Sensing-Performance Tradeoff in Cognitive Radio Enabled Smart Grid

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Abstract—Smart grid is widely considered to be the next generation of power grid, where power generation, management, transmission, distribution, and utilization are fully upgraded to improve agility, reliability, efficiency, security, economy, and environmental friendliness. Demand response management (DRM) is recognized as a control unit of the smart grid, with the attempt to balance the real-time load as well as to shift the peak-hour load. Communications are critical to the accuracy and optimality of DRM, and hence at the core of the control performance of the smart grid. In this paper, we introduce cognitive radio into the smart grid to improve the communication quality. By means of spectrum sensing and channel switching, smart meters can decide to transmit data on either an original unlicensed channel or an additional licensed channel, so as to reduce the communication outage. Considering the energy cost taxed by spectrum sensing together with the control performance degradation incurred by imperfect communications, we formulate the sensing-performance tradeoff problem between better control performance and lower communication cost, paving the way towards a green smart grid. The impact of the communication outage on the control performance of DRM is also analyzed, which reduces the profit of power provider and the social welfare of the smart grid, although it may not always decrease the profit of power consumer. By employing the energy detector, we prove that there exists a unique optimal sensing time which yields the maximum tradeoff revenue, under the constraint that the licensed channel is sufficiently protected. Numerical results are provided to validate our theoretical analysis.

Index Terms—Cognitive radio enabled smart grid, demand response management, sensing-performance tradeoff.

I. INTRODUCTION

T

HE POWER GRID is a large interconnected infrastructure for delivering electricity from power plants to end users. New challenges are emerging in the traditional grid, e.g., rising demand, aging infrastructure, and increasing greenhouse gas emission, which have become an urgent global concern. As widely considered to be the next generation of power grid, the smart grid fully upgrades power generation, management, transmission, distribution and utilization to improve agility, reliability, efficiency, security, economy, and environmental friendliness [2], [3], [4]. It is envisioned as a promising technology to integrate with renewable green energy resources such as wind and solar power. Besides, it is an open market for electricity providers and consumers with flexible pricing strategies and load shifting capabilities. For example, the widespread use of plug-in hybrid electric vehicles with vehicle-to-grid capacity in the smart grid can help alleviate overload during peak hours [5].

Advances in smart metering and digital communications promote the smart grid to be an intelligent closed-loop system where power plants and end users interact closely to achieve efficient and economical power generation, distribution, and utilization. As shown in Fig. 1, input of the system is supply provided by power plants, while feedback is demand of end users measured by smart meter. Demand response management (DRM) acts as a control unit to balance and shape the real-time load. Output of the system is electricity delivered to each user through transmission and distribution. The forward path is power flow, and the bidirectional path is information flow, which provides two-way communications in the smart grid.

There are several studies on DRM, with the attempt to reduce and shift the peak-hour load [6]–[11]. Real-time pricing is known as one of the most common tools that can encourage power utilization in an efficient and economical way. A distributed and iterative algorithm is proposed in [8], [9] to balance and shape the real-time load. In [10], [11], game theory is used to address DRM via price predication and energy consumption scheduling. However, most of these studies assume...
perfect two-way communications which is too strong for practical applications, whereas the impact of communication unreliability on the control performance of DRM has not been revealed in the literature.

Communications are at the core of realization and performance of the smart grid. Due to low installation cost and high flexibility, wireless communications are prevalent in the smart grid. As depicted in Fig. 1, there are three main types of wireless networks different in size and location, i.e., home area networks (HAN), neighborhood area networks (NAN), and wide area networks (WAN) [12]. In HAN, smart meters from home are connected to the gateway based on wireless protocols such as ZigBee. Gateways transmit meter data to data aggregate unit (DAU) through NAN using WiFi. WAN connects all NANs in the smart grid to meter data management system (MDMS) by employing broadband wireless communication technologies (e.g., 3G and WiMAX) [13].

With the rapid development of the smart grid, more and more smart meters are applied, requiring tremendously increasing amount of meter data to be transmitted. Therefore, more frequency bands are required to support wireless communications in the smart grid, which poses a significant challenge on originally scarce spectrum resources. According to a report by the U.S. Federal Communications Commission, the allocated spectrum, however, is heavily under-utilized in vast temporal, spatial, and spectral dimensions [14]. The main reason is that under the current regulatory policy, frequency bands are statically restricted to the licensed users [primary users (PU)] and no reutilization is permitted for the unlicensed users [secondary users (SU)]. A novel technology to tackle the conflict between spectrum scarcity and under-utilization is cognitive radio, which enables SU to opportunistically utilize the channel when PU is absent, and to vacate it instantly when PU returns in order to avoid interfering with PU [15], [16].

