucratic discretion. For example, in societies where bureaucrats are more vulnerable to accepting bribes, the government mitigates bureaucratic bias using a floor to avoid too low quality. In other societies, the government may prefer to cap the bias to avoid too high quality.

### 6.2 Restrictions on transfers

In this section, we relax the assumption of no constraints on transfers. The bureaucrat can, however, choose the quality and the marginal cost of the project without any constraints.\(^{10}\) We show that the government’s problem of setting bounds on transfers can be reformulated as a problem of setting bounds on quality. To see this, suppose that the bureaucrat chooses regulatory contracts \( \alpha (\theta) = (q (\theta), c (\theta), t (\theta)), \theta \in \{ \theta, \bar{\theta} \} \), under the constraint that \( t (\theta) \in [t_1, t_2] \). Her optimal contract for type \( \theta \) now solves, instead of the one in (21), the following problem:

\[
\max_{\alpha} U_B (\theta, \alpha) \quad \text{subject to } U_P (\theta, \alpha) \geq 0 \text{ and } t \in [t_1, t_2].
\] (27)

Let \( \alpha_{BI}^{RT} (\theta, t_1, t_2) \) denote the bureaucrat’s optimal contract. Then, the government’s choice of bounds \( t_1 \) and \( t_2 \) solves the following problem:

\[
\max_{t_1, t_2} \nu U_G (\bar{\theta}, \alpha_{BI}^{RT} (\bar{\theta}, t_1, t_2)) + (1 - \nu) U_G (\bar{\theta}, \alpha_{BI}^{RT} (\bar{\theta}, t_1, t_2)).
\] (28)

In general, (27) can have multiple locally optimal solutions depending on which constraint binds, and the global solution depends on the parameters in a non-trivial way. However, at the

\(^{10}\)Having constraints on transfers, quality, and marginal costs at the same time would not be possible to discuss in any interesting way without allowing for more than two firm types.
solution of (28), the firm’s participation constraint binds, which allows us to restrict attention to a specific class of local solutions of (27) in order to study the government’s choice of $t_1$ and $t_2$ in (28). The following proposition documents our findings.

**Proposition 7.** At the solution of (28), $\alpha^RT_{BI} (\theta, t_1, t_2)$ can be expressed as $\alpha^P_{BI} (\theta, \sqrt{2kt_1}, \sqrt{2kt_2})$, where $\alpha^P_{BI} (\theta, q_1, q_2)$ is the bureaucrat’s choice of contract in case of delegation with constraint on quality.

From Proposition 7, it follows that the optimization problem in (28), where the government chooses optimal bounds on transfers, can be reformulated as the optimization problem in (22), where the government chooses optimal bounds on quality.

## 7 Concluding remarks

In this paper, we develop a simple model to study a government’s incentives to delegate to a bureaucrat the regulation of an industry. While the bureaucrat has more industry-specific knowledge, her interest may not align completely with that of the government. Our analysis shows how constrained delegation, i.e., delegation followed by laws and rules to restrict bureaucratic discretion, improves the government’s benefit from delegation.

The key result of the paper is the characterization of the optimal constrained-delegation rule. In particular, we point to the occurrence of strict delegation, in which the government caps the bureaucrat’s actions based on expected costs. Such strict delegation has interesting implications. It gives rise to a regulatory contract featuring distortion at the top; and it leads to a modification of the uncertainty principle. We describe how various factors, including bureaucratic drift and the government’s lack of information, affect the delegation rule and, subsequently, the equilibrium regulation policy. We show that, while bureaucratic discretion typically reduces with bureaucratic drift, it is affected in a non-trivial way by changes in the government’s beliefs about the firm’s technology. In particular, allowing bureaucratic discretion is more interesting the more likely it is that the firm is low-cost.

While our analysis provides some normative suggestions for the design of delegation rules, many interesting questions remain unanswered. First, the delegation framework assumes no contractual relationship between the principal and the delegates. While the assumption properly reflects the relationship between a politician and a bureaucrat, there are other situations where the assumption may not be appropriate. Secondly, we do not address the bureaucrat’s incentives for acquiring information. Again, this assumption seems appropriate in situations in which bureaucrats can possibly be hired based on their industry-specific knowledge. In other situations, we might expect that the delegation rule could have a direct effect on her incentive to acquire information. For example, low bureaucratic discretion can demotivate a bureaucrat from a detailed investigation of the firm. In such a situation, the government must take the issue of information acquisition into consideration when designing the delegation rule. Finally, our search
for the optimal delegation rule has been limited in that we have imposed on the constrained delegation that it be an interval and on top of that have allowed constrained delegation in only one dimension: quality. A richer discussion of whether the optimal delegation rule is an interval or, rather, a closed set in our setting of three-dimensional regulation contracts, would require a continuous type space for firms’ private information on technology. We leave these questions for future research.

Appendix A

Appendix A contains the proofs and additional results applied in the proofs. We will begin with a useful concept that is applied in some of these proofs.

**Definition.** Define

\[ f(\theta, q) := U_G(\theta, \alpha), \]

where \( \alpha = \left(q, \theta - \frac{q^2}{2}, \frac{q^2}{2}k\right) \), as the government’s full-information payoff when requiring quality \( q \) from a firm of type \( \theta \).

It follows that

\[ f(\theta, q) = S(q) - (1 + \lambda) \left[ (\theta - \frac{q}{k}) q + \frac{q^2}{2k} \right] = S(q) - (1 + \lambda) \left( \theta q - \frac{q^2}{2k} \right). \quad (A.1) \]

Furthermore, \( \frac{df(\theta, q)}{dq} = S'(q) - (1 + \lambda) \left( \theta - \frac{q}{k} \right) \), and \( \frac{df(\theta, q)}{dq^2} = S''(q) + \frac{k}{2} < 0 \), where the inequality follows from Assumption 1. From (9), we have that

\[ \frac{df(\theta, q)}{dq} \geq 0 \quad \text{if} \quad q \leq q_{GI}(\theta). \quad (A.2) \]

Furthermore, applying the envelope theorem, we get

\[ \frac{d}{d\theta} f(\theta, q_{GI}(\theta)) = -(1 + \lambda) q_{GI}(\theta) < 0, \]

implying that

\[ f(\theta, q_{GI}(\theta)) > f(\overline{\theta}, q_{GI}(\overline{\theta})). \quad (A.3) \]

The following Lemma characterizes the government’s choice of contract under asymmetric information.

**Lemma A.1.** Consider the case of no delegation. The contract pair

\[ (\alpha_{GN}(\overline{\theta}), \alpha_{GN}(\overline{\theta})) = ((q_{GN}(\theta), c_{GN}(\theta), t_{GN}(\theta)), (q_{GN}(\overline{\theta}), c_{GN}(\overline{\theta}), t_{GN}(\overline{\theta}))) \]
that the government offers to the firm is given as follows: \( q_{GN}(\theta) = q_{GI}(\theta) \) uniquely solves
\[
S'(q) = (1 + \lambda) \left( \theta - \frac{q}{k} \right); \quad (A.4)
\]
\( q_{GN}(\theta) \) uniquely solves
\[
S'(q) = \begin{cases} 
(1 + \lambda) \left( \theta - \frac{q}{k} + \eta \Delta \theta \right) & \text{if } \nu \leq \nu^* \\
(1 + \lambda) \left( \theta - \frac{q}{k(n+1)} \right) & \text{if } \nu > \nu^*;
\end{cases} \quad (A.5)
\]
and
\[
c_{GN}(\theta) = \theta - \frac{q_{GN}(\theta)}{k}, \quad (A.6)
\]
\[
c_{GN}(\theta) = \begin{cases} 
\theta - \frac{q_{GN}(\theta)}{k} + \eta \Delta \theta & \text{if } \nu \leq \nu^* \\
\theta - \frac{q_{GN}(\theta)}{k(n+1)} & \text{if } \nu > \nu^* ;
\end{cases} \quad (A.7)
\]
\[
t_{GN}(\theta) = \begin{cases} 
\frac{k}{2} (\theta - c_{GN}(\theta))^2 + \frac{k}{2} (\theta + \theta - 2c_{GN}(\theta)) & \text{if } \nu \leq \nu^* \\
\frac{k}{2} (\theta - c_{GN}(\theta))^2 + \frac{k}{2} (\theta - c_{GN}(\theta))^2 & \text{if } \nu > \nu^* ;
\end{cases} \quad (A.8)
\]
\[
t_{GN}(\theta) = \frac{k}{2} (\theta - c_{GN}(\theta))^2. \quad (A.9)
\]
Furthermore, this contract pair is continuous at \( \nu = \nu^* \).

