The TV Industry: Advertising and Programming*

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Abstract:
The key to an understanding of the TV industry is the market for TV advertising. We present a model of this market that also encompasses the product markets and the viewer market. Because viewers dislike commercials, there is congestion in advertising, and TV channels offer complementary goods to advertisers. A move from a TV monopoly to a TV duopoly, we find, may reduce both the total number of viewers and the total amount of TV advertising. A softening of competition in each product market results in more investment in programming, higher price per advertising slot, and more advertising. (98 words)

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1. INTRODUCTION

The television industry is often referred to as part of the entertainment business. In that respect, it is an important industry, for example in terms of the time people spend watching TV. At the same time, it is important as a transmitter of advertising. The purpose of this article is to investigate the two-fold role of television, both as a provider of entertainment and as a transmitter of advertising. Our main focus is on the interplay between the TV market and the product markets through the market for advertising. We examine how the rivalry between TV channels and the profit potential in the product markets affect TV channels’ prices on advertising slots, their investments in programming, and the producers’ purchase of advertising on TV.

A basic feature in the model we set up is that viewers are attracted to a TV channel that invests in its programming, but they dislike TV advertising. A TV channel, on the other hand, earns its revenues by selling advertising slots to producers in the product market and attracts viewers for this advertising by investing in programming. The producers in the product markets expand sales by advertising. Since an increase in the amount of advertising tends to reduce the number of TV viewers, there are diminishing returns to TV advertising. In addition, there is congestion in TV advertising: The more one producer advertises its own products on a particular TV channel, the fewer viewers are available there for other producers to advertise to.

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1See, for example, Robinson and Godbey (1999).

2The amounts spent on TV advertising are significant. In one survey, TV advertising in the US amounted to $41.1 billion, out of a total advertising of $79.5 billion; i.e., more than half of all advertising (see http://adage.com/dataplace/archives/dp394.html). In another survey, TV advertising in the US was projected to $52 billion, 39% of total advertising (see http://adage.com/dataplace/topmarkets/). According to the latter survey, TV’s share of total advertising varies considerably between countries. While its share is 60% in Brazil, its share of total advertising is only 23% in Germany.
Despite the important role of the TV industry, there are relatively few studies of it in the economics literature. Early studies of the industry focused on how rivalry between TV channels affects program diversity. Recently, work has appeared in which the market for TV (or radio) advertising has been modeled explicitly. The study by Anderson and Coate (2000) relates closely to the existing literature on program diversity, since they analyze a TV channel’s choice between two types of programs. They view advertising as a link between the product markets and the TV market, and their main concern is the market’s ability to provide an efficient outcome. On the one hand, viewers dislike commercial breaks. On the other hand, viewers, as consumers, receive information about new products from advertisements on TV. They find that the market in some cases leads to under-provision of advertisements and/or programming and in some cases to over-provision.

The work by Gabszewicz et al. (2000) has many similarities with the modeling approach in Anderson and Coate (2000). They introduce advertising in a Hotelling-type model of the TV industry. Each TV firm chooses a program which consists of a mix of entertainment and culture. In contrast to our model, neither the product market nor the programming investment is explicitly modeled.

Steiner (1952), writing on radio broadcasting, was concerned with whether competing radio stations would air identical type of programs at the same time. For elaborations on his model, see Owen and Wildman (1992). Spence and Owen (1977) use a model of monopolistic competition to compare the program diversity of pay-TV and advertising-financed TV. In Nilssen and Sørgard (1998), we discuss a TV duopoly where TV channels choose both programs’ contents and their time scheduling. See Gal-Or and Dukes (2001) for a recent addition to the literature on program diversity. Zhou (2000) examines the timing of commercial breaks. Empirical studies of program diversity, such as Rust and Eechambadi (1989), Rust et al. (1992), Goettler (1999), and Goettler and Shachar (2001), primarily focus on how to estimate the viewers’ demand for TV programs and the implications for TV stations’ program choice; see also Berry and Waldfogel (1999) on radio broadcasting.
Dukes and Gal-Or (2001), like Anderson and Coate (2000), take into account that viewers dislike advertising. Their novelty is the modeling of transactions on the TV-advertising market as based on contract negotiations. In particular, they put emphasis on the importance of exclusivity clauses in such contracts.

Motta and Polo (1997a) examine how TV channels’ investments in programming to attract viewers affect the structure in the TV market. In line with Sutton (1991), they find that, even in a large market, the number of TV channels can be limited in a free-entry equilibrium. The reason is that a large market size triggers intense rivalry in programming and thereby a high endogenous fixed cost per firm.

Like the work cited above, our model encompasses the two-fold role of television. However, our study is different from these in many other respects. One important distinction between our model and the others, except Motta and Polo (1997a), is that we let investments in programming be a choice variable. Although our modeling approach thus is much more closely related to Motta and Polo (1997a), it still has a focus distinctly different from theirs. We examine how product-market competition affects the equilibrium outcome in the TV industry. The profit potential in the product market depends on the toughness of price competition, the number of producers, as well as other factors. Among the above-cited authors, only Dukes and Gal-Or discuss how details of the product market affect the market for TV advertising.

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4 See also Dukes (2000).
5 Among the top ten advertisers on network TV in the US in 1999, there are three automobile producers (GM, Ford and DaimlerChrysler) and five producers of dominant brands in consumer goods industries such as, for example, cosmetics and beer (Procter & Gamble, Johnson & Johnson, Philip Morris, Unilever and Diageo). On spot TV in the US, six out of the top ten advertisers are automobile firms. See the data reported on http://adage.com/dataplace/archives. This suggests that the major advertisers on TV are dominant firms in what we typically characterize as oligopolistic industries. In line with this, we find it plausible to assume strategic interaction in the product markets that we model.
6 See also our other papers, Nilssen and Sørgard (2001a, 2001b).
In our model of the TV industry, the TV channels set the amounts of programming investment and the prices (or quantities) of advertising, while the producers determine their demand for advertising and the product price. There are, however, questions that arise that have not been discussed in the existing literature on the industry. For example, it is not clear whether the TV channels set the quantity or price of advertising. In Section 2, where we present our model and its equilibrium, we therefore report how different assumptions affect the equilibrium outcomes and the interaction between different choice variables (strategic substitutes versus strategic complements). This enables us to better understand the implications of some idiosyncratic characteristics of the TV industry.