Spectrum sensing, which enables SU to detect the state of channel occupancy, is a core technique for cognitive radio [17]. Since spectrum sensing and data transmission cannot be performed at the same time, several works have been devoted to sensing time optimization to tradeoff between interference avoidance and sensing efficiency [18], [19]. It has been widely recognized that cognitive radio can dramatically improve channel utilization as well as SU’s communication quality. Considering the energy consumption by spectrum sensing, the authors in [20], [21] propose an energy-efficient algorithm to reduce the taxed cost.

The pioneering work of applying cognitive radio to smart grid communications is proposed in [22], [23]. By means of spectrum sensing and channel switching, the smart meter can decide to transmit its data on either an original unlicensed channel or an additional licensed channel, in order to improve communication quality [1]. However, the sensing-dependent energy consumption has not been taken into account. In [24], the impact of communication unreliability on an independent price-demand model is considered. However, electricity price and power supply/demand are always coupled with each other in the smart grid.

Considering the cost taxed by spectrum sensing together with the performance degradation incurred by imperfect communications, we attempt to achieve better control performance with lower communication cost, paving the way towards a green smart grid. Our contributions are summarized as follows.

1) We introduce cognitive radio with spectrum sensing and channel switching into the smart grid to improve communication reliability.
2) We assess the impact of communication quality on the control performance of DRM.
3) We formulate the sensing-performance tradeoff problem in view of the sensing cost, and provide both theoretical analysis and simulation verification.

The remainder of this paper is organized as follows. The sensing-performance tradeoff problem is formulated in Section II. We analyze in Section III how cognitive radio improves the communication quality, and hence the control performance of DRM. The existence of the optimal solution is proved in Section IV, followed by numerical results in Section V to validate our theoretical analysis. We conclude this paper with future work in Section VI.

II. PROBLEM FORMULATION

A. Demand Response Management

We consider a smart grid consisting of one power provider, a total of $N$ power consumers, and a control unit. The cycle of a day is divided into several time slots. In each slot, let $s$ denote the supply of power provider, and $d_i$ be the demand of power consumer $i$ (where $i = 1, 2, \ldots, N$). We consider the cost function $C(s)$ indicating the expense of supplying power $s$ by the provider, which is increasing and convex [11]. The power demand of each consumer depends on electricity price and consumer type. Specifically, for each consumer $i$, the gain function $G_i(d_i)$ represents the obtained satisfaction as a function of its power demand $d_i$, which is non-decreasing and concave [8].

For the power provider, under electricity price $p$, the profit by supplying power $s$ is defined as $P_p(s) = ps - C(s)$. Its goal is to adjust its supply to maximize its profit, i.e.,

$$\max_s ps - C(s).$$

(1)

For each consumer $i$, under electricity price $p$, the profit by demanding power $d_i$ is calculated as $P_i(d_i) = G_i(d_i) - pd_i$. Its goal is to adjust its demand to maximize its profit, i.e.,

$$\max_{d_i} G_i(d_i) - pd_i.$$  

(2)

From the social perspective, it is desirable that the expense of provider is minimized and the aggregate satisfaction of consumers is maximized. Therefore, the social welfare can be defined as $\Phi(s, \{d_i\}) = \sum_{i=1}^{N} G_i(d_i) - C(s)$ with $\sum_{i=1}^{N} d_i < s$, which can be considered as the control performance of DRM. Therefore, under the constraint that supply should satisfy the total demand, the global optimization problem is

$$\max_{s, \{d_i\}} \sum_{i=1}^{N} G_i(d_i) - C(s)$$

s.t. $s \geq \sum_{i=1}^{N} d_i$.  

(3)

Although (3) can be solved by convex optimization in a centralized manner [25], the challenge is that the control unit needs to know the exact cost function of the provider and the gain function of each consumer. However, since the information is private, the control unit may not be able to solve the problem.
In [8], the authors have shown it is possible to approach the optimal solution of (3) in a distributed and iterative way as follows.

1) The control unit begins with any initial electricity price $p_1 \geq 0$ and announces it to the provider and each consumer.

2) On receiving $p_k$ (where $k \in \mathbb{N}^+$ denotes the number of iterations), the provider updates its supply $s^*_k$ by solving (1) and feeds back to the control unit; at the same time, each consumer also updates its demand $d^*_k$ by solving (2) and feeds back too.

