Sensing-Energy Tradeoff in Cognitive Radio Networks With Relays
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I. Introduction

With a fast-growing demand of wireless communications, crowded spectrum cannot accommodate all users if each application requires an exclusive frequency band. The cognitive radio (CR) technique was proposed [1], [2] to improve the utilization of spectrum resources by allowing multiple users to share a frequency band with different priorities. The user with a higher priority, known as the primary user (PU), should be sufficiently protected, while the user with a lower priority, known as the secondary user (SU), may access the frequency band that is licensed to the PU, given that SU will not cause harmful interference to the PU.

Spectrum sensing is one of the key algorithms to realize CR’s vision, in which a SU detects the activities of a PU in a frequency band and seeks the opportunities to access the spectrum. A variety of spectrum sensing methods have been proposed, such as matched-filtering detection, energy detection [3], [4], and cyclostationary feature detection [5], [6]. The authors in [7] analyzed these methods and identified the challenges associated with spectrum sensing. Usually, single-user spectrum sensing methods cannot attain reliable sensing results, especially at a low signal-to-noise-ratio region. Therefore, cooperative spectrum sensing methods were proposed to improve the sensing performance [8]–[10]. The cooperation among the SUs helps combat the detrimental effects of shadowing and fading in wireless communication environments. Relay works as a type of cooperation, which can increase diversity order [11] and can be applied to improve sensing performance. However, relay cooperation increases system complexity with minor loss of sensing performance, compared to cooperative spectrum sensing without relay selection [12]. In [13], two relaying models (amplify-and-relay and detect-and-relay) for cooperative spectrum sensing were proposed to improve detection performance. In [14] and [15], the authors showed that sensing time can be reduced by allowing relays to assist transmission for a SU that tries to detect the activities of a PU. In [16], transmit and relay diversity schemes were proposed for improving the performance of cooperative spectrum sensing in CR networks. In [17], the performance of energy detection-based cooperative spectrum sensing over both multipath fading and shadowing channels was investigated. These works, however, did not consider the tradeoff between sensing performance and energy consumption, even though...
they have taken into account cooperative spectrum sensing with relaying.

Spectrum sensing consumes time and energy. Sensing operations may perform better if more time and energy are used. However, more time and energy mean higher overhead that decreases the effective throughput of a SU. Therefore, there may exist a tradeoff between sensing performance and throughput of a SU (or energy consumption). Some earlier works investigated the tradeoff between sensing performance and throughput of the SU. In [18], the tradeoff between sensing performance and throughput of the SU was analyzed and an optimal sensing time was given. Furthermore, the authors in [19] discussed the effect of the PU traffic on sensing-throughput tradeoff for CR networks. In [20], the authors adopted a k-out-of-N fusion rule for spectrum sensing and identified an optimal pair of sensing time and k value under the constraint that the PU is sufficiently protected. In addition, an appropriate SU frame duration was found to maximize the throughput while keeping the collision probability for a PU below a given threshold [21]. The overhead for spectrum sensing has also been well investigated [22]–[24]. Three data fusion policies were proposed to achieve superior sensing performance with minimum overhead [22]. The overhead in sensing measurement and reporting was described, and a low overhead energy-detection scheme based on cooperative spectrum sensing was proposed in [23]. In [24], the impact of spectrum sensing overhead on the outage probability of SU transmission was studied. These works focused on the tradeoff, but did not consider the energy consumption for cooperative spectrum sensing with relay.

There are some works on studying energy-efficient spectrum sensing, but they did not consider the tradeoff between sensing performance and energy consumption for cooperative spectrum sensing with relay. In [25], distributed spectrum sensing and access strategies were designed for dynamic spectrum access under an energy constraint on the SU. In [26], sensing-access strategies and sensing order were designed jointly for sequential sensing and access in CR networks to achieve a maximum energy efficiency. In [27], a spectrum sensing and access mechanism was suggested for its application in a CR system with one SU that accesses multiple channels. The aim was to minimize the average energy cost of the SU, while still satisfying the constraints on sensing reliability, throughput, and delay of the SU. In [28], sensing and transmission durations were determined jointly for energy-efficient spectrum sensing and transmission in a CR system. In [29], a sleeping and censoring scheme was proposed for energy-efficient spectrum sensing in cognitive sensor networks, where the objective was to minimize the energy consumed for collaborative spectrum sensing under given detection performance constraint.

