Strategic Informative Advertising on TV*

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Abstract:

In order to get a grasp of the market for advertising, one needs to understand why firms demand advertising, and how suppliers of advertising set their prices. In the present analysis, firms buy advertising in order to inform consumers about their products. TV channels offer advertising on a per-viewer basis. We find that the performance of this market for advertising depends on features of both the demand side and the supply side. In particular, the price of advertising is increasing both in the degree of consumer heterogeneity in the product market, and in the difference between the TV channels' audiences. (100 words)

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1. Introduction

A considerable amount of advertising is channeled through TV.[1] Still, there is little economic research focusing particularly on TV advertising. In the present analysis, we fill this gap by modeling firms that compete with each other in the product market and therefore demand advertising on TV in order to increase consumer awareness of their products. At the other side of the advertising market, we model TV channels selling advertising on a per-viewer basis. This captures the widespread practice of relating the price of an advertising slot to the number of viewers watching it. This way of selling advertising creates a scope for a TV station to become capacity constrained: It cannot sell advertising at a rate higher than 100% of its viewers. Thus, even if a small TV station has the lowest advertising price, it may be necessary for an advertiser to turn also to higher-priced TV stations when the desired advertising reach goes beyond the low-price TV station's audience. This leads to a softening of competition between the TV stations, as even the higher-priced station may get some advertising business. We find that the more unequal the TV channels are by audience size, the higher is the equilibrium advertising price. More expectedly, this price is also increasing in the extent of differentiation in the product market: The more heterogeneous consumers' preferences are, the more profitable it is to advertise.

Although the interest in the market for TV advertising has increased in the recent literature, only a few other papers include strategic interaction among the advertisers in the analysis.[2] Of particular relevance is the work of Gal-Or and Dukes (2001). They discuss informative advertising, like we do here. However, they are more concerned with how the presence of strategic advertising strengthens the conclusion from the early literature on the TV industry[3] that competition among TV stations tends to result in too similar TV programs. In our companion papers [Nilssen and Sørgard (2001a, 2001b)], we consider various aspects of competition between TV stations with respect to pricing of advertising and investments in programming, in a setting where TV stations' viewers are endogenous and where there are

[1]In the US in 1998, for example, TV advertising amounted to $ 41.1 billion, out of a total advertising of $ 79.5 billion; *i.e.*, more than half of all advertising, in terms of value, was on TV. See the data reported by *Advertising Age* on http://adage.com/dataplace/archives/dp394.html.
[2] Other recent models of the TV industry disregard the strategic interaction among advertisers. See Masson, *et al.* (1990), Motta and Polo (1997), Anderson and Coate (2000), and Gabszewicz, *et al.* (2000).
[3] The early literature is summarized in Owen and Wildman (1992).

multiple product markets. This comes at the cost of a product-market model that is more simplistic than the one we use here.

In the analysis below, we first present, in Section 2, a model of product-market competition with advertising. We focus our analysis on informative advertising and present a version of a model due to Grossman and Shapiro (1984). This version takes into account that advertising is offered by a pair of TV stations. We go on, in Section 3, to discuss how the TV stations set their prices of advertising and present our main results on the equilibrium price of advertising, as discussed above. Section 4 contains concluding comments. Precise statements of some of our results, as well as all proofs, are collected in an Appendix.

2. Strategic Informative Advertising on TV

Actual advertising on TV is in part informative and in part persuasive: Firms advertise in order for their products to be known to the public, but they also advertise in order to persuade consumers to buy their product rather than some other product. At present, we want to focus on informative TV advertising.[4] A seminal analysis of informative advertising is by Grossman and Shapiro (1984). Their analysis clarifies the basic effects of firms' informative advertising in an oligopoly: First, the more it advertises, each firm obtains an increase in the number of consumers aware of its product. Thus, effectively, advertising increases the potential market for a firm's product and therefore is a good thing for the firm. Secondly, the more the firms in the market advertise in total, the higher is the number of consumers being aware of two or more products. Such segments of informed consumers are marked by more fierce competition than other segments, since these consumers are able to pick from all the offers they are aware of. Thus, more advertising means more competition, which harms firms' profits. In equilibrium, one will see a trade-off between these two effects of advertising.