**Proof of Lemma A.1**

*Proof.* We prove the lemma in the following steps. First, observe that (IRH) and (ICL) imply (IRL). Next, we relax the problem by omitting one incentive-compatibility constraint, solve the relaxed problem, and then check that the optimal solution indeed satisfies the omitted constraint. Since the full-information optimal contract pair satisfies (ICH), but does not satisfy (ICL), we omit (ICH) and consider the following relaxed problem:
\[
\max_{\alpha, \alpha} \nu U_G(\theta, \alpha) + (1 - \nu) U_G(\bar{\theta}, \bar{\alpha}) \quad (A.10)
\]
subject to (IRH) and (ICL).

Observe that both (IRH) and (ICL) are binding at the optimum; otherwise, the government can increase payoff by reducing transfers to both types without affecting the constraints. Binding (IRH) implies \( U_P(\bar{\theta}, \bar{\alpha}) = 0 \), or, equivalently, \( \bar{t} = \frac{k}{2} \left( \max \{ \bar{\theta} - \bar{\alpha}, 0 \} \right)^2 \). Binding (ICL) implies \( U_P(\bar{\theta}, \bar{\alpha}) = U_P(\bar{\theta}, \bar{\alpha}) = \frac{k}{2} \left( \max \{ \bar{\theta} - \bar{\alpha}, 0 \} \right)^2 - \frac{k}{2} \left( \max \{ \bar{\theta} - \bar{\alpha}, 0 \} \right)^2 \). The government, therefore, shares an information rent with the low-cost firm, and this rent is given by
\[
U_P(\theta, \pi) = \begin{cases} 
\frac{k \Delta \theta}{2} (\bar{\theta} - \theta - 2\pi) & \text{if } \pi \leq \theta \\
\frac{k}{2} (\bar{\theta} - \pi)^2 & \text{if } \theta < \pi \leq \bar{\theta} \\
0 & \text{if } \bar{\theta} < \pi 
\end{cases}
\]

After replacing \(U_P(\theta, \pi)\) and \(U_P(\bar{\theta}, \alpha)\) in (A.10), the objective function is strictly decreasing in \(\pi\) for \(\pi > \theta\) and strictly decreasing in \(\pi\) for \(\pi > \bar{\theta}\). Therefore, \(\pi \leq \theta\) and \(\pi \leq \bar{\theta}\) at the optimum, and so we optimize only over this range of parameter values. The expression of the objective function, however, differs depending on whether \(\pi\) is below or above \(\theta\). We first consider the case \(\pi \leq \theta\), solve for the optimal \(\pi\), and derive the condition under which the optimal \(\pi\) is below \(\theta\).

Case 1: If \(\pi \leq \theta\) and \(\pi \leq \bar{\theta}\), then we can rewrite the optimization problem as

\[
\max_{q, \pi} \nu \left[ S(q) - (1 + \lambda) \left( cq + \frac{k}{2} (\theta - c)^2 \right) - \frac{\lambda k \Delta \theta}{2} (\bar{\theta} + \theta - 2\pi) \right] + (1 - \nu) \left[ S(\pi) - (1 + \lambda) \left( c\pi + \frac{k}{2} (\bar{\theta} - \pi)^2 \right) \right].
\]  

(A.11)

The first-order conditions with respect to \(c\) and \(\pi\) are

\[
\frac{q - k(\theta - c)}{\pi - k(\bar{\theta} - \pi) - \eta k \Delta \theta} = 0,
\]

(A.12)

We rewrite (A.11), inserting for \(c\) and \(\pi\) from (A.12) and (A.13) and simplifying, as

\[
\max_{q, \pi} \nu \left[ S(q) - (1 + \lambda) \left( \frac{\theta q}{2k} - \frac{q^2}{2k} \right) \right] - \frac{\nu \lambda k \Delta \theta (\bar{\theta} + \theta)}{2}
\]

\[
+ \nu \lambda k \Delta \theta \left( \frac{\bar{\theta} - \pi}{k} + \eta \Delta \theta \right) + (1 - \nu) \left[ S(\pi) - (1 + \lambda) \left( \frac{\theta q}{2k} - \frac{q^2}{2k} + \frac{k \eta^2 (\Delta \theta)^2}{2} \right) \right].
\]

(A.14)

The first-order conditions with respect to \(q\) and \(\pi\) are

\[
S'(q) - (1 + \lambda) \left( \frac{\theta q}{k} - \frac{q}{k} \right) = 0,
\]

(A.15)

\[
S'(\pi) - (1 + \lambda) \left( \frac{\theta \pi}{k} - \frac{\pi}{k} + \eta \Delta \theta \right) = 0.
\]

(A.16)

By Assumption 1, \(q_{\text{GN}}(\theta)\) and \(q_{\text{GN}}(\bar{\theta})\) are unique solutions of (A.15) and (A.16), respectively. Inserting for \(q_{\text{GN}}(\theta)\) and \(q_{\text{GN}}(\bar{\theta})\) in (A.12) and (A.13), we get the optimal marginal costs. Further,

\[
c_{\text{GN}}(\bar{\theta}) \leq \theta \iff k(\eta + 1) \Delta \theta \leq q_{\text{GN}}(\bar{\theta}),
\]

which, by Assumption 1 and (A.16), gives us \(S'(k(\eta + 1) \Delta \theta) \geq (1 + \lambda) \theta\). As \(\eta\) is strictly increasing in \(\nu\), the above condition is violated for sufficiently high values of \(\nu\), and at \(\nu = \nu^*\),
defined in (14), the condition holds with equality. Therefore, for \( \nu \leq \nu^* \), the solution of the problem is given by (A.15) and (A.16).

Case 2: If \( c \leq \theta \) and \( c > \theta \), we can rewrite the optimization problem as

\[
\max_{\alpha, \alpha' \nu} \left[ S(q) - (1 + \lambda) \left( \alpha q + \frac{k}{2} (\theta - \alpha' \nu)^2 \right) - \frac{\lambda k}{2} (\bar{\theta} - \bar{\nu})^2 \right]
\]

(A.17)

\[
+ (1 - \nu) \left[ S(\bar{q}) - (1 + \lambda) \left( \alpha \bar{q} + \frac{k}{2} (\theta - \alpha' \nu)^2 \right) \right].
\]

The first-order conditions with respect to \( c \) and \( c' \) are

\[
q - k (\theta - \nu) = 0, \quad (A.18)
\]

\[
\bar{q} - k (\eta + 1) (\theta - \nu) = 0. \quad (A.19)
\]

We rewrite (A.17), inserting for \( c \) and \( c' \) from (A.18) and (A.19) and simplifying, as

\[
\max_{\bar{q}, q} \nu \left[ S(\bar{q}) - (1 + \lambda) \left( \bar{q} \theta - \frac{\bar{q}^2}{2k} \right) \right]
\]

(A.20)

\[
+ (1 - \nu) \left[ S(\bar{q}) - (1 + \lambda) \left( \bar{q} \theta - \frac{\bar{q}^2}{2k (\eta + 1)} \right) \right].
\]

The first-order conditions with respect to \( \bar{q} \) and \( q \) are

\[
S'(\bar{q}) - (1 + \lambda) \left( \bar{\theta} - \frac{\bar{q}}{k} \right) = 0, \quad (A.21)
\]

\[
S'(q) - (1 + \lambda) \left( \theta - \frac{q}{k} \right) = 0. \quad (A.22)
\]

By Assumption 1, \( q_{GN} (\theta) \) and \( q_{GN} (\bar{\theta}) \) are unique solutions of (A.21) and (A.22), respectively. Inserting for \( q_{GN} (\theta) \) and \( q_{GN} (\bar{\theta}) \) in (A.18) and (A.19), we get the optimal marginal costs. Further,

\[
c_{GN} (\bar{\theta}) > \theta \iff k (\eta + 1) \Delta \theta > q_{GN} (\bar{\theta}).
\]

By Assumption 1 and (A.22), this gives us \( S'(k (\eta + 1) \Delta \theta) < (1 + \lambda) \theta \), which is equivalent to \( \nu > \nu^* \). Therefore, for \( \nu > \nu^* \), the solution of the problem is given by (A.21) and (A.22).

Together, the two cases characterize the optimal solution as stated in Lemma A.1. Note that the solution also satisfies the omitted constraint (ICH).