Among the results reported in this section, we find that advertising in the two TV channels are complementary goods for the advertisers. Moreover, we find that a TV channel’s two strategic variables, programming investment and either quantity or price, reinforce each other: Increasing one also increases the marginal profit with respect to the other. The outcome is that, when the price of advertising is high, so is also investment in programming. In all cases we consider, the positive effect of the latter on the demand for advertising dominates the negative effect of the former, so that also advertising is high when the price of advertising is high. Advertising and programming investments are the highest when advertising the TV channels compete in prices instead of quantities.

In Sections 3 and 4, we apply the model to investigate two different issues. We start out, in Section 3, by asking how rivalry in the TV market affects the equilibrium outcome. This is done by an analysis of the transition from monopoly to duopoly. We find that adding a TV channel may actually decrease the amount of TV advertising as
well as the total number of viewers. This happens when, in a duopoly situation, viewer leakage between the TV channels is large.

In Section 4, we ask how product-market competition affects the equilibrium outcome. A change in product market competition can come about either through a change in the number of firms in a market or through a change in market conduct. In our model, the two sources interact: We find that an increase in the number of firms in each product market decreases advertising when producers compete but increases advertising when producers collude.

In Section 5, we summarize our results and point to some issues for future research. Results and proofs not stated in the text are collected in the Appendix.

2. THE MODEL AND ITS EQUILIBRIUM

Consider \( n \) advertising firms and a TV industry with two TV channels, where \( n \geq 2 \). The \( n \) advertising firms may or may not belong to the same product market. We assume simply that the product markets are identical, so that firms are symmetric in terms of their gains from advertising. In a companion article, we investigate the case where product markets differ (Nilssen and Sørgard, 2001a).

A standard modeling issue is whether TV channels compete in the market for TV advertising by choosing quantities or prices. On the one hand, there are programs on any TV channel's schedule where the quantity of advertising is very flexible. For example, when transmitting newscasts and sports events, a TV channel may be able to air large quantities of advertising. In order to accommodate a small amount of advertising sold, it

\[ \text{For the discussion in Sec. 3, we need to extend our model in a straightforward way to the case of a single-channel TV monopoly.} \]
can fill in with advertising for its own programs.\footnote{Such advertising for own programs are called "tune-ins". According to Shachar and Anand (1998), tune-ins constituted about one-sixth of total advertising on US TV networks in 1995.} This points in the direction of letting the price of advertising be a TV channel's decision variable, with its quantity of advertising being determined by how much it can sell at its chosen price. On the other hand, a TV station's time scheduling will in many cases put restrictions on the quantity of advertising on a channel. If, for example, a TV channel transmits a series of 25-minute sit-coms during an evening, there will only be time for 5 minutes of advertising per half-hour. Such a capacity constraint points, along the lines suggested by Kreps and Scheinkman (1983) and Tirole (1988), in the direction of treating the quantity of advertising as the actual decision variable for a TV channel. In line with this latter reasoning, we assume that the TV channels set quantities of advertising. However, in the discussion of the model in this Section, we also explore the effects of letting price rather than quantity being the decision variable.

A modeling issue more idiosyncratic to the TV industry is the unit of pricing of advertising. Despite TV channels’ use of viewer meters in order to keep records of actual viewer attendance, we choose here to model an advertising slot as the pricing unit, rather than a viewer of that advertising slot. With price being set per advertising slot, we are closer to a standard model. Moreover, it has been reported that TV channels, even with viewer meters in use, actually have not been able to set a price per viewer of advertising.\footnote{According to Goettler (1999), a typical contract between an advertiser and a TV channel specifies the prices to be paid for an advertising slot and minimum guaranteed ratings. When the guaranteed ratings are not attained, the advertiser’s ad is aired later, in another show. However, the additional ad slot is typically aired on a less popular show and does not fully compensate advertisers. In an earlier version of our paper (Nilssen and Sørgard, 2000), we explored the effect of letting the firms set the price of advertising per viewer rather than per slot. It turns out that the assumption matters for the comparative statics results. The advertising price is also set on a per-viewer basis in the model we present in Nilssen and Sørgard (2001b).}
With respect to the sequencing of decisions, it is crucial that TV viewers make their decisions knowing the benefit they gain from each TV channel. Thus, TV channels' programming decisions, as well as advertising firms' advertising decisions, are made before TV viewers make their choices in our model. At the same time, the effect of advertising on the product markets is only felt after the advertising has been actually aired and watched by the viewers-consumers. Thus, product-market competition takes place after the TV viewers' decisions are made. Finally, we assume that the advertising firms make their decisions about how much to advertise on each channel only after the TV channels have committed, not only to their programming investments, but also to their quantities (or prices) of advertising. These considerations give rise to the following four-stage game:

Stage 1: Each TV-channel chooses its quantity (or price) of advertising and its investments in programming.

Stage 2: Each producer determines how much to advertise in each TV channel.

Stage 3: Each viewer decides whether or not to watch TV and, if so, which TV channel to watch.

Stage 4: The producers compete in the product market.

For the sake of analytical simplicity, we will represent a TV channel’s decision on programming investments by the resulting attractiveness of the channel’s programs. We will denote our measure of attractiveness by *quality*, in line with Motta and Polo (1997a) and Sutton (1991), although there is arguably only a weak connection between popularity and quality of TV programs.\(^{10}\)

\(^{10}\)A different approach is taken by Cabizza and De Fraja (1998), who let quality be a measure of the regulator’s taste for the programming, rather than of its popularity among viewers.
Since we are interested in finding the subgame perfect equilibrium of this game, we proceed by backward induction and start out with describing and analyzing stage 4.

**Stage 4: The product markets**

In Section 4, we will discuss the product markets in detail. For the moment, let us simply assume that a firm's profits, gross of advertising costs, are proportional to its level of advertising. Thus, in our model, there are constant returns to scale in advertising when the product market is viewed in isolation. As will be clear shortly, diminishing returns to advertising are introduced through the effect of advertising on TV viewers' behavior.

Let firm $i$'s advertising on channel $k$ be denoted $a_{ik}$. Define $Z_{ik}$ as firm $i$'s gross profit per viewer of channel $k$. We assume that the effect of advertising on gross profit is multiplicatively separable from the other effects, *i.e.*, that there exists some $K > 0$ such that:

$$Z_{ik} = Ka_{ik},$$

(1)

A foundation for this assumption is provided in Sec. 4, where we present a model of the product markets with the property that (1) holds in equilibrium.

