Optimal Charging Schemes for Electric Vehicles in Smart Grid: A Contract Theoretic Approach

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Abstract—Due to their environment friendliness, electric vehicles (EVs) are anticipated to form a considerable fraction of vehicles for transportation in smart cities. It is essential to design an electricity charging scheme that takes the utilities of both the charging stations and the EVs into consideration. However, the self-interested nature of the EVs together with the information asymmetry between the energy demand and supply sides makes the design a significant challenge. In this paper, we propose a queuing network-based model to characterize the charging process of the multiple EVs in a renewable energy-aided charging station. Based on the model, we adopt a contract theoretic approach to design an optimal charging policy in an information asymmetry scenario. Furthermore, we propose the new contract-based charging rate assignment and admission control schemes that maximize the utility of the charging station under certain charging constraints. To derive the optimal contract, we present a two-step iterative algorithm and prove its convergence. We evaluate the proposed schemes based on the IEEE 69-bus distribution test system. Results indicate that the contract-based charging schemes can effectively benefit both the charging stations and the EVs and concurrently improve the load level of the smart grid.

Index Terms—Electric vehicle, charging scheme, queuing model, contract theory, admission control.

I. INTRODUCTION

BEING a paradigm of green transportation, Electric Vehicles (EVs) form one of the main components for sustainable smart cities [1]. In order to enhance the energy efficiency of the grid as well as provide reliable operation of EVs, it is of paramount importance to study the characteristics and control policies of EVs charging. In recent years, vehicle platoons have gained increasing attention with innovative capabilities for dealing with traffic congestion and improving energy efficiency. Some research and demonstration have been taken on the application of vehicle platoons [2]. An EV platoon is a group of EVs composed of a head vehicle and a number of followers traveling the same route [3]. As an EV platoon always arrives collectively at a charging station, the feature of the arriving EV platoons challenges the charging service policy of the stations, which is always designed for the independently arriving EVs. Moreover, the EVs charging scheduling schemes should consider the Quality of Service (QoS) which can be characterized by the charging rates, the electricity price and the waiting time. Nonetheless, uncoordinated charging of a high number of EVs may significantly increase burden on the local neighborhood circuits of the grid [4]. Thus, an efficient charging policy should consider both the constraints of the grid and customer satisfaction.

Intuitively, the willingness of the EVs to coordinate in the charging process may be of great importance to the viability of an optimal charging service. However, in practice, EVs are self-interested. It is unrealistic to assume that they follow the control instructions from the charging station unconditionally. In addition, there may exist information asymmetry between the charging station and the EVs which is caused by the station’s unawareness of the actual charging preference of the EVs. These factors pose a significant challenge on designing the optimal charging scheme.

A contract theoretic approach is a powerful tool from microeconomics that brings two self-interested and rational entities to agreements by providing economic incentives [5]. In this paper, we design an EV charging mechanism which maximizes the utility of the charging station and concurrently enhances the QoS of the charging process. To the best of our knowledge, this is the first work that applies contract theoretic approach in arranging power resources of charging stations for
serving EV platoons. In addition, unlike traditional contract-based strategies, which mainly focus on contract adjustment to improve service performance, we combine contract design with service QoS requirement and propose optimal contracts incorporating charging admission control policies. The main contributions of this paper are as follows:

- We introduce a contract theoretic approach for resource management at charging stations that serve EV platoons. In order to maximize the utilities of the charging stations while also satisfying certain charging QoS constraints, we incorporate EV platoon admission control into charging contract design.
- To cope with the variable characteristics of different charging operation scenarios, we propose an efficient two-step iterative algorithm to obtain the optimal contract and prove the algorithms convergence.
- We present a queuing network based performance analysis framework for the charging process of EV platoons served at a renewable aided charging station. Moreover, by implementing the proposed contract-based charging schemes into charging process management, we obtain the steady-state distribution of the queuing network.

The rest of the paper is organized as follows. In Section II, we review related work. A platoon-based charging queuing model and the problem formulation are derived in Section III. The contract-based charging rates assignment and admission control schemes are described in Section IV. In Section V, we derive the steady-state distribution of the charging queuing system. Performance evaluation is presented in Section VI. Finally, we conclude our work in Section VII.

II. RELATED WORK

EV platoon is a promising transportation way with significant environmental and safety benefits. There are several projects and experiments focusing on demonstrating platoon in general or the EV platoon. For instance, the SARTRE project investigates and trials technologies for platoon driving of road vehicles [6]. In [7], experimental results have shown that platooning of trucks improves vehicle energy efficiency. Several practical experiments have also been conducted on real EV platoons. For instance, the autonomous platoon driving system was tested in the experiment shown in [8]. The performance of the platooning control scheme designed for urban electric vehicles was investigated via full-scale experiments. Furthermore, some studies have carried out on the EV platoons. For example, Yu et al. [9] proposed a predictive control system for Hybrid Electric Vehicle (HEV) platoons.

As battery charging technologies play a critical role in the support and proliferation of EVs, EVs’ charging systems have been extensively studied. Wang et al. [10] presented a comprehensive overview of coordinated EV charging mechanism from an algorithmic perspective. The study in [11] focused on the spatial-temporal random dynamics of EVs, and proposed a probabilistic model for charging demands of moving EVs. Tang and Zhang [12] formulated optimal EV charging scheduling as a finite-horizon dynamic programming, and presented a low complexity online algorithm to solve the program. The work in [13] studied the charging schemes for large populations of EVs, and provided a hierarchical charging control framework. Coping with the growth of wind power generation, He et al. [14] designed a bi-layer optimization scheduling of generators, electric vehicles as well as wind power in the two dimensions of time and space. Aiming to establish an optimal load pattern, Alonso et al. [15] introduced a genetic algorithm based scheme, which coordinates electric vehicle charging with various characteristics of the smart grid.

Through a hierarchical game approach, Tan and Wang [16] proposed a charging navigation framework for electric vehicles, where both power system and transportation system were considered. Saad et al. [17] gave an overview of applying game-theoretic methods in managing microgrid systems, power demand response coordination and smart grid communications. Rigas et al. [18] focused on the utilization of artificial intelligence in managing electric vehicles in smart grid, including elaboration of challenges and comparison of technical approaches.

The queuing theory is a powerful mathematical tool to construct the model of an operation system. Through this model, some statistical characteristics, such as queue lengths and waiting time, can be obtained. These characteristics are very helpful to improve the design of the system operation schemes. We note that a few recent studies have adopted queuing theory to study the charging process and improved the quality of charging service. For example, in [19], the queuing theory was adopted to model the EV aggregation behavior. In [20], the capacity of an EV charging station was determined through a queuing theoretic approach. In [21], the process of charging multiple EVs at a charging facility was modeled as a queuing network. Based on the proposed queuing model for battery swapping stations, Tan et al. [22] and Sun et al. [23] introduced some valuable performance indicators of the charging system, and presented an optimal charging scheme by dynamic programming. The work in [24] modeled fast charging as a queuing system where both the direct current fast charging model and the revenue model of the station are incorporated into the queuing analysis. However, none of the aforementioned work has considered the influence of EV platoons charging in the stations.