In this paper, the problem of cooperative spectrum sensing in CR networks with amplify-and-forward relay is studied. Particularly, the tradeoff between sensing performance and energy consumption is investigated. The CR network consists of one PU, one SU, and M relays (which work as the other SUs). Any of the relays can be selected to assist the SU for cooperative spectrum sensing (denoted as Rk). The spectrum sensing duration τ is divided into two slots, i.e., τ1 and τ2. In the first slot τ1, the SU and the relay Rk receive the signal from the PU. In the second slot τ2, the relay Rk amplifies its received signal with an amplification gain βk, and then directly forwards the amplified signal to the SU. The SU receives the signals from relay Rk and PU at the same time. The SU utilizes its received signal in both slots τ1 and τ2 to determine the activity of the PU using an energy detector.

When a different relay is selected, the energy consumed for spectrum sensing may not be identical. Therefore, it is necessary to find an optimal relay such that the energy consumption is minimized. The key issue in this scenario is to minimize the energy consumed for spectrum sensing while maintaining sensing performance above a certain threshold. In order to minimize relay energy consumption while keeping sensing performance above a certain level, an optimization problem is formulated. Specifically, energy consumed for spectrum sensing under consideration is the energy consumed in relay Rk for communications, which dominates the overall energy consumption. The sensing performance is measured by detection probability pd and false alarm probability pf. Thus, the optimization problem becomes energy consumption minimization under the constraints that the detection probability pd is not less than a given threshold  \( \gamma \) and the false alarm probability pf is not greater than a certain threshold  \( \gamma \). An optimal pair of the number of samples N (proportional to the sensing time) and the amplification gain βk are found to achieve the minimum energy consumption under the constraints.

The contributions in this paper can be summarized as follows:

1) the performance and energy consumption for spectrum sensing in a CR network with relays are evaluated;
2) the tradeoff between sensing performance and energy consumption is formulated as an optimization problem;
3) an optimal pair of the number of samples and the amplification gain are identified to achieve the minimum energy consumption under the sensing performance constraint.

The remainder of this paper is outlined as follows. In Section II, a model of cooperative spectrum sensing with amplify-and-forward relay is described and the energy consumption for spectrum sensing is calculated. In Section III, the tradeoff between sensing performance and energy consumption is investigated. In Section IV, simulation results are presented, followed by the concluding remarks drawn in Section V.

II. SYSTEM MODEL

In this section, we first describe a model of cooperative spectrum sensing in CR networks with amplify-and-forward relay, and then calculate the energy consumption for spectrum sensing.

A. Cooperative Spectrum Sensing With Amplify-and-Forward Relaying

Fig. 1 shows the model of spectrum sensing with relay, where there are one PU, one SU, and M relays (which serve as the other SUs).
An SU wants to detect whether the PU is active or inactive. One of the relays will be selected to help the SU to perform spectrum sensing in order to enhance sensing performance.

The spectrum sensing phase is further divided into two slots, i.e., \( \tau_1 \) and \( \tau_2 \). During the slot \( \tau_1 \), the SU and the relay \( R_0 \) (assume that the relay \( R_0 \) is selected for cooperating with the SU) receive the signal from the PU. Then, during the next slot \( \tau_2 \), the SU receives the signals from the PU and the relay \( R_0 \) that amplifies its received signal in the slot \( \tau_1 \) and directly forward the amplified signal. It is noted that the signal transmitted by \( R_0 \) may cause interference to the PU. However, in spectrum sensing process, usually this problem is ignored. The random activity of a PU is not considered, as spectrum sensing is performed based on the received signal, which is only relevant to the presence and absence of the PU.

In slot \( \tau_1 \), when the PU is active, SU’s received signal originated from the PU can be expressed as

\[
H_1: y_s(n) = h_{p,s} x_p(n) + u_s(n), \quad n = 1, \ldots, N/2
\] (1)

where \( h_{p,s} \) denotes the channel fading coefficient from the PU to the relay \( R_0 \), and \( u_s(n) \) is an additive white Gaussian noise, and \( N \) denotes the number of samples. When the PU is inactive, the received signal at the relay \( R_0 \) can be given by

\[
H_2: y_s(n) = u_s(n), \quad n = 1, \ldots, N/2
\] (2)

Similarly, when the PU is active, the received signal at the relay \( R_k \) can be given by

\[
H_k: y_k(n) = h_{p,k} x_p(n) + u_k(n), \quad n = 1, \ldots, N/2
\] (3)

where \( h_{p,k} \) denotes the channel fading coefficient from the PU to the relay \( R_k \) and \( u_k(n) \) is additive white Gaussian noise.