Finally, \( \lim_{\nu \rightarrow \nu^*} c_{GN} (\bar{\theta}) = \theta \) and \( \lim_{\nu \rightarrow \nu^*} q_{GN} (\bar{\theta}) = k (\eta + 1) \Delta \theta \), and therefore, \( q_{GN} (\bar{\theta}), c_{GN} (\bar{\theta}) \), and subsequently \( t_{GN} (\bar{\theta}) \) and \( t_{GN} (\bar{\theta}) \), are continuous at \( \nu = \nu^* \), showing that this contract pair is continuous at \( \nu = \nu^* \).

\[\Box\]

Proof of Proposition 1

Proof. Comparing (9) and (A.5) and using Assumption 1, we have that \( q_{GN} (\bar{\theta}) < q_{GI} (\bar{\theta}) \), which,
in turn, implies that $c_{GN} (\bar{\theta}) > c_{GI} (\bar{\theta})$. Comparing (11) and (A.8), we have that $t_{GN} (\bar{\theta}) < t_{GI} (\bar{\theta})$; the difference between them is the information rent given in the proposition. 

The following Lemma characterizes the bureaucrat’s choice of contract.

**Lemma A.2.** Assume that the government delegates the decision-making authority to a bureaucrat. The contract $\alpha_{BI} (\theta) = (q_{BI} (\theta), c_{BI} (\theta), t_{BI} (\theta))$ that the bureaucrat offers to a producer of type $\theta \in \{\underline{\theta}, \bar{\theta}\}$ is given as follows: $q_{BI} (\theta)$ uniquely solves

$$S' (q) = \frac{1 + \lambda}{1 + \beta} \left( \theta - \frac{q}{k} \right) ;$$

(A.23)

and

$$c_{BI} (\theta) = \theta - \frac{q_{BI} (\theta)}{k} ;$$

(A.24)

$$t_{BI} (\theta) = \frac{k}{2} (\theta - c_{BI} (\theta))^2 = \frac{q_{BI}^2 (\theta)}{2k} .$$

(A.25)

**Proof of Lemma A.2**

*Proof.* The bureaucrat’s objective function is decreasing in $t$, so that the firm’s participation constraint will be binding and we can write $U_P = 0$. After replacing $U_P$ in (16), the objective function is strictly decreasing in $c$ for $c > \theta$. Therefore, $c \leq \theta$ at the optimum, and the first-order condition with respect to $c$ is

$$q - k (\theta - c) = 0 .$$

(A.26)

We rewrite (16), inserting for $c$ from (A.26), as

$$\max_q \ (1 + \beta) S (q) - (1 + \lambda) \left( \theta q - \frac{q^2}{2k} \right) .$$

(A.27)

By Assumption 1, (A.27) has a unique solution $q_{BI} (\theta)$ that solves the first-order condition, which is given by (A.23). Using (A.26), the optimal marginal cost is given by (A.24). Further, the binding participation constraint implies (A.25). 

**Proof of Proposition 2**

*Proof.* Comparing (9) and (A.23) and applying Assumption 1, we find that $q_{BI} (\theta) > q_{GI} (\theta)$. Then, by comparing (10) and (A.24), we get that $c_{BI} (\theta) < c_{GI} (\theta)$. Since the participation constraint of a firm of type $\theta$ is binding, there is no information rent. 

The following lemma documents the convexity property of $U^N_D G$. The lemma will be useful in the proofs of Propositions 3 and 5.

**Lemma A.3.** $U^N_D G$, as defined in (15), is strictly convex in $\nu \in [0,1]$. 

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The proof of Lemma A.3 is given in Appendix B. Next, we prove Proposition 3.

**Proof of Proposition 3**

**Proof.** Part (i): We prove part (i) by showing that $\Delta D < 0$ for $\beta = 0$ and $\frac{d\Delta D}{d\beta} > 0$ for $\beta > 0$.

For $\beta = 0$, $U^{FD}_G = U^{FI}_G > U^{ND}_G$, and therefore, $\Delta D < 0$.

To see that $\frac{d\Delta D}{d\beta} > 0$, first note that $U^{ND}_G$ is independent of $\beta$. The government’s expected payoff in the full-delegation regime is given by

\[
U^{FD}_G = \nu \left[ S(q_{BI}(\bar{\theta})) - (1 + \lambda) \left\{ \left( \bar{\theta} - \frac{q_{BI}(\bar{\theta})}{k} \right) q_{BI}(\bar{\theta}) + \frac{q_{BI}^2(\bar{\theta})}{2k} \right\} \right] \\
+ (1 - \nu) \left[ S(q_{BI}(\bar{\theta})) - (1 + \lambda) \left\{ \left( \bar{\theta} - \frac{q_{BI}(\bar{\theta})}{k} \right) q_{BI}(\bar{\theta}) + \frac{q_{BI}^2(\bar{\theta})}{2k} \right\} \right]
\]

\[
= \nu f(\bar{\theta}, q_{BI}(\bar{\theta})) + (1 - \nu) f(\bar{\theta}, q_{BI}(\bar{\theta}))
\]

where $f(\theta, q)$ is given in (A.1). Therefore,

\[
\frac{dU^{FD}_G}{d\beta} = \nu \left[ \frac{df(\bar{\theta}, q_{BI}(\bar{\theta}))}{dq} \cdot \frac{dq_{BI}(\bar{\theta})}{d\beta} \right] + (1 - \nu) \left[ \frac{df(\bar{\theta}, q_{BI}(\bar{\theta}))}{dq} \cdot \frac{dq_{BI}(\bar{\theta})}{d\beta} \right].
\]

We have $\frac{df(\theta, q_{BI}(\theta))}{dq} < 0$, by (17), (18), and (A.2), and $\frac{dq_{BI}(\theta)}{d\beta} = \frac{-S'(q_{BI}(\theta))}{(1 + \beta)S''(q_{BI}(\theta)) + \frac{\lambda}{k}} > 0$ by Assumption 1, part (iv). Together, the two inequalities imply $\frac{dU^{FD}_G}{d\beta} < 0$. Therefore,

\[
\frac{d\Delta D}{d\beta} = \frac{dU^{ND}_G}{d\beta} - \frac{dU^{FD}_G}{d\beta} > 0.
\]

Part (ii): We prove part (ii) by showing that $\Delta D > 0$ for $\nu = 0, 1$, and $\Delta D$ is strictly convex in $\nu$ for $\nu \in [0, 1]$. These properties of $\Delta D$ ensure that $\Delta D$ can be negative only at an interval, if at all.

For $\nu = 0, 1$, $U^{ND}_G = U^{FI}_G > U^{FD}_G$, and therefore, $\Delta D > 0$. To see that $\Delta D$ is strictly convex, note that $\frac{dU^{FD}_G}{d\beta^2} = 0$, and by Lemma A.3, $U^{ND}_G$ is strictly convex in $\nu \in [0, 1]$.

The following Lemma describes the bureaucrat’s optimal choice of contracts under constrained delegation.

**Lemma A.4.** Assume that the government delegates the decision making authority with the constraint that $q \in [q_1, q_2] \subseteq [0, k\theta]$. The bureaucrat’s preferred regulation contract for type
\[ \theta \in \{ \theta, \overline{\theta} \} \] is given by

\[ \alpha^{P}_{BI}(\theta, q_1, q_2) = (q^{P}_{BI}(\theta, q_1, q_2), c^{P}_{BI}(\theta, q_1, q_2), t^{P}_{BI}(\theta, q_1, q_2)), \]

where

\[ q^{P}_{BI}(\theta, q_1, q_2) = \begin{cases} 
q_1, & \text{if } q_1 \geq q_{BI}(\theta); \\
q_2^{P}_{BI}(\theta, q_1, q_2), & \text{if } q_1 < q_{BI}(\theta) < q_2; \\
q_2, & \text{if } q_{BI}(\theta) \geq q_2. 
\end{cases} \] (A.28)

\[ c^{P}_{BI}(\theta, q_1, q_2) = \theta - \frac{q^{P}_{BI}(\theta, q_1, q_2)}{k}; \] (A.29)

\[ t^{P}_{BI}(\theta, d_1, d_2) = \frac{k}{2} (\theta - c^{P}_{BI}(\theta, q_1, q_2))^2 = \frac{(q^{P}_{BI}(\theta, q_1, q_2))^2}{2k}. \] (A.30)

**Proof of Lemma A.4**

Proof. Replacing \( U_{P} = 0 \) in (21), the first-order condition with respect to \( c \) gives \( c = \frac{q}{k} \). Replacing \( c \) by \( \frac{q}{k} \), we rewrite (21) as

\[ \max_{q \in [q_1, q_2]} (1 + \beta) S(q) - (1 + \lambda) \left( \theta q - q^2 \frac{k}{2k} \right). \] (A.31)

By Lemma A.2, there is a unique solution of the unconstrained problem given by \( q_{BI}(\theta) \). Further, strict concavity of the objective function in (A.31) implies that the optimal solution of the constrained problem is \( q_1 \) if \( q_{BI}(\theta) < q_1 \) and \( q_2 \) if \( q_{BI}(\theta) > q_1 \). The optimal transfer and the marginal cost follow from the binding participation constraint and the first-order condition with respect to \( c \).