Due to their suitability for modeling market mechanisms of electricity trading, economic theories have now been pervasively and successfully applied in the studies of smart grid. For instance, Zeng et al. [25] used group selling based auction for motivating EVs to feedback power to the grid. The studies in [26] proposed a deadline differentiated pricing scheme, which incentivizes charging EVs to defer their electric power consumption. Shuai et al. [27] focused on economic and incentive aspects of electric vehicle charging process, and provided a comprehensive survey of charging economic models as well as charging management schemes.

Being a promising economic theoretic approach, contract theory is widely used in various resource management problems. For instance, Duan et al. [28] investigated cooperative spectrum sharing under incomplete information, and proposed contract-based optimal sharing schemes between primary and secondary users. In [29], contract theory was adopted to address the problem of relay selection in OFDM-based wire-
less systems. To mitigate the interference between remote radio heads and macro base stations, Peng et al. [30] introduced a contract-based interference coordination framework. Asheralieva and Miyanaga [31] focused on joint user association and inter-cell interference mitigation in heterogeneous LTE-A networks, and proposed an efficient contract-based mechanism. Applying contract theory in the field of power management, Gao et al. [32] proposed a contract-based mechanism which is helpful in matching the aggregated energy rate to the service request while also maximizing the EVs’ profits. Namerikawa et al. [33] utilized a real time pricing contract to guarantee the participation of the energy suppliers and consumers in the energy market. To improve power transmission efficiency, Zhang et al. [34] proposed an energy exchange mechanism between electric vehicles, where the energy trading process was modeled and managed in a contract theoretic approach. Nevertheless, most of these studies only considered the quantity of the required energy and the profits gained by both sides. Few studies of them have taken into account the QoS of the charging process in the energy exchange.

Serving as an important means of transportation in the modern society, EVs are expected to have short charging duration. There are a few studies focused on the charging scheme with charging duration limitations. For example, Xu et al. [35] formulated the EV charging scheduling problem as a Markov decision process, where both the charging task deadlines and the random electricity cost have been considered. Yu et al. [36] proposed an intelligent energy management system with charging deadline constraints and energy source choices. You et al. [37] proposed a cooperative charging scheme for a charging station, which enables EVs to economically be charged within the given deadlines. Zhou et al. [38] formulated an EV charging optimization problem to minimize the supply costs of charging stations, which takes into account the individual charging deadline constraint of each EV. However, none of these works have incorporated the charging QoS guarantee strategies into the economic schemes.

Different from these studies, in this paper we concentrate on the charging process of EV platoons and propose the optimal contract-based charging schemes to improve the utilities of the charging station while guarantee the charging QoS.

III. SYSTEM MODEL AND PROBLEM FORMULATION

Fig. 1 shows a renewable energy aided charging station with a total of c parallel chargers. The charging station is modeled as a queuing network, where the input is the platoons needing electrical power charging and the output is the fully charged platoons. We consider that the arrival of the EV platoons follows a Poisson process with arrival rate \( \lambda_p \) [22]. In practice, the Poisson arrival model of vehicles on highways has been verified in [39]. As vehicle platoons have not been widely used in our real life, there is scarcely any statistical data for modeling the arriving platoons. For an EV platoon, a group of EVs may travel together in the same route, and we can take each platoon as a special EV. Thus, it is not exceptional to assume that the arrival process for EV platoons follows a Poisson process.

The maximum capacity of the charging station for accommodating vehicles is limited to \( N \), i.e., besides the \( c \) charging vehicles the station can at most provide \( N - c \) parking lots. The size of each platoon is a random variable denoted as \( z \). The probability that a platoon consists of \( z \) vehicles is denoted as \( P_{t,z} \), where \( \sum_{z=1}^{\max} P_{t,z} = 1 \). We consider EV platoons charging fairly with the first-come-first-serve policy. The State of Charge (SoC) of each arriving EV follows an i.i.d. random variable.

The energy charging schedules of the platoons and the charging station are considered to operate in a discrete time model with fixed length time slots. For ease of analysis, we consider that the length of each time slot, denoted as \( \tau \), is short, and that no more than one platoon arrives at the station during one time slot. Let time slot \( t \) denote the time interval \((t\tau, (t+1)\tau]\), \( t = \{0, 1, 2, \ldots\}\).

Let \( P_t(e(t) = m) \) denote the probability that the renewable energy source \( e(t) \) generates \( m \) units of electricity in time slot \( t \), \( m \geq 0 \). Here, one unit is defined as the average energy consumed for charging one EV. The generated renewable energy is divided into two parts. The first part is imported into the station for the charging service, and the remaining part is sold to the grid. As each time slot is a short time interval, the number of EVs in the charging station at time slot \( t + 1 \) mainly depends on that at time slot \( t \). The charging demand from the EVs changes slightly between these two consecutive slots, especially in the charging station at a steady state. Thus, we can predict the charging demand of each time slot based on that of the last slot. We consider that for each time slot, the amount of the imported renewable energy depends on the charging demand of the EVs. Then, the imported renewable energy in time slot \( t + 1 \) is defined as

\[
\tau_{re}(t + 1) = \min(e(t + 1), s(t), M),
\]
where \( s(t) \) is the number of EVs that have been served by the station in time slot \( t \), \( s(t) \geq 0 \). Due to the definition of the unit of the renewable energy, the amount of consumed energy in time slot \( t \) is numerically equal to \( s(t) \). \( M \) is the limitation of the energy transmission capacity of the line connecting the renewable generator and chargers in one time slot.

We consider there are \( G \) types of EV platoons according to their preferred charging rates, with different willingness-to-pay parameters of \( \theta_1, \theta_2, \ldots, \theta_G \) [32], [40]. Here, charging rate indicates the amount of electrical energy supplied by the charging station to the EV platoons per unit of time. The charging preference type is private information of each platoon which is not known to the charging station. However, we assume that the charging station has the knowledge of the probability distribution of platoon types based on statistical information. The probability of the EV platoons belonging to type-\( \theta_i \) is denoted as \( P_{r,i} \) with \( \sum_{i=1}^{G} P_{r,i} = 1 \). Without loss of generality, we consider that \( \theta_1 < \theta_2 < \ldots < \theta_G \) and the higher \( \theta \) implies higher preference for faster charging. The utility of a type-\( \theta \) platoon which charges at rate \( r \) can be expressed as

\[
U_p(\theta, r, q) = v(\theta, r) - q,
\]

where \( q \) is the cost a platoon pays for choosing charging rate \( r \). \( v(\theta, r) \) is the evaluation function of a platoon according to the archived charging rate, defined as \( v(\theta, r) = \sigma \log(1 + \theta r), r \in \mathbb{R}^+ \). Here \( \sigma \) is a coefficient with the same unit as \( q \). For analysis simplicity, the value of \( \sigma \) is set to 1. In practical charging operations, the evaluation function should satisfy two properties [41]. First, a driver obtains more utility when the driver’s EV served with faster charging rate, especially for the driver of the higher \( \theta \) type EVs. Second, the EV drivers may have less interest to charge when the charging rate increases. Logarithmic function is one of the functions that satisfy these two properties. Due to its properties and mathematical simplicity, logarithmic function utility has been widely used in many different fields [42]–[44]. In our paper, we also adopt the logarithmic representation in the evaluation function. It is noteworthy that any increasing concave function can be used as the evaluation function, and the change of these functions does not affect the design of the charging schemes.