3) On receiving the local optimal supply $s^*_k$ and demand $d^*_k$, the control unit updates the electricity price $p_{k+1}$ for next iteration using a gradient projection method, i.e., $p_{k+1} = \left[ p_k - \theta \left( s^*_k - \sum_{i=1}^{N} d^*_k \right) \right]$, where $\theta > 0$ is the step size which adjusts the convergence rate, and $\{x^t\} = \max \{x, 0\}$.

4) Repeat steps 2)–3) until the electricity price remains unchanged.

The interactions in DRM are shown in Fig. 2. The above control process is based on perfect two-way communications. Usually the control unit is located at the supply side, and reliable communications are often assumed. However, the control unit is far away from the demand side, and power consumers are connected to it through wireless communications. Due to the unreliability of wireless communication (e.g., packet loss and delay), outage events may occur in the two-way communications between consumers and the control unit, as shown by the dashed lines in Fig. 2. Let $\zeta$ denote the average outage, which is assumed to be uniform and independent for all consumers. The control performance with outage $\zeta$ is defined as $\Phi(\zeta)$. We show in Section III-B that the outage is critical to the accuracy and optimality of DRM, and analyze the control performance degradation incurred by the outage.

### B. Cognitive Radio

In order to improve the communication quality and reduce the outage, we integrate cognitive radio into DRM. In HAN, smart meters periodically transmit data to the gateway. In this paper, we consider that the data can be transmitted through two channels: one is from the unlicensed spectrum, referred to as the original channel $Ch_2$, the other lies in the licensed spectrum, referred to as the cognitive channel $Ch_3$, which is randomly occupied by PU. The smart meter can opportunistically switch data transmission to $Ch_2$, let $H_1$ denote the state that $Ch_2$ is busy (PU is in operation), and $H_0$ be that $Ch_2$ is idle (PU is absent). The traffic pattern of PU can be modeled as a two-state independent and identically distributed process [26]. Let $P_1$ denote the probability of $H_1$, and $P_0$ be that of $H_0$, where $P_1 + P_0 = 1$.

 Spectrum sensing, incorporated into the transceiver of smart meters, is able to detect the state of $Ch_2$ for opportunistic reutilization. A collection of spectrum sensing techniques are proposed in [27], among which the energy detector is popular and optimal for detecting a weak unknown signal from a known zero-mean constellation [28].

With the energy detector, by integrating signal in bandwidth $W$ over sensing time $\tau$, the smart meter compares the collected energy with a predefined threshold $\epsilon$ to decide whether $Ch_2$ is occupied by PU or not. There are two important metrics used to evaluate the performance of spectrum sensing. The detection probability $P_d$ is that the channel is detected busy when PU is in operation; while the false alarm probability $P_f$ is that the channel is detected busy when PU is absent. They can be calculated in terms of $Q$-function as [19]

$$P_d = Q \left( \frac{\epsilon}{2\sqrt{W\tau(\gamma+1)\sigma^2_n}} - \sqrt{W\tau} \right)$$

$$P_f = Q \left( \frac{\epsilon}{2\sqrt{W\tau\sigma^2_n}} \right)$$

where $Q(z) = \frac{1}{\sqrt{2\pi}} \int_{\infty}^{\infty} e^{-\frac{x^2}{2}} dx$, $\sigma^2_n$ is the variance of additive white Gaussian noise (AWGN), and $\gamma$ is signal to noise ratio (SNR) at the smart meter. The higher the $P_d$, the better the PU is protected; while the lower the $P_f$, the more efficiently $Ch_2$ can be reutilized by the smart meter.

**Proposition 1:** With any detection and false alarm probabilities, spectrum sensing before channel switching reduces collision probability on $Ch_2$.

**Proof:** See Appendix A.

Proposition 1 leads to two advantages: i) less collision with PU’s transmission guarantees that PU is better protected; ii) lower collision probability means lower outage. As shown in Fig. 3, in each sampling period, before data transmission, the smart meter performs spectrum sensing on $Ch_2$ to detect whether it is occupied by PU. If it is detected idle, the smart meter can switch data transmission to $Ch_2$ for lower outage; otherwise, the smart meter remains on $Ch_1$ to avoid interfering with PU.

### C. Sensing-Performance Tradeoff Problem

Though cognitive radio is able to improve communication quality, it involves energy cost for spectrum sensing. Therefore, the system should control the outage requirement using a...
sensing-dependent cost, assumed to be an increasing and convex function $\varphi(\tau)$ of sensing time $\tau$. Although the energy consumed for channel switching is rather small compared with that for spectrum sensing, we incorporate it into $\varphi(\tau)$. The control performance of DRM is dependent on outage, while the outage is dependent on sensing time. Thus control performance is a function of sensing time, which can be rewritten as $\Phi(\tau)$. Considering the cost taxed by spectrum sensing together with the performance degradation incurred by outage, our aim is to achieve better control performance with lower communication cost, towards a green smart grid.

