Price competition with capacity constraints

Consumers are rationed at the low-price firm. But who are the rationed ones?

As before: two firms; homogeneous goods.

Efficient rationing

If \( p_1 < p_2 \) and \( \bar{q}_1 < D(p_1) \), then the residual demand facing firm 2 is:

\[
\tilde{D}_2(p_2) = \begin{cases} 
  D(p_2) - \bar{q}_1, & \text{if } D(p_2) > \bar{q}_1, \\
  0, & \text{otherwise}
\end{cases}
\]

This is the rationing that maximizes consumer surplus: The consumers with the highest willingness to pay get the low price.
Proportional rationing

Let $p_1 < p_2$ and $\bar{q}_1 < D(p_1)$.

Instead of favouring the consumers with the highest willingness to pay, all consumers have the same chance of getting the low price.

Probability of being supplied by the low-price firm 1 is:

$$\frac{\bar{q}_1}{D(p_1)}$$

The residual demand facing the high-price firm 2 is:

$$\tilde{D}_2(p_2) = D(p_2) \left[1 - \frac{\bar{q}_1}{D(p_1)}\right]$$

Not efficient – some consumers get supplies despite having a willingness to pay below $p_2$, consumers’ marginal cost.
Example:

Two firms, homogeneous demand: \( D(p) = 1 - p \)

Zero marginal costs of production: \( c = 0 \).

High investment costs have led to low capacity: \( \bar{q}_1 = \bar{q}_2 \leq \frac{1}{3} \).

Assume efficient rationing.

Define: \( p^* = 1 - (\bar{q}_1 + \bar{q}_2) \)

Is \( p_1 = p_2 = p^* > 0 \) an equilibrium?

Note that \( D(p^*) = \bar{q}_1 + \bar{q}_2 \); total capacity exactly covers demand at this price.

Can another price be preferable for firm 1 to \( p^* \), if firm 2 sets \( p_2 = p^* \)?

(i) Consider \( p_1 < p_2 = p^* \). A lower price for firm 1 without any increase in sales.

(ii) Consider \( p_1 > p_2 = p^* \). Firm 1’s sales less than before:

\[
q_1 = \tilde{D}_1(p_1) = D(p_1) - \bar{q}_2 = 1 - p_1 - \bar{q}_2
\]

\( \Rightarrow p_1 = 1 - q_1 - \bar{q}_2 \)
Profit of firm 1:
$$\pi_1 = p_1 \tilde{D}_1(p_1)$$
Equivalently:
$$\pi_1 = (1 - q_1 - \bar{q}_2)q_1$$

$$\frac{d\pi_1}{dq_1} = 1 - 2q_1 - \bar{q}_2$$

Is it profitable for firm with a price above $p^*$?
Equivalently: profitable with a quantity below $\bar{q}_1$?
Second-order condition: $\frac{\partial^2 \pi_1}{\partial q_1^2} < 0$. 
$$\frac{d\pi_1}{dq_1}|_{q_1=\bar{q}_1} = 1 - 2\bar{q}_1 - \bar{q}_2 \geq 0$$
Optimum is at $\bar{q}_1$.

Thus, the optimum price for firm 1 is $p^*$. Equivalently for firm 2. Thus, $p_1 = p_2 = p^*$ in equilibrium.

Is this equilibrium unique? Yes.

Larger capacities: No equilibria in pure strategies.

[Exercise 5.2]
Capacity a more long-term decision than price?

Consider the two-stage game:

Stage 1: Firms choose capacities
Stage 2: Firms choose prices

Investment costs: $c_0$ per unit of capacity

Suppose $c_0$ is so high that, in equilibrium, capacities will be low. We can then make use of our analysis of the price game: Prices equal $p^*$.

Profit net of investment costs:

$$\pi^1(q_1, q_2) = \{[1 - (q_1 + q_2)] - c_0\} q_1.$$ 

Now, the game is equivalent to a one-stage game in capacities where demand = total capacity = total supply.

That is, a one-stage game in quantities. (Cournot, 1838)

With efficient rationing and a concave demand function, the two games are equivalent in equilibrium outcome, for all $c_0$.

Therefore, a model of one-stage quantity competition, with prices coming from nowhere, can be understood as a simple substitute for a more realistic but more complex model where firms compete in capacities and thereafter in prices.
The Cournot model

Two firms choose quantities simultaneously.