Besides the charging rate cost, another critical issue in the charging system is the charging QoS. In this paper, we mainly focus on the study of the service loss caused by impatient drivers waiting in the queue for a certain length of time, and propose an admission control scheme for the arriving platoons to reduce the service loss probability. We assume that the platoons of type-\( \theta \) will leave the charging station if they are kept waiting for longer than \( T_0 \). As long as a platoon has an overview of the battery status of its vehicles, it can drive to an alternative and feasible charging station where it will have to wait for less time, or wait at the current station and charge at a slower rate if the remaining power is insufficient to drive to the alternative stations. For these platoons that reduce their charging rate requirements due to the unavailability of alternative stations, we can model them as new types of platoons with reduced charging rates and different willingness-to-pay parameters. It is noteworthy that our contract-based approach can be directly applied even with the additional types of platoons. Recall that a larger \( \theta \) means higher charging rate preference, which in turn implies less tolerance on waiting. Thus we can get \( T_{\theta,1} > T_{\theta,2} > \ldots > T_{\theta,G} \). We assume that the charging station could provide \( G \) different charging rates \( \{r_1, r_2, \ldots, r_G\} \), whose charging times are exponentially distributed due to the different initial EV battery’s SoC.

The charging station should pay cost to the grid for the energy it consumes, which is affected by two factors. The first one is the renewable energy imported into the charging station. The other factor is the load of the power grid. In smart grid, the load level information can be delivered to each charging station through cable or wireless communications. Thus, the unit charging rate cost for the station is defined as

\[
\frac{\zeta}{d} = \rho_0 - \alpha_1 r e(t) + \alpha_2 \left( \frac{d}{d_{\text{ideal}}} \right)^{\gamma},
\]

where \( \rho_0 \) is the base cost of unit charging rate. \( \alpha_1, \alpha_2 \) are coefficients, and \( \gamma \) is a constant, \( \gamma > 1 \). \( d \) is the electrical
energy demand of the district where the charging station is located excluding the charging consumption. \(d_{\text{ideal}}\) is the optimal demand of the district set by the power grid company. Typically, \(d_{\text{ideal}}\) is set as 80% of the nominal capacity of the transformer in a district [45].

In order to improve the revenue obtained by the charging station and the charging QoS while ensuring that each EV platoon can obtain the best match charging rate according to its type, we propose an optimization problem incorporated with charging rates assignment and charging admission control, which is formulated as

\[
\max_{\{q_i,r_i,P_{ac,i}\}} U_{\text{sta}} = \sum_{i=1}^{G} (1 - P_{\text{loss},i})P_{ac,i}P_{r,i}(q_i - r_i\xi)
\]

s.t.\(\quad\)C1 : \(v(\theta_i, r_i) - q_i \geq v(\theta_i, r_j) - q_j, \quad i \neq j,\)
\(\quad\)
C2 : \(v(\theta_i, r_i) - q_i \geq 0, \quad r_{\min} \leq r_i \leq r_{\max},\)
C3 : \(q_i \geq 0, \quad P_{\text{loss},i} \leq \varepsilon, \quad P_{ac,i} = \{0, 1\}, \quad i, j \in G.\)

(4)

In (4), \(U_{\text{sta}}\) is the expected utility of the charging station. \(P_{\text{loss},i}\) is type-\(\theta_i\) platoon service loss probability due to long waiting time. \(P_{ac,i}\) is the admission policy of type-\(\theta_i\) platoons, where the policy is either “always reject” with \(P_{ac,i} = 0\) or “always accept” with \(P_{ac,i} = 1.\) \(U_{\text{sta}}\) can be maximized through optimizing the design of charging rates \(\{r_i\}\) and corresponding costs \(\{q_i\}\) as well as the admission policy set \(\{P_{ac,i}\}\) for each type of platoons. In C2, \(r_{\min}\) and \(r_{\max}\) are constants, which are the limitation of the charging rate of each charger in practice. \(\varepsilon\) in C3 is a constant QoS threshold, and \(G = \{1, 2, \ldots, G\}.\) Constraint C1 in (4) is non-linear which makes the solution of the optimization problem challenging.

IV. CONTRACT-BASED CHARGING RATE ASSIGNMENT AND ADMISSION CONTROL

In this section, we model the charging rate assignment and admission control schemes as a contract problem. Based on the analysis of the relation between the admission control policies and the assigned charging rates, a two-step iterative algorithm to obtain the optimal solution of the problem is presented.

A. Feasible Contracts Formulation

The charging station effectively specifies a rate-cost bundle contract denoted as \((r_i, q_i)\) for type-\(\theta_i\) platoons, \(i \in G.\) To ensure its feasibility, the contract should satisfy both the Individual Rational (IR) constraint and the Incentive Compatibility (IC) constraint. In the considered problem, the IR constraint could be denoted as \(U'_{\theta_i} = v(\theta_i, r_i) - q_i \geq 0, \quad i \in G,\) which motivates the participation of the self-interested EV platoons. Due to the IR constraint, if the type-\(\theta_i\) platoons are prohibited by the admission control policy, i.e., \(P_{ac,i} = 0,\) correspondingly, the charging cost \(q_i\) of contract \((r_i, q_i)\) could be set to be a sufficiently large number. The IC constraint makes the platoons of type-\(\theta_i\) prefer the contract \((r_i, q_i)\) over all other options, i.e., \(v(\theta_i, r_i) - q_i \geq v(\theta_i, r_j) - q_j, \quad i, j \in G, \quad i \neq j.\)

Considering the feasibility of the contracts, the optimization problem (4) can be rewritten as

\[
\max_{\{q_i,r_i,P_{ac,i}\}} U_{\text{sta}} = \sum_{i=1}^{G} (1 - P_{\text{loss},i})P_{ac,i}P_{r,i}(q_i - r_i\xi)
\]

s.t.\(\quad\)C1 : \(v(\theta_i, r_i) - q_i \geq 0, \quad 1 \leq i \leq G,\)
\(\quad\)
C2 : \(v(\theta_i, r_i) - q_i \geq v(\theta_i, r_{i+1}) - q_{i+1}, \quad 1 \leq i < G,\)
C3 : \(v(\theta_i, r_i) - q_i \geq 0, \quad 1 \leq i \leq G.\)

\(P_{\text{loss},i} \leq \varepsilon, \quad P_{ac,i} = \{0, 1\}, \quad i \in G.\)

(5)

In (5), constraints C2 and C3 are called Local Upward Incentive Constraint (LUIC) and Local Downward Incentive Constraint (LDIC), respectively [5].