The following lemma describes the government’s optimal choice of the lower bound in case of constrained delegation.

**Lemma A.5.** Fix \( q_2 \in [0, k\theta] \). Suppose the government delegates with a constraint that \( q(\theta), q(\overline{\theta}) \in [q_1, q_2] \), for some \( q_1 \in [0, q_2] \). The government’s payoff is maximized at any \( q_1 \leq \min \{q_2, q_{BI}(\overline{\theta})\} \).

**Proof of Lemma A.5**

Proof. Step 1: Consider the case \( q_{BI}(\overline{\theta}) \leq q_2 \). By Lemma A.4, if \( q_1 \leq q_{BI}(\overline{\theta}) \), then \( B \) sets \( q^{P}_{BI}(\overline{\theta}, d_1, d_2) = q_{BI}(\overline{\theta}) \), and the government’s payoff is independent of \( q_1 \) in this range. If \( q_1 \geq q_{BI}(\overline{\theta}) \), then \( B \) sets \( q^{P}_{BI}(\overline{\theta}, d_1, d_2) = q_1 > q_{GI}(\overline{\theta}) \), and the government’s payoff decreases with \( q_1 \) in this range. Hence, the government’s payoff is maximized at any \( q_1 \leq q_{BI}(\overline{\theta}) \). Step 2: Consider the case \( q_{BI}(\overline{\theta}) > q_2 \). By Lemma A.4, for any \( q_1 \leq q_2 \), \( B \) sets \( q^{P}_{BI}(\overline{\theta}, d_1, d_2) = q_2 \), and the government’s payoff is independent of \( q_1 \) for all \( q_1 \leq q_2 \). Together, the two steps complete the proof.

The following lemma describes the government’s optimal choice of the upper bound in case of constrained delegation.
Lemma A.6. Let $q_1 = q_{GI}(\bar{\theta})$. Suppose the government delegates with a constraint that $q(\bar{\theta}), q(\bar{\theta}) \in [q_1, q_2]$, for some $q_2 \in [q_{GI}(\bar{\theta}), k\bar{\theta}]$. If $q_{BI}(\bar{\theta}) \geq q_{GI}(\bar{\theta})$, then, among all $q_2 \in [q_{GI}(\bar{\theta}), k\bar{\theta}]$, the government’s payoff is maximized at $q_2 = q_{GI}(E_{\theta}\theta)$. If $q_{BI}(\bar{\theta}) < q_{GI}(\bar{\theta})$, then, among all $q_2 \in [q_{GI}(\bar{\theta}), q_{BI}(\bar{\theta})]$, it is maximized at $q_2 = q_{GI}(E_{\theta}\theta)$.

Proof of Lemma A.6

Proof. In this proof, we make use of Lemma A.4 repeatedly while determining the bureaucrat’s optimal response for a given $q_2$.

Consider first $q_{BI}(\bar{\theta}) \geq q_{GI}(\bar{\theta})$. For $q_2 > q_{BI}(\bar{\theta})$, the bureaucrat sets $q^p_{BI}(\bar{\theta}, q_1, q_2) = q_{BI}(\bar{\theta})$ and $q^p_{BI}(\bar{\theta}, q_1, q_2) = q_{BI}(\bar{\theta})$ and the government’s payoff is independent of $q_2$. For $q_{BI}(\bar{\theta}) \leq q_2 \leq q_{BI}(\bar{\theta})$, the bureaucrat sets $q^p_{BI}(\bar{\theta}, q_1, q_2) = q_{BI}(\bar{\theta})$ and $q^p_{BI}(\bar{\theta}, q_1, q_2) = q_2$, and the government’s payoff is decreasing in $q_2$, since $q_{BI}(\bar{\theta}) \geq q_{GI}(\bar{\theta})$. For $q_1 \leq q_2 \leq q_{BI}(\bar{\theta})$, the bureaucrat sets $q^p_{BI}(\bar{\theta}, q_1, q_2) = q^p_{BI}(\bar{\theta}, q_2, q_2) = q_2$, resulting in a uniform quality level for both types of firms. In such a case, by Lemma A.4, $c^p_{BI}(\bar{\theta}, q_1, q_2) = \theta - \frac{q_1}{k}$, $t^p_{BI}(\bar{\theta}, q_1, q_2) = \frac{q_1^2}{2k}$, and the government’s most preferred choice of $q_2$ solves

$$\max_{q \in [q_{GI}(\bar{\theta}), q_{BI}(\bar{\theta})]} \nu \left[ S(q) - (1 + \lambda) \left( \theta q - \frac{q^2}{2k} \right) \right] + (1 - \nu) \left[ S(q) - (1 + \lambda) \left( \bar{\theta} q - \frac{q^2}{2k} \right) \right]. \quad (A.32)$$

By Assumption 1, $q_{GI}(E_{\theta}\theta)$ uniquely solves the first-order condition of the unconstrained problem

$$S'(q) = (1 + \lambda) \left( E_{\theta}\theta - \frac{q}{k} \right).$$

Note that $q_{GI}(\bar{\theta}) \leq q_{GI}(E_{\theta}\theta) \leq q_{GI}(\bar{\theta}) = q_{BI}(\bar{\theta})$, and so $q_{GI}(E_{\theta}\theta)$ is the solution of (A.32).

Consider next $q_{BI}(\bar{\theta}) < q_{GI}(\bar{\theta})$. For $q_2 > q_{BI}(\bar{\theta})$, the bureaucrat sets $q^p_{BI}(\bar{\theta}, q_1, q_2) = q_{BI}(\bar{\theta})$ and $q^p_{BI}(\bar{\theta}, q_1, q_2) = q_{BI}(\bar{\theta})$, and the government’s payoff is independent of $q_2$. For $q_{BI}(\bar{\theta}) \leq q_2 \leq q_{BI}(\bar{\theta})$, the bureaucrat sets $q^p_{BI}(\bar{\theta}, q_1, q_2) = q_2$ and $q^p_{BI}(\bar{\theta}, q_1, q_2) = q_{BI}(\bar{\theta})$, and the government’s payoff increases with $q_2$ for $q_2 \leq q_{GI}(\bar{\theta})$ and decreases thereafter. For $q_1 \leq q_2 \leq q_{BI}(\bar{\theta})$, the bureaucrat sets $q^p_{BI}(\bar{\theta}, q_1, q_2) = q^p_{BI}(\bar{\theta}, q_2, q_2) = q_2$, resulting in a uniform quality level for both types of firms. In such a case, the government’s payoff increases with $q_2$ for $q_2 \leq q_{GI}(E_{\theta}\theta)$ and decreases thereafter.

Proof of Proposition 4

Proof. The proof follows directly from Lemmas A.4, A.5, and A.6.

Proof of Lemma 1

Proof. We prove the lemma in the following steps.

Step 1: $U^W_G$ is increasing and linear in $\nu$, is decreasing in $\beta$ for $\beta < \beta$, and is independent of $\beta$ for $\beta \geq \beta$, for some $\beta > 0$. 

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Proof of Step 1: Consider (23). Since $U_G(\theta, \alpha_{GI}(\theta))$ and $U_G(\theta, \alpha_{BI}(\theta))$ are independent of $\nu$, $U_G^{WD}$ is linear in $\nu$. Further,

$$U_G(\theta, \alpha_{GI}(\theta)) = f(\theta, q_{GI}(\theta)) > f(\theta, q_{GI}(\nu)) > f(\theta, q_{BI}(\theta)) = U_G(\theta, \alpha_{BI}(\theta)), $$

where the first inequality follows from (A.3), and the second inequality follows from (18) and (A.2). Also, differentiation of (A.23) with respect to $\beta$ shows that $q_{BI}(\theta)$ is increasing in $\beta$. Using this together with (A.2) and (18), we have that $U_G(\theta, \alpha_{BI}(\theta))$ is decreasing in $\beta$. Since $q_{BI}(\theta)$ is increasing in $\beta$ and $q_{GI}(\theta)$ is independent of $\beta$, $q_{BI}(\theta) \geq q_{GI}(\theta)$ for sufficiently high values of $\beta$. Define $\beta := \min \{ \beta > 0 : q_{BI}(\theta) \geq q_{GI}(\theta) \}$. Then, for $\beta \geq \beta$, the bureaucrat will implement $q_{GI}(\theta)$ for both types. Therefore, in such a case, $U_G^{WD} = \nu U_G(\theta, \alpha_{GI}(\theta)) + (1 - \nu) U_G(\theta, \alpha_{GI}(\theta))$, which is independent of $\beta$.