Intuitively, the longer the sensing time, the lower the outage. The control performance degradation incurred by outage will decrease, but the communication cost taxed by spectrum sensing will increase. Thus there exists a fundamental tradeoff between sensing time and system performance in cognitive radio enabled smart grid. The net revenue can be defined as

$$R(\tau) = \Phi(\tau) - \kappa \varphi(\tau)$$

where the parameter $\kappa > 0$ is used to indicate the importance of $\Phi(\tau)$ and $\varphi(\tau)$ in the revenue. Besides, the application of cognitive radio should not violate the protection of PU. The objective of sensing-performance tradeoff is to find the optimal sensing time such that the total revenue is maximized while the PU is sufficiently protected, i.e.,

$$\max_{\tau} R(\tau) \quad \text{s.t.} \quad P_d \geq \bar{P}_d.$$  \hspace{1cm} (7)

where $\bar{P}_d$ is the low bound of detection probability with which the interference incurred to PU is tolerable. In practice, $\bar{P}_d$ is chosen close to but less than 1, because if PU requires full protection, the smart meter is no longer allowed to transmit data on $Ch_2$. Besides, we suppose $P_f$ is small, such that it is economically advisable for the smart meter to switch channels.

### III. Theoretical Analysis

#### A. Cognitive Radio Improves Communication Quality

Under cognitive radio framework for two-way communications, the smart meter performs spectrum sensing before channel switching. However, channel switching probability $P_{sw}$ is not exactly equal to $P_0$, due to the inaccuracy of spectrum sensing. $P_{sw}$ can be evaluated considering two cases: i) when $H_1$ but the smart meter misses to detect it, i.e., $P_1(1 - \bar{P}_d)$; ii) when $H_0$ and no false alarm is generated, i.e., $P_0(1 - P_f)$.

$$P_{sw} = 1 - P_1 \bar{P}_d - P_0 P_f.$$  \hspace{1cm} (9)

If spectrum sensing is perfect, i.e., $P_d = 1$ and $P_f = 0$, then $P_{sw} = P_0$.

Let $\zeta_1$ and $\zeta_2$ denote the outages of $Ch_1$ and $Ch_2$ respectively. The resultant outage $\zeta$ with cognitive radio can be evaluated considering two cases: i) when $Ch_2$ is detected busy, the smart meter remains data transmission on $Ch_1$, i.e., $(1 - P_{sw}) \zeta_1$; ii) when $Ch_2$ is detected idle, the smart meter switches data transmission to $Ch_2$, i.e., $P_{sw} \zeta_2$.

$$\zeta = (\zeta_2 - \zeta_1) P_{sw} + \zeta_1.$$  \hspace{1cm} (10)

If $\zeta < \zeta_1$, it is economically advisable for the smart meter to perform spectrum sensing and channel switching. Thus we obtain $\zeta < \zeta_1$, so it makes sense to assume the communication quality of $Ch_2$ to be better than that of $Ch_1$.

Using the energy detector, since the collected energy depends on sensing time $\tau$, it makes sense to have the energy threshold $\varepsilon$ also dependent on $\tau$. We keep $P_d = \bar{P}_d$ to satisfy the constraint of (8). From (4), the relationship between $\varepsilon$ and $\tau$ can be formulated as

$$\varepsilon = 2\sqrt{W\tau} \left( Q^{-1}\{\bar{P}_d\} + \sqrt{W\tau} \right) (\gamma + 1)\sigma^2_n.$$  \hspace{1cm} (11)

Substituting (11) into (5), we obtain the false alarm probability

$$P_f = Q \left[ \gamma \sqrt{W\tau} + Q^{-1}\{\bar{P}_d\} (\gamma + 1) \right].$$  \hspace{1cm} (12)

Since $Q$-function is decreasing, $P_f$ decreases with an increase of $\tau$, which means that increasing sensing time can improve spectrum sensing in terms of $P_f$.

Furthermore, keeping $P_d = \bar{P}_d$ and substituting (12) into (9), we obtain the channel switching probability

$$P_{sw} = 1 - P_1 \bar{P}_d - P_0 Q \left( \gamma \sqrt{W\tau} + Q^{-1}\{\bar{P}_d\} (\gamma + 1) \right).$$  \hspace{1cm} (13)

It is observed that $P_{sw}$ increases with an increase of $\tau$, which is because longer sensing time can make spectrum sensing more accurate and provide the smart meter more chances to switch data transmission to $Ch_2$.