B. Optimal Contract Simplification

The maximum utility of the charging station will be obtained under the condition that LDICs are binding for the optimization problem. Given the monotonicity condition \(r_{i-1} < r_i, \quad 1 \leq i \leq G,\) and that all the LDICs are binding, then the LUICs can be drawn from the LDICs [5]. Thus, we can say that with the monotonicity condition and binding LDICs, LUICs could be reduced. Then, the optimization problem in (5) can be simplified as

\[
\max_{\{q_i,r_i,P_{ac,i}\}} U_{\text{sta}} = \sum_{i=1}^{G} (1 - P_{\text{loss},i})P_{ac,i}P_{r,i}(q_i - r_i\xi)
\]

s.t.\(\quad\)C1 : \(v(\theta_i, r_i) - q_i = 0, \quad 1 \leq i \leq G,\)
\(\quad\)
C2 : \(v(\theta_i, r_i) - q_i = v(\theta_i, r_{i-1}) - q_{i-1}, \quad 1 < i \leq G,\)
C3 : \(r_{\min} \leq r_i \leq r_2 \leq \ldots \leq r_G \leq r_{\max},\)
C4 : \(q_i \geq 0, \quad P_{\text{loss},i} \leq \varepsilon, \quad P_{ac,i} = \{0, 1\}, \quad i \in G.\)

(6)

Here, we define the notations \(\Delta_1 = 0, \quad \Delta_i = v(\theta_i, r_i) - v(\theta_i, r_{i-1}), \quad 1 < i \leq G\) [46]. Then, according to the constraints of (6), we get \(q_i = v(\theta_i, r_i) + \sum_{j=1}^{\xi} \Delta_j, \quad i \in G.\) As \(q_i\) always can be substituted by a function with variables \(\{\theta_1, \ldots, \theta_i, r_1, \ldots, r_i\}, \quad i \in G,\) we only use a symbol \(q_i\) here to represent the payment, but not any form of functions. By substituting this into the object function of (6), the function can be rewritten as

\[
\max_{\{r_i,P_{ac,i}\}} U_{\text{sta}} = \sum_{i=1}^{G} (1 - P_{\text{loss},i})P_{ac,i}P_{r,i}\left(v(\theta_i, r_i) - r_i\xi\right)
\]

\[
+ (v(\theta_i, r_i) - v(\theta_{i+1}, r_i))\sum_{k=i+1}^{G} (1 - P_{\text{loss},k})P_{ac,k}P_{r,k},
\]

s.t.\(\quad\)C1 : \(r_{\min} \leq r_i \leq r_2 \leq \ldots \leq r_G \leq r_{\max},\)
\(\quad\)
C2 : \(P_{\text{loss},i} \leq \varepsilon, \quad P_{ac,i} = \{0, 1\}, \quad i \in G.\)

(7)

(8)
C. Optimal Contract Solution

To solve (8), we propose a two-step iterative algorithm. In the first step, given \( \{P_{ac,i}\} \), we derive the optimal charging rates for each type denoted as \( \{r_i^*\} \). In the second step, based on the \( \{r_i^*\} \), we obtain the optimal \( \{P_{ac,i}^*\} \). The algorithm converges under certain conditions, which will be discussed in the later part of this subsection.

1) Optimal Charging Rates: In the charging system, the waiting time of a newly arrived platoon may be affected by the service times of the EVs which are before it in the queue. As these EVs can belong to any type-\( \theta \), the charging rates of different types may impact the waiting time, and further may impact \( P_{loss} \) of platoons. Furthermore, due to the stochastic characteristics of the charging times of all types of platoons, the optimization of the charging rates is an NP-hard problem in terms of computational complexity. To solve the problem efficiently, we rewrite the objective function of (8) as

\[
U_{sta}' = (1 - \varepsilon) \sum_{i=1}^{G} P_{ac,i}(v(\theta_i, r_i) - r_i \xi) + (v(\theta_i, r_i) - v(\theta_{i-1}, r_i)) \sum_{k=1}^{G} P_{ac,k} P_{r,k}.
\]

As \( 1 - P_{loss,i} \geq 1 - \varepsilon, i \in G \), then \( U_{sta}' \leq U_{sta} \). Thus we improve the utility of the charging station by optimizing its lower bound.

Let \( H_i = \sum_{k=1}^{G} P_{ac,k} P_{r,k} \), \( H_i \) is a concave function on \( r_i \), \( 1 \leq i < G \).

**Proof:** Let \( A = \sum_{k=i}^{G} P_{ac,k} P_{r,k} \) and \( B = P_{ac,i} P_{r,i} \). Then the second derivative of \( H_i \) can be given as

\[
d^2H_i/dr_i^2 = -B \theta_i^2(1 + \theta_i + r_i)^2 + A \theta_i^2 - \theta_i^2 + 2 \theta_i r_i - 2 \theta_i^2 r_i.
\]

It is clear that \( B \theta_i^2(1 + 2 \theta_i + r_i) \leq B \theta_i^2(1 + \theta_i + r_i)^2 \). Recall that \( \theta_i > \theta_{i-1} \), we have \( A \theta_i^2(1 + 2 \theta_i + r_i) > A \theta_{i-1}^2(1 + 2 \theta_{i-1} + r_{i-1}) > A \theta_{i-1}^2(1 + \theta_{i-1} + r_{i-1}) \). According to the sufficient condition given in Lemma 1, \( A \theta_i^2(1 + 2 \theta_i + r_i) \leq B \theta_i^2(1 + \theta_i + r_i) \). By combining the above inequalities, we can prove \( d^2H_i/dr_i^2 < 0 \) which indicates \( H_i \) is a concave function on \( r_i \).