**Step 2: $U_G^{SD}$ is increasing and convex in $\nu$, and is independent of $\beta$.**

Proof of Step 2: Note that $U_G^{SD} = U_G(E_0\theta, \alpha_{GI}(E_0\theta)) = \max_q S(q) - (1 + \lambda) \left( E_0q - \frac{q^2}{2k} \right)$, which is independent of $\beta$. As $\frac{dE_0\theta}{d\nu} = -\Delta \theta$, applying the envelope theorem, we get that

$$\frac{dU_G^{SD}}{d\nu} = (1 + \lambda) \Delta \theta q_{GI}(E_0\theta),$$

(A.33)

which is strictly positive. Further, $q_{GI}(E_0\theta)$ uniquely solves the equation $S'(q) = (1 + \lambda) \left( E_0q - \frac{q^2}{2k} \right)$, and differentiation of the equation with respect to $\nu$ shows that $q_{GI}(E_0\theta)$ is increasing in $\nu$. Therefore, $\frac{dU_G^{SD}}{d\nu} = (1 + \lambda) \Delta \theta q_{GI}(E_0\theta) > 0$, which completes the proof of Step 2.

**Step 3: $U_G^{ND}$ is convex in $\nu$, and is independent of $\beta$.**

Proof of Step 3: Convexity of $U_G^{ND}$ follows from Lemma A.3.

**Step 4: For any $\beta > 0$, $U_G^{SD} = U_G^{ND} > U_G^{WD}$ at $\nu = 0$, and $U_G^{SD} = U_G^{ND} = U_G^{WD}$ at $\nu = 1$.**

Proof of Step 4: At $\nu = 0$, for $\beta > 0$, $U_G^{SD} = U_G^{ND} = U_G(\theta, \alpha_{GI}(\theta)) = f(\theta, q_{GI}(\theta)) > f(\theta, q_{BI}(\theta)) = U_G(\theta, \alpha_{BI}(\theta)) = U_G^{WD}$, where the inequality follows from (A.2) and (18). At $\nu = 1$, $U_G^{SD} = U_G^{ND} = U_G^{WD} = U_G(\theta, \alpha_{GI}(\theta))$.

**Step 5: $\exists \nu^{SD}(\beta) \in (0, 1]$, increasing in $\beta$, such that $U_G^{SD} \geq U_G^{WD}$ if and only if $\nu \leq \nu^{SD}(\beta)$. Further, $\nu^{SD}(\beta) < 1$ for $\beta < \beta$ and $\nu^{SD}(\beta) = 1$ for $\beta \geq \beta$.**

Proof of Step 5: From Steps 1, 2, and 4, it follows that two possibilities can arise: First, $U_G^{SD} > U_G^{WD}$ for $\nu \in [0, 1]$, and second, $U_G^{SD}$ intersects $U_G^{WD}$ at most once, at some $\nu^{SD} \in (0, 1)$. It follows that $U_G^{SD} \geq U_G^{WD}$ if and only if $\nu \leq \nu^{SD} \leq 1$. The first possibility arises when $\nu^{SD} = 1$. Further, $U_G^{SD}$ intersects $U_G^{WD}$ from above, and therefore, $\frac{dU_G^{WD}}{d\nu} > \frac{dU_G^{SD}}{d\nu}$ at $\nu = \nu^{SD}$. To see how $\nu^{SD}$ changes with $\beta$, write $U_G^{SD} = U_G^{SD}(\nu)$ and $U_G^{WD} = U_G^{WD}(\beta, \nu)$, and differentiate $U_G^{SD}(\nu) = U_G^{WD}(\beta, \nu^{SD})$ with respect to $\beta$: 
\begin{align*}
\frac{\partial U_G^{SD} (ν^{SD})}{\partial ν} \frac{dν^{SD}}{dβ} &= \frac{\partial U_G^{WD} (β, ν^{SD})}{\partial ν} \frac{dν^{SD}}{dβ} + \frac{\partial U_G^{WD} (β, ν^{SD})}{\partial β},
\end{align*}

which is positive as both the numerator and the denominator are negative. Therefore, ν^{SD} (β) is increasing in β.

Further, as shown in Step 1, for β ≥ \overline{β}, the bureaucrat implements a uniform quality q_{GI} (\overline{θ}) for both types. Since U_G^{SD} is the maximum payoff that G gets from implementing a uniform quality level, U_G^{SD} ≥ U_G^{WD} for \beta ≥ \overline{β}. Therefore, ν^{SD} (β) = 1 for \beta ≥ \overline{β}.

Finally, to prove that ν^{SD} (β) < 1 for \beta < \overline{β}, we show that, for any \beta < \overline{β}, there exists some ν, sufficiently close to 1, for which U_G^{WD} > U_G^{SD}. To see this, note that, for any 0 < \beta < \overline{β}, q_{GI} (\overline{θ}) < q_{BI} (\overline{θ}) < q_{GI} (\overline{θ}), and \lim_{ν\to1} q_{GI} (E_θ) = q_{GI} (\overline{θ}).

Therefore, there exists ν < 1 such that, for ν ∈ [ν, 1], q_{BI} (\overline{θ}) < q_{GI} (E_θ), and consequently, by (A.2), \int \{ \overline{θ}, q_{BI} (\overline{θ}) \} > \int (\overline{θ}, q_{GI} (E_θ)). This and the fact that \int (\overline{θ}, q_{GI} (E_θ)) < \int (\overline{θ}, q_{GI} (\overline{θ})) for all ν < 1, give us that, for ν ∈ [ν, 1],

\begin{align*}
U_G^{SD} &= νf (\overline{θ}, q_{GI} (E_θ)) + (1 - ν) f (\overline{θ}, q_{GI} (E_θ)) \\
&< νf (\overline{θ}, q_{GI} (\overline{θ})) + (1 - ν) f (\overline{θ}, q_{BI} (\overline{θ})) \\
&= U_G^{WD}.
\end{align*}

Step 5 proves part (i) of the lemma.

**Step 6**: \exists ν^{ND} (β) ∈ (0, 1], increasing in β, such that U_G^{ND} ≥ U_G^{WD} if and only if ν ≤ ν^{ND} (β).

Proof of Step 6: The existence and monotonicity properties of ν^{ND} (β) follow from arguments similar to the ones we used in characterizing ν^{SD} (β) in the proof of Step 5, as both U_G^{SD} and U_G^{ND} are convex and coincide at ν = 0, 1.

Step 6 proves part (ii) of the lemma.

**Proof of Proposition 5**

Proof. Lemma 1 implies that U_G^{WD} > max \{ U_G^{ND}, U_G^{SD} \} if and only if ν ≥ max \{ ν^{SD} (β), ν^{ND} (β) \}, from which part (i) of the proposition follows. As both U_G^{ND} and U_G^{SD} are independent of β, if for some β, U_G^{ND} ≤ U_G^{SD}, then this relation holds for all β. This implies part (ii) of the proposition.

The following Lemma characterizes an extrinsically-motivated bureaucrat’s choice of contract.

**Lemma A.7.** Assume that the government delegates the decision-making authority to an extrinsically-motivated bureaucrat. The contract c_{BI} (θ) = (q_{BI} (θ), c_{BI} (θ), t_{BI} (θ)) that the bureaucrat
offers to a producer of type \( \theta \in \{ \varrho, \bar{\varrho} \} \) is given as follows: \( q_{BI}^B (\theta) \) uniquely solves

\[
S' (q) = \frac{1 + \lambda}{1 + \beta} \left( \theta - \frac{q - \Lambda b}{k} \right),
\]

(A.34)

where \( \Lambda := \frac{1 - \lambda}{1 + \lambda} \in (0, 1) \), and

\[
c_{BI}^B (\theta) = \theta - \frac{q_{BI}^B (\theta) - \Lambda b}{k},
\]

(A.35)

\[
t_{BI}^B (\theta) = \frac{k}{2} \left( \theta - c_{BI}^B (\theta) \right)^2 + bc_{BI}^B (\theta).
\]

(A.36)

**Proof of Lemma A.7**

*Proof.* Note that, at the solution, the firm’s participation constraint is binding, which gives \( U_P = b (c) \). After replacing \( U_P \) in (26), the bureaucrat maximizes

\[
(1 + \beta) S(q) - (1 + \lambda) \left( cq + \frac{k}{2} \max \{ \theta - c, 0 \} \right)^2 + (1 - \lambda) b (c),
\]

which is strictly decreasing in \( c \) for \( c > \theta \). Therefore, \( c \leq \theta \) at the optimum, and thus \( U_P = bc \).