**Stage 3: The viewers**

At stage 3, viewers decide whether or not to watch a TV channel. A typical viewer is attracted by the quality of TV programs but dislikes commercial breaks.\footnote{It is documented that viewers try to escape from advertising breaks, see, e.g., Moriarty and Everett (1994) and Danaher (1995). In this respect, TV advertising may be distinctly different from advertising in other media. In particular, readers may actively look for certain advertisements in newspapers or magazines. Accordingly, Häckner and Nyberg (2000), in their analysis of the newspaper industry, assume that newspaper readers like advertising. Other analyses of media valued by their consumers for their advertising include Rysman (2000) on Yellow Pages, and Baye and Morgan (1999) on information gateways on the Internet.} In line with
this, we assume that a channel’s number of viewers is increasing (decreasing) in own (rival’s) program quality and decreasing (increasing) in own (rival’s) number of advertising slots.

Let $q_k$ denote program quality in channel $k$. Moreover, define total advertising on channel $k$ as $\alpha_k := \sum_i a_{ik}$. We specify the following audience function for TV channel $k$, i.e., the channel's number of viewers:\(^{12}\)

$$v_k = [q_k - \alpha_k] - d[q_h - \alpha_h], \quad d \in (0, \frac{1}{2}], \ k, h \in \{1, 2\}, k \neq h. \quad (2)$$

The parameter $d$ captures the extent to which viewers switch TV channel because of a difference in the net program quality, $q - \alpha$. If $d = \frac{1}{2}$, then a second channel entering the market with identical programming investment and advertising amount does not change the total number of viewers. It turns out that TV channels’ profits would be negative if we were to allow for $d > \frac{1}{2}$. Therefore, we restrict the analysis to $d \in (0, \frac{1}{2}]$. The number of viewers in a TV monopoly is found by putting $d = 0$ in (2).

Note that our audience function, with an increase in advertising reducing a channel's number of viewers, introduces diminishing returns to a producer's advertising: The more a firm advertises on a TV channel, the fewer viewers the channel has, and the lower gross profits the firm earns. But this also creates a congestion effect from advertising: The reduction in the number of viewers caused by one firm’s advertising affects negatively, not only this firm’s, but also other firms’ advertising on the same TV channel.

\(^{12}\)This audience function resembles, and is inspired by, the one in Motta and Polo (1997a). THEIRS, Unlike ours, is derived from a discrete-choice model of viewer behavior. Their formulation is, however, not analytically tractable for a number of our purposes.
Stage 2: Producers choose advertising

At stage 2, the producers in the product markets decide how much to advertise on each TV channel. This is a special kind of congestion game between the advertisers: When one advertiser increases its advertising on a TV channel, this will reduce the number of viewers on this channel for all its advertisers. Moreover, since viewers may switch between the TV channels as a result of differences in net quality, an advertiser may help its own (and all other advertisers') advertising on one channel by increasing its advertising on the other channel. This causes advertising on the two channels to be complementary goods. We have:

Proposition 1: The demand for advertising.

Advertising on the two channels are complementary goods for the advertising firms, and advertising demand at each TV channel is a decreasing function of the two channels’ prices per advertising slot.

Proof: Let $r_k$ denote the price per advertising slot charged by channel $k$. Producer $i$ has the following maximization problem at stage 2:

$$\text{Max } \pi_i = \sum_{k=1}^{2} Z_{ik} v_k - \sum_{k=1}^{2} r_k a_{ik} = \sum_{k=1}^{2} (Kv_k - r_k)a_{ik}$$

(3)

Total gross profits are the per-capita gross profits times the number of viewers. Producer $i$'s advertising on the two channels is determined by the following first-order conditions:

$$\frac{d\pi_i}{da_{ik}} = K\left[(q_k - dq_h) - 2(a_{ik} - da_{ih}) - \left(\alpha_{i,k} - d\alpha_{i,h}\right)\right] - r_k = 0, \ k, h \in \{1, 2\},$$
where $\alpha_{i,k} = \sum_{j \neq i} a_{jk}$.

In a symmetric equilibrium, this gives rise to a system of two equations, which we solve for a producer's demand for advertising in each channel. We find that a producer's demand for advertising space is determined by the TV channels' program quality and advertising prices in the following way:

$$a_k = \frac{1}{n+1} \left[ q_k - \frac{r_k + dr_h K}{1 - d^2} \right], \quad k, h \in \{1, 2\},$$

where $a_k$ denotes a producer's demand for advertising on channel $k$. From this expression, we see that advertising on one channel is complementary to advertising on the other, and demand is decreasing in the prices. QED.

Total advertising on channel $k$ is simply

$$\alpha_k := na_k, \quad k \in \{1, 2\}.\quad (5)$$

To see why advertising in the two channels are complements, note that an increase in the advertising price of one channel will decrease the amount of advertising there. This decrease in advertising makes the channel more attractive for viewers, and some viewers move over from the other channel. This reduction in the number of viewers on the other channel leads to a reduction in advertising in that channel as well.

\[\text{We restrict attention to symmetric equilibria.}\]
Stage 1: TV channels choose advertising quantities (or prices) and programming investments

A TV channel’s profit is the difference between its revenue from advertising and its investments in programming. The latter is modeled as a cubic function of the program quality. TV channel $k$’s problem at Stage 1 is to maximize its profits with respect to its programming and its other strategic variable, either the quantity or the price of advertising.

The concepts of strategic complements and strategic substitutes, introduced by Bulow et al. (1985), are useful for understanding the nature of the competition in a market. Let TV channel $k$'s profit be denoted $H_k$ and a generic strategic variable for the TV channels be denoted $u_k$. The TV channels' $u$s are strategic complements if channel $k$'s marginal profits with respect to $u_k$ is increasing in $u_h$, $k \neq h$, formally, if $\frac{\partial^2 H_k}{\partial u_k \partial u_h} > 0$, and they are strategic substitutes if the opposite relation holds, i.e., if $\frac{\partial^2 H_k}{\partial u_k \partial u_h} < 0$. In most textbook models, prices are strategic complements and quantities are strategic substitutes [see, e.g., Tirole (1988)]. This is not so in the present model. We have:

**Proposition 2:** Strategic variables – prices and quantities

Advertising prices are strategic substitutes and advertising quantities are strategic complements.