It is worth noting that, if the concave condition in Lemma 1 is not satisfied, we can first solve the optimization problem (8) without the constraint \( r_{min} \leq r_1 \leq r_2 \leq \cdots \leq r_G \leq r_{max} \) by Lagrangian relaxation. Then, we need to check whether the solution of the relaxed problem satisfies this constraint. In the following sections, we consider the case that \( P_{r,i}, i \in G \), satisfy the condition specified in Lemma 1. Thus, each \( r_i \) which maximizes \( H_i \) can be obtained at the boundary points, i.e., 0 or \( r_{max} \), or at the critical point by setting \( dH_i/dr_i = 0 \) according to Fermat’s theorem. However, the obtained set \( \{r_i\} \) may not satisfy the first constraint in (8). The sub-sequences of the set which are not in the increasing order, are called infeasible sub-sequences. As \( \{H_i\} \) are concave functions, the infeasible sub-sequences can be replaced by feasible sub-sequences \( \{r_i^*\} \) iteratively. The algorithm is presented as follows [46].

a) Initialize: \( \hat{r}_i = \arg \max_{r_i} H_i, i \in G \).

b) Repeat: While there exists an infeasible sub-sequence \( \{\hat{r}_i, \hat{r}_{i+1}, \ldots, \hat{r}_j\} \), set \( \hat{r}_k = \arg \max_{\hat{r}_k} \sum_{i=k}^{j} H_i(q) \), \( k \in \{i, i + 1, \ldots, j\} \).

c) Output: The optimal charging rates \( \{r_i^*\} = \{\hat{r}_i\} \).

2) Optimal Admission Probabilities: Now, having obtained the charging rates \( \{r_i^*\} \), we derive the optimal \( \{P_{ac,i}^*\} \). Considering \( G \) types of charging rates in the charging station, i.e., \( \{r_1^*, r_2^*, \ldots, r_G^*\} \), and the charging times are exponentially distributed with means \( \{1/\mu_1, 1/\mu_2, \ldots, 1/\mu_G\} \), respectively. Let \( L \) be the number of EVs in the charging system when a new EV platoon arrives at the station. For the waiting time of a new platoon, there are two cases. The first one is \( L < c \), where the waiting time for the new platoon equals to 0 due to the spare chargers. In the second case where \( L \geq c \), the new platoon should wait \( L - c + 1 \) EVs having been served before starting its own service. Thus, the probability of the waiting time \( w \) can be expressed as

\[
P_w(w \leq \xi) = \begin{cases} 1, & 0 \leq L < c \\ \int_0^\infty \frac{e^{-cx}/\Gamma(L - c + 1)}{L - c + 1} dx, & L \geq c \\ \end{cases}
\]

(10)

where \( \Gamma(\hat{y}) = \int_0^\infty e^{-x}/\hat{y} dx \). \( \hat{y} \) is the average service time for a charger to finish charging one EV, which is given as

\[
\hat{y} = \sum_{i=1}^{G} \frac{P_{ac,i} P_{r,i}/\mu_i}{\sum_{i=1}^{G} P_{ac,i} P_{r,i}}.
\]

(11)

Based on (10), the service loss probability of type-\( \theta \) platoons can be presented as

\[
P_{loss,i} = P_w(w > T_{\theta,i}).
\]

(12)

Recall that \( T_{\theta,1} > T_{\theta,2} > \cdots > T_{\theta,G} \), which means if \( P_{loss,1} \leq \varepsilon \), then \( P_{loss,j} < \varepsilon \), where \( j < i \) and \( j \in G \). Thus, we can obtain the optimal admission probabilities \( \{P_{ac,i}^*\} \) by searching for the critical \( P_{loss,k} \) which satisfies that \( P_{loss,k} > \varepsilon \) and \( P_{loss,k-1} \leq \varepsilon \). As the charging station is rational, \( q_i = r_i \), then \( \{P_{ac,i}^*\} \) is given as \( \{P_{ac,1}^* = 1, P_{ac,2}^* = 1, \ldots, P_{ac,k-1}^* = 1, P_{ac,k}^* = 0, \ldots, P_{ac,G}^* = 0\} \).

The complete optimal contract-based charging rate assignment and admission control are illustrated in Algorithm 1. The maximum number of iterations of Algorithm 1 is \( O(G + 1) \).

**Theorem 1:** Under the concave condition stated in Lemma 1, Algorithm 1 converges monotonically to the optimal contract-based charging rate assignment and admission control policy.

**Proof:** Let us consider two iterations \( j \) and \( j + 1 \). Let \( \{r_i^*\} \) and \( \{r_i^{*+1}\}, i \in G \), denote the optimal charging rates obtained from these two iterations, respectively. Furthermore, let \( G_j \) and \( G_{j+1} \) denote the highest type index of platoons admitted charging in the iteration \( j \) and \( j + 1 \), respectively.
The Optimal Charging Rate Assignment and Admission Control Schemes

**Initialization:**

1. **Step 1:** Based on the given \( \{ P^*_a, i \} \), compute the optimal charging rates \( \{ r^*_i \} \), \( i \in G \).
2. **Step 2:** According to the obtained \( \{ r^*_i \} \), searching for the critical \( P_{loss,k} \):
   
   3. If \( P_{loss,k} = \emptyset \) then
      
      4. Break;
   
   5. else
      
      6. Let \( \{ P^*_a, i = 1, P^*_a, 2 = 1, \ldots, P^*_a, k-1 = 1, P^*_a, k = 0, \ldots, P^*_a, G = 0 \} \);
      
      7. Set \( G = k - 1 \);
      
      8. Go to Step 1;
   
   9. end if
10. return \( \{ r^*_i \} \) and \( \{ P^*_a, i \} \), \( i \in G \).

Then, the first derivative of \( H_i \) of these two iterations can be expressed as following.

\[
H'_{i,j} = dH_{i,j}/dr_i = P_{ac,i} P_r,i (\frac{\theta_i}{1 + \theta_i r_i} - \zeta) + (\frac{\theta_i}{1 + \theta_i r_i} - \frac{\theta_i + 1}{1 + \theta_i (r_i + 1)} \sum_{k=i+1}^{G_j} P_{ac,k} P_{r,k} \),
\]

\[
H'_{i,j+1} = dH_{i,j+1}/dr_i = P_{ac,i} P_r,i (\frac{\theta_i}{1 + \theta_i r_i} - \zeta) + (\frac{\theta_i}{1 + \theta_i r_i} - \frac{\theta_i + 1}{1 + \theta_i (r_i + 1)} \sum_{k=i+1}^{G_j} P_{ac,k} P_{r,k}. \]