In our analysis, we use Tirole’s (1988) version of the Grossman-Shapiro (1984) model. This version is based on the Hotelling (1929) set-up: Consumers are uniformly distributed on [0, 1]; the number of consumers is normalized to 1. Each consumer has a unit demand with a gross benefit equal to s, and his transportation costs are linear: A distance d on [0, 1] between a

[4] Empirical work by Eckard (1991), Leahy (1991), and Gallet (1999) indicates that TV advertising provides a downward pressure on prices, as is predicted by theories of informative advertising.

consumer's favorite and the actually consumed product entails a utility loss equal to *td*. In order to simplify our subsequent analysis, we make the assumption that $t \geq 0.3$.

On the supply side of the product market, there is a duopoly, with the two firms, *A* and *B*, located at the two extreme positions 0 and 1. A consumer is able to consume a particular firm's product if and only if he receives an ad from this firm. Let φ_i be firm *i*'s *advertising reach*, the fraction of the consumers in the market who receive an ad from the firm.

If firm *A*'s advertising reach is φ_A, then this is also the potential demand for firm *A*'s product. A fraction φ_B of the φ_A consumers receive, in addition to an ad from firm *A*, also an ad from firm *B* and therefore know about both firms. The residual fraction $(1 - \varphi_B)$ of consumers receive an ad from firm *A* only and do not know about the presence of firm *B* in this market. There is no targeting of the advertising, nor anything else present that could create a correlation between the characteristics of a firm's product and those of the consumers reached by the firm's advertising.[5] Thus, the φ_A consumers aware of firm *A*'s product are evenly spread on [0, 1]. So is also that subgroup of $\varphi_A\varphi_B$ consumers aware of both products, and so on. It is for the consumers who have received ads from both firms that competition prevails.

Advertising is offered by TV channels competing on advertising prices. This makes producers' cost of advertising determined by the TV stations' rivalry. How much an advertiser will have to pay for its advertising depends on how many viewers the advertising has. We assume that TV channels, through the use of viewermeters, are able to monitor the number of viewers at any time and therefore sell advertising space at a price per viewer of the advertising.[6] An advertising firm decides not only how much to advertise, but also how to distribute its advertising efforts among the two TV channels.

The audience is not the same for the two TV channels. In particular, *v* is the size, in terms of viewers, of the bigger channel, and $(1 - v)$ the size of the smaller channel, with $v \in [½, 1)$. Throughout, we let channel 1 be the bigger one.[7] At each point in [0, 1], a fraction *v* of the

[5]An interesting extension would be to take into account that TV channels are attractive to different viewer groups, *e.g.*, because of differences in programming. However, while we assume there to be no scope for targeting, existing models of targeting in advertising, such as Bester and Petrakis (1996) and Roy (2000), tend to make the opposite extreme assumption that targeting is perfect.
[6] The alternative would be to assume that advertising is priced per advertising slot; this is the assumption made in Nilssen and Sørgard (2001a, 2001b).
[7]We treat a TV station's number of viewers as an exogenous variable. An alternative, pursued in Nilssen and Sørgard (2001a, 2001b), would be to assume that viewers respond negatively to an increase in a TV station's advertising.

consumers preferring this particular product variety watch the bigger channel while the rest watch the smaller one.

We assume that the only costs of advertising for the firms on the product market are related to what they have to pay the TV channels for the advertising space, or more accurately, for the viewers reached by the advertising.[8]

Each producer has a total of three decisions to make: the price of its product; advertising in the big TV channel 1; and advertising in the small channel 2. Let φ_{ij} denote producer j's advertising reach among the viewers of TV station i, $i \in \{1, 2\}$, $j \in \{A, B\}$. Producer j's total advertising reach is $\varphi_j = v\varphi_{1j} + (1 - v)\varphi_{2j}$, $j \in \{A, B\}$. The price of advertising per viewer at TV station i is b_i. A producer's advertising costs depend on how many viewers are reached by advertising on each channel: $A(\varphi_{1j}, \varphi_{2j}) = b_1\varphi_{1j}v + b_2\varphi_{2j}(1 - v)$, $j \in \{A, B\}$.

Among the consumers being informed about both producers' existence, the indifferent one is given by:

$$\frac{1}{2}+\frac{p_B - p_A}{2t};$$

see, *e.g.*, Tirole (1988, Sec. 7.3.2.2); we assume throughout that the gross benefit s is so large that the indifferent consumer prefers buying to not buying.