When the PU is inactive, the received signal at the relay \( R_k \) can be expressed as

\[
H_k: y_k(n) = u_k(n), \quad n = 1, \ldots, N/2
\] (4)

In the slot \( \tau_2 \), the relay \( R_0 \) amplifies its received signal with an amplification gain \( \beta_k \) and directly forward the amplified signal to the SU. The SU receives the signals from the PU and the relay \( R_0 \) simultaneously. When the PU is present, the signal received by the SU can be written as

\[
H_1: y_s(n) = h_{p,s} x_p(n) + \beta_k h_{p,k} y_k(n - N/2) + u_s(n)
\]

\[
\text{for } n = N/2 + 1, \ldots, N
\]

where \( h_{p,k} \) denotes the channel fading coefficient from the relay \( R_k \) to the SU.

When the PU is absent, the signal received by the SU can be written as

\[
H_2: y_s(n) = \beta_k h_{p,k} y_k(n - N/2) + u_s(n)
\]

\[
\text{for } n = N/2 + 1, \ldots, N.
\]

For analytical simplicity, the following assumptions are made:

1) The transmitted signals experience Rayleigh fading. The channel fading coefficients \( h_{p,k} \) and \( h_{p,s} \) are constant within each spectrum sensing period.

2) The noise \( u_s(n) \) is an independently and identically distributed (i.i.d.) circularly symmetric complex Gaussian (CSCG) random sequence with zero mean and variance \( \sigma_s^2 \). The noise \( u_k(n) \) is also an i.i.d., CSCG random sequence with zero mean and variance \( \sigma_k^2 \).

3) The transmitted PU signal \( x_p(n) \) is an i.i.d. random sequence with zero mean and variance \( \sigma_p^2 \).

4) The noises, \( u_s(n) \) and \( u_k(n) \), and the transmitted PU signal \( x_p(n) \) are statistically independent.

An energy detector is adopted at the SU for spectrum sensing. The test statistics are given by

\[
\Gamma = \sum_{n=1}^{N} |y_s(n)|^2 = \sum_{n=1}^{N/2} |y_s(n)|^2 + \sum_{n=N/2+1}^{N} |y_s(n)|^2
\] (7)

Let us define

\[
\Gamma_1 = \sum_{n=1}^{N/2} |y_s(n)|^2
\] (8)

\[
\Gamma_2 = \sum_{n=N/2+1}^{N} |y_s(n)|^2
\] (9)

Thus, the mean and the variance of the test statistics \( \Gamma \) can be expressed as

\[
E(\Gamma) = E(\Gamma_1) + E(\Gamma_2)
\] (10)
Var(Γ) = Var(Γ₁) + Var(Γ₂) + 2Cov(Γ₁, Γ₂) \quad (11)

where \( E(Γ₁) \) and \( E(Γ₂) \) denote the mean of the test statistics \( Γ₁ \) and \( Γ₂ \), respectively, \( \text{Var}(Γ₁) \) and \( \text{Var}(Γ₂) \) denote the variances of the test statistics \( Γ₁ \) and \( Γ₂ \) respectively, and \( \text{Cov}(Γ₁, Γ₂) \) denotes the covariance of \( Γ₁ \) and \( Γ₂ \).

Under the hypotheses \( H₀ \), we have

\[
E(Γ₁) = \frac{N}{2} \left( |h_{p,s}|^2 |Ω|^2 + σ^2_p \right) \quad (12)
\]
\[
E(Γ₂) = \frac{N}{2} \left( |h_{p,s}|^2 |Ω|^2 + B_k |h_{p,s}|^2 |Ω|^2 \right) + 4(ab + ac + ad + bc + bd + cd) \quad (13)
\]
\[
\text{Var}(Γ₁) = \frac{N}{2} \left( |h_{p,s}|^2 E(|x_p(n)|^2) + E(|x_o(n)|^2) \right) \quad (14)
\]
\[
\text{Var}(Γ₂) = \frac{N}{2} \left( |h_{p,s}|^2 E(|x_p(n)|^2) + E(|x_o(n)|^2) \right) + 4(ab + ac + ad + bc + bd + cd) \quad (15)
\]
\[
\text{Cov}(Γ₁, Γ₂) = \frac{N}{2} \left( \text{Var}(Γ₁) - σ^2_p \right) \quad (16)
\]

where \( a = |h_{p,s}|^2 |Ω|^2, b = |h_{p,s}|^2 |Ω|^2, c = |h_{p,s}|^2 |Ω|^2, d = σ^2_p, E(|x_o(n)|^2) = 2σ^2_s, \) and \( E(|x_o(n)|^2) = 2σ^2_s \). Because the primary signal \( x_p(n) \) is i.i.d., CSCG random sequence, we have \( E(|x_o(n)|^2) = 2σ^2_s \).