As both \( H_{i,j} \) and \( H_{i,j+1} \) are concave functions, the optimal rates \( r^*_i \) and \( r^*_i, j+1 \) are obtained by setting \( H'_{i,j} = 0 \) and \( H'_{i,j+1} = 0 \), respectively. Let \( H'_{i,j}(r^*_i) \) represent substituting \( r^*_i \) into \( H'_{i,j} \). To prove \( r^*_i < r^*_i, j+1 \) by contradiction, we assume \( r^*_i > r^*_i, j+1 \). Note that \( \sum_{k=i+1}^{G_j} P_{ac,k} P_{r,k} > \sum_{k=i+1}^{G_j} P_{ac,k} P_{r,k} \), due to \( G_j > G_j+1 \). If \( r^*_i = r^*_i, j+1 \), it is obvious that \( H'_{i,j}(r^*_i) = H'_{i,j+1}(r^*_i, j+1) \), which contradicts with \( H'_{i,j}(r^*_i) = H'_{i,j+1}(r^*_i, j+1) = 0. \) If \( r^*_i > r^*_i, j+1 \), as \( H'_{i,j} \) is a concave function, \( H'_{i,j}(r^*_i, j+1) > 0 \). Comparing \( H'_{i,j}(r^*_i, j+1) \) with \( H'_{i,j+1}(r^*_i, j+1) \), due to the second term of \( H'_{i,j}(r^*_i, j+1) \) being less than that of \( H'_{i,j+1}(r^*_i, j+1) \), we can conclude that \( H'_{i,j+1}(r^*_i, j+1) > H'_{i,j}(r^*_i, j+1) = 0 \), which contradicts with \( H'_{i,j}(r^*_i, j+1) = H'_{i,j+1}(r^*_i, j+1) = 0. \) Thus we have proved \( r^*_i < r^*_i, j+1 \). As the charging rates increase, the average charging time gets shorter which will in turn decrease the service loss probabilities. As a result, the algorithm will come to a convergence.

Under the condition that the number of EVs in the charging system is \( L \), we have obtained the optimal charging rates and access control probabilities which are denoted as \( \{ r^*_i(L) \} \) and \( \{ P^*_a, i(L) \} \), respectively. The optimal charging price \( \{ q^*_i(L) \} \) for platoons can be easily drawn based on \( \{ r^*_i(L) \} \). It should be noted that the value of \( L \), which is affected by the platoon arrival rate, the size of platoon and the charging service policy, etc., will be analyzed in the next section.

**V. Steady-State Distribution of Charging System**

In this section, we will incorporate the proposed contract-based charging schemes into the queuing model, and obtain the steady-state distribution of the drawn equilibrium equations of the charging system, which will be used by the performance analysis of the proposed charging schemes.

**A. Dynamics of Imported Renewable Energy**

Let \( f_1(b,s) \) denote the probability of the renewable energy imported at \( b \) units in time slot \( t+1 \) conditioned by \( s \) EVs that have been served in the charging station during time slot \( t. \) According to (1), \( f_1(b,s) \) can be derived as follows.

1) **Case 1:** If \( b = 0 \), which means no renewable energy will be imported in the charging station in time slot \( t+1 \), this situation can be caused by two reasons. One is no EV has been charged during time slot \( t. \) The other is that there have been some EVs served in time slot \( t, \) but no renewable energy is generated in time slot \( t+1. \) Thus, the probability of this case can be written as

\[
f_1(b,s) = \begin{cases} 
1, & b = 0, s = 0 \\
\Pr(e(t+1) = 0), & b = 0, s > 0.
\end{cases}
\]

2) **Case 2:** If \( 0 < b < M \), which requires both the number of the served EVs in time slot \( t \) and the renewable energy generated in time slot \( t+1 \) are no less than \( b, \) then we have

\[
f_1(b,s) = \begin{cases} 
\Pr(e(t+1) = b), & 0 < b < M, s > b \\
\Pr(e(t+1) = b), & 0 < b < M, s = b \\
0, & 0 < b < M, 0 < s < b.
\end{cases}
\]

3) **Case 3:** If \( b = M, \) i.e., both the number of the served EVs and the generated renewable energy should be no less than \( M, \) then we get

\[
f_1(b,s) = \begin{cases} 
\Pr(e(t+1) \geq M), & b = M, s \geq b \\
0, & b = M, 0 < s < M.
\end{cases}
\]

4) **Case 4:** If \( b > 0, \) then \( f_1(b,s) = 0. \)

**B. Dynamics of EVs in the Charging System**

In this subsection, we study the dynamics of the number of EVs as well as the imported renewable energy in the charging system. Let \( P(n,i,m,b) \) be the probability of the charging station holding \( i \) EVs with \( b \) renewable energy imported at time slot \( t+1 \) under the condition that it has \( n \) EVs with \( m \) units renewable energy imported at time slot \( t. \) Recall that time-slot duration \( \tau \) is short, and there is no more than one platoon arrival during one time slot. We denote the probability of \( z \) new EVs arrivals at the charging station during a time slot as

\[
\beta_z = \left\{ \begin{array}{ll}
0, & z = 0 \\
e^{-p_{c}^* \tau}, & z = 0 \\
e^{-p_{c}^* \tau} q_{t} \cdot P_{i,t}, & z > 0.
\end{array} \right.
\]

For each time slot \( t, \) the arriving time of each platoon is i.i.d. random variable. The probability of the time of the platoon arrival in the charging station at time \( t + t_d \) given that the
platoon arrival has occurred, is a conditional probability, and can be expressed as

\[
P_{\text{arr}}(t_a|A) = \begin{cases} 1/\tau, & 0 \leq t_a \leq \tau \\ 0, & \text{otherwise} \end{cases},
\]

(19)

where \( A \) denotes the event of a platoon arrival.

With the above results, two cases are considered as follows.

1) Case 1: No new platoons join the charging system in time slot \( t \). Since there is no EV joining, comparing the number of EVs at time slot \( t + 1 \) and \( t \), we can find that \( n - i \) EVs have finished charging during time slot \( t \). This case can be further divided into two scenarios.

   a) Scenario 1: No platoon arrives during this time slot. The probability of this scenario can be denoted as

\[
P_{1a}(n, i, m, b) = \beta_0 \cdot \left( \frac{\min[c, n]}{n - i} \right) (1 - e^{-\tau/\tilde{\eta}^n})^{n-i} \cdot e^{-\tau(\min[c, n] - n + i)/\tilde{\eta}} f_1(b, n - i),
\]

(20)

where \( \tilde{\eta} \) is the average charging time for an EV served by one charger according to the contract charging rates \( \{\tau^i(n)\} \), given the condition that there are \( n \) EVs in the queueing system. \( \tilde{\eta} \) can be obtained from (11).

b) Scenario 2: The second scenario is that a platoon arrives during time slot \( t \), but it cannot join the charging system either due to the insufficient available capacity of the charging station or because of the admission control scheme. Given that there are \( n \) EVs in the charging system at the beginning of time slot \( t \), we consider that a platoon arrives at the charging system at time \( t + t_a \). If there are \( n \) EVs at time \( t + t_a \), which means \( n - i \) EVs have been served in time \( (t, t + t_a) \), and \( n - i \) EVs will be served during time \( (t + t_a, (t + 1)t) \). Under these conditions, we can get the probability of the platoon getting admission as

\[
P_{1a}(t + t_a) = \begin{cases} 0, & z > N - v \\ \sum_{k=1}^{G} \Pr_{\text{arr}}(t_a|A) \cdot P_{\text{acc}, x}(v) \cdot P_{r,x}, & z \leq N - v \end{cases}.
\]