The two producers make their pricing and advertising decisions simultaneously.[9] Producer i's problem thus is:

$$\max_{\{p_i,\varphi_{1i},\varphi_{2i}\}} \pi_i = \left\{\varphi_i\left[\left(1-\varphi_j\right)+\varphi_j\left(\frac{1}{2}+\frac{p_j - p_i}{2t}\right)\right]\left(p_i - c\right)-b_1\varphi_{1i}v-b_2\varphi_{2i}\left(1-v\right)\right\},$$

$$i, j \in \{A, B\},\ i \neq j.$$

The first-order condition with respect to price is:

$$p_i = \frac{p_j + c + t}{2}+\frac{t\left(1-\varphi_j\right)}{\varphi_j}, \quad i, j \in \{A, B\},\ i \neq j. \tag{1}$$

Each firm has two first-order conditions with respect to advertising, one for each TV channel. Note, however, that there is a linear relationship between a firm's marginal costs of advertising

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[8]An alternative would be to add a cost to cover the creative component of the advertising. Assuming that this added cost is convex, we would obtain convexity also in total costs of advertising. However, this extension is not analytically tractible.

[9] As it turns out, prices are determined recursively; thus, results would not be altered if producers' demand for advertising was determined before their product prices.

on a particular TV station and its marginal benefit from it. The marginal benefit per viewer from advertising is independent of which channel the marginal advertising is put on, since both stations' viewers are evenly distributed as consumers in product space. Denote this marginal benefit per viewer for firm *A* by K_A, *i.e.*:

$$K_A = (p_A - c)\left[(1-\varphi_B) + \varphi_B\left(\frac{1}{2} + \frac{p_B - p_A}{2t}\right)\right]$$

The net marginal profit for firm *A* from advertising on the big channel 1 can now be expressed as:

$$\frac{d\pi_A}{d\varphi_{1A}} = K_A v - b_1 v = [K_A - b_1]v.$$

The corresponding expression for advertising on channel 2 is:

$$\frac{d\pi_A}{d\varphi_{2A}} = [K_A - b_2](1-v).$$

The firm will spend the additional funds on advertising on the channel where the net marginal profit is the greater. Thus, it chooses channel 1 if:

$$[K_A - b_1]v > [K_A - b_2](1 - v),$$

i.e., if:

$$b_1 < \left(2 - \frac{1}{v}\right)K_A + \left(\frac{1}{v} - 1\right)b_2.$$

A corresponding condition holds for firm *B*.

We focus on symmetric equilibria. We let $p_A = p_B = p$, $\varphi_A = \varphi_B = \varphi$, and, therefore, $K_A = K_B = K$. Also, $\varphi_{iA} = \varphi_{iB} = \varphi_i$, $i \in \{1, 2\}$. In our simple model, a firm will choose the cheaper TV channel for the whole of its basis of viewer, if necessary, before considering making use of the other channel's services. It may have to use the latter channel, however, if the desired level of advertising cannot be covered through one channel. For example, if the small channel 2 is the cheaper one but each firm wants to obtain an advertising reach in excess of $(1 - v)$, the number of channel 2's viewers, then they both will have to turn to the other channel.

In equilibrium, the marginal benefit of advertising is set equal to marginal costs of advertising for the firms. Since the latter is equal to either b_1 or b_2, we have that, either $K = b_1$, or $K = b_2$. In both these cases, however, the condition for a firm preferring channel 1 over channel 2, which with symmetry reads:

$$b_1 < \left(2 - \frac{1}{v}\right)K + \left(\frac{1}{v} - 1\right)b_2,$$

reduces to:

$$b_1 < b_2.$$

With symmetry, the equilibrium price can be expressed as a function of the level of advertising, from (1):

$$p = c + t\frac{2-\varphi}{\varphi}.$$

Using this, we can write the marginal benefit of advertising as:

$$K = \frac{t(2-\varphi)^2}{2\varphi}.$$

The producers' advertising decisions, as functions of the TV channels advertising prices b_1 and b_2, are detailed in Proposition A1, in the Appendix. An illustration is provided in Figure 1. Notice that, if both TV channels raise their prices, so that we move in a northeast direction in the Figure, then advertising goes down and therefore also competition is weakened in the product market. If one TV channel alone raises its price, so that we move horizontally or vertically in the Figure, then that channel gets less advertising.

< FIGURE 1 ABOUT HERE>

3. The market for advertising on TV

In the analysis of the previous Section, the prices that the TV channels charge for advertising were exogenous. In this Section, we analyse the TV channels' decisions on advertising prices.[10]

We envisage a two-stage game, in which the TV channels simultaneously determine their respective prices of advertising at stage 1, while the advertising firms' price and advertising decisions are made simultaneously, as in the previous Section, at stage 2.