Proof: The profit of TV channel $k, k \in \{1, 2\}$, is:

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14This appears to be the simplest specification ensuring interior equilibrium levels of programming investments; in particular, a quadratic function is not convex enough. The cubic programming investment function is also used by Motta and Polo (1997a).
\[ H_k = r_k \alpha_k - \frac{q_k^3}{3}. \quad (6) \]

Suppose first that TV channels set prices in addition to program quality. From (4) and (5), we find TV channel \( k \)'s residual demand for advertising as:

\[ \alpha_k = \frac{n}{n+1} \left[ q_k - \frac{r_k + dr_h}{K(1-d^2)} \right], \quad k, h \in \{1, 2\}. \]

Inserting this into (6) and differentiating, we find that:

\[ \frac{\partial^2 H_k}{\partial r_k \partial r_h} = -\frac{nd}{K(n+1)(1-d^2)} < 0. \]

Suppose next that TV channels set quantities of advertising in addition to program quality. The prices of advertising are those that clear the market, \( i.e. \ r_1 \) and \( r_2 \) must solve:

\[ \alpha_k = na_k, \quad k \in \{1, 2\}, \quad \text{with } a_k \text{ given in (4)}. \]

Thus, channel \( k \)'s inverse residual demand for advertising is:

\[ r_k = K \left[ (q_k - dq_h) - \frac{n+1}{n} \left( \alpha_k - \alpha_h \right) \right], \quad k, h \in \{1, 2\}. \quad (7) \]

Inserting this into (6) and differentiating, we now have:

\[ \frac{\partial^2 H_k}{\partial \alpha_k \partial \alpha_h} = \frac{K(n+1)d}{n} > 0 \]

\[ QED. \]

We know from Proposition 1 that advertising in the two channels are complementary goods. This feature of the competition between the TV channels explains
why prices are strategic substitutes and quantities are strategic complements in that case.  

An interesting feature of the model is that a TV channel’s two strategic variables reinforce each other: An increase in one makes it profitable for the TV channel also to increase the other.

**Proposition 3: Strategic variables – reinforcement**

A TV channel’s two strategic variables are reinforcing each other, *i.e.*,  

\[
\frac{\partial^2 H_k}{\partial q_k \partial u_k} > 0, \quad k \in \{1, 2\},
\]

where \( u_k \in \{r_k, q_k\} \), depending on what is the TV channel’s other strategic variable in addition to program quality.

**Proof:** Follows from straightforward differentiations in each case. *QED.*

To illustrate the mechanism reported in the Proposition, consider the case of a positive, exogenous shift in the total number of viewers in the TV market. This would trigger more investment in programming in each TV channel in order to capture a larger share of the viewers and, in turn, a larger share of the advertising on TV. By also increasing the value of an advertising slot, each TV channel can then increase both the quantity and the price of advertising.

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\(^{15}\)Results on strategic substitutes and strategic complements are regularly reversed when we have complementary rather than substitute products. In particular, with price competition and complementary products, prices are typically strategic substitutes. For more details, see Vives (1999, Section 6.3).
The equilibrium outcome can, in each of the cases considered here, be found by solving the system of first-order conditions for the two channels. Details of this calculation and the various equilibrium expressions can be found in the Appendix (Proposition A1). Here, we note some overall features of the equilibrium outcomes.

The equilibrium variables of interest are: the price of advertising, $r$; the programming investment of each TV channel, represented by the program quality, $q$; the quantity of advertising on each channel, $\alpha$; the number of viewers on each channel, $v$; the profits earned by each TV channel, $H$; and the profits earned by each advertiser, $\pi$.

Define $X := \{r, q, \alpha, v, H, \pi\}$ as this list of variables. Let superscript, $P$ or $Q$, denote whether TV channels compete in prices or in quantities.

**Proposition 4: Comparing outcomes.**

All variables are higher when TV channels compete in prices than when they compete in quantities: $x^P > x^Q$, $x \in X$.

*Proof:* Follows from straightforward comparisons of expressions in Proposition A1 in the Appendix. *QED.*

The implication of this Proposition is that it is unclear whether price or quantity competition is the more competitive regime. Both the prices of advertising as well as the investments in programming are higher with price setting than with quantity setting. Thus, price setting results in more intense rivalry on programming and less intense rivalry on advertising than quantity setting does.
3. COMPARING TV MONOPOLY AND TV DUOPOLY

An important policy issue in broadcasting in many countries is whether to allow further entry into the TV industry by advertising-financed TV stations. Our model is suitable for performing an analysis of the effects of such an entry. In order to illustrate this, we discuss here how a TV monopoly fares relative to a TV duopoly. The TV monopoly we focus on is one with one TV channel.\footnote{This seems the relevant question in countries, like Norway, where the TV industry has seen a liberalization in recent years. In other circumstances, the relevant question may concern a merger between already established TV channels. When this is the case, the relevant comparison is between a two-channel TV duopoly and a two-channel TV monopoly.} Moreover, we stick, from now on, to the assumption that TV channels set quantities of advertising, rather than prices.\footnote{The results reported in this Section and the next are not affected by the TV channels’ choice variable; see Nilssen and Sørgard (2000).}

We introduce subscripts $M$ and $D$ to capture the distinction between a TV monopoly ($M$) and a TV duopoly ($D$). Let $\alpha_s, q_s, v_s,$ and $H_s$ (resp., $A_s, Q_s, V_s,$ and $TH_s$) denote equilibrium per-channel (resp., total) spending on advertising, program quality (representing investments in programming), viewer attendance, and TV-channel profit, respectively, in the industry, when the TV market structure is $s, s \in \{M, D\}$. Moreover, let $r_s$ denote equilibrium price per advertising slot, $s \in \{M, D\}$. Finally, each producer's gross profit per channel (resp., in total) is denoted $\pi_s$ (resp., $\Pi_s$), $s \in \{M, D\}$. Note that, in the duopoly case, $A_D = 2A_D, Q_D = 2Q_D, V_D = 2V_D, TH_D = 2H_D,$ and $\Pi_D = 2\pi_D$.