Under the hypotheses \( H₁ \), we have

\[
E(Γ₁) = \frac{N}{2} σ^2_p \quad (17)
\]
\[
E(Γ₂) = \frac{N}{2} \left( B_k |h_{p,s}|^2 |Ω|^2 \right) \quad (18)
\]
\[
\text{Var}(Γ₁) = \frac{N}{2} \left( E(|x_o(n)|^2) - σ^2_p \right) \quad (19)
\]
\[
\text{Var}(Γ₂) = \frac{N}{2} \left( B_k |h_{p,s}|^2 E(|x_o(n)|^2) \right) + E(|x_o(n)|^2) \quad (20)
\]
\[
\text{Cov}(Γ₁, Γ₂) = 0. \quad (21)
\]

When the number of samples \( N \) is sufficiently large, using the central limit theorem, we can show that the test statistics under either hypotheses \( H₀ \) or hypotheses \( H₁ \) approximately follows a Gaussian distribution. Therefore, the detection probability and the false alarm probability are calculated by

\[
p_d = Q \left( \sqrt{\text{Var}(Γ₁)} \right) \quad (22)
\]

\[
p_f = Q \left( \sqrt{\text{Var}(Γ₂)} \right) \quad (23)
\]

respectively, where \( \lambda \) is the decision threshold, \( E(Γ₁|H₁) \) and \( \text{Var}(Γ₁|H₁) \) are the mean and the variance of the test statistics \( Γ₁ \) under hypotheses \( H₁ \), \( E(Γ₂|H₀) \) and \( \text{Var}(Γ₂|H₀) \) are the mean and the variance of \( Γ₂ \) under the hypotheses \( H₀ \), and \( Q(·) \) is the complementary cumulative distribution function of the standard Gaussian random variable, that is

\[
Q(s) = \frac{1}{\sqrt{2\pi}} \int_s^{\infty} \exp \left( -\frac{t^2}{2} \right) dt. \quad (24)
\]

B. Energy Consumption

Total energy consumed for spectrum sensing includes the energy consumed by circuitry inside three terminals (PU, Rᵢ, and SU), and the energy for transmission. The SU cannot control the energy consumed by the PU, since it is not involved in the scenario. Moreover, the SU and the relay \( Rᵢ \) spend energy in receiving signals, which is much less than the energy consumed in signal transmission and is hence ignored. Here, we focus on the energy consumed by the relay \( Rᵢ \), which can be expressed as

\[
\mathcal{T} = \mathcal{T}_f \quad (25)
\]

where \( \mathcal{T} \) denotes the average transmission power of the relay \( Rᵢ \), and it is given by

\[
\mathcal{T} = \beta^2 E \left( \left| y(n) - \frac{N}{2} |Ω|^2 \right|^2 \right) = \beta^2 \left( |h_{p,s}|^2 σ^2_p + σ^2_s \right). \quad (26)
\]

We designate that \( t_n = (N/2)Ts, \) where \( Ts \) denotes the sampling interval. Then, the energy consumption \( \mathcal{T} \) is calculated by

\[
\mathcal{T} = \beta^2 \left( |h_{p,s}|^2 σ^2_p + σ^2_s \right) \frac{Nγ}{2}. \quad (27)
\]

III. SENSING-ENERGY TRADEOFF

In the previous section, we have derived the detection probability and the false alarm probability for cooperative spectrum sensing with amplify-and-forward relay. We also calculated the energy consumed for spectrum sensing. In this section, the tradeoff between sensing performance and energy consumption will be investigated.

From (22), for a given target detection probability \( \mathcal{P}_d \), the decision threshold can be determined by

\[
\lambda_d = \mathcal{Q}^{-1}(\mathcal{P}_d) \sqrt{\text{Var}(Γ₁|H₁)} + E(Γ₁|H₁). \quad (28)
\]

Then, from (23), the corresponding false alarm probability can be expressed as

\[
p_f = \mathcal{Q} \left( \frac{\lambda_d - E(Γ₂|H₀)}{\sqrt{\text{Var}(Γ₂|H₀)}} \right). \quad (29)
\]

On the other hand, from (23), for a given target false alarm probability \( \mathcal{P}_f \), the decision threshold can be determined by

\[
\lambda_f = \mathcal{Q}^{-1}(\mathcal{P}_f) \sqrt{\text{Var}(Γ₂|H₀)} + E(Γ₂|H₀). \quad (30)
\]

From (22), the corresponding detection probability can be expressed as

\[
p_d = \mathcal{Q} \left( \frac{\lambda_f - E(Γ₁|H₁)}{\sqrt{\text{Var}(Γ₁|H₁)}} \right). \quad (31)
Since $Q(x)$ monotonically decreases with $x$, noting that both $E(\gamma)$ and $\text{Var}(\gamma)$ have the constant term $(N/2)$, from (28) and (29) we can easily obtain the following proposition.