It is clear from Proposition A1, in the Appendix, and the illustration of it in Figure 1 that a channel's audience works as a form of capacity: A TV channel cannot sell more advertising reach than it has viewers. Thus, even the higher-priced channel may get some of the advertising business, precisely because the lower-priced channel cannot provide the full service that the advertising firms want. However, in contrast to other models of capacity constraints, notably Kreps and Scheinkman (1983), this does not give rise to non-existence of a pure-strategy equilibrium. In the present model, each TV channel's best-response price is a continuous function of the other channel's price: If the other channel's price is low, then it pays to keep one's own price a bit higher. Otherwise, the best response is to undercut. The precise statement is found in Proposition A2 in the Appendix. An illustration of the best-response functions of the two TV channels is provided in Figure 2.

< FIGURE 2 ABOUT HERE>

When the rival's price on advertising is sufficiently small, it does not pay for a TV channel to undercut it. Rather, the optimum price in such a case is higher than the rival's price, but otherwise independent of it. The equilibrium prices are always equal and determined by which of the two, β_1 or β_2 in Figure 2, is the larger. The interesting issue, thus, is whether $\beta_1 > \beta_2$ or the opposite. It turns out that $\beta_1 > \beta_2$ under the maintained restriction that $t \geq 0.3$.[11]

Theorem 1: In equilibrium, the two TV channels set the same price on advertising reach: $b_1^* = b_2^* = b^*$. In particular:

(i) If $0.3 \leq t < \frac{3-v}{2(1-v)}$, then $b^* = \frac{1}{8}\left[(1-v-20t)+(4t+1-v)\sqrt{1+\frac{16t}{1-v}}\right]$.

(ii) If $t \geq \frac{3-v}{2(1-v)}$, then $b^* = \frac{(1+v)^2 t}{2(1-v)}$.

Proof: It follows from Proposition A2 in the Appendix that, with ε negligibly small, $b_1^* = b_2^* = b^* = \max[\beta_1, \beta_2]$ in equilibrium. Case-by-case comparisons of the two reveal that, under the assumption that $t \geq 0.3$, $\beta_1 > \beta_2$. Now, the Theorem follows from Proposition A2(i). *QED.*

We see from Theorem 1 how the introduction of a market for TV advertising has made the price of advertising, and therefore the advertising costs of the advertisers, endogenous. While

[10] In doing this, we disregard any effect on advertising prices that may come about from the presence of multiple product markets. See Nilssen and Sørgard (2001a, 2001b) for discussions of advertising pricing with multiple product markets.

[11] There exist cases in which $\beta_1 < \beta_2$; however, in all those cases, $t < 0.3$. The restriction $t \geq 0.3$ serves to greatly facilitate the presentation of our results. Also our comparative-statics results in Theorem 2 would be affected by a relaxation of this restriction.

there is some competitive pressure towards low prices, this pressure is restricted by the advertising reach obtained through one single channel being limited by the size of this channel's viewer base. Even if one channel prices advertising lower than the other one does, it may still be necessary for advertising firms to make use of both channels in order to reach the desired fraction of consumers. This is the main mechanism by which the competition between the TV channels is softened.

Note that, although the two TV channels have the same advertising price, their marginal profits are not the same: The price is for advertising per viewer, and since channel 1 has more viewers than channel 2, it also has a greater marginal profit in equilibrium.

The equilibrium in Theorem has essentially the feature that the larger channel 1 finds it unprofitable to undercut the other channel and therefore lets it have as much advertising as it can take. This leaves a residual demand for channel 1 that, at the equilibrium price of case (i) of the Theorem, exactly balances the profit that it could have had by undercutting. In case (ii), the decision by channel 1 not to undercut is restricted by the advertising firms' behaviour: It cannot choose an advertising price greater than $\frac{(1+v)^2 t}{2(1-v)}$ or it will be totally without business, according to Proposition A1(vi) in the Appendix.

It is also straightforward to get out some comparative statics:

Theorem 2:

(i) The equilibrium price of advertising, b^*, increases with either an increase in the differentiation in the product market, or the size of the larger TV channel, *i.e.*,

$$\frac{db^*}{dt} > 0, \text{ and } \frac{db^*}{dv} > 0.$$

(ii) Firms' advertising, φ, decreases with the size of the larger TV channel, while the effect of an increase in product differentiation is ambiguous, *i.e.*,

$$\frac{d\varphi}{dt} < or > 0, \text{ and} \frac{d\varphi}{dv} < 0.$$

Proof: (i) Follows from differentiating the expressions for b^* in Theorem 1. (ii) Total advertising equals $v\varphi_1 + (1 - v)$, since, in equilibrium, we are in case (vii) of Proposition A1 in the

Appendix. Substituting the expressions for b^* in Proposition 2 into the expression for φ_1 given in Proposition A1(vii) and differentiating the total give the result. *QED*.