From the analysis above, the longer the sensing time, the lower the false alarm probability, and then the higher the channel switching probability. We see from (10) that $\zeta$ decreases with an increase of $\tau$, which indicates that cognitive radio can reduce the communication outage.

#### B. Communication Quality Affects Control Performance

First assuming the two-way communications in the smart grid are perfect, DRM converges to the global optimal price $p^*$, which balances between supply and demand

$$s^* = \frac{\sum_{i=1}^{N} d_i^*}{N}.$$  \hspace{1cm} (14)

By jointly solving (1), (2) and (14), we can obtain the optimal electricity price, supply and demand as $p^* = f^{-1}(1)$, $s^* = (C')^{-1}(p^*)$, and $d_i^* = (G_i')^{-1}(p^*)$, where $f \triangleq (C')^{-1}/\sum_{i=1}^{N} (G_i')^{-1}$.

However, when the two-way communications in the smart grid suffer from outage $\zeta$, the power demand of each consumer $i$ estimated at the control unit becomes $(1 - \zeta) d_{i,k}^*$.

The electricity price updated at the control unit becomes $p_{k+1} = \left[ p_k - \tilde{\theta} \left( s_k^* - (1 - \zeta) \sum_{i=1}^{N} d_{i,k}^* \right) \right]_{+}$ and the iteration converges to a suboptimal price $\tilde{p}$ when

$$s = (1 - \zeta) \sum_{i=1}^{N} d_i.$$  \hspace{1cm} (15)
By jointly solving (1), (2) and (15), we can obtain the suboptimal electricity price, supply and demand as

\[ \hat{p} = f^{-1}(1 - \zeta) \]
\[ \hat{s} = (G_t')^{-1}(\hat{p}) \]
\[ \hat{d}_i = (G_i')^{-1}(\hat{p}). \]

Note that the cost function \( C \) is increasing and convex, hence \( (C')^{-1} \) is positive and increasing; while the gain function \( G_i \) is non-decreasing and concave, thus \( (G_i')^{-1} \) is non-negative and decreasing. Therefore \( f^{-1} \) is positive and increasing, and it is clear that \( \hat{p} < p^*, \hat{s} < s^* \) and \( \hat{d}_i > d_i^* \).

Usual operation of the smart grid is made in two stages [29]. In the first stage called unit commitment, the provider reserves power from power plants according to the estimated demand. Then, in the second stage called economic dispatch, the reserved power is supplied to each consumer. However, if the reserved power is not enough, additional power will be bought from the spot market to meet the demand [30]. Since the decision in unit commitment stage is made in advance, consumers can choose and buy power with a cheaper forward price than that in economic dispatch stage with an option price \( p_o \).

Firstly, the profit of provider is calculated as

\[ \hat{P}_p = \hat{p}\hat{s} - C(\hat{s}). \]  

Next, all consumers demand a total of \( \sum_{i=1}^{N} \hat{d}_i \) power, but only get \( \hat{s} \) from the provider with a cheaper forward price \( p_f = \hat{p} \). The additional \( \sum_{i=1}^{N} \hat{d}_i - \hat{s} = \zeta \sum_{i=1}^{N} \hat{d}_i \) power will be bought from the spot market with a more expensive optional price \( p_o > \hat{p} \). Therefore the aggregate profit of consumers is calculated as

\[ \hat{P}_c = \sum_{i=1}^{N} G_i(\hat{d}_i) - \hat{p}\hat{s} - p_o \zeta \sum_{i=1}^{N} \hat{d}_i. \]  

Finally, the social welfare (control performance) is calculated as

\[ \hat{\phi} = \hat{P}_p + \hat{P}_c. \]  

Proposition 2:
1) The outage in two-way communications reduces the profit of power provider.
2) The imperfect communication may not always decrease the aggregate profit of power consumers.
3) The control performance of DRM drops with an increase of outage.

Proof: See Appendix B.

IV. EXISTENCE OF OPTIMAL SOLUTION

When the energy detector is employed, substituting (13) into (10), we have

\[ \zeta(\tau) = (\zeta_1 - \zeta_2) P_0 Q \left( \gamma \sqrt{W} + Q^{-1}(P_d) (\gamma + 1) \right) + (\zeta_1 - \zeta_2) P_1 P_d + \zeta_2. \]  

which indicates that outage is a function of sensing time.

Theorem 1: There exists a unique optimal sensing time \( \tau^* \) which yields the maximum revenue.

