On Stability and Robustness of Demand Response in V2G Mobile Energy Networks

Weifeng Zhong, Rong Yu, Member, IEEE, Shengli Xie, Senior Member, IEEE, Yan Zhang, Senior Member, IEEE, and David K.Y. Yau, Senior Member, IEEE

Abstract—Demand response (DR) plays a significant role in enhancing the reliability of future smart grids. Electric vehicles (EVs) can be exploited to facilitate DR because their batteries, as a form of flexible energy storage, can be controlled to consume energy from or feed energy back to the grid depending on user needs. However, EVs’ mobility is inherently probabilistic, which presents a challenge for system stability particularly. This paper analyzes the stability of DR in which mobile EVs participate. Using the methodology of dynamical complex networks, we present a DR model of vehicle-to-grid (V2G) mobile energy network in which the EVs generally move across different districts represented as network nodes. EV fleets, therefore, transport energy and energy storage capacity among these nodes in general. A difference equation system is developed to model the DR dynamics of the nodes, which mutually affect each other. A DR algorithm is proposed to control the demand for EV charging and discharging. It is proved that the stability of the algorithm is robust to internode coupling. Numerical results show that incoming EVs that bring new energy and storage into a district can impact the DR stability. Real-world traces of vehicle mobility are used in simulations to illustrate the DR model.

Index Terms—Smart grid, demand response, stability, robustness, vehicle-to-grid energy system, mobility, dynamical complex network

I. INTRODUCTION

Smart grid, as an emerging form of urban power grid, promises to admit renewable energy integration and intelligent energy management at high efficiency and reliability [1]. Demand response (DR), in particular, is an effective method for reducing energy wastage, balancing energy supply and demand, and enhancing the reliability of power systems [2].

In recent years, electric vehicles (EVs) are being adopted increasingly worldwide due to their environmental and economic benefits [3]. In practice, many personal vehicles are parked for up to 22 hours per day. Currently, the batteries of these parked vehicles mostly represent idle energy resources [4]. But they can be exploited for flexible DR through vehicle-to-grid (V2G) technology that enables bidirectional energy flows among the EVs and the grid.

The implementation of DR for EVs requires effective energy control and battery management. Dynamic pricing provides a basic control signal [5], under which the EV owners, driven by economics, adjust their charge or discharge schedule according to their needs and the current real-time prices. An aggregator [6], which acts as an agent between the grid and EVs, aggregates the EVs’ batteries into a resource pool that participates in the energy market subject to user requirements. The EVs in aggregate thus serve as a large and flexible energy storage device, which can in turn serve as a buffer that smooths fluctuating demand and intermittent renewable generation [7].

Although a fleet of EVs acts like an energy storage device conceptually, its detailed behavior can be quite different from any conventional devices. In particular, the capacity of EV-aggregated energy storage is continually changing according to the usage of vehicles, particularly their mobility which affects the charging states. Most existing papers consider DR for EVs within a single geographical area only, where EV mobility is typically characterized by a summary arrival rate or a random process such as the Gaussian distribution [7], [8], Poisson process [9], and Markov chain [10]. When an EV arrives, the initial state-of-charge (SOC) of its battery is taken to the usage of vehicles, particularly their mobility which affects the charging states. Most existing papers consider DR for EVs within a single geographical area only, where EV mobility is typically characterized by a summary arrival rate or a random process such as the Gaussian distribution [7], [8], Poisson process [9], and Markov chain [10]. When an EV arrives, the initial state-of-charge (SOC) of its battery is taken
follows.

- We propose a new DR model of V2G mobile energy networks using the methodology of dynamical complex networks. In the model, each network node represents a district having a V2G system that operates the DR locally. The nodes have coupled DR dynamics through the movements of EV fleets among them.

- A DR algorithm is proposed to control the charging and discharging of EVs within each district. Based on this algorithm, a difference equation system is formulated to model the behavior of nodes in the coupled network over time.

- It is proved that the proposed DR algorithm is stable for an isolated network node. Further, the algorithm is proved to be robust, in that it remains stable when the nodes are coupled through the EV fleets moving across them.