An increase in the product differentiation makes the products more different in the view of consumers and firms get more interested in advertising since now competition is more relaxed. This increased demand for advertising, in turn, accounts for the increased price of advertising. But it also explains the ambiguous total effect of an increase in t on advertising: In addition to the positive direct effect, there is also a negative effect through the increase in advertising price.

Under the restriction we use here, that t is not very low, it is the smaller TV channel that has the higher incentives to set a low price. An increase in the (relative) size of the larger channel means that a larger fraction of total advertising will have to be made through the high-price large channel. Of course, in equilibrium, there is a negligible price difference between the two channels. However, as the larger channel gets even larger, it has even lesser incentives to compete with the aggressive small channel, and the equilibrium price of advertising increases. This feeds into less advertising being sold.

We thus find that asymmetry between TV channels dampens price competition. The more limited the small TV channel's audience, the higher are the prices on advertising. A natural question, then, is whether it can be profitable for the 'small' TV channel to deliberately act so that its audience is limited. One natural interpretation is that the 'small' TV channel is an entrant. If it invests only a limited amount in program quality, its audience is limited. By such a strategy, it both saves investment in program quality and dampens price competition.

If we, as above, interpret our game as an entry game, our result shares some similarities with what Gelman and Salop (1983) labeled *judo economics*. They showed that a producer may have incentives to restrict its sales in order to stop its rival from cutting its price: A producer with a large market share would rather maintain high prices to serve all its existing consumers than cut prices to compete with the small producer. However, in their model with sequential price setting, the small firm sets a lower price than the large firm in equilibrium. Given that prices are typically flexible, we find it more natural to assume simultaneous price setting, and we reproduce the main result in Gelman and Salop (1983) - capacity limitation dampens price competition - although equilibrium prices are now identical.

4. Concluding remarks

Our starting point was that the price of advertising on TV is determined by the rivalry between TV channels. In line with this observation, the producers' cost of advertising on TV should be endogenously determined. Surprisingly, though, no studies have, as far as we know, followed such an approach when modeling the advertising technology. The purpose of this paper has been to help fill in this gap.

Although our model is highly stylized, we point to one mechanism suggesting that price rivalry on TV advertising may not result in cut-throat competition. The driving force is that the advertising firms may need to use more than one TV channel to reach its consumers. The larger the asymmetry between TV channels concerning the number of viewers, the less is the rivalry on prices on advertising. The large TV channel sets a high price and lets the small TV channel slightly undercut its price and therefore lets the small channel have as much advertising as it can. The large channel then becomes the residual supplier of advertising, setting a high price to maximize its profit from the residual demand for advertising. This suggests that it can be more beneficial to be a small scale TV channel than to challenge a dominant TV channel. A small-scale TV channel saves on investments in program quality to attract viewers, and it dampens the rivalry on prices of advertising slots. One interpretation might be that an entrant has incentives to enter on a small scale in order to keep the post-entry advertising price high.

Appendix

In this Appendix, we provide precise statements of some of our results. First, in Proposition A1, is our result on how firms' decisions to advertise depend on the TV channels' advertising prices.[12] Thereafter, in Proposition A2, we provide details of the TV channels' best-response functions.

Proposition A1: Given TV channels' advertising prices b_1 and b_2, firms' advertising decisions are as follows:

(i) If $\frac{(2-v)^2 t}{2v} < b_1 < b_2$, then:

$$\varphi_1 = \frac{1}{tv}\left[b_1 + 2t - \sqrt{b_1^2 + 4b_1 t}\right], \text{ and } \varphi_2 = 0.$$

(ii) If $b_1 \leq \dfrac{(2-v)^2 t}{2v} < b_2$, then:

$\varphi_1 = 1$, and $\varphi_2 = 0$.

(iii) If $b_1 < b_2$ and $b_2 \in \left(\dfrac{[(1+v)t-v]^2}{2[(1-v)t+v]}, \dfrac{(2-v)^2 t}{2v}\right]$, then:

$$\varphi_1 = 1, \text{ and } \varphi_2 = \frac{1}{t(1-v)}\left[b_2 + 2t - v - \sqrt{b_2^2 + 4b_2 t}\right].$$

(iv) If $b_1 < b_2 \leq \min\left\{\dfrac{[(1+v)t-v]^2}{2[(1-v)t+v]}, \dfrac{(2-v)^2 t}{2v}\right\}$, then:

$\varphi_1 = \varphi_2 = 1$.