**Proposition 1:** For a fixed amplification gain $\beta_k$, under the target detection probability $\gamma_0$, as the number of samples $N$ increases, the false alarm probability $p_f$ decreases.

Similarly, from (30) and (31), we can also have the following proposition.

**Proposition 2:** For a fixed amplification gain $\beta_k$, under the target false alarm probability $\gamma_f$, as the number of samples $N$ increases, the detection probability $p_d$ increases.

Based on the two aforementioned propositions, we can obtain an important proposition, stated as follows.

**Proposition 3:** For a fixed amplification gain $\beta_k$ and a given pair of target probability $(\gamma_0, \gamma_f)$, there exists a minimal number of samples $N_{\text{min}}$ to achieve the target probabilities.

From (28) and (30), let $\lambda_d = \lambda_f$. The minimal number of samples is given by

$$N_{\text{min}} = 2 \left( \frac{A - B}{C - D} \right)^2 \tag{32}$$

where $A = Q^{-1}(\gamma_0)/\sqrt{2 \text{Var}(\gamma_0)/N}$, $B = Q^{-1}(\gamma_f)/\sqrt{2 \text{Var}(\gamma_f)/N}$, $C = 2E(\gamma_0)/N$, and $D = 2E(\gamma_f)/N$.

It should be noted that from Section II-A, $\text{Var}(\gamma_H0)$, $\text{Var}(\gamma_H1)$, $E(\gamma_H0)$, and $E(\gamma_H1)$ all contain a constant term $(N/2)$.

From Propositions 1 and 2, we can see that, for a fixed amplification gain $\beta_k$, as the number of samples $N$ increases, the performance of spectrum sensing will be improved. On the other hand, from (27), when the number of samples $N$ increases, more energy would be consumed. Hence, there exists a tradeoff between sensing performance and energy consumption. Note that, from (22) and (23), since both $p_f$ and $p_d$ are nonlinear functions of $N$ ($Q(x)$ is nonlinear), when the sensing performance has reached at a sufficiently high level (e.g., the detection probability $p_d$ is close to one), it will not improve much even if the number of samples further increases. However, from (27), the energy consumption $\mathcal{E}$ increases linearly with the number of samples $N$. Therefore, we want to minimize energy consumption while guaranteeing that sensing performance is kept above a certain level. The amplification gain $\beta_k$ can be a variable, so that the aforementioned tradeoff problem can be formulated as

$$\min \mathcal{E}(\beta_k, N) = p_f \left( \frac{1}{2} \sum \sigma_i^2 + \sigma_c^2 \right) \frac{NT_s}{2}$$

s.t. $p_f(\beta_k, N) \geq \gamma_0$ \hspace{1cm} $p_d(\beta_k, N) \leq \gamma_f$ \hspace{1cm} \hspace{1cm} $p_f(\beta_k, N) < p_d(\beta_k, N)$, i.e., $p_f(\beta_k, N) < \gamma_f$ \hspace{1cm} \hspace{1cm} (33)

Let us take two numbers of samples $N_1$ and $N_0$, as an example. For a fixed $\beta_k$, under the detection probability constraint $\gamma_f$, according to Proposition 1, we let $N_1 > N_0$ and $p_f(\beta_k, N_1) > p_f(\beta_k, N_0)$, and thus $p_f(\beta_k, N_1) < p_d(\beta_k, N_0)$, i.e., $p_f(\beta_k, N_1) < \gamma_f$. On the other hand, from (33), we have $\mathcal{E}(\beta_k, N_1) < \mathcal{E}(\beta_k, N_0)$. Therefore, the energy consumption $\mathcal{E}$ reaches to its minimum when the false alarm probability satisfies the equality constraint in (35). Similarly, under the false alarm probability constraint $\gamma_f$, according to Proposition 2, we let $p_f(\beta_k, N_0) = \gamma_f$, and then $p_d(\beta_k, N_1) > \gamma_f$, while $\mathcal{E}(\beta_k, N_1) > \mathcal{E}(\beta_k, N_0)$. Therefore, the minimum energy consumption can also be achieved with the equality constraint in (34). Meanwhile, we define $\mathcal{E} = \frac{1}{2}N$. Note that $(\beta, \gamma_0) = (\gamma_f, \gamma_f)$ is a constant term. Then, the tradeoff problem can be simplified into

$$\min \mathcal{E}(\beta_k, N) = \frac{1}{2}N \tag{36}$$

s.t. $p_f(\beta_k, N) = \gamma_f$ \hspace{1cm} $p_d(\beta_k, N) = \gamma_f$ \hspace{1cm} \hspace{1cm} $p_f(\beta_k, N) < \gamma_f$ \hspace{1cm} \hspace{1cm} (37)