Costs: $C_i(q_i)$

Total production: $Q = q_1 + q_2$

Inverse demand: $P(Q), \quad P' < 0.$

Profit, firm 1:

$$\pi^1(q_1, q_2) = q_1P(q_1 + q_2) - C_1(q_1).$$

First-order condition:

$$\frac{\partial \pi^1}{\partial q_1} = P(q_1 + q_2) + q_1P'(q_1 + q_2) - C_1'(q_1) = 0$$

$q_1P'(q_1 + q_2)$ – the *infra-marginal effect* of an increase in quantity

Equilibrium: $\frac{\partial \pi^1}{\partial q_1} = 0; \quad \frac{\partial \pi^2}{\partial q_2} = 0.$
For firm 1:

\[ P - C_1' = -q_1P' = -\frac{q_1}{Q}P'Q = -\frac{Q}{P'} \frac{1}{P'} \frac{1}{Q} \]

\[ \Rightarrow \frac{P - C_1'}{P} = \frac{q_1}{Q} \frac{1}{P'} \frac{1}{P'} \frac{1}{Q} \]

\[ L_1 = \frac{P - C_1'}{P} \quad \text{– the Lerner index} \]

\[ \alpha_1 = \frac{q_1}{Q} \quad \text{– firm 1’s market share} \]

\[ D(p) \quad \text{– the market demand} \]

\[ D(P(Q)) \equiv Q \]

\[ \Rightarrow D'(p) \quad P'(Q) = 1 \]

Demand elasticity:

\[ \varepsilon = -\frac{D'}{D} = -\frac{1}{P'} \frac{P}{Q} \]

\[ \Rightarrow L_1 = \frac{\alpha_1}{\varepsilon} \]

Note: (i) \( \alpha_1/\varepsilon > 0 \Rightarrow L_1 > 0 \Rightarrow P > C_1' \)。
(ii) Monopoly: \( L_1 = 1/\varepsilon \), and \( \alpha_1 = 1 \).
\( n \) firms: \( Q = \sum_{i=1}^{n} q_i \)

\[ \pi^i(q_1, \ldots, q_n) = q_i P(Q) - C_i(q_i) \]

\[ \frac{\partial \pi^i}{\partial q_i} = P(Q) + q_i P' \frac{dQ}{dq_i} - C_i' = 0 \]

Example: \( P(Q) = a - Q; \ C_i(q_i) = C(q_i) = cq_i \).

Assume: \( a > c \).

First-order condition firm \( i \): \( a - Q - q_i - c = 0 \).

All firms identical \( \Rightarrow q_1 = \ldots = q_n = q, \ Q = nq \)

Applied to the first-order condition:

\[ a - nq - q - c = 0 \]

\[ q = \frac{a-c}{n+1} \]

\[ P = a - nq = a - \frac{n(a-c)}{n+1} = a + nc = c + \frac{a-c}{n+1} > c \]

\[ Q = nq = \frac{n}{n+1} (a-c) \]

\[ \pi = q \left( c + \frac{a-c}{n+1} \right) - cq = \frac{a-c}{n+1} q = \left( \frac{a-c}{n+1} \right)^2 \]

\[ n \to \infty \Rightarrow P \to c, \ Q \to a - c, \ \pi \to 0. \]

[Exercises 5.3, 5.4, 5.5]
Bertrand vs. Cournot

Competing models? – No.

Firms set prices.
When capacity constraints are of little importance, the Bertrand model is the preferred one.
When capacity constraints are present to an important extent (decreasing returns to scale), the Cournot model is the best choice.

Measuring concentration

A substitute for measuring price-cost margins, since costs are unobservable. (Econometric studies?)

A popular measure: the Herfindahl index.

\[ R_H = \sum_{i=1}^{n} \alpha_i^2 \]

Model: \( n \) firms, \( C_i(q_i) = c_i q_i \), quantity competition

Total industry profits:

\[ \sum_i \pi_i = \sum_i (P - c_i) q_i = \sum_i \frac{P \alpha_i q_i}{\varepsilon} = \frac{P Q}{\varepsilon} \sum_i \alpha_i^2 = \frac{D^2}{-D'} R_H \]

Assume: \( D(p) = k/p \Rightarrow D^2/(− D') = k \Rightarrow \sum_i \pi_i = k R_H \)

The Herfindahl index proportional to total industry profits.

[Exercises 5.6, 5.7]