Proof: It can be verified from (22) that

\[ \partial \zeta / \partial \tau = (\zeta_1 - \zeta_2) P_0 \partial Q / \partial z \partial z / \partial \tau, \]  

where

\[ z = \gamma \sqrt{W} + Q^{-1}(P_d) (\gamma + 1) \]  

is an increasing function of \( \tau \). Recall that the derivative of \( Q \)-function is \( \partial Q / \partial z = -1/(2\pi e^{-z^2/2}) \), so

\[ \partial \zeta / \partial \tau = -A \sqrt{\tau} e^{-\tau^2/2} \]  

where \( A = (\zeta_1 - \zeta_2) P_0 \gamma \sqrt{W} / 2 \sqrt{2\pi} \) is a positive constant. Therefore,

\[ \partial R / \partial \tau = \partial \hat{\phi} / \partial \zeta \left( -A \sqrt{\tau} e^{-\tau^2/2} \right) - \kappa \phi' \]

1) if \( \tau \to 0 \), that is, \( z \to 0 \), we have

\[ \lim_{\tau \to 0} \partial R / \partial \tau \to -\kappa \phi' > 0 \]

2) if \( \tau \to +\infty \), that is, \( z \to +\infty \), we have

\[ \lim_{\tau \to +\infty} \partial R / \partial \tau \to -\kappa \phi' < 0. \]

In Appendix C, we further prove that \( R(\tau) \) is concave. \( R(\tau) \) first increases when \( \tau \) is small and then decreases when \( \tau \) is large, which makes the maximum point to be unique.

V. NUMERICAL RESULTS

A. Demand Response Management

In order to evaluate the distributed and iterative approach to DRM, a simple one-provider and one-consumer case is considered. However, the result can be trivially extended to the one-provider and multi-consumer case. We consider a quadratic cost function \( C(s) = as^2 + bs + c \), and a quadratic gain function

\[ G(d) = \begin{cases} \omega d - \alpha/2d^2, & 0 \leq d < \omega/\alpha \\ \omega^2/2\alpha, & d > \omega/\alpha \end{cases} \]

The parameters are set as \( a = 0.1, b = 0.5, c = 0 \) and \( \omega = 3, \alpha = 0.5 \). The social welfare is defined as \( \phi\{s, d\} = G(d) - C(s) \) when \( s > d \), and \( \phi\{s, d\} = G(s) - C(s) \) when \( s < d \).

In Fig. 4, we fix step size at \( \theta = 0.1 \), and set the initial price at \( p_1 = 0 \) and 3 respectively. It is shown that in both cases the electricity price converges to the global optimum which balances between supply and demand. The social welfare achieves the best, where the locally optimal solution of the provider and consumer becomes globally optimal at the converged price.

B. Cognitive Radio Improves Communication Quality

We conduct extensive simulations to understand the capability of cognitive radio in terms of improving communication quality. PU on \( C_h \) has bandwidth \( W_\gamma = 6 \) MHz. The noise is AWGN. We are interested in a low SNR scenario where \( \gamma = -15 \) dB. The lower bound of the detection probability is chosen to be \( P_d = 0.9 \) in order to protect PU.

Firstly, we compare the false alarm probability between simulation and theoretical results from (12). In the simulation, for each sample under \( H_1 \), we find out the energy threshold \( \kappa \) to
achieve the target detection probability based on 600 test statistics. Then $\varepsilon$ is applied under $H_0$ to obtain the false alarm probability. As shown in Fig. 5, the false alarm probability decreases with an increase of sensing time.

Next, we compare the channel switching probability between simulation and theoretical results from (13). For the sake of simplicity, we assume that the durations of $H_1, H_0$ are exponentially distributed with the means of $t_1, t_0$ respectively. The value of $t_0$ is chosen to be three times $t_1$, then $P_s = t_1 / t_0 + t_3 = 0.25$. As shown in Fig. 6, the channel switching probability increases with an increase of sensing time, which is because the false alarm probability decreases as sensing time increases.

Finally, we compare outage between simulation and theoretical results from (22). We have assumed that $\zeta_2 < \zeta_1$, for instance, $\zeta_1 = 0.5$ and $\zeta_2 = 0.2$. Fig. 7 shows that outage decreases with an increase of sensing time.

C. Communication Quality Affects Control Performance

We conduct simulations to verify the performance degradation when DRM suffers from the communication outage. We first compare the electricity price, provider supply and consumer demand under perfect communications with those suffering from outage. As shown in Fig. 8, $\bar{p} < p^*$, $\bar{s} < s^*$ and $\bar{d} > d^*$, which is the same as our theoretical analysis.