It is straightforward to establish that a TV monopolist sets a higher price, invests more in programming, obtains more advertising and more viewers, and earns more profit, than does each TV duopolist. However, in assessing the two TV-market structures, what
we need to know is whether total programming investment, advertising, and so on, are higher in a monopoly than in a duopoly. We have:

**Proposition 5: Comparing TV monopoly and TV duopoly.**

Suppose TV channels in a duopoly compete in quantities of advertising. Then:

(i) \( r_D < r_M \);

(ii) \( q_D < q_M \), and \( Q_M < Q_D \);

(iii) \( \alpha_D < \alpha_M \), and \( A_M < \alpha_d A_D \), if \( d < \alpha \) \( d_A \equiv 0.48 \);

(iv) \( v_D < v_M \), and \( V_M < \alpha v \) \( V_D \), if \( d < \alpha v \) \( d_V \), where \( 0.35 < d_V < 0.45 \);

(v) \( H_D < H_M \), and \( TH_M < \alpha H \) \( TH_D \), if \( d < \alpha H \) \( d_H \equiv 0.19 \); and

(vi) \( \pi_D < \pi_M \), and \( \Pi_M < \alpha \) \( \Pi_D \), if \( d < \alpha \) \( d_\Pi \equiv 0.20 \).

*Proof:* See the Appendix.

We see from this Proposition that the price of advertising is always lower with duopoly than with monopoly in the TV market. This should be no surprise: The introduction of a second TV channel results in rivalry on prices.

There are two effects on total investments in programming from adding a second TV channel. On the one hand, a second channel triggers competition for the advertising slots and thereby reduces the incentives to invest in programming. On the other hand, a second channel introduces a business stealing effect: Higher own programming investment will not only increase the total number of viewers in the market, but also shift some viewers from watching the rival’s programs to watching the channel’s own
programs. We find that the business-stealing effect dominates, causing total investment in programming to rise as a result of the introduction of a second TV channel. However, each duopoly channel’s investment in programming is always lower than the monopoly channel’s investment.

The total number of viewers may drop following the introduction of a second TV channel. To see this, consider the case of \( d = \frac{1}{2} \). If now a second channel enters, its investment in programming merely duplicates the first channel’s investment, seen from the viewers’ point of view. If \( d = \frac{1}{2} \) and the entering channel has the same programming investment and advertising amount as the incumbent, then the entry of a new channel does not affect the total number of TV viewers. In such a case, therefore, total programming investment in the industry must be more than doubled following the introduction of a second channel for the number of viewers to increase. However, each duopoly channel’s programming investment is lower than that of a monopoly channel. Therefore, the total number of viewers drops when a second channel enters. On the other hand, if the second channel were independent of the first channel (\( d = 0 \)), then it would be as if there are two monopoly channels, and the introduction of a second channel will surely increase the total number of viewers. By continuity, then, there must be some critical value of \( d \) between 0 and \( \frac{1}{2} \) at which the total number of viewers is equal among monopoly and duopoly.

Surprisingly, the total spending on advertising may drop when a second TV channel is introduced. All else equal, a lower price per advertising slot will result in more advertising. On the other hand, as explained above, the total number of viewers may drop
as a result of entry. If \( d \) is sufficiently high, then the reduction in the total number of viewers is so large that it offsets the effect of lower price on advertising.

Finally, we see that, even if total investment in programming increases and price per advertising slot drops as a result of an introduction of a second TV channel, producers may be better off with a monopoly than with a duopoly in the TV industry. Interestingly, this occurs when the viewers are sufficiently prone to switch channels (a high \( d \)). When this is the case, the number of viewers on each of the two channels is low, and therefore the producers advertise less. Despite a lower advertising price, the combined effect is a preference for a TV monopoly among advertising firms, even at a modest degree of viewer leakage. The driving force is the reduction in the number of viewers following a transition from a monopoly to a duopoly TV industry.

4. THE PRODUCT MARKETS

Let us now extend the basic model to take into account the rivalry in the product markets. We assume that all product markets are identical, with the same demand conditions and the same number of producers.\(^{18}\)

There are a total of \( m \) product markets, with \( f \) firms in each, \( m \geq 1 \) and \( f \geq 2 \), so that the total number of advertisers is: \( n = mf \). Furthermore, we assume that the products sold in each market are identical, and we let \( p \) denote the price per unit. By way of normalization, we set production costs equal to zero.

In general, both price and advertising are expected to affect sales in the product market: A price reduction expands sales, and so does an increase in advertising.

\(^{18}\)In Nilssen and Sørgard (2001a), we relax this assumption by letting product markets differ with respect to the number of firms.
However, it is not obvious how price and advertising interact. On the one hand, advertising may increase each existing consumer’s loyalty to one's product, or increase the number of loyal consumers relative to that of other consumers. If so, a producer’s optimal response to more own advertising may be to raise price to exploit the loyal consumers. On the other hand, advertising makes consumers aware of one's product. To the extent this is the case, we may observe more intense rivalry on prices because the informed consumers are able to pick from all those offers that they are aware of. Hence, in theory, advertising has an ambiguous effect on prices. Empirical studies report ambiguous effects of advertising as well. We side-step from the question of whether advertising has a price-increasing or price-reducing effect by developing here a model where a firm’s advertising in equilibrium affects its sales only, not the price.

Although prices are not affected by the amount of advertising, the number of firms in the product market may affect product prices. We investigate two different regimes: Cournot competition and collusion on prices (semi-collusion). As it turns out, the two regimes are sufficient to show that the market outcome depends crucially on the toughness of competition.

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19Eckard (1991) studies the effect of the 1970 ban on TV advertising for cigarettes in the US and concludes that the ban had an anti-competitive effect, implying that TV advertising as such would have increased price rivalry. Also Leahy (1991) reports a negative relation between TV advertising and prices. Kanetkar, et al. (1992) examine how TV advertising affects consumers’ price sensitivity for two frequently purchased consumer goods. They find that, for high levels of advertising exposure, price sensitivity drops, while the opposite is true for lower levels of advertising exposure. This implies that, at high levels of TV advertising, further advertising dampens price competition, while the opposite is true for lower levels. According to their study, then, there is a U-shaped relation between the level of advertising and the product price. Moreover, studies of advertising in general find ambiguous results as well. See, for example, Vakratsas and Ambler (1999) for a review of the marketing literature.

20Our modeling approach shares some similarities with Schmalensee (1992), who develops two simple models complementing the analytical framework introduced in Sutton (1991). He does not insist on a particular regime. In one model, he uses a parameter to capture the degree of competition. In another
Cournot competition

Each viewer of channel $k$ has the following individual inverse demand in each product market:

$$p_k = 1 - \frac{1}{B} \sum_{i} \left( \frac{y_{ik}}{a_{ik}} \right),$$

(8)

where $y_{ik}$ is the per-capita quantity offered by firm $i$ to viewers of channel $k$, with $Y_k := \sum_i y_{ik}$ being the total sales in each product market. The parameter $B$ can be interpreted as a scale parameter. Recall that $a_{ik}$ denotes producer $i$'s advertising on channel $k$.