II. V2G Mobile Energy Network

The proposed V2G mobile energy network is shown in Fig. 1. Districts are denoted by nodes, and they are connected by EV fleets. Thus, the DR dynamics of all the districts are coupled through the mobile EVs. The V2G mobile energy network is sometimes also referred to simply as a V2G network in this paper.

A. Nodes of districts

Each district has consumers and producers, as well as hybrid prosumers. DR is performed within each district to implement local load control through dynamic pricing. Consumers adjust their energy consumptions according to the local electricity prices. The EVs are prosumers in that they can both charge (consume energy from the grid) and discharge (provide energy to the grid). Regarding battery management, a V2G system aggregates all the parked EVs in a district to form a flexible energy storage device conceptually, which we call a battery pool. The demand of the battery pool represents the total demand of all the EVs in a district on average. This demand is affected by the battery’s SOC, in addition to the electricity prices. A DR manager recalculates electricity prices periodically based on the demand by the EVs, as well as that by other (background) consumers in the grid. We assume that in the V2G network, all the districts adopt an identical DR mechanism. A DR model will be constructed to track changes in total demand driven by the real-time pricing. It is assumed that energy supply will follow the controlled demand to maintain power balance. The detailed problem of how to control generation to follow the load is beyond the scope of this paper. We will develop a DR algorithm that can stabilize the EVs’ demand, consumers’ demand, and electricity prices.

B. Links of EV fleets

The size and SOC of the battery pool in a district change continually, driven by arrivals and departures of the EVs, which in turns changes the battery pool’s demand and the district’s DR dynamics. The vehicle mobility data for experiments in this paper comes from the 2010–2011 New York Regional Household Travel Survey [13] and the 2010–2012 California Household Travel Survey [14]. The datasets provide detailed travel information of the subject vehicles in one day. We select analysis intervals (of duration 15 minutes in our simulations) within that day and log the numbers of vehicles arriving at certain locations within each interval. It is assumed that the EV fleets’ rates of entering or leaving a district hold steady during each interval, but these rates may change between the intervals. Particularly, in a given analysis interval, let \( G_{ij} \) denote the number of EVs that start from district \( j \) and end up (stopping) in district \( i \) in a time slot. If \( G_{ij} = G_{ji} \), the EV fleets between the two districts are considered symmetrical. It is shown [15] that symmetrical EV fleets can balance the power demand among districts synchronously. However, real-world vehicle fleets are usually quite asymmetrical among districts, i.e., \( G_{ij} \neq G_{ji} \). This asymmetry is particularly pronounced during rush hours. The asymmetry impacts the SOCs and sizes of battery pools, as well as their charge and discharge behaviors. This paper will show that in the V2G network, the stability of the proposed DR algorithm is robust to asymmetrical EV fleets among the districts.

The EVs can participate in the DR and affect the grid only when they stop and are connected to the grid. The stopping information can be obtained from our datasets. It is further assumed that an EV will be connected to the local grid after it has stopped at a place for enough time [10], [16]. It is because (i) EV users may wish to maximize their opportunities for participating in the energy market through their vehicles; and (ii) in the future it may be common for parking spaces to be equipped with a charger so that connection to the grid is convenient and will not require a trip to a remote charging station. In addition, note that (i) an EV that arrives at a district but leaves it soon after are not considered to be connected to the grid; and (ii) EVs are not always charged whenever they are grid-connected – rather, any charge or discharge is determined by additional factors including the EVs’ SOCs, electricity prices, and parameters of the DR algorithm.

III. DYNAMICS OF DEMAND RESPONSE

To analyze the dynamics of the V2G network, we develop a difference equation system to describe the DR dynamics of the coupled nodes. The formulation considers the following components: (i) electricity price, (ii) demand of background
customers, (iii) demand of battery pool corresponding to the EV charging/discharging, (iv) change in energy in a battery pool, (v) change in size of a battery pool, and (vi) SOC of a battery pool.

A. Components of DR dynamics

1) Electric price: An analysis interval is divided into a series of time slots, indexed by \( t = 1, 2, \ldots \). Let \( P_t(i) \) denote the electricity price in district \( i \) in time slot \( t \). A local DR manager adjusts the electricity price according to the following formula [17]:

\[
P_t(i + 1) = a \left( \frac{D_t(i)}{C} \right)^K,
\]

where \( a, C, \) and \( K \) are constants; and \( D_t(i) \) denotes the total demand in district \( i \) in time slot \( t \). Let \( D_{c,i}(t) \) denote the aggregate consumer’s demand in district \( i \) in time slot \( t \). In the case of \( D_t(i) = D_{c,i}(t) \), there are only consumers in district \( i \) consuming energy. This case has been analyzed in [17]. This paper considers both consumers and prosumers in a district. The prosumers are denoted by EV-aggregated battery pools.