Supposing an interior solution, we have the following first-order condition with respect to \( c \):

\[
q - k (\theta - c) - \Lambda b = 0,
\]

(A.37)

where \( \Lambda := \frac{1 - \lambda}{1 + \lambda} \in (0, 1) \), which gives

\[
c = \theta - \frac{q - \Lambda b}{k}.
\]

Inserting for \( c \), we rewrite the optimization problem (26) as

\[
\max_q (1 + \beta) S (q) - (1 + \lambda) \left( \theta - \frac{q - \Lambda b}{k} \right) q + \frac{(q - \Lambda b)^2}{2k} + (1 - \lambda) b \left( \theta - \frac{q - \Lambda b}{k} \right).
\]

(A.38)

By Assumption 1, (A.38) has a unique solution \( q_{BI}^B (\theta) \) given by\(^{11}\)

\[
S' (q) = \frac{1 + \lambda}{1 + \beta} \left( \theta - \frac{q - \Lambda b}{k} \right).
\]

(A.39)

\(^{11}\)We find that, if \( S' (\Lambda b) < \frac{1 + \lambda}{1 + \beta} \theta \), then the solution of the first-order condition of (A.38) is less than \( \Lambda b \), and therefore the optimal marginal cost is indeed an interior solution. On the other hand, if \( S' (\Lambda b) \geq \frac{1 + \lambda}{1 + \beta} \theta \), then the optimal contract is given by \( (q_{BI}^B (\theta), c_{BI}^B (\theta), t_{BI}^B (\theta)) \), where \( q_{BI}^B (\theta) \) uniquely solves \( S' (q) = \frac{1 + \lambda}{1 + \beta} \theta \), \( c_{BI}^B (\theta) = \theta \), and \( t_{BI}^B (\theta) = b \theta \).
The optimal transfer and the marginal costs are given by

\[ c_{BI}^B (\theta) = \theta - \frac{q_{BI}^B (\theta)}{k} - \Lambda b, \quad \text{(A.40)} \]

\[ t_{BI}^B (\theta) = \frac{k}{2} (\theta - c_{BI}^B (\theta))^2 + b c_{BI}^B (\theta). \quad \text{(A.41)} \]

**Proof of Proposition 6**

*Proof.* It follows from (A.39), (A.23), and by Assumption 1 that \( q_{BI}^B (\theta) < q_{BI} (\theta) \). Further, total differentiation of (A.39) shows that \( q_{BI}^B (\theta) \) decreases with \( b \). \( \square \)

**Proof of Proposition 7**

*Proof.* We prove the proposition in the following two steps.

**Step 1:** At the solution of (28), \( U_P (\theta, \alpha) = 0 \).

Proof of Step 1: We prove step 1 by contradiction. Let \( (\bar{t}_1, \bar{t}_2) \) denote the solution of (28) and suppose \( U_P (\theta, \alpha_{RT}^B (\theta, \bar{t}_1, \bar{t}_2)) > 0 \). Note that the possibility of \( U_P (\theta, \alpha_{BI}^B (\theta, \bar{t}_1, \bar{t}_2)) < 0 \) is ruled out as \( \alpha_{RT}^B (\theta, \bar{t}_1, \bar{t}_2) \) is a solution of (27).

Since \( U_B (\theta, \alpha) \) is strictly decreasing in \( t \), \( B \) prefers to reduce \( t \) as much as possible, for any given \( q \) and \( c \), which implies that \( t_{BI}^{RT} (\theta, \bar{t}_1, \bar{t}_2) = \bar{t}_1 \). Further, since the producer’s participation is not binding at \( \alpha_{BI}^B (\theta, \bar{t}_1, \bar{t}_2) \), this contract is also a solution of the following problem:

\[
\max_{\alpha} U_B (\theta, \alpha) \quad \text{(A.42)}
\]

subject to \( t = \bar{t}_1 \).

By Assumption 1, (A.42) has a unique solution \( \alpha^* = (q^*, c^*, t^*) \), where \( q^* \) solves

\[
S' (q) = \frac{1 + \lambda}{1 + \beta} \left( \frac{\theta - (1 + \lambda) q}{k} \right); \quad \text{(A.43)}
\]

and

\[
c^* = \theta - \frac{(1 + \lambda) q^*}{k}, \quad \text{(A.44)}
\]

\[
t^* = \bar{t}_1. \quad \text{(A.45)}
\]

Therefore, \( \alpha^* = \alpha_{BI}^B (\theta, \bar{t}_1, \bar{t}_2) \), and so \( U_P (\theta, \alpha^*) > 0 \). Further,

\[
U_P (\theta, \alpha^*) > 0 \iff \bar{t}_1 > \frac{k}{2} \left( \max \{ \theta - c^*, 0 \} \right)^2 \Rightarrow \bar{t}_1 > \frac{(1 + \lambda)^2 (q^*)^2}{2k}. \]
Choose $\epsilon > 0$ such that $\bar{t}_1 - \epsilon > \frac{(1+\lambda)^2(q^*)^2}{2k}$ and consider the government’s payoff from setting the following bound on transfer: $t \in [\bar{t}_1 - \epsilon, \bar{t}_2]$. Then, the bureaucrat’s choice of regulation contract for type $\theta$ solves a problem similar to (A.42), with the constraint now replaced by $t = \bar{t}_1 - \epsilon$. From (A.43) and (A.44), we see that $q^*$ and $c^*$ do not depend on $t^*$. Therefore, the bureaucrat’s choice of contract will be 

$$\alpha_{BI}^RT(\theta, \bar{t}_1 - \epsilon, \bar{t}_2) = (q^*, c^*, \bar{t}_1 - \epsilon).$$

Further, the government’s gets a strictly higher payoff from implementing $\alpha_{BI}^RT(\theta, \bar{t}_1 - \epsilon, \bar{t}_2)$ than $\alpha_{BI}^RT(\theta, \bar{t}_1, \bar{t}_2)$, since the former contract is associated with less transfer; the quality level and the marginal costs are the same in both contracts; and both contracts satisfy the producer’s participation constraint. This observation contradicts our supposition that $(\bar{t}_1, \bar{t}_2)$ is the solution of (28) when $U_P(\theta, \alpha_{BI}^RT(\theta, \bar{t}_1, \bar{t}_2)) > 0$.

**Step 2:** If $U_P(\theta, \alpha) = 0$, then $\alpha_{BI}^RT(\theta, t_1, t_2) = \alpha_{BI}^P(\theta, \sqrt{2kt_1}, \sqrt{2kt_2})$, where $\alpha_{BI}^P(\theta, q_1, q_2)$ is described in Lemma A.4.

Proof of Step 2: When $U_P(\theta, \alpha) = 0$, (27) can be rewritten as follows:

$$\max_{q,c} (1 + \beta) S(q) - (1 + \lambda) \left( cq + \frac{k}{2} (\max \{\theta - c, 0\})^2 \right)$$

subject to $t_1 \leq \frac{k}{2} (\max \{\theta - c, 0\})^2 \leq t_2$.

The objective function is strictly decreasing in $c$ for $c > \theta$. Therefore, $c \leq \theta$ at the optimum. Note that the solutions of the unconstrained problem are given by $q_{BI}(\theta)$ and $c_{BI}(\theta)$. Further, at that solution, we have $\frac{k}{2} (\max \{\theta - c_{BI}(\theta), 0\})^2 = \frac{q_{BI}(\theta)^2}{2k}$, and so the constraint can be rewritten as $\sqrt{2kt_1} \leq q_{BI}(\theta) \leq \sqrt{2kt_2}$. The concavity of the objective function in (A.46) implies that the optimal solution of the constrained problem is given by $\alpha_{BI}^P(\theta, \sqrt{2kt_1}, \sqrt{2kt_2})$.

Step 2 proves Proposition 7. \hfill \Box

**Appendix B**

Appendix B contains the proof of Lemma A.3, which is introduced in Appendix A, and an additional result, documented in Lemma B.2, about the effect of $\nu$ on the difference in payoff between the two regimes, ND and SD. In addition, we will begin by stating and proving a lemma that will be useful in the proof of Lemma A.3.