We allow for prices offered to consumers to differ according to which TV channel the consumers are viewing; this is done for analytical convenience. More significantly, we allow a firm's advertising to affect demand: The more a producer advertises, the less sensitive is the market price to an increase in its offered quantity.

Firm $i$'s per-capita profit, gross of advertising costs, equals $p_k y_{ik}$ among channel $k$'s viewers, since production costs are assumed to be zero. This gives rise to the following first-order condition for firm $i$ with respect to its offered quantity:

$$\left[ 1 - \frac{1}{B} \sum_{i} \left( \frac{y_{ik}}{a_{ik}} \right) \right] - \frac{y_{ik}}{a_{ik} B} = 0, \quad k \in \{1, 2\}.$$  

Summing over all $f$ firms' first-order conditions in each market, we obtain:

$$f \left[ 1 - \frac{1}{B} \sum_{i} \left( \frac{y_{ik}}{a_{ik}} \right) \right] - \frac{1}{B} \sum_{i} \frac{y_{ik}}{a_{ik}} = 0, \quad k \in \{1, 2\},$$

implying that, in equilibrium,
\[
\frac{1}{B} \sum_i \left( \frac{y_{ik}}{a_{ik}} \right) = \frac{f}{f+1}, \quad k \in \{1, 2\}.
\]

Thus, the equilibrium price in each market does not depend on how much firms advertise or which channel consumers are viewing:

\[
p_k = 1 - \frac{1}{B} \sum_i \left( \frac{y_{ik}}{a_{ik}} \right) = \frac{1}{f+1}, \quad k \in \{1, 2\}.
\]

There are two effects of a firm's advertising: Its sales increase, leading to an increase in total sales and thus a reduction in price. But this, in turn, entails a reduction in the sales of rival firms, which leads to an increase in price. The two effects balance each other off exactly in this particular model.

The above expression may be inserted in each firm's first-order condition to obtain firm \(i\)'s equilibrium per-capita sales among viewers of channel \(k\):

\[
y_{ik} = \frac{Ba_{ik}}{f+1}.
\]

The per-capita gross profits of firm \(i\) among the viewers of channel \(k\) amount to:

\[
Z_{ik} = Ka_{ik} = \frac{B}{(f+1)^2} a_{ik}.
\] (9)

Thus, \(K\), the marginal gross profits per viewer with respect to a firm's advertising, is a specific decreasing function of the number of firms in the product market.

We are now in a position to investigate how the equilibrium outcome is affected by a change in the number of advertisers, \(n\). This number may increase, either through an increase in the number of firms in each market, \(i.e.,\) a decrease in market concentration throughout the economy, or through an increase in the number of product markets.
Proposition 6: Changing the number of advertisers: Cournot competition.

Suppose producers compete in quantities in each product market. Then, equilibrium variables in the market for TV advertising decrease if the number of firms in each product market, $f$, increases, and increase if the number of product markets, $m$, increases:

$$\frac{\partial x}{\partial f} < 0 < \frac{\partial x}{\partial m},$$

where $x \in \{r_s, q_s, \alpha_s, v_s, H_s, \pi_s\}$, and $s \in \{M, D\}$.

Proof: In the equilibrium values shown in Proposition A1(ii), in the Appendix, we substitute: $K = B/(f + 1)^2$, and $n = mf$. Now, the results can easily be verified. QED.

According to this Proposition, total spending on advertising increases as a result of a reduction in the number of firms, keeping constant the number of product markets. Note that there are two opposing forces at work. On the one hand, a reduction in the number of firms makes each remaining firm more concerned about the fact that own advertising tends to reduce the number of viewers. This dampens the incentive for each firm to increase advertising, and would all else equal result in a reduction in total advertising. On the other hand, fewer firms result in a higher price-cost margin. This would encourage firms to advertise more. The latter effect turns out to dominate, and it is reinforced by the TV channels’ responses. They invest more in programming, thereby attracting more viewers and even more advertising. The result is that both total advertising and total investment in programming increase following a reduction in the number of firms.
Also the total number of viewers increases following a reduction in the number of firms. Since advertising increases as well, which tends to reduce the number of viewers, the driving force behind this result is the TV channels’ increased investment in programming.

However, total spending on advertising can also increase as a result of an increase in the number of advertising firms, if this latter increase is solely due to an increase in the number of product markets. In such a case, price-cost margins are unaffected by a change in the number of firms. Now, an increase in the number of firms makes each firm less concerned about own advertising’s effect on the number of viewers. This spurs an increase in total advertising. Again, the TV channels’ response reinforces the initial effect. They invest more in programming, thereby increasing the total advertising even more.

**Semi-collusion**

Suppose now that firms collude on prices at stage 4. The collusion is restricted to the pricing, though; thus, ours is a case of semi-collusion, with firms colluding on price while behaving non-cooperatively in their stage-2 advertising decisions, foreseeing the collusion in price further on.²¹

Suppose each viewer on channel \( k \) has the following demand function:

\[
Y_k = (1 - p)B\alpha_k, \tag{10}
\]

²¹Since prices are more flexible than most other choice variables, it is easier to collude on prices than on other variables. Therefore, most of the literature on semi-collusion assumes collusion on prices and competition along another dimension, such as for example advertising, capacity, or location. For a review of the semi-collusion literature, see Phlips (1995).
where, as above, $\alpha_k$ is total advertising on channel $k$ and $B$ is a scale parameter. Maximizing their total profits $pY_k$ on each viewer, the colluding firms set $p = 1/2$, so that $pY_k = B\alpha_k/4$. Note that the collusive price at stage 4 again is independent of the amount of advertising. The sale of each firm is assumed to be determined by its amount of advertising. In particular, we assume that each member of the colluding group of firms obtains a market share equal to its share of total advertising. It follows that, in this case of semi-collusion, $Z_{ik} = Ba_{ik}/4$. Thus, $K = B/4$; i.e., the marginal gross profit from advertising is now independent of the number of firms in each market, contrary to the case of Cournot competition above. We have:

**Proposition 7: Changing the number of advertisers: Semi-collusion.**

Suppose the product markets are characterized by semi-collusion. Then, the effect of an increase in the number of advertisers is to increase equilibrium variables in the market for TV, irrespective of whether it is the number of firms in each product market, $f$, or the number of product markets, $m$, that increases:

$$\frac{\partial x}{\partial f} = \frac{\partial x}{\partial m} > 0,$$

where $x \in \{r_s, q_s, \alpha_s, v_s, H_s, \pi_s\}$, and $s \in \{M, D\}$.