2) Consumer demand: One aggregate consumer is modeled to represent the behaviors of all the consumers in the one district on average. For district \( i \), the consumer adapts its demand \( D_{c,i} \) according to the following formula [17]:

\[
D_{c,i}(t + 1) = D_{c,i}(t) + \gamma (\omega - D_{c,i}(t) P_t(i)),
\]

where \( \gamma \) is the control rate; \( \omega \) is the consumer’s willingness to pay; \( A \) consumer who adjusts its demand based on (2) can maximize its utility eventually [17].

3) Battery pool demand: In a district, a battery pool’s demand reflects the total demand of all the EVs on average. Let \( SOC_t(i) \) denote the SOC of battery pool \( i \) in time slot \( t \). The depth-of-discharge (DOD) of battery pool \( i \) is given by \( DOD_t(i) = 1 - SOC_t(i) \). A battery pool computes its \( DOD_t(i) \) to maximize its utility [18] given by

\[
\max_{DOD_t(i)} DOD_t(i) P_t(i) - \beta DOD_t^2(i),
\]

where \( \beta \) is a positive constant. In [18], the objective function in (3) represents the utility of an energy storage device. The first term \( DOD_t(i) P_t(i) \) is the profit of discharging, and the second term \( \beta DOD_t^2(i) \) is the cost of discharging. Let \( P_t^* \) stand for the equilibrium price value. We can calculate the optimal solution of (3), which is given by \( DOD_t^* = P_t^*/\beta \), i.e., \( SOC_t^* = 1 - P_t^*/\beta \).

The battery pool \( i \) will adapt its demand \( D_{b,i} \) according to

\[
D_{b,i}(t + 1) = \alpha S_{b,i}(t) \{ \beta [1 - SOC_t(i)] - P_t(i) \},
\]

where \( \alpha \) is a positive constant. The term \( \beta [1 - SOC_t(i)] \) denotes users’ willingness to charge. If the willingness is higher than the electricity price \( P_t(i) \), the battery pool performs charging, i.e., \( D_{b,i}(t + 1) > 0 \). Otherwise, it discharges, i.e., \( D_{b,i}(t + 1) < 0 \). Note that the battery pool’s demand is proportional to the number of EVs and thus proportional to the battery pool size \( S_{b,i}(t) \). If a battery pool adapts its energy demand according to (4), it will have \( \lim_{t \to \infty} SOC_t^*(i) = SOC_t^* = 1 - P_t^*/\beta \) for maximizing its utility. This will be proved in Section IV.

The proposed DR algorithm is given by (1), (2), and (4). Their parameters are determined to provide stability for the algorithm and its equilibrium points. The parameter selection will be discussed in Section IV. In addition to (1), (2), and (4), there are other indicators in the DR model, provided as follows.

4) Change in energy of battery pool: There are two factors that influence the amount of energy in battery pool \( i \). One factor is the charge and discharge of EVs, i.e., \( D_{b,i} \). Another is the energy taken away by outgoing EVs from, as well as brought in by incoming EVs to, district \( i \). The change in energy of battery pool \( i \) is denoted by

\[
Q_t(i) = D_{b,i} + Q_{m,i}(t),
\]

where \( Q_{m,i}(t) \) denotes the energy transportation caused by the EV mobility. In this paper, as mentioned before, the EV mobility is given by a \( N \times N \) matrix \( G \), which we call the EV fleet matrix. \( G \) denotes the total number of districts in the V2G network. \( G_{ij} \) gives the number of EVs that originate from district \( j \) and arrive at district \( i \) in each time slot, for \( i, j \in \{1, 2, \ldots, N\} \). This paper considers steady states of the EV fleets in an analysis interval. Thus, \( G \) does not change with \( t \). However, \( G \) can be different in different analysis intervals. As mentioned before, \( G \) is, in general, an asymmetrical matrix, i.e., \( G_{ij} \) may not be equal to \( G_{ji} \) for \( j \neq i \). The diagonal elements \( G_{ii} \) are negative and denote the number of outgoing EVs from district \( i \) in each time slot. For the \( j \)th column of \( G \), we have \( \sum_i G_{ij} = 0 \). Therefore, the change in energy of battery pool \( i \) caused by mobile EV fleets is given by