(v) If $b_1 \geq b_2 > \dfrac{(1+v)^2 t}{2(1-v)}$, then:

$$\varphi_1 = 0, \text{ and } \varphi_2 = \frac{1}{t(1-v)}\left[b_2 + 2t - \sqrt{b_2^2 + 4b_2 t}\right].$$

(vi) If $b_1 > \dfrac{(1+v)^2 t}{2(1-v)} \geq b_2$, then:

$\varphi_1 = 0$, and $\varphi_2 = 1$.

(vii) If $b_1 \geq b_2$ and $b_1 \in \left(\dfrac{[(2-v)t-(1-v)]^2}{2(tv+1-v)}, \dfrac{(1+v)^2 t}{2(1-v)}\right]$, then:

$$\varphi_1 = \frac{1}{tv}\left[b_1 + 2t - (1-v) - \sqrt{b_1^2 + 4b_1 t}\right], \text{ and } \varphi_2 = 1.$$

(viii) If $\min\left\{\dfrac{[(2-v)t-(1-v)]^2}{2(tv+1-v)}, \dfrac{(1+v)^2 t}{2(1-v)}\right\} \geq b_1 \geq b_2$, then:

$\varphi_1 = \varphi_2 = 1$.

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[12] In order to resolve a technicality associated with the possibility of one of the two TV channels having to choose its price from an open set, we assume that, at equal prices, *i.e.*, when $b_1 = b_2$, the small channel 2 serves all the advertising demand up to its full capacity $(1 - v)$ of viewers.

Proof: Suppose first that $b_1 < b_2$. In order for each firm's equilibrium advertising to be so low that $\varphi < v$, implying that only the cheaper channel 1 is used, it must be true that: $K = b_1$, or, inserting for K,

$$\frac{t(2-\varphi)^2}{2\varphi} = b_1.$$

Solving for the only root that takes value between 0 and 1, we obtain:

$$\varphi = \frac{1}{t}\left[b_1 + 2t - \sqrt{b_1^2 + 4b_1 t}\right]$$

In order for this to satisfy the premise $\varphi < v$, calculations reveal that we must have:

$$b_1 > \frac{(2-v)^2 t}{2v}.$$

Likewise, in order to have $\varphi > v$, we must have:

$$\frac{t(2-\varphi)^2}{2\varphi} = b_2,$$

or:

$$\varphi = \frac{1}{t}\left[b_2 + 2t - \sqrt{b_2^2 + 4b_2 t}\right]$$

In order for this to satisfy $\varphi > v$, we must have:

$$b_2 < \frac{(2-v)^2 t}{2v}.$$

In the intermediate case, we have $\varphi = v$. In addition, the solution must obey the restriction $\varphi \leq 1$. Together, this gives parts (i)-(iv) of the Proposition.

Suppose next that $b_1 \geq b_2$. If $\varphi < 1 - v$, then it is given by $K = b_2$, or:

$$\varphi = \frac{1}{t}\left[b_2 + 2t - \sqrt{b_2^2 + 4b_2 t}\right]$$

In order to satisfy $\varphi < 1 - v$, we must have:

$$b_2 > \frac{(1+v)^2 t}{1-v}.$$

The proof for parts (v)-(viii) now continues as for the case of $b_1 < b_2$. *QED*.

Proposition A2: Let ε be a small, positive number. There exists a $t^* > 0.3$ such that:[13]

(i) channel 1's best response, $B_1(b_2)$, is:

$B_1(b_2) = \beta_1$, if $b_2 < \beta_1$, and

$B_1(b_2) = b_2 - \varepsilon$, otherwise,

where

$$\beta_1 = \frac{1}{8}\left[(1-v-20t)+(4t+1-v)\sqrt{1+\frac{16t}{1-v}}\right], \text{ if } t < \frac{3-v}{2(1-v)}; \text{ and}$$

$$\beta_1 = \frac{(1+v)^2 t}{2(1-v)}, \text{ if } t \geq \frac{3-v}{2(1-v)}; \text{ and}$$

(ii) channel 2's best response, $B_2(b_1)$, is:

$B_2(b_1) = \beta_2$, if $b_1 < \beta_2$,

$B_2(b_1) = b_1$, otherwise,

where

$$\beta_2 = \frac{[(1+v)t-v]^2}{2[(1-v)t+v]}, \text{ if } t \in [0.3, t^*] \text{ or}$$

$$\text{if } v \geq 4\sqrt{2}-5 \text{ and } t \in \left(\frac{v\left(3v-1-\sqrt{v^2+10v-7}\right)}{4(1-v)^2}, \min\left[\frac{v}{2v-1}, \frac{2+v}{2v}\right]\right);$$

$$\beta_2 = \tfrac{1}{8}\left[(v-20t)+(4t+v)\sqrt{1+\tfrac{16t}{v}}\right], \text{ if } v < 4\sqrt{2}-5 \text{ and } t \in \left(t^*, \frac{2+v}{2v}\right), \text{ or}$$

$$v \geq 4\sqrt{2}-5 \text{ and } t \in \left(t^*, \frac{v\left(3v-1-\sqrt{v^2+10v-7}\right)}{4(1-v)^2}\right); \text{ and}$$

$$\beta_2 = \frac{(2-v)^2 t}{2v}, \text{ if } t \geq \min\left[\frac{v}{2v-1}, \frac{2+v}{2v}\right].$$

[13]The precise expression for t^* is:

$$t^* = \frac{v}{16}\left[\frac{1}{9}\left(2-v+\sqrt[3]{H}+\frac{v^2-4v+7}{\sqrt[3]{H}}\right)^2-1\right],$$

where: $H = 206 - 3v + 6v^2 - v^3 + 3\sqrt{4677-72v+222v^2-24v^3-3v^4}$.

Proof: We sketch here a proof of part (i). Part (ii) can be proved the same way. Channel 1's profit is:

$$\Pi_1 = b_1 v \varphi_1.$$

Whenever $\varphi_1 = 1$, the channel's profit increases with an increase in b_1. Whenever $\varphi_1 = 0$, profit is zero. Thus, there are two cases left to consider, in which $\varphi_1 \in (0,1)$.

Consider first case (i) of Proposition A1, where $\varphi_1 = \frac{1}{tv}\left[b_1 + 2t - \sqrt{b_1^2 + 4b_1 t}\right]$. Inserting this into the profit expression, the channel's marginal profit with respect to price is, after some rearrangement, found to be equal to:

$$\frac{d\Pi_1}{db_1} = \left[\sqrt{b_1^2 + 4b_1 t} - b_1\right]\left[\frac{b_1 + 2t}{\sqrt{b_1^2 + 4b_1 t}} - 1\right] > 0,$$

where the inequality holds because both terms inside brackets are positive. Thus, it pays for channel 1 to increase the price in case (i), which means that the optimum response to a high price by channel 2 is to slightly undercut it.

Consider next case (vii) of Proposition A1, where $\varphi_1 = \frac{1}{tv}\left[b_1 + 2t - (1-v) - \sqrt{b_1^2 + 4b_1 t}\right]$, creating scope for an interior solution. In particular, the first-order condition of channel 1 in this case can be written as:

$$\left[\sqrt{b_1^2 + 4b_1 t} - b_1\right]\left[\frac{b_1 + 2t}{\sqrt{b_1^2 + 4b_1 t}} - 1\right] = \frac{1-v}{t}.$$

The solution to this equation has two roots, one of which is always negative and thus can be excluded. The other root is:

$$b_1 = \frac{1}{8}\left[(v - 20t) + (4t + v)\sqrt{1 + \frac{16t}{v}}\right].$$

We have to check for combinations of t and v such that either case (vii) is empty or the interior solution is outside the bounds of case (vii). The result of this check is the definitions of β_1 and β_2 in the Proposition. *QED.*

References

Anderson, S.P., Coate, S., 2000. Market provision of public goods: the case of broadcasting. Working Paper 7513, National Bureau of Economic Research.

Bester, H., Petrakis, E., 1996. Coupons and oligopolistic price discrimination. International Journal of Industrial Organization 14, 227-242.

Eckard, E.W., 1991. Competition and the cigarette TV advertising ban. Economic Inquiry 29, 119-133.

Gabszewicz, J., Laussel, D., Sonnac, N., 2000. TV-Broadcasting Competition and Advertising. Discussion Paper 00/6, CORE, Université Catholique de Louvain; available at http://server.core.ucl.ac.be/services/psfiles/dp00/dp2000-6.pdf.

Gallet, C.A., 1999. The effect of the 1971 advertising ban on behavior in the cigarette industry. Managerial and Decision Economics 20, 299-303.

Gal-Or, E., Dukes, A. 2001. Minimum differentiation in commercial media markets. Unpublished manuscript, University of Pittsburgh.

Gelman, J.R., Salop, S.C. 1983. Judo economics: capacity limitation and coupon competition. Bell Journal of Economics 14, 315-325.