Next, we investigate how the outage will affect the control performance of DRM. The optional price is chosen to be $p_o = p^*$. The outage $\zeta_2$ grows from 0 to 1, with the step size of 0.1. As shown in Fig. 9, the electricity price and supply decrease, while the demand increases, with an increase of the outage. From Fig. 10, we can draw such conclusion that the outage in
two-way communications reduces the profit of power provider and the social welfare of the smart grid, however it may not always decrease the profit of power consumer.

D. Sensing-Performance Tradeoff

Numerical results are provided to demonstrate the sensing-performance tradeoff in cognitive radio enabled smart grid. We consider the communication cost function to be quadratic as \( \varphi(\tau) = \tau^2 \). The parameter \( \kappa \) is set to be \( 10^4 \). Sensing time varies from 0.3 ms to 2.7 ms, with the step size of 0.2 ms. Fig. 11 shows the comparison between simulated and theoretical tradeoff. It is shown that both the control performance and communication cost increase with sensing time. The simulation results match with the theoretical analysis, where the revenue first increases when sensing time is small and then decreases when it becomes large. There exists a unique optimal sensing time \( \tau^* \) which yields the maximum revenue.

VI. CONCLUSION

By introducing cognitive radio with spectrum sensing and channel switching into the smart grid, we show that the communication outage can be reduced. How the communication quality affects the control performance of DRM is also analyzed. We claim that the outage in two-way communications reduces the profit of power provider and the social welfare of the smart grid, however it may not always decrease the profit of power consumer. Given two channels, there exists a unique optimal sensing time which achieves the best sensing-performance tradeoff, while the PU on the licensed channel is sufficiently protected. This paper provides the guidelines of achieving better control performance with lower communication cost, paving the way towards a green smart grid. In our future work, the impact of both supply uncertainty and communication unreliability on the control performance of DRM will be considered.

APPENDIX A

PROOF OF PROPOSITION 1

Let \( Y \) denote the duration of \( H_1 \), and the meter data transmission time be \( t_x \). If the smart meter switches data transmission to \( Ch_2 \) without spectrum sensing in advance, the collision probability \( P_{ns} \) can be evaluated considering two cases: i) when \( H_1 \), collision occurs; ii) when \( H_0 \), the smart meter’s transmission collides with PU if and only if PU returns during \( t_x \).

\[
P_{ns} = P_1 (1 - P_d) + P_0 (1 - P_f) Pr\{Y < t_x\}.
\]

However, if the smart meter switches data transmission to \( Ch_2 \) only when it is detected idle, the collision probability \( P_s \) can be evaluated considering two cases: i) when \( H_1 \) but the smart meter misses to detect it, collision occurs; ii) when \( H_0 \) and no false alarm is generated, the smart meter’s transmission collides with PU if and only if PU returns during \( t_x \).

\[
P_s = P_1 (1 - P_a) + P_0 (1 - P_f) Pr\{Y < t_x\}.
\]

The difference of the collision probability on \( O_h \) by spectrum sensing before channel switching is calculated as

\[
\Delta - P_{ns} - P_s = P_1 P_d + P_0 P_f Pr\{Y < t_x\} > 0
\]

where the inequality always holds because \( P_1, P_0, P_d, P_f, Pr\{Y < t_x\} > 0 \).

APPENDIX B

PROOF OF PROPOSITION 2

Firstly, the derivative of (19) is \( \partial P_2 / \partial \zeta = \partial \tilde{P} / \partial C \tilde{\zeta} + \tilde{P} \partial \tilde{\zeta} / \partial \zeta - \partial C / \partial \tilde{\zeta} \). Note that from (16) and (17) we have \( \partial \tilde{P} / \partial \zeta = - (f^{-1})^t = -1 / f \) and \( \partial C / \partial \tilde{\zeta} = \partial C / \partial \zeta = f^{-1} \).
where \( f' = \left( (C')^{-1} \right)' \sum_{i=1}^{N} (G'_i)^{-1} - (C')^{-1} \sum_{i=1}^{N} (G'_i)^{-1} \), and then \( \partial \dot{p} / \partial \zeta < 0 \). Therefore
\[
\frac{\partial \dot{P}_p}{\partial \zeta} = \frac{\partial \dot{p}}{\partial \zeta} \dot{s} < 0
\]
which means that the profit of power provider decreases with an increase of outage.