\[
Q_{m,i}(t) = \sum_{j=1}^{N} G_{ij} [S_{EV} SOC_j(t) - L_{ij}],
\]

where \( S_{EV} \) denotes the average battery size of an EV. The SOC of an EV from district \( j \) is equal to the SOC of battery pool \( j \) on average. Thus, \( S_{EV} SOC_j(t) \) represents the amount of energy of an EV that started from district \( j \). \( L_{ij} \) denotes the average energy cost of a trip from district \( j \) to district \( i \).

5) Change in size of battery pool: Asymmetrical EV fleets impact the sizes of battery pools. The change in size of battery pool \( i \) is given by

\[
S_t(i) = 0 + \sum_{j=1}^{N} G_{ij} S_{EV}.
\]

The change of battery pool size can be influenced by both internal and external EV mobility, i.e., within a district and among districts, respectively. This paper mainly focuses on the external mobility and assumes that the internal effect is equal to 0.
6) Battery pool SOC: The SOC of battery pool \( i \) is influenced by \( Q_i \), \( S_i \), and their initial states, given by

\[
SOC_i(t) = \frac{Q_{0,i} + \sum_{\tau=1}^{t} Q_i(\tau)}{S_{0,i} + \sum_{\tau=1}^{t} S_i(\tau)},
\]

where \( Q_{0,i} \) and \( S_{0,i} \) are the initial energy and size of battery pool \( i \), respectively. The formula of battery pool size in (4) is given by \( S_{b,i}(t) = S_{0,i} + \sum_{\tau=1}^{t} S_i(\tau) \).

B. Equation of DR dynamics

Here, \( t+1 \) is used on the left-hand side of (5), (7), and (8) instead of \( t \). This specification can be regarded as a case of delayed reading, i.e., the current states at \( t \) are read during the next time slot \( t+1 \). It gives a difference equation system to describe the DR dynamics of district \( i \), as shown in (10). Let

\[
\begin{pmatrix}
P_i(t+1) \\
D_{ci}(t+1) \\
D_{bi}(t+1) \\
Q_i(t+1) \\
S_i(t+1) \\
SOC_i(t+1)
\end{pmatrix} =
\begin{pmatrix}
\alpha\{D_{ci}(t) + D_{bi}(t)/C\}^K \\
D_{ci}(t) + \gamma[\omega - D_{ci}(t)]P_i(t) \\
\alpha S_{bi}(t)\{\beta[1 - SOC_i(t)] - P_i(t)\} \\
D_{bi}(t) \\
[Q_{0,i} + \sum_{\tau=1}^{t} Q_i(\tau)]/[S_{0,i} + \sum_{\tau=1}^{t} S_i(\tau)]
\end{pmatrix} +
\begin{pmatrix}
0 \\
0 \\
0 \\
\sum_{j=1}^{N} G_{ij}[S_{EV}SOC_j(t) - L_{ij}] \\
\sum_{j=1}^{N} G_{ij}S_{EV}
\end{pmatrix}
\]

In addition, we use the original definitions of (5), (7), and (8) in the following analysis.

A. Stability

First, the stability of the DR algorithm in an isolated node is analyzed without considering EV fleet movements. For an isolated node, the DR dynamics equation does not have the second coupling term. The equilibrium values of the consumer demand, battery pool demand, battery pool SOC, and price are denoted respectively by \( D_{c}^*, D_{b}^*, SOC^* \), and \( P^* \). The subscript \( i \) that denotes a district is omitted for simplicity.

**Theorem 1:** Without EV mobility, for an isolated node, the proposed DR algorithm reaches stable equilibrium values of \( D_{c}^*, D_{b}^*, SOC^* \), and \( P^* \).