**Lemma B.1.** Let $g(\underline{x}, \nu) : C \times [0, 1] \to \mathbb{R}$ be a differentiable function where $C \subseteq \mathbb{R}^n$ is closed and convex. Assume that $g$ is strictly concave in $\underline{x} = (x_1, \ldots, x_n) \in C$. Define

$$\underline{x}^*(\nu) := \arg \max_{\underline{x} \in C} g(\underline{x}, \nu),$$

$$G(\nu) := \max_{\underline{x} \in C} g(\underline{x}, \nu) = g(\underline{x}^*(\nu), \nu).$$
Then,
\[ G''(\nu) = \sum_{i=1}^{n} \frac{\partial^2 g(x^*(\nu), \nu)}{\partial x_i \partial \nu} \cdot \frac{dx^*(\nu)}{d\nu} + \frac{\partial^2 g(x^*(\nu), \nu)}{\partial \nu^2} \]  

(B.1)

**Proof of Lemma B.1**

Proof. Since \( C \) is closed and convex, strict concavity of \( g \) in \( x \) ensures that \( x^*(\nu) \) is unique and \( G \) is well-defined. Define \( h(x, \nu) := \frac{\partial g(x, \nu)}{\partial \nu} \). Applying the envelope theorem, we get

\[ G'(\nu) = \frac{\partial g(x^*(\nu), \nu)}{\partial \nu} = h(x^*(\nu), \nu). \]

Therefore,

\[ G''(\nu) = \sum_{i=1}^{n} \frac{\partial h(x^*(\nu), \nu)}{\partial x_i} \cdot \frac{dx^*(\nu)}{d\nu} + \frac{\partial h(x^*(\nu), \nu)}{\partial \nu}, \]

where \( h(x, \nu) := \frac{\partial g(x, \nu)}{\partial \nu} \).

\[ \square \]

**Proof of Lemma A.3**

Proof. Observe that \( U_{ND}^G = G(\nu) = \max_{\bar{x} \in C} g(\bar{x}, \nu) \) where \( \bar{x} = (\bar{q}, \bar{c}, \bar{q}, \bar{c}) \), \( C = [0, k\bar{\theta}] \times [0, \bar{\theta}] \times [0, k\bar{\theta}] \times [0, \bar{\theta}] \), and

\[ g(x, \nu) = \begin{cases} 
\nu \left[ S(q) - (1 + \lambda) \left( eq + \frac{k}{2} (\theta - \bar{\theta})^2 \right) \right] - \frac{\nu \lambda k \Delta \theta}{2} (\bar{\theta} + \theta - 2\bar{\theta}) \\
+ (1 - \nu) \left[ S(q) - (1 + \lambda) \left( eq + \frac{k}{2} (\theta - \bar{\theta})^2 \right) \right] 
\end{cases} \]

if \( \nu \leq \nu^* \)

\[ \begin{cases} 
\nu \left[ S(q) - (1 + \lambda) \left( eq + \frac{k}{2} (\theta - \bar{\theta})^2 \right) \right] - \frac{\nu \lambda k}{2} (\bar{\theta} - \bar{\theta})^2 \\
+ (1 - \nu) \left[ S(q) - (1 + \lambda) \left( eq + \frac{k}{2} (\theta - \bar{\theta})^2 \right) \right] 
\end{cases} \]

if \( \nu > \nu^* \)

(B.2)

By Lemma A.1, we get that \( \arg \max_{\bar{x} \in C} g(\bar{x}, \nu) = (q_{GN}(\bar{\theta}), c_{GN}(\bar{\theta}), q_{GN}(\bar{\theta}), c_{GN}(\bar{\theta})) \). By Lemma B.1, we can, therefore, write

\[ G''(\nu) = \sum_{i=1}^{n} \frac{\partial^2 g(x^*(\nu), \nu)}{\partial x_i^2} \cdot \frac{dx^*(\nu)}{d\nu} + \frac{\partial^2 g(x^*(\nu), \nu)}{\partial \nu^2} \]

(B.3)

where

\[ x^*(\nu) = (q_{GN}(\bar{\theta}), c_{GN}(\bar{\theta}), q_{GN}(\bar{\theta}), c_{GN}(\bar{\theta})). \]
From (B.2), we have $\overline{\partial^2 g(x^*(\nu),\nu)} = 0$. From (A.4) and (A.6), we have $\overline{dq_{GN}(\theta)} = 0$ and $\overline{dc_{GN}(\theta)} = 0$. Further,

$$\overline{\frac{\partial^2 g(\bar{\eta}, \bar{\xi}, q, c, \nu)}{\partial \bar{\eta} \partial \nu}} = - \left[ S'(\bar{\eta}) - (1 + \lambda) \bar{\pi} \right]$$

for all $\nu \in [0, 1]$. By (A.5) and (A.7), we get $\overline{\partial^2 g(x^*(\nu),\nu)} = 0$ for all $\nu \in [0, 1]$. We simplify (B.3), inserting for $\overline{dq_{GN}(\theta)}$, $\overline{dc_{GN}(\theta)}$, $\overline{\partial^2 g(x^*(\nu),\nu)}$, and $\overline{\partial^2 g(x^*(\nu),\nu)}$, to get

$$G''(\nu) = \overline{\frac{\partial^2 g(x^*(\nu),\nu)}{\partial c \partial \nu}} \cdot \overline{\frac{dc_{GN}(\theta)}{d\nu}}.$$  \hspace{1cm} (B.4)

Note that

$$\overline{\frac{dc_{GN}(\theta)}{d\nu}} = \begin{cases} \frac{\lambda \Delta \theta}{(1+\lambda)(1-\nu)} - \frac{1}{k} \overline{dq_{GN}(\theta)} & \text{if } \nu \leq \nu^* \\ \frac{\lambda \Delta \theta}{k(\eta+1)^2(1+\lambda)(1-\nu)} - \frac{1}{k(\eta+1)} \overline{dq_{GN}(\theta)} & \text{if } \nu > \nu^* \end{cases}$$

and it is strictly positive since

$$\overline{\frac{dq_{GN}(\theta)}{d\nu}} = \begin{cases} \frac{\lambda \Delta \theta}{(1-\nu)^2[\lambda q_{GN}(\theta) + \frac{1+\lambda}{\kappa}] + q_{\theta}} & \text{if } \nu \leq \nu^* \\ \frac{(\eta+1)^2(1-\nu)^2(q_{\theta})}{\lambda q_{GN}(\theta) + \frac{1+\lambda}{\kappa(\eta+1)}} & \text{if } \nu > \nu^* \end{cases}$$

is strictly negative by Assumption 1, part (iv). Further,

$$\overline{\frac{\partial^2 g(\bar{\eta}, \bar{\xi}, q, c, \nu)}{\partial \bar{\xi} \partial \nu}} = \begin{cases} \lambda k \Delta \theta + (1 + \lambda) \left[ \bar{q} - k \left( \bar{\theta} - \bar{\xi} \right) \right] & \text{if } \nu \leq \nu^* \\ \lambda k \left( \bar{\theta} - \bar{\xi} \right) + (1 + \lambda) \left[ \bar{q} - k \left( \bar{\theta} - \bar{\xi} \right) \right] & \text{if } \nu > \nu^* \end{cases}$$

and by (A.7),

$$\overline{\frac{\partial^2 g(x^*(\nu),\nu)}{\partial \bar{\eta} \partial \nu}} = \begin{cases} \lambda k \Delta \theta + \eta (1 + \lambda) k \Delta \theta & \text{if } \nu \leq \nu^* \\ (1 + \lambda) \bar{\eta} - \frac{\eta}{\eta+1} & \text{if } \nu > \nu^* \end{cases}$$

is strictly positive. Therefore, the signs of $\overline{\frac{dc_{GN}(\theta)}{d\nu}}$ and $\overline{\frac{\partial^2 g(x^*(\nu),\nu)}{\partial \bar{\eta} \partial \nu}}$, together with (B.4), imply $G''(\nu) > 0$ for all $\nu \in [0, \nu^*]$ and $(\nu^*, 1]$, and we have that $G(\nu)$ is a piecewise-convex function.

Next, we claim that $G'(\nu)$ is continuous at $\nu^*$. To prove this, we apply the envelope theorem to derive
Then, \[
\frac{dU_{ND}^G}{d\nu} = G'(\nu) = \left\{ \begin{array}{ll}
& \left[ S(q_{GN}(\theta)) - (1 + \lambda) \left( c_{GN}(\theta) q_{GN}(\theta) + \frac{k}{2} (\theta - c_{GN}(\theta))^2 \right) \right] \\
& \left( \frac{\lambda k \theta}{2} (\overline{\theta} + \theta - 2c_{GN}(\overline{\theta})) \right) \\
& \left[ S(q_{GN}(\overline{\theta})) - (1 + \lambda) \left( c_{GN}(\overline{\theta}) q_{GN}(\overline{\theta}) + \frac{k}{2} (\overline{\theta} - c_{GN}(\overline{\theta}))^2 \right) \right] \\
& \left( \frac{\lambda k}{2} (\overline{\theta} - c_{GN}(\overline{\theta}))^2 \right)
\end{array} \right.
\]
if \(\nu \leq \nu^*\).