For each fixed amplification gain $\beta_k$, according to Proposition 3, there exists a minimum number of samples $N_{\text{min}}$ to achieve a local minimum energy consumption $\mathcal{E}$, while satisfying the constraints (37) and (38). In order to achieve the global minimum $\mathcal{E}$, we substitute the formula (32) into (36) and obtain an equivalent unconstrained optimization function, which can be expressed as

$$\mathcal{E}(\beta_k) = 2 \beta_k^2 \left( \frac{A - B}{C - D} \right)^2 \tag{39}$$

We can theoretically solve the optimal amplification gain $\beta_k$ through derivative. That is, we first compute the derivative $\frac{\partial \mathcal{E}(\beta_k)}{\partial \beta_k}$ of $\mathcal{E}(\beta_k)$ with respect to $\beta_k$. Then, we have $\frac{\partial \mathcal{E}(\beta_k)}{\partial \beta_k} = 0$ and solve the equation. However, in practice, it is difficult to obtain an analytic expression of the optimal amplification gain $\beta_k$. Hence, we will find an approximately optimal solution through a search algorithm.

The amplification gain $\beta_k$ is subject to the maximum average transmission power $\mathcal{P}_{\text{max}}$, and from (26), its upper bound can be calculated by

$$\beta_{\text{max}} = \sqrt{\frac{\mathcal{P}_{\text{max}}}{\frac{1}{2} \sigma_c^2 + \gamma_f}} \tag{40}$$

For a given frame duration $T$, the number of samples $N$ is also limited. The reason is that the spectrum sensing time $\tau$ is subject to the required minimum throughput $R_{\text{min}}$ of the SU, i.e., $R(\tau) \geq R_{\text{min}}$. $R(\tau)$ denotes the practical throughput, which is given by

$$R(\tau) = \frac{T - \tau}{T} C_0 (1 - p_f(\tau)) p_f(\tau) \frac{\gamma_0}{\gamma_f} \tag{41}$$

In the above formula, $C_0$ denotes the throughput of the SU when the PU is active. $p_f(\gamma_0)$ and $p_f(\gamma_f)$ represent the probabilities for which PU is inactive and for which PU is active, respectively. Then, $p_f(\gamma_0) + p_f(\gamma_f) = 1$. From the constraints (37) and (38), we have $p_f(\tau) = \gamma_f$ and $p_d(\tau) = \gamma_f$. 


Fig. 3. Simulated and theoretical detection probability, target detection probability, and normalized energy consumption ($h_{p,s} = 0.1 + 0.1 \cdot j$, $\beta_k = 1$, and $p_f = 0.05$).

Fig. 4. Simulated and theoretical false alarm probability, target false alarm probability, and normalized energy consumption ($h_{p,s} = 0.1 + 0.1 \cdot j$, $\beta_k = 1$, and $p_d = 0.95$).

Then, from $R(\tau) \geq R_{\text{min}}$, we can get the maximum spectrum sensing time $\tau_{\text{max}}$ as

$$\tau_{\text{max}} = T \left( 1 - \frac{R_{\text{min}}}{R_0 + R_1} \right)$$

(42)

where $R_0 = C_0 (1 - p_f P(H_0))$ and $R_1 = C_1 (1 - p_d P(H_1))$.

We denote $T = LT_s$, $\tau = NT_s$. Then, the maximum number of samples $N_{\text{max}}$ can be expressed as

$$N_{\text{max}} = L \left( 1 - \frac{R_{\text{min}}}{R_0 + R_1} \right)$$

(43)

From the above discussions, the amplification gain $\beta_k$ can vary in a range from $0$ to $\beta_k_{\text{max}}$, and the number of samples can vary in a range from zero to $N_{\text{max}}$, i.e., $\beta_k \in [0, \beta_k_{\text{max}}]$, $N \in [0, N_{\text{max}}]$. $\beta_k = 0$ corresponds to the case without a relay for spectrum sensing. For each of $\beta_k \in [0, \beta_k_{\text{max}}]$ from the formula (32), we can compute the corresponding minimum number of samples $N_{\text{min},k}$.

From (36), we can obtain the corresponding local minimum energy consumption $E'_{\text{min},k}$. That is to say, for a set of $(\beta_k, \ldots, \beta_k, \ldots, \beta_k)$, we have the corresponding set of $\{E'_{\text{min},1}, \ldots, E'_{\text{min},k}, \ldots, E'_{\text{min},J}\}$, from which the approximately global minimum energy consumption $E'_{\text{min}}$ can be found and the corresponding pair of $(\beta_k, N_{\text{min}})$ can also be found.