**Proof:** As shown in Derivation 1 in the Appendix, the SOC of an isolated battery pool can be expressed by

\[
SOC(t+1) = (1 - \alpha \beta) SOC(t) + \alpha (\beta - P^*).
\]

Therefore, if \( |1 - \alpha \beta| < 1 \), \( SOC \) will reach an equilibrium value, i.e., \( \lim_{t \to \infty} SOC(t) = SOC^* = 1 - P^*/\beta \), which means that a battery pool can maximize its utility (3) by using the proposed DR algorithm. Since \( 0 < SOC < 1 \) and \( P > 0 \), \( \beta > P^* \) should be kept.

As shown in Derivation 2 in the Appendix, the demand of an isolated battery pool is given by

\[
D_{b}(t+1) = (1 - \alpha \beta) D_{b}(t).
\]

If \( |1 - \alpha \beta| < 1 \), \( D_{b} \) will stabilize at its equilibrium value, i.e., \( \lim_{t \to \infty} D_{b}(t) = D_{b}^* = 0 \).

For the consumer demand, (2) can be derived as follows:

\[
D_{c}(t+1) = (1 - \gamma P^*) D_{c}(t) + \gamma \omega.
\]

When \( |1 - \gamma P^*| < 1 \) is held, we have the equilibrium of the consumer demand as \( \lim_{t \to \infty} D_{c}(t) = D_{c}^* = \omega/P^* \). Substituting \( D_{c}^* \) and \( D_{b}^* \) into (1), we derive the equilibrium of the price as \( P^* = (\alpha \omega/C)^K/(1+K) \).

In the case of an isolated node without the effects of EV mobility, \( D_{c}^* \) and \( SOC^* \) can be reached by choosing \( \alpha \) and \( \beta \) appropriately. Since \( D_{c}^* \) is reachable, \( D_{c}^* \) and \( P^* \) are also achievable for a suitable \( \gamma \).
B. Robustness of stability

Next, we show that the stability of the DR dynamics is robust to the coupling of nodes by EV mobility in the V2G mobile energy network.

Theorem 2: When the nodes are coupled by EV fleets, the DR algorithm achieves stable equilibrium values of $D^*_b$, $D^*_c$, $SOC^*$, and $P^*$ for each of the nodes.

Proof: We define the following variables. $S_i$ denotes the change of battery pool size caused by the incoming EVs, given by $S_{in} = \sum_{j \neq i} G_{ij} S_{EV}$; $S_{out}$ denotes the change of battery pool size caused by the outgoing EVs, given by $S_{out} = G_{ij} S_{EV}$; $S_m$ denotes the amount of energy brought into a district by EVs from other districts, given by $Q_{in} = S_{in} + S_{out}$; $Q_{in}$ is the amount of energy brought into a district by EVs from other districts, given by $Q_{in} = \sum_{j \neq i} G_{ij} S_{EV} S_j - L_{ij}$.

EV mobility directly impacts the state of a battery pool. In this case, as shown by Derivation 3 in the Appendix, a battery pool’s SOC is given by

$$SOC(t + 1) = p_{i}(t)SOC(t) + q_{i}(t),$$

$$p_{i}(t) = \left(1 + \frac{1}{S_{0} + tS_{in} + S_{in}}\right)(1 - \alpha\beta),$$

$$q_{i}(t) = \left(1 + \frac{1}{S_{0} + tS_{in} + S_{in}}\right)\alpha(\beta - P^*) + \frac{Q_{in}}{S_{0} + tS_{in} + S_{in}}.$$ 

If $|S_m| \geq 1$, we have $\lim_{t \to \infty} p_{i}(t) = 1 - \alpha\beta$ and $\lim_{t \to \infty} q_{i}(t) = \alpha(\beta - P^*)$. Thus, the equilibrium solution of (15) is stable at $SOC^* = 1 - P^*/\beta$, which is similar to the solution of (12). Therefore, in the case of general EV mobility, a battery pool can likewise maximize its utility (3) under the proposed DR algorithm.