Since \(c_{GN}(\theta)\) and \(q_{GN}(\theta)\) are independent of \(\nu\), \(G'(\nu)\) is continuous at \(\nu^*\) if \(c_{GN}(\overline{\theta})\) and \(q_{GN}(\overline{\theta})\) are continuous at \(\nu^*\) and \(\lim_{\nu \searrow \nu^*} \frac{\lambda k \Delta \theta}{2} (\overline{\theta} + \theta - 2c_{GN}(\overline{\theta})) = \lim_{\nu \searrow \nu^*} \frac{\lambda k}{2} (\overline{\theta} - c_{GN}(\overline{\theta}))^2\). By Lemma A.1, \(c_{GN}(\overline{\theta})\) and \(q_{GN}(\overline{\theta})\) are continuous at \(\nu^*\), and

\[
\lim_{\nu \searrow \nu^*} c_{GN}(\overline{\theta}) = \lim_{\nu \searrow \nu^*} q_{GN}(\overline{\theta}) = 0.
\]

Further,

\[
\lim_{\nu \searrow \nu^*} \frac{\lambda k \Delta \theta}{2} (\overline{\theta} + \theta - 2c_{GN}(\overline{\theta})) = \frac{\lambda k (\Delta \theta)^2}{2},
\]

and

\[
\lim_{\nu \searrow \nu^*} \frac{\lambda k}{2} (\overline{\theta} - c_{GN}(\overline{\theta}))^2 = \frac{\lambda k (\Delta \theta)^2}{2}.
\]

Therefore, \(\lim_{\nu \searrow \nu^*} G'(\nu) = G'(\nu^*)\) and so \(G'(\nu)\) is continuous at \(\nu^*\).

So far, we have shown that \(G'(\nu)\) is strictly increasing in \(\nu\) for \(\nu \in [0, \nu^*]\), continuous at \(\nu^*\), and strictly increasing in \(\nu\) for \(\nu \in (\nu^*, 1]\). These three observations together imply that \(G'(\nu)\) is strictly increasing in \(\nu\) for \(\nu \in [0, 1]\). Therefore, \(G(\nu)\) is strictly convex in \(\nu\) for \(\nu \in [0, 1]\).
Furthermore, \( \frac{d \delta (1)}{d \lambda} > 0 \), while \( \frac{d \delta (0)}{d \lambda} \) can be of either sign.

**Proof of Lemma B.2**

*Proof.* Note that

\[
\delta (1) = \lim_{\nu \to 1} \left[ \frac{d U_{G}^{N (\nu)}}{d \nu} - \frac{d U_{G}^{S (\nu)}}{d \nu} \right] \quad \text{and} \quad \delta (0) = \lim_{\nu \to 0} \left[ \frac{d U_{G}^{N (\nu)}}{d \nu} - \frac{d U_{G}^{S (\nu)}}{d \nu} \right].
\]

Consider (A.33). Observe that

\[
\lim_{\nu \to 1} q_{G I} (E_{\theta} \theta) = q_{G I} (\overline{\theta}) \quad \text{and} \quad \lim_{\nu \to 0} q_{G I} (E_{\theta} \theta) = q_{G I} (\overline{\theta}).
\]

This gives us

\[
\lim_{\nu \to 1} \frac{d U_{G}^{S D (\nu)}}{d \nu} = (1 + \lambda) \Delta \theta q_{G I} (\overline{\theta}), \quad (B.9)
\]

\[
\lim_{\nu \to 0} \frac{d U_{G}^{S D (\nu)}}{d \nu} = (1 + \lambda) \Delta \theta q_{G I} (\overline{\theta}). \quad (B.10)
\]

Next, consider (B.5). To find the limit of \( \frac{d U_{G}^{N D(\nu)}}{d \nu} \) as \( \nu \) goes to 1. From (B.5) (the expression when \( \nu > \nu^{*} \)), using the fact that \( q_{G N} (\overline{\theta}) = q_{G I} (\overline{\theta}) \) and \( c_{G N} (\overline{\theta}) = c_{G I} (\overline{\theta}) \), as well as (B.11) and (B.12), we have that

\[
\lim_{\nu \to 1} \frac{d U_{G}^{N D (\nu)}}{d \nu} = \left[ S (q_{G I} (\overline{\theta})) - (1 + \lambda) \left( c_{G I} (\overline{\theta}) q_{G I} (\overline{\theta}) + \frac{k}{2} \left( \overline{\theta} - c_{G I} (\overline{\theta}) \right)^{2} \right) \right] - \left[ S (q^{*}) - (1 + \lambda) \overline{q} \overline{q} \right] = f (\overline{\theta}, q_{G I} (\overline{\theta})) - f (\overline{\theta}, \overline{q}) + (1 + \lambda) \frac{q_{G I}^{2}}{2} \overline{q}. \quad (B.13)
\]

Consider next the limit of \( \frac{d U_{G}^{N D (\nu)}}{d \nu} \) as \( \nu \) goes to 0. From (B.5) (the expression when \( \nu \leq \nu^{*} \)), using the fact that \( q_{G N} (\overline{\theta}) = q_{G I} (\overline{\theta}) \) and \( c_{G N} (\overline{\theta}) = c_{G I} (\overline{\theta}) \), as well as (B.11) and (B.12), we have that

\[
\lim_{\nu \to 0} \frac{d U_{G}^{N D (\nu)}}{d \nu} = \left[ S (q_{G I} (\overline{\theta})) - (1 + \lambda) \left( c_{G I} (\overline{\theta}) q_{G I} (\overline{\theta}) + \frac{k}{2} \left( \overline{\theta} - c_{G I} (\overline{\theta}) \right)^{2} \right) \right] - \left[ \frac{\lambda k \Delta \theta}{2} \left( \overline{\theta} + \overline{\theta} - 2 \left( \overline{\theta} - q_{G I} (\overline{\theta}) k \right) \right) \right].
\]
\[
- \left[ S(q_{GI} (\bar{\theta})) - (1 + \lambda) \left( c_{GI} (\bar{\theta}) q_{GI} (\bar{\theta}) + \frac{k}{2} (\bar{\theta} - c_{GI} (\bar{\theta}))^2 \right) \right] \\
= f (\theta, q_{GI} (\bar{\theta})) - f (\bar{\theta}, q_{GI} (\bar{\theta})) + \lambda \Delta \theta \left[ \frac{k \Delta \theta}{2} - q_{GI} (\bar{\theta}) \right].
\] (B.14)

We find (B.6) and (B.7) by subtracting (B.9) and (B.10) from (B.13) and (B.14), respectively.

Next, we differentiate (B.6) and (B.7) with respect to \( \lambda \). We rewrite (B.6) as
\[
\delta (1) = \max_q \left[ S (q) - (1 + \lambda) \left( \bar{\theta} q - \frac{q^2}{2k} \right) \right] - \max_q \left[ S (q) - (1 + \lambda) \bar{\theta} q \right] - (1 + \lambda) \Delta \theta q_{GI} (\bar{\theta}).
\]

Therefore, applying the envelope theorem, we have
\[
\frac{d\delta (1)}{d\lambda} = \bar{\theta} (q_{GI} (\bar{\theta}) + \hat{q}) + \frac{q_{GI}^2 (\bar{\theta})}{2k} - (1 + \lambda) \Delta \theta \frac{dq_{GI} (\bar{\theta})}{d\lambda}.
\]

Differentiation of (8) with respect of \( \lambda \) gives
\[
\frac{dq_{GI} (\bar{\theta})}{d\lambda} = \frac{k \theta - q_{GI} (\bar{\theta})}{k (S'' (q_{GI} (\bar{\theta})) + \frac{\theta}{k})},
\]
which is negative by Assumption 1. Therefore, \( \frac{d\delta (1)}{d\lambda} > 0 \).

We rewrite (B.7) as
\[
\delta (0) = \max_q \left[ S (q) - (1 + \lambda) \left( \bar{\theta} q - \frac{q^2}{2k} \right) \right] - \max_q \left[ S (q) - (1 + \lambda) \bar{\theta} q \right] + \frac{\lambda k (\Delta \theta)^2}{2} - (1 + 2\lambda) \Delta \theta q_{GI} (\bar{\theta}).
\]

Applying the envelope theorem, we have
\[
\frac{d\delta (0)}{d\lambda} = \left[ (q_{GI} (\bar{\theta}) - q_{GI} (\bar{\theta})) \left( \frac{(q_{GI} (\bar{\theta}) + q_{GI} (\bar{\theta}))}{2k} - \bar{\theta} \right) \right] + \left[ -\Delta \theta \left( q_{GI} (\bar{\theta}) - \frac{k (\Delta \theta)}{2} \right) \right] + \left[ - (1 + 2\lambda) \Delta \theta \frac{dq_{GI} (\bar{\theta})}{d\lambda} \right].
\]

In the above expression, the first term is negative, the third term is positive, and the second term can take either sign.

\[\square\]

References