**Proof:** In this case, $K$ is substituted with $B/4$, and, as in the case of Proposition 6, $n$ is substituted with $mf$ in the equilibrium values in Proposition A1(ii), in the Appendix. The results are now easily verified. *QED*.
The results concerning $f$, the number of firms in each market, are now reversed compared to the case of Cournot competition. A reduction in the number of firms results in lower prices on advertising, less total advertising, less investment in programming, and fewer viewers. The main distinction between semi-collusion and Cournot competition is that, now, product prices are unaffected by a reduction in the number of firms. The incentive to increase advertising and, in turn, sales, due to higher product prices, is no longer present. The driving force is that fewer firms result in less intense rivalry on advertising. Both prices and quantities of advertising drop as a result of a reduction in the number of firms. The reduction in the amount of advertising dampens the TV channels’ incentives to invest in programming. Note also that the lower investment in programming reduces the number of viewers, despite the fact that also the amount of advertising is lower.

Finally, let us examine how the toughness of competition affects the market outcome. We do this by comparing our two cases of Cournot competition and semi-collusion. Let superscripts $S$ and $C$ denote the semi-collusion and Cournot regimes, respectively.

**Proposition 8:** The toughness of competition

All equilibrium values are higher with semi-collusion than with Cournot:

$$x^S > x^C$$

where $x \in \{r_s, q_s, \alpha_s, v_s, H_s, \pi_s\}$, and $s \in \{M, D\}$. 
Proof: $K$ enters as a multiplicative term in all the equilibrium values in Proposition A1(ii) in the Appendix. We know that $K = B/(1+f)^2$ with Cournot competition and $K = B/4$ with semi-collusion. It follows straightforwardly that the equilibrium values are always higher with semi-collusion than with Cournot competition, since $f \geq 2$. QED.

There is a larger profit potential in the product market under collusive price setting than under Cournot competition. Each TV channel exploits this by setting a higher price per slot of advertising, and by increasing its investment in programming, thereby attracting more viewers.

Also advertising is higher under price collusion than what is the case when Cournot competition prevails. In a TV duopoly, price collusion results in more investment in programming and a higher price of advertising. Programming and the price of advertising have opposite effects on producers’ demand for advertising. However, the effect of higher price on each advertising slot is not large enough to offset the effect of more investment in programming.

Note also that the number of viewers is higher under price collusion than under Cournot competition. There is more advertising in semi-collusion, which tends to reduce the number of viewers. On the other hand, the large investment in programming in collusion attracts viewers. According to the Proposition, the latter effect dominates.

The results reported here indicate that there are two successive battles over profit potentials in the product markets, one battle among the producers and one among the TV channels, and that these two battles may mutually reinforce each other. An escalation of advertising by the producers spurs more investment in programming, and vice versa.
5. CONCLUDING REMARKS

There are few studies in the economics of the two-fold role of the TV industry as both a provider of entertainment and a transmitter of advertising. To help fill this gap, we have presented a stylized model that encompasses some of the TV industry’s idiosyncratic characteristics. Most importantly, we assume that viewers are attracted by TV channels’ investments in programming but dislike their advertising, and we model advertising as a link between the product markets and the TV market.

Since the TV industry has some idiosyncratic features, it is of interest to elaborate on some of the basic mechanisms that are in force. It turns out that the strategic interaction in this particular industry can be distinctly different from other industries. For example, we find that advertising prices are strategic substitutes. Moreover, we find that price, instead of quantity, as a choice variable can trigger more intense rivalry on programming but less intense rivalry on the price of advertising. Thus, it is unclear whether price or quantity setting in advertising gives rise to the more competitive environment. This suggests that one should be careful with applying standard IO results to the TV industry, but rather draw conclusions only from models that are tailor-made for it.

We have applied our model to two different issues, the first one being how rivalry in the TV market affects the market outcome. By comparing monopoly and duopoly in the TV market, we found that rivalry between TV channels can lead to a reduction in the total number of viewers. The reason is that TV channels partly duplicate each other's
programming investments, and each duopoly TV channel invests less in programming than a monopoly TV channel does. If the viewers’ propensity to switch TV channels is high enough, then the total increase in the investment in programming is not large enough to offset the duplication of programming investments among the TV channels, and the number of viewers drops. The amount of advertising may drop as well. This happens if the effect on advertising of a reduction in the number of viewers offsets the effect of a reduction in the price of advertising.

The second issue is how product-market competition affects the equilibrium outcome. We found that the profit potential in the product market is of importance for the amount of programming investments as well as for the amount and price of advertising. The less intense product-market rivalry is, the larger is the potential revenue generated by advertising. A TV channel exploits this in two ways. First, it reduces its supply of advertising slots. Second, it invests more in programming to attract more viewers and thereby to encourage the producers to advertise more. As a result, a relaxation of price competition in the product markets results in higher prices of advertising, more advertising, and more investment in programming. This suggests that there are two successive battles over the profit potential in the product markets: one among the producers and one among the TV channels. An escalation of advertising by the producers spurs more investment in programming, and vice versa.

Product market competition may also be affected by a change in the number of firms. We found that the effect of increasing the number of advertising firms depends on whether the increase is by increasing the number of firms in each market, making the markets less concentrated, or by increasing the number of markets. The former way of
increasing the number of advertising firms reduces the price-cost margin and thereby the profit potential in the product markets. Thus, while there now are more firms demanding advertising, they also earn less from advertising. When firms compete a la Cournot in the product market, we find that the latter effect dominates. In that case, total advertising drops when the number of firms in each market increases. We showed that the investment in programming drops as well, something that, in turn, reduces the number of viewers.

The result with Cournot competition is reversed if the firms in each market are able to collude on price, and/or if the number of advertising firms increases through an increase in the number of product markets. In those two cases, there are no price effects in the product market due to a change in the number of firms. More firms would then trigger more intense rivalry on advertising, increasing the total amount of advertising.