As shown by Derivation 4 in the Appendix, a battery pool’s demand under EV mobility is given by

$$D_{b}(t + 1) = q_{i}(t)D_{b}(t) + q_{i}(t),$$

$$p_{i}(t) = \left(1 + \frac{S_{out}}{S_{0} + tS_{in} + S_{out}}\right)(1 - \alpha\beta),$$

$$q_{i}(t) = \left(1 + \frac{S_{out}}{S_{0} + tS_{in} + S_{out}}\right)[\alpha S_{m}(\beta - P^*) - \alpha\beta Q_{in}].$$

We hold $|S_m| \geq 1$. Then, we have $\lim_{t \to \infty} p_{i}(t) = 1 - \alpha\beta$ and $\lim_{t \to \infty} q_{i}(t) = \alpha S_{m}(\beta - P^*) - \alpha\beta Q_{in}$. Thus, the solution of (18) is $\lim_{t \to \infty} D_{b}(t) = D_{b}^* = (1 - P^*/\beta)S_{m} - Q_{in}$, which shows that the incoming energy $Q_{in}$ and incoming storage capacity $S_{m}$ will influence a battery pool’s demand. Consider a special case of $SOC^*_j = SOC^*_i = 1 - P^*/\beta$ and $L_{ij} = 0$ for $j \neq i$, which means that all the battery pools in the V2G network reach the same equilibrium value of the SOC, and the energy costs of all the trips are zero. In this case, we have $Q_{in} = (1 - P^*/\beta)S_{m}$ and $q_{i}(t) = 0$. Hence, the demand of the battery pool is stable at $D_{b}^*$, which is similar to the solution of (13). Since $D_{b}^*$ is achievable, $D_{c}^*$ can be also reached if $|1 - \gamma P^*| < 1$ is kept. To satisfy the condition of $|S_m| < 1$, the DR model requires additional constraints for stability, i.e., $S_{i} + S_{m} > \max\{1/\alpha\beta - 1, 1/(2 - \alpha\beta) - 1\}$ and $S_{in}/S_{0} < \min\{\alpha\beta, 2 - \alpha\beta\}$. ■

It is shown that the stability of the proposed DR algorithm is robust to general movements of EVs among the districts. From (20), it is known that $D_{b}^*$ is related to the incoming EVs (i.e., $Q_{in}$ and $S_{in}$). On the other hand, the outgoing EVs (i.e., $Q_{out}$ and $S_{out}$) do not influence the equilibrium values of the DR. This is because the DR algorithm has knowledge of the outgoing EVs, but the impact of the incoming EVs is unknown in advance. To maintain stable DR, we need to control the charge ($D^*_b > 0$) and discharge ($D^*_b < 0$) of a battery pool to compensate for any external impact. In summary, for an isolated network node, the demand of a battery pool is stable at $D_{b}^* = 0$. In the case of general mobility in the V2G mobile energy network, $D_{b}^*$ is decided by the incoming energy $Q_{in}$ and incoming storage capacity $S_{in}$ caused by the EV mobility.

V. NUMERICAL RESULTS

This section presents numerical results to show how EV mobility may influence the DR dynamics of the V2G mobile energy network. First, a special case of unidirectional EV fleets is adopted in the V2G network to verify the convergence properties of the DR algorithm. Second, real-world traces of vehicle mobility are used in simulations to illustrate the EV charge/discharge performance under the DR algorithm and statistical characteristics of the V2G network.

A. Unidirectional EV fleets

The following parameter settings are used in the case of unidirectional EV fleets. We consider a V2G network with four nodes, i.e., $N = 4$. The battery pools’ initial sizes are $S_{0,1} = S_{0,2} = S_{0,3} = S_{0,4} = 1000$, and their initial amounts of energy are $Q_{0,1} = Q_{0,2} = 600$ and $Q_{0,3} = Q_{0,4} = 700$. To manifest the effects of EV mobility, we set $\beta_1 = 167$, $\beta_2 = 200$, $\beta_3 = 250$, $\beta_4 = 333$, and $L_{ij} = 0$. V2G networks with non-negligible $L_{ij}$ and more nodes will be considered in the next simulation. Other parameters are set as follows: $\alpha = 0.01$, $C = 1$, $K = 3$, $\gamma = 0.001$, $\omega = 855$, $\alpha = 2 \times 10^{-5}$, and $S_{EV} = 1$. The superscripts $\circ$ and $\prime$ are used to denote equilibrium value for the case of no EV fleet mobility and the case of unidirectional EV fleets, respectively.