Grossman, G.M., Shapiro, C. 1984. Informative advertising with differentiated products. Review of Economic Studies 51, 63-81.

Hotelling, H., 1929. Stability in competition. Economic Journal 39, 41-57.

Kreps, D.M., Scheinkman, J.A., 1983. Quantity precommitment and Bertrand competition yield Cournot outcomes. Bell Journal of Economics 14, 326-337.

Leahy, A.S., 1991. The effect of television advertising on prices. Advances in Econometrics, Vol. 9: Econometric Methods and Models for Industrial Organization (G.F. Rhodes, ed.), Greenwich, CT: JAI Press, 255-265.

Masson, R.T., Mudambi, R., Reynolds, R.J., 1990. Oligopoly in advertiser-supported media. Quarterly Review of Economics and Business 30, 3-16.

Motta, M., Polo, M., 1997. Beyond the spectrum constraint: concentration and entry in the broadcasting industry. Working Paper 115, IGIER, Università Bocconi; available at ftp://ftp.igier.uni-bocconi.it/wp/1997/115.pdf.

Nilssen, T., Sørgard, L. 2001a. The TV industry: advertising and programming. Unpublished manuscript, University of Oslo and Norwegian School of Economics and Business Administration.

Nilssen, T., Sørgard, L. 2001b. Who are the advertisers? Unpublished manuscript, University of Oslo and Norwegian School of Economics and Business Administration.

Owen, B.M., Wildman, S.S. 1992, Video Economics. Cambridge, MA: Harvard University Press.

Roy, S., 2000. Strategic segmentation of a market. International Journal of Industrial Organization 18, 1279-1290.

Tirole, J., 1988. The Theory of Industrial Organization. Cambridge, MA: MIT Press.

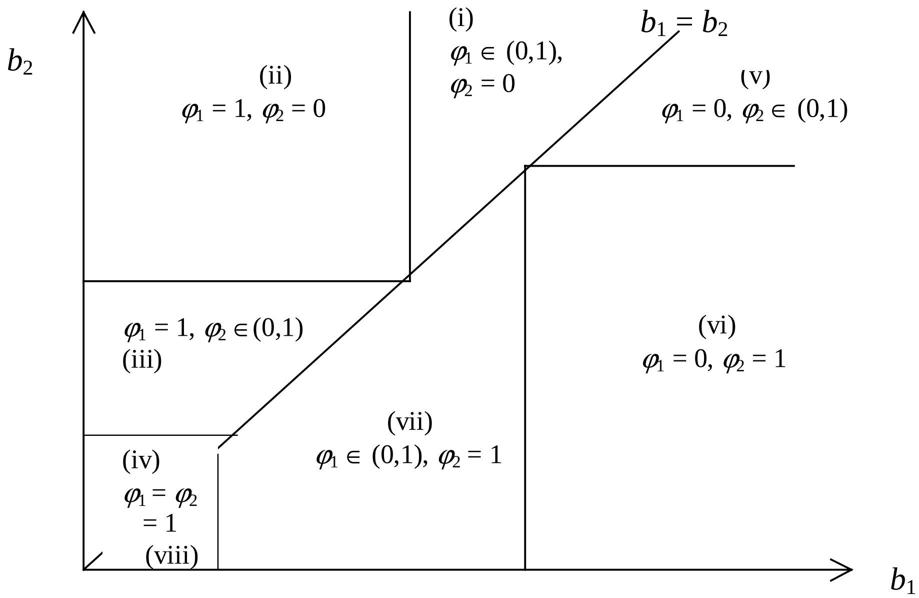

Figure 1

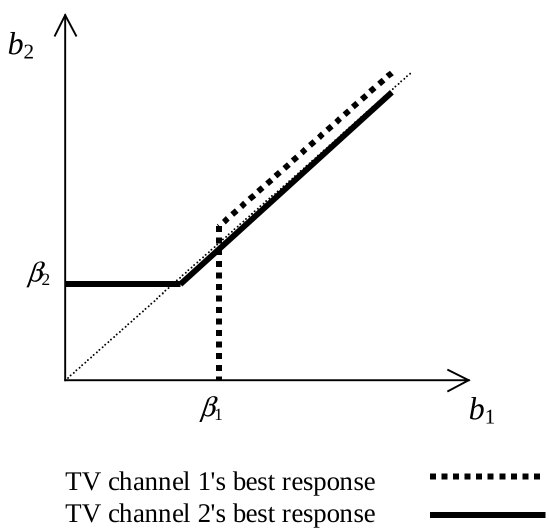

Figure 2