A topic left unexplored in this paper is the welfare properties of an unregulated market for TV advertising and how, if necessary, regulation should be done. In Norway and France, for example, there are regulations on the amount of TV commercials. In the U.S., there used to be a similar regulation, self-imposed by the TV industry itself, through the National Association of Broadcasters. In the early 1980's, however, the U.S. Department of Justice filed an antitrust suit against the N.A.B. code, and its restrictions on the amount of TV advertising was lifted. As far as we know, Anderson and Coate (2000) and Gabszewicz et al. (2000) are the only studies that discuss the question whether there is too much or too little TV advertising. Because we feel that aspects of TV advertising are left out of their analysis, such as the roles of programming investments

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22 In Norway, TV commercials are restricted to a maximum of 12 minutes per hour and to a maximum of 15% of daily transmission time. There are also restrictions on how various programs can be interrupted by commercial breaks (see http://www.smf.no/4.2.0.asp for details). In France, TV commercials are also restricted to a maximum of 12 minutes per hour (Desmoulins, 1998).
and advertisers’ profits, we believe applying our model to the welfare issue would provide more insight about the efficiency of an unregulated market for TV advertising.

APPENDIX

Proposition A1: Equilibrium outcomes in TV duopoly.

(i) If TV channels compete in prices, then the equilibrium outcome is given by:

\[ r = \frac{K^2 n(1-d^2)^2}{(n+1)(d+2)^2}; \]

\[ q = \frac{Kn(1-d^2)}{(n+1)(d+2)}; \]

\[ \alpha = \frac{n}{n+1} \left[ q - \frac{r}{K(1-d)} \right] = \frac{Kn^2 (1-d^2)}{(n+1)^2 (d+2)^2}; \]

\[ \nu = (1-d)(q-\alpha) = \frac{Kn(1-d)^2(d+1)(d+1)(n+1)+1}{(n+1)^2 (d+2)}; \]

\[ H = r\alpha - \frac{q^3}{3} = \frac{K^3 n^3 (1-d)^4 (1+d)^3}{3(n+1)^3 (d+2)^4}; \]

\[ \pi = \frac{\alpha(K\nu - r)}{n} = \frac{K^3 n^2 (1-d)^3 (1+d)^2}{(n+1)^4 (d+2)^4}. \]

(ii) If TV channels compete in quantities, then the equilibrium outcome is given by:

\[ r = K(1-d) \left[ q - \frac{n+1}{n} \alpha \right] = \frac{K^2 n(1-d)^2}{(n+1)(2-d)^2}; \]

\[ q = \frac{Kn(1-d)}{(n+1)(2-d)}; \]

\[ ^{23}U.S. \ v. \ National \ Association \ of \ Broadcasters, \ 553 \ F. \ Supp. \ 621 \ (1982). \ Descriptions \ of \ the \ case \ can \ be \ found \ in \ Hull \ (1990) \ and \ Owen \ and \ Wildman \ (1992, \ ch. \ 5). \]
\[ \alpha = \frac{Kn^2(1-d)^2}{(n+1)^2(2-d)^2}; \]

\[ v = (1-d)(q-\alpha) = \frac{Kn(1-d)^2[n+2-d]}{(n+1)^2(2-d)^2} ; \]

\[ H = r\alpha - \frac{q^3}{3} = \frac{K^3n^3(1-d)^3(1-2d)}{3(n+1)^3(d-2)^4}; \]

\[ \pi = \frac{\alpha(Kv-r)}{n} = \frac{K^3n^2(1-d)^5}{(n+1)^3(d-2)^4}. \]

*Proof of Proposition 5:* By differentiations in the expressions in Proposition A1(ii), we obtain:

\[ \frac{\partial r}{\partial d} = \frac{2K^2n(1-d)}{(n+1)(d-2)^3}; \]

\[ \frac{\partial q}{\partial d} = -\frac{Kn}{(n+1)(d-2)^2}; \]

\[ \frac{\partial \alpha}{\partial d} = -\frac{2Kn^2(d-1)}{(n+1)^2(d-2)^3}; \]

\[ \frac{\partial v}{\partial d} = \frac{Kn(1-d)[2n+6-5d+d^2]}{(n+1)^2(d-2)^3}; \]

\[ \frac{\partial H}{\partial d} = -\frac{K^3n^3(1-d)^3(-2+3d)}{(n+1)^3(d-2)^5}; \]

\[ \frac{\partial \pi}{\partial d} = -\frac{K^3n^2(d-1)^5}{(n+1)^3(d-2)^4}. \]

We see that they are all negative for \( d \in (0, \frac{1}{2}] \). Since monopoly corresponds to a duopoly with \( d = 0 \), the inequality in part (i) and the first inequality in each of parts of (ii)-(vi) follow from the above expressions. Furthermore, using the expressions in Proposition A1(ii), we have that:

\[ Q_M - Q_D = \frac{Kn(2-3d)}{2(n+1)(d-2)} = 0 \text{ for } d = 2/3. \]
\[ A_M - A_D = \frac{K n^2 (4 - 12d + 7d^2)}{4(n + 1)^2 (d - 2)^2} = 0 \text{ for } d = \frac{6 - \sqrt{2}}{7} \equiv 0.45. \]

\[ V_M - V_D = -\frac{K n^3 [4n - 12nd + 7nd^2 + 8 - 32d + 30d^2 - 8d^3]}{4(1 + n)^2 (d - 2)^3} = 0 \text{ for a value of } d \text{ which depends on } n \text{ but which is always within (0.35, 0.45)}. \]

\[ H_M - TH_D = -\frac{K n^3 (16 - 128d + 264d^2 - 216d^3 + 63d^4)}{48(n + 1)^3 (d - 2)^3} = 0 \text{ for } d \equiv 0.19. \]

\[ \Pi_M - \Pi_c = \frac{K n^3 (-16 + 128d - 296d^2 + 312d^3 - 159d^4 + 32d^5)}{16(n + 1)^4 (d - 2)^4} = 0 \text{ for } d \equiv 0.20. \]

Recall from the above that \( X_D \) is decreasing in \( d \) for all \( d \in (0, \frac{1}{2}) \) and for \( X \in \{Q, A, V, TH, \Pi\} \). Since the monopoly variables are independent of \( d \), there is therefore at most one value of \( d \in (0, 1/2) \) for which each difference is zero, and if such a critical value exists, then the difference is negative for \( d \) below this value and positive above it. \( \text{QED} \).

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