The above parameter settings let the four nodes have the same charge/discharge performance under the DR algorithm and the case of unidirectional EV fleets, respectively. In Fig. 2, the following EV fleet matrix is used to generate unidirectional EV fleets among the four nodes, which indicates a highly asymmetrical setting of the fleets.

$$G = \begin{pmatrix}
0 & 0 & 0 & 10 \\
0 & -10 & 0 & 0 \\
0 & 10 & 0 & 0 \\
0 & 0 & 0 & -10
\end{pmatrix}$$

The $G$ indicates two fleets. In $fleet 1$, 10 EVs travel from node 4 to node 1 in each time slot, given by $G_{4,1} = -10$ and $G_{1,4} = 10$. In $fleets 2$, 10 EVs travel from node 2 to node 3 in each time slot, given by $G_{2,3} = -10$ and $G_{3,2} = 10$.

Fig. 3 shows the performance of the DR algorithm under the unidirectional EV fleets of (21). It is shown that the consumer demand, price, battery pool demand, and battery pool SOC are
stabilized, which confirms that the stability of the proposed DR algorithm is kept under the EV fleet movements. Comparing Fig. 3 with Fig. 2, it is found that equilibrium values of the DR change in nodes 1 and 3, but they remain unchanged in nodes 2 and 4. Take fleet 1 as an example to understand the result. The EVs in fleet 1 travel from node 4 to node 1. Node 4 has no incoming EV, so that $D_{b,4} = D_{c}$. Moreover, it has $SOC_{4} = SOC_{4}'$, $P'_{4} = P_{c}$, and $D'_{c,4} = D_{c}$. On the other hand, node 1 has incoming EVs from node 4 and $SOC_{1} = SOC_{1}'$. The EVs with high SOCs add significant energy to the battery pool of node 1, leading to $SOC_{1} > SOC_{1}' = 0.70$. The battery pool of node 1 has continual energy input, so that we need discharging to keep the stability of the SOC. Thus, node 1 has that $D'_{c,1} < D_{c} = 0$, which indicates that the EVs are discharging. The discharge energy mitigates the energy demand in node 1. Hence, the price is reduced, i.e., $P'_{1} < P_{c}$. The low price stimulates the demand of consumers and we have $D'_{c,1} > D_{c}$. The effects of fleet 2 from node 2 to node 3 can be analyzed similarly.

Fig. 2(e) and Fig. 3(e) show the energy in a battery pool $Q_{b}$ during demand control. In Fig. 3(e), node 2 and node 4 keep losing EVs, which takes energy away. Hence, $Q_{b,2}$ and $Q_{b,4}$ are decreasing. Nodes 1 and 3 experience incoming EVs that bring in energy. Hence, $Q_{b,1}$ and $Q_{b,3}$ keep increasing. This means that the proposed DR algorithm’s stability is kept even if the battery pools have varying sizes and energy levels due to the EV mobility.

Fig. 2 and Fig. 3 show that the DR algorithm under the aforementioned parameter settings can reach convergence after about 80 time slots. In this case, if the DR manager desires convergence within a given time duration, say $T_{c}$, $T_{c}$ has to contain at least 80 time slots, so that the duration of one time slot should be less than $T_{c}/80$. The setting of $T_{c}$ is based on the following two practical considerations. First, power systems may have backup resources (e.g., energy storage) to compensate for power variations caused by uncontrolled

---

Fig. 2. DR dynamics in the case of no EV mobility, where the equilibrium values are indicated by superscript $\circ$. (a) Consumer demand. (b) Electricity price. (c) Battery pool demand. (d) Battery pool SOC. (e) Battery pool energy quantity.

Fig. 3. DR dynamics in the case of unidirectional EV fleets, where the equilibrium values are indicated by superscript $\prime$. (a) Consumer demand. (b) Electricity price. (c) Battery pool demand. (d) Battery pool SOC. (e) Battery pool energy quantity.
demand during $T_c$. Thus, $T_c$ should be made small enough to avoid excessive cost of the backup. Second, the DR algorithm in implementation will incur inevitable delay of message exchanges via communications channels. $T_c$ should be large enough to be feasible under this delay.