For each relay $R_k$, there exists an approximately global minimum energy consumption $E'_{\text{min},k}$, which can be solved by comparing $E'_{\text{min},k}$ one by one. Then, we can find the optimal relay that consumes the minimal energy $E'_{\text{min}}$ for spectrum sensing. Based on the above discussion, an energy-efficient relay selection scheme for cooperative spectrum sensing can be proposed.

IV. SIMULATION RESULTS

In this section, simulation results are presented to show the tradeoff between sensing performance and energy consumption (using MATLAB). Without loss of generality, we assume that the fading coefficients are $h_{p,k} = h_{s,k} = 0.5 + 0.5 j$, and the variance of the noises and the transmitted PU signal are...
Fig. 7. Normalized minimum number of samples and normalized minimum energy consumption ($h_{p,s} = 0.1 + 0.1j$, $\overline{p}_d = 0.95$, and $\overline{p}_f = 0.05$).

Fig. 8. Normalized minimum number of samples and normalized minimum energy consumption ($h_{p,s} = 0.05 + 0.05j$, $\overline{p}_d = 0.95$, and $\overline{p}_f = 0.05$).

Fig. 9. Normalized minimum number of samples and normalized minimum energy consumption ($h_{p,s} = 0.2 + 0.2j$, $\overline{p}_d = 0.95$, and $\overline{p}_f = 0.05$).

Fig. 10. Normalized minimum number of samples and normalized minimum energy consumption for varying channel fading coefficients $h_{p,s}$ ($\overline{p}_d = 0.95$ and $\overline{p}_f = 0.05$).

$\sigma_s^2 = \sigma_k^2 = \sigma_p^2 = 1$. The target detection probability and target false alarm probability are set to be $\overline{p}_d = 0.95$ and $\overline{p}_f = 0.05$, respectively.

First, we show the sensing performance with relay. Under the target false alarm probability ($\overline{p}_f = 0.05$), the detection probability in terms of the number of samples is shown in Fig. 3. Under the target detection probability ($\overline{p}_d = 0.95$), the false alarm probability in terms of the number of samples is shown in Fig. 4. It is observed that when the target false alarm probability is satisfied, the detection probability increases with the number of samples $N$. When the target detection probability is achieved, the false alarm probability decreases with $N$. This demonstrates that the sensing performance improves with the number of samples. Meanwhile, we can also observe that the theoretical detection probability and false alarm probability are rather close to the simulated results.

We normalize each energy consumption by the maximum value of all, i.e., $\overline{E} = E / \max(E)$, such that $\overline{E} \in [0, 1]$. Fig. 3 shows the normalized energy consumption $\overline{E}$ and detection probability $p_d$ in terms of the number of samples $N$. Fig. 4 shows $\overline{E}$ and false alarm probability $p_f$ versus $N$.

We can see that, for a fixed amplification gain $\beta_k$, the detection probability $p_d$ increases slowly with the number of samples $N$. When it is higher than the target detection probability (i.e., $p_d > \overline{p}_d$), the false alarm probability $p_f$ decreases slowly with $N$. On the other hand, the normalized energy consumption always increases linearly with the number of samples for a fixed amplification gain $\beta_k$. Therefore, it is necessary to decrease the energy consumption as much as possible while keeping the sensing performance above a certain level.

In the text followed, we will show how to obtain an optimal pair of the parameters, i.e., the number of samples and the amplification gain, which can achieve a minimum energy consumption under sensing performance constraints. Before doing so, we need to determine their upper bounds with the following simulation parameters. Assume that the maximum average transmission power of relay $R_k$ is $P_{\text{max}} = 100W$. Then, from (40), the maximum amplification gain is $\beta_{k,\text{max}} = 8.1650$. On the other hand, let $R_{\text{max}} = 1 \text{ b/s}$, $C_0 = 2 \text{ b/s}$, $C_1 = 1.5 \text{ b/s}$, $p(\overline{H}_0) = 0.7$, $p(\overline{H}_1) = 0.3$, $T = 20 \text{ ms}$, and $T_s = 1 \mu \text{s}$. Then, from (42), the longest spectrum sensing time
is $t_{\text{max}} = 5.2126$ ms, and the upper bound of the number of samples is $N_{\text{max}} = t_{\text{max}}/T_s \approx 5212$, given that the required minimum throughput $R_{\text{max}}$ for the SU can be satisfied. Fig. 5 shows the minimum number of samples $N_{\text{min}}$ in terms of the amplification gain $\beta_k$. For $\beta_k = 1$, the minimum number of samples $N_{\text{max}}$ computed from (32) is $N_{\text{max}} = 952.7$. Moreover, from Figs. 3 and 4, for $\beta_k = 1$, the minimum number of samples $N_{\text{max}}$ obtained by simulation is $N_{\text{sim}} \approx 950$. Hence, the minimum numbers of samples $N_{\text{min}}$ obtained from analysis and simulation match to each other very well. In this figure, $\beta_k$ denotes the critical point of the amplification gain $\beta_k$ above which the corresponding minimum number of samples $N_{\text{min}}$ can satisfy the required minimum throughput $R_{\text{max}}$ for the SU (i.e., $N_{\text{min}} < N_{\text{sim}}$). From the figure, it can also be seen that the required minimum number of samples $N_{\text{min}}$ decreases with the amplification gain $\beta_k$. It is worth noting that $N_{\text{max}}$ drops quickly with $\beta_k$ when $\beta_k < 1$, but $N_{\text{min}}$ decreases slowly with $\beta_k$ when $\beta_k > 1$.