Next, the derivative of (20) is \( \partial \dot{P}_c/\partial \zeta = \sum_{i=1}^{N} \partial G_i \left( \dot{a}_i \right) / \partial \dot{a}_i / \partial \zeta - \dot{p} \partial s / \partial \zeta - p_o \sum_{i=1}^{N} \left( \dot{a}_i + \zeta \partial \dot{a}_i / \partial \zeta \right) \). Note that from (18) and (15) we have \( \partial G_i \left( \dot{a}_i \right) / \partial \dot{a}_i = \dot{p} \) and \( \partial \dot{s} / \partial \zeta = - \sum_{i=1}^{N} \dot{a}_i + (1 - \zeta) \sum_{i=1}^{N} \partial \dot{a}_i / \partial \zeta \). Therefore
\[
\frac{\partial \dot{P}_c}{\partial \zeta} = (\dot{p} - p_o) \sum_{i=1}^{N} \left( \dot{a}_i + \zeta \frac{\partial \dot{a}_i}{\partial \zeta} \right) - \frac{\partial \dot{p}}{\partial \zeta} \dot{s}
\]
where \( \partial \dot{a}_i / \partial \zeta = \left( (G'_i)^{-1} \right)' \dot{p} \partial \zeta / \partial \dot{a}_i > 0 \).

1) When \( \zeta \to 0 \), then \( \dot{p} \to \dot{p}^* \) and \( \dot{s} \to \sum_{i=1}^{N} \dot{a}_i \). If choose \( p_o = p^* \), we have
\[
\lim_{\zeta \to 0} \frac{\partial \dot{P}_c}{\partial \zeta} = - \frac{\partial \dot{p}}{\partial \zeta} \dot{s} > 0
\]
2) When \( \zeta \to 1 \), then \( \dot{s} \to 0 \), and we have
\[
\lim_{\zeta \to 1} \frac{\partial \dot{P}_c}{\partial \zeta} = (\dot{p} - p_o) \sum_{i=1}^{N} \left( \dot{a}_i + \frac{\partial \dot{a}_i}{\partial \zeta} \right) < 0
\]
from which we can never infer that the aggregate profit of power consumers always decreases with an increase of outage.

Finally, the derivative of (21) is
\[
\frac{\partial \Phi}{\partial \zeta} = \frac{\partial \dot{P}_c}{\partial \zeta} + \frac{\partial \dot{P}_c}{\partial \zeta} = (\dot{p} - p_o) \sum_{i=1}^{N} \left( \dot{a}_i + \zeta \frac{\partial \dot{a}_i}{\partial \zeta} \right) < 0 \quad (25)
\]
which means that the control performance of DRM decreases with an increase of outage.

**APPENDIX C**

**CONCAVITY OF**

**Lemma 1:** \( R\{\tau\} \) is concave.

**Proof:** Assume the cost function \( C \) and the gain function \( G_i \) are at most quadratic, thus we have \( f'' = -2 \left( \sum_{i=1}^{N} (G'_i)^{-1} \right)' \sum_{i=1}^{N} (G'_i)^{-1} - (C')^{-1} \left( \sum_{i=1}^{N} (G'_i)^{-1} \right)' \sum_{i=1}^{N} (G'_i)^{-1} 3 > 0 \). Further, \( \partial^2 \dot{p} / \partial \zeta^2 = -f''(f')^2 < 0 \) and \( \partial^2 \dot{a}_i / \partial \zeta^2 = \left( \sum_{i=1}^{N} (G'_i)^{-1} \right) \partial^2 \dot{p} / \partial \zeta^2 > 0 \).

It can be verified from (25) that
\[
\frac{\partial^2 \Phi}{\partial \zeta^2} = \frac{\partial \dot{p}}{\partial \zeta} \sum_{i=1}^{N} \left( \dot{a}_i + \zeta \frac{\partial \dot{a}_i}{\partial \zeta} \right) + (\dot{p} - p_o) \sum_{i=1}^{N} \left( 2 \frac{\partial \dot{a}_i}{\partial \zeta} + \zeta \frac{\partial^2 \dot{a}_i}{\partial \zeta^2} \right) < 0
\]
and further from (24) we have
\[
\frac{\partial^2 R}{\partial \tau^2} = \frac{\partial^2 \Phi}{\partial \zeta^2} \left( - \frac{A}{\sqrt{2 \pi \tau^2}} \right)^2 + \frac{\partial \Phi}{\partial \zeta} \frac{A}{2 \tau \sqrt{2 \pi \tau}} \left( 1 + z \gamma \sqrt{W} \right) e^{-z^2/2 - k \varphi''} < 0
\]
It says that \( \partial R / \partial \tau \) is decreasing, which means \( R\{\tau\} \) is concave.

**REFERENCES**


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