(21)

The probability of the dynamics of the platoons in this scenario can be divided into two parts, namely \( P_{1b1} \) and \( P_{1b2} \). \( P_{1b1} \) is the probability that the platoon cannot join the station due to the lack of available capacities. \( P_{1b2} \) is the probability that the leaving of the platoon is caused by the admission control policy. \( P_{1b1} \) and \( P_{1b2} \) are respectively expressed as follows.

\[
P_{1b1} = f_1(b, n - i) \int_0^\tau \Pr_{\text{arr}}(t_a|A) \sum_{v=n-min[c,n]}^{n} \beta_k \cdot \left( \min[c, n] \right) (1 - e^{-\tau/\tilde{\eta}^n})^{n-i} \cdot f_2(n, n - v, t_a) f_2(v, v - i, t - t_a) dt_a.
\]

(22)

\[
P_{1b2} = f_1(b, n - i) \int_0^\tau \Pr_{\text{arr}}(t_a|A) \sum_{v=n-min[c,n]}^{n} \beta_k \cdot \min[0, Z_{\max} - v] (1 - \sum_{k=1}^{G} P_{\text{acc}, x}(v) \cdot P_{r,x}) \cdot \left( \min[0, Z_{\max} - v] \right) \cdot f_2(n, n - v, t_a) f_2(v, v - i, t - t_a) dt_a.
\]

(23)

In (22) and (23), \( f(x) \) is an indicator function which equals 1 if \( x \) is true and 0 otherwise. The function \( f_2(w, s, \Delta t) \) denotes the probability that on average \( s \) EVs have been charged during time \( \Delta t \), if there were \( w \) EVs at the beginning, which could be described as

\[
f_2(w, s, \Delta t) = \left( \frac{\min[c, w]}{s} \right) (1 - e^{-\Delta t/\tilde{\eta}^w}) \cdot e^{-\Delta t(\min[c, w] - s)/\tilde{\eta}^w}.
\]

(24)

The probability \( P(n, i, m, b) \) of this case can be stated as

\[
P_{1}(n, i, m, b) = \left\{ \begin{array}{ll} P_{1a} + P_{1b1} + P_{1b2}, & (n - i) \in [0, \min[c, n]] \\ 0, & \text{otherwise.} \end{array} \right.
\]

(25)

2) Case 2: In this case, one platoon arrives and joins the charging station during time slot \( t \). Similar to the second scenario of Case 1, we assume that when the platoon consisting of \( k \) EVs arrives at the station at time \( t + t_a \), there are \( v \) EVs in the charging system. Recall that there is a capacity limitation of the charging station, then \( k + v \leq N \) should be satisfied. As a result, \( n - v \) EVs have finished charging during time \( (t, t + t_a) \), and \( i - (k + v) \) EVs will be fully charged during time \( (t + t_a, (t + 1)t) \). The probability of this case can be expressed as

\[
P_{2}(n, i, m, b) = \int_0^\tau \Pr_{\text{arr}}(t_a|A) \sum_{k=1}^{K} \beta_k \cdot \left( \frac{\min[c, n]}{n - i} \right) (1 - \sum_{k=1}^{G} P_{\text{acc}, x}(v) \cdot P_{r,x}) \cdot f_1(b, n + k - i) \sum_{v=V_1}^{V_2} \left( 1 - \sum_{i=1}^{G} P_{\text{acc}, x}(v) \cdot P_{r,x} \right) \cdot f_2(n, n - v, t_a) f_2(k + v, k + v - i, t - t_a) dt_a.
\]

(26)

where \( K = \min[Z_{\max}, N - n - \min[c, n]], V_1 = n - \min[n, c], \) and \( V_2 = \min[n, N - l] \). Based on (25) and (26), the probability \( P(n, i, m, b) \) can be denoted as

\[
P(n, i, m, b) = P_{1}(n, i, m, b) + P_{2}(n, i, m, b),
\]

(27)

C. Steady-State Distribution

Now, we shall study the steady-state distribution of the EVs and the imported renewable energy in the charging system. Let \( p(i, b) \) denote the steady-state probability that the system is in state \( S = (i, b) \) where the charging station holds \( i \) EVs and the amount of the imported renewable energy is \( b \). The steady-state equations are

\[
p(i, b) = \sum_{n=0}^{N} \sum_{m=0}^{M} p(n, m) P(n, i, m, b),
\]

(28)

By solving a set of \( (N + 1)(M + 1) \) linear equations in (28), together with the normalization equation \( \sum_{n=0}^{N} \sum_{m=0}^{M} p(n, m) = 1 \), we can get the steady-state distributions of the charging system. As the corresponding embedded Markov chain is ergodic, the charging system has a unique steady-state solution [22].
Based on the obtained steady-state probabilities, we can get some performance indicators of the charging system. For instance, the average EVs in the system and the average imported renewable energy can be given as 
\[ \bar{L} = \sum_{n=0}^{N} n \sum_{m=0}^{M} p(n, m), \bar{r} = \sum_{m=0}^{M} m \sum_{n=0}^{N} p(n, m), \]
respectively. According to these indicators, the optimal contracts in the steady-states are derived. Then, the effects of the contact-based scheme on both the utility maximization of the station and the adjustment of grid load level can be obtained.

To introduce the main flow of our proposed charging schemes more clearly, we illustrate the relationship between the main system components in Fig. 2.

VI. NUMERICAL RESULTS

In this section, we evaluate the performance of the proposed schemes through simulation on the IEEE 69-bus distribution test system as shown in Fig. 3, where the topology of the grid as well as typical parameters of lines and devices are set according to [48]. In the test system, each bus belongs to a district which is randomly categorized into a residential, commercial or industrial district. According to electricity consumption profiles of various districts, base load of each district is added to transformers that connect districts to the grid [47]. As platoons are often used for industrial transportation, we consider the charging stations located in the industrial districts. We add the charging stations and the renewable energy generators into industrial districts, and connect them to the corresponding transformers. The renewable energy is generated by photovoltaic panels equipped in the stations. We adopt the climate data set of Seattle [49]. Each charging station is considered to have \( N = 7 \) parking lots and \( c = 4 \) chargers. The duration of each time slot \( \tau \) is assumed to be 1 minute. The maximal size of each platoon is \( Z_{\text{max}} = 5 \) and the arrival rate of EV platoons is set \( \lambda_p = 1/10 \) platoon/minute. According to the requirement of charging rates, these platoons are classified into five categories with \( \theta = \{1.0, 1.2, 1.5, 1.8, 2.0\} \), respectively. The charging QoS threshold \( \varepsilon \) is 0.1.