Fig. 6 shows the minimum energy consumption $T_{\text{max}}$ in terms of the amplification gain $\beta_k$. It is clear that there exists a local maximum energy consumption when the amplification gain is about 0.4, and a local minimum energy consumption when the amplification gain is about 1.4. This demonstrates that the energy consumption does not always increase with the amplification gain.

Fig. 7 shows the minimum number of samples $N_{\text{min}}$ and minimum energy consumption $T_{\text{max}}$ in terms of amplification gain $\beta_k$. Here, we have $N_{\text{min}} = N_{\text{max}}/\max(\beta_k)$ and $N_{\text{max}} = N_{\text{max}}/\max(\beta_k)$. When $\beta_k = 0$, corresponding to the single-user sensing case, the energy consumption is equal to zero, but the minimum number of samples $N_{\text{min}}$ cannot satisfy the required throughput of the SU. The minimum energy consumption can be achieved when the amplification gain is about 1.4, while the required minimum throughput of the SU can also be satisfied. When $\beta_k < 1.4$, the number of samples drops quickly with $\beta_k$, while the energy consumption fluctuates with $\beta_k$. When $\beta_k > 1.4$, the number of samples drops slowly with $\beta_k$, while the energy consumption decreases quickly with $\beta_k$. Therefore, considering both the number of samples (sensing time) and the energy consumption, we know that it is appropriate to choose $\beta_k = 1.4$ in this exemplary case. This phenomenon is similar as shown in Fig. 8, where $h_{\beta_k} = 0.05 + 0.05j$.

When the channel coefficient is $h_{\beta_k} = 0.02 + 0.2j$, the values of $N_{\text{max}}$ and $T_{\text{min}}$ are shown in Fig. 9. Although all different amplification gains $\beta_k$s can make the corresponding minimum numbers of samples $N_{\text{min}}$ satisfy the condition of $N_{\text{min}} < N_{\text{max}}$, it is still necessary to balance the number of samples and the energy consumption. When $\beta_k > 2$, $N_{\text{max}}$ decreases slowly with $\beta_k$, while the normalized energy consumption $T_{\text{min}}$ increases quickly with $\beta_k$. Fig. 10 shows $N_{\text{min}}$ and $T_{\text{min}}$ for varying channel fading coefficients $h_{\beta_k}$. It can be seen that the minimum number of samples does not always decrease with $\beta_k$ when the fading coefficient $h_{\beta_k}$ increases to a certain value, e.g., $h_{\beta_k} = 0.5 + 0.5j$. On the contrary, the minimum number of samples increases with $\beta_k$. This is because the channel condition $h_{\beta_k}$ is better than the relay channel condition $h_{\beta_k}$ and $h_{\beta_k}$ (i.e., $|h_{\beta_k}|^2 < |h_{\beta_k}|^2|h_{\beta_k}|^2$). Hence, for $h_{\beta_k} = 0.5 + 0.5j$, we should adopt the single-user spectrum sensing ($\beta_k = 0$) such that the energy consumption can be minimized.

V. CONCLUSION

In this paper, the issues on cooperative spectrum sensing in CR networks with amplify-and-forward relay have been investigated. In particular, the tradeoff between sensing performance and energy consumption has been analyzed. In particular, when the amplification gain is a constant, we have identified the number of samples to achieve minimum energy consumption while satisfying the sensing performance constraint. Moreover, when the amplification gain varies, an appropriate amplification gain can be found in this paper to minimize the energy consumption while guaranteeing a satisfactory sensing performance.

REFERENCES


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