In practice, each district is served by a substation equipped with transformers, which requires the total demand of the customers belonging to the district under the nominal capacities of the transformers. In Fig. 4, we show the impact of different charging rate assignment schemes on the demand load of the transformers in the industrial district \( S_1 \). The fixed rate scheme delivers 14.4 kW of electricity to the EVs constantly, which is specified as the level 2 of EV charging in the U.S. National Electric Code (NEC). The other two schemes are the Contract-Based (CB) rate assignment with incorporated Renewable Energy (RE) source and without it, respectively. As shown in Fig. 4, significant overloads are experienced from 8:00-11:00 with the fixed charging rate. In contrast, both the two CB rate assignments can effectively moderate the stress of the electricity demand under the capacity of the transformer during the peak hours. Especially the CB rate with RE, which reduces approximately 20% of the load level compared with the fixed rate scheme when the solar generator equipped in the charging station could create adequate power at noon time. Furthermore, the two CB schemes achieve a good valley-filling effect in the grid, which improves the energy utilization.

Fig. 5 indicates the load level performance of \( S_2 \) which reflects the performance of these schemes on the branch of the 69-bus system. The load level of this branch is different from that of district \( S_1 \), as the branch consists of different types of districts, and each type has distinct load profiles.
These mixed characteristics of the load profiles may undermine the effectiveness of the load level adjustment given by the proposed scheme. For example, the load level of this branch is close to 97% at 12:00 adopting the CB scheme without RE. However, it could be reduced to 91% by applying the CB scheme with RE, which shows the effect of the renewable energy in peak load shaving.

Fig. 6 and Fig. 7 show the electricity price and the utility of the charging station which is located in $S_1$ and has no renewable energy, respectively. The electricity price depends on the load level of the power grid. As the grid is the main power source of the charging station, the three charging schemes in Fig. 7 have the identical charging cost at a given time.

By comparing Fig. 6 and Fig. 7, we get the following observations. The utility plot of the fixed rate scheme has a similar shape as the charging cost. The reason is that the charging rate is 14.4kW and the price is fixed under a given grid load level. In the variable rate scheme, the charging price is proportional to the rate chosen by a platoon, and the scaling factor depends on the grid load. Given a scaling factor, each platoon determines the optimal charging rate that mostly benefits itself. A platoon chooses high rate when the price is low, and vice versa. Thus, the utility gained by this scheme has a flat shape. From Fig. 7, it is clear that our proposed CB scheme yields higher profit to the station compared to other schemes, especially in the valley load time. The reason is that the CB scheme can raise the charging price to make LDIC binding. Thus, compared to the other schemes which cannot adjust the price according to the type of platoon, our scheme makes part of the charging utilities transfer from the platoons to the station. The less electricity cost at the valley load time, the greater the gap between the contract-based charging price and the actual electricity cost. Therefore, larger profits can be obtained by the station. The characteristics of the CB scheme make the shape of the utility plot roughly opposite to the shape of the charging cost curve shown in Fig. 6. It should be noted that for a given time, the increase in the charging price in our proposed algorithm is on the electricity cost at that moment. Compared to the charging price at daytime, the resulting charging price at night is still lower. Thus, the rational platoons still prefer to be charged at night.

Fig. 8 illustrates the comparison of the optimal utility of problem (8) with its lower bound. In the relaxed problem, we replace $1 - P_{\text{loss,}_i}$ with $1 - \varepsilon$. Since the charging station is modeled as a complicated stochastic queuing system, the steady-state probabilities cannot be obtained explicitly. Thus, we cannot qualify the tightness of this relaxation through any mathematical expressions. However, we illustrate the quality of this bound with different power grid load levels through simulation as shown in Fig. 8. The average difference between the optimal utility and its lower bound is 5.9%.

We compare the impacts of the three schemes on the service loss probabilities with different power grid load levels through simulation as shown in Fig. 9. Due to information asymmetry, it is hard to distinguish the types of the platoons by applying the first two schemes. Accordingly, it is hard to implement any efficient admission...
Fig. 10. Charging rates for different types of EV platoons with various load level of grid.

Fig. 11. Average waiting time of EV platoons in the charging station.

control strategies by these schemes. However, this shortcoming can be overcome by our proposed CB scheme, which adjusts the admission control policies according to the characteristics of different types of platoons together with the electricity cost. It is worth noting that by using the CB scheme the loss probability decreases at the point where the grid load level is 60%. This is caused by the adjustment of the contract-based admission policy where the highest type-θ platoons are driven to leave the charging station due to their profit-driven characteristics.

Fig. 10 shows the charging rates for different types of EV platoons with various load levels of the grid. With the increase of load level, all the rates decrease to make both the charging power consumption to better match power supply and the grid operates smoothly. It is noteworthy that when the load level increases to 60% and 70%, the charging rates of type-5, 4 and 3 platoons become zero, respectively. This is caused by our proposed charging admission control strategies. As charging rates decrease, the charging time for platoons becomes longer. To guarantee charging QoS requirements, the platoons with low tolerance to waiting time are not allowed to be charged in the station, and their corresponding charging service is suspended.

Fig. 11 gives the average waiting time of EV platoons in the charging station. Applying our proposed admission control strategies in platoon charging scheduling, the average waiting time can be greatly reduced when the load level is above 50%. The reason is that some delay sensitive platoons are not allowed to be charged in the station when the load level reaches 60%, which is shown in Fig. 1. Due to the reduced arrival rates of the platoons, the waiting time also decreases.

Fig. 12 shows the average charging time of EVs belonging to various types of platoons. It can be found that the type of platoons with less tolerance on waiting time has shorter average charging time. It is worth noting that there is no charging time record for type-3 platoons when the load level is above 60% and for type-4, 5 platoons when the load level is above 50%. Due to the charging admission control schemes, the station never provides charging service to the platoons in these cases.

VII. Conclusion and Future Work

In this paper, we have proposed a queuing-based network model for studying EV platoons charging process. We first design the contract-based charging rate assignment and admission control schemes. Then, we incorporate the schemes into the queuing system and derive its steady-state probabilities. Through simulations, we have demonstrated that the proposed schemes result in the optimal utility for the charging station, while respecting the charging QoS requirements. Furthermore, the schemes greatly improve the regulation of the grid’s peaks and valleys. The analytical results prove the validity of the queuing model and the contract-based scheme for the design and control of the charging stations.

As well as our proposed contract-based charging schemes, there are further ways to enhance the charging of EV platoons, which can be covered in future work. One possible research direction is ensuring the fairness of various types of EV platoons served in capacity-constrained charging stations. Furthermore, in charging stations with large parking areas, alternatives to FIFO charging policy can be deployed. The challenge is how to propose an incentive mechanism that improves charging efficiency through the order adjustment of queuing platoons. In addition, the design of cooperative platoon charging schemes among multi-stations is still an unexplored problem. In the case where some EVs with plenty of energy in their batteries, they could potentially act as energy sources instead of being energy consumers, incentive-driven vehicle-to-vehicle charging mechanism is also an interesting topic for future study.
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