Generally, for a district, the stable points of the DR will not be influenced by the outgoing EVs, as shown by nodes 2 and 4. It is because the outgoing EVs do not change the battery pools’ SOCs. However, the incoming EVs may significantly impact the DR dynamics, as shown by nodes 1 and 3. They bring external energy and storage into the local battery pools, so that charge/discharge control, which adapts $D'_b$, is required for achieving stability.

B. Real-world vehicle mobility

In this subsection, real-world traces of vehicles, from the 2010–2011 New York (NY) Regional Household Travel Survey [13] and the 2010–2012 California (CA) Household Travel Survey [14], are used to drive the simulations. The following experiments on DR performance present mainly average EV charging/discharging demand, indicated by $D'_b$. The parameter settings of the V2G network are the same as in the previous simulations, but all the nodes now have the same $\beta = 167$, namely $SOC^* = 0.70$. First, we present results for the two cities under the dynamic traffic traces. Second, statistical characteristics of the V2G network are presented. Third, the service range of a central node is analyzed.

1) Time-variant mobility: As presented in Section IV, the equilibrium values of the DR are determined by the EV fleets (i.e., $G$). In other words, if $G$ changes over time slot $t$, the equilibrium values will also change with $t$. Hence, for simplicity, a $G$ that remains unchanged over the analysis interval is used in the convergence analysis in Fig. 3. Here, we investigate the equilibrium of a battery pool’s demand while $G$ changes over different analysis intervals. The duration of an interval is fixed to be 15 minutes. Within this interval, the states of EV fleets are considered to be steady. In the NY dataset, New York city is selected as a service node, and the numbers of incoming and outgoing vehicles are counted every 15 minutes. In the CA dataset, we choose San Francisco city as a service node and count the vehicle numbers in the same way.

Fig. 4(a) and Fig. 4(b) show the incoming and outgoing vehicle numbers, respectively, over the whole day. Fluctuations in the plots may be caused by random driving behaviors of the vehicles on that day. Take New York city as an example. It is observed that the city clearly has two rush hour periods. One occurs around 8:00–9:00, with a large number of arriving vehicles. The other occurs around 18:00–19:00, with a lot of vehicles leaving the city. Variations of the vehicle traffic lead to variations of the EV energy demand in Fig. 4(c). Comparing Fig. 4(c) with Fig. 4(a), we can see that the EV demand is contributed mainly by the incoming EVs. The SOCs of the incoming EVs are estimated based on driving distances according to the datasets. When the average SOC of incoming vehicles is higher than $SOC^*$, the battery pool is controlled to discharge. Thus, the battery pools have $D'_b < 0$ sometimes in Fig. 4(c). In New York city, the two rush hour periods, as well as the corresponding EV demand, cause significant changes of energy quantity of the battery pool, as shown in Fig. 4(d). Although the energy of the pool changes over time during that day, the DR algorithm is able to control the charging/discharging of the battery pool to keep it stable at $SOC^*$.

2) Statistical characteristics: To investigate statistical characteristics of the V2G network, we divide the survey area of a dataset into districts according to zip codes. The analysis interval is extended to one whole day. For each district, the numbers of departing and arriving vehicles are recorded in an EV fleet matrix to construct a large V2G network. The V2G network corresponding to the NY dataset is called the NY network, with 497 nodes, 30404 links, and a zip code range of 10001–12780. The V2G network corresponding to the CA dataset is called the CA network, with 1841 nodes, 61749 links.
and a zip code range of 90001–96204. The statistical results of the NY network and CA network are shown in Fig. 5 and Fig. 6, respectively.

Take the NY network as an example. Fig. 5(a) shows the distribution of trip distances, which is Poisson-like. Most household vehicles are used within 4–5 km in a trip. Notice that the distance is measured as the actual driving distance according to the datasets. In Fig. 5(b), it is observed that only a small number of nodes serve a large number of EVs. In particular, most of the nodes have less than 10 incoming vehicle numbers. Fig. 5(c) shows also that only a few of the nodes have high EV charging load. This observation reflects the driving behaviors of the household drivers. They usually live in the suburb (corresponding to a large number of leaf nodes) and often go downtown (a few central nodes). The CA network has similar characteristics, as shown in Fig. 6. In summary, the V2G mobile energy networks exhibit a scale-free feature, in which many of nodes are leaf nodes that connect to a few central nodes (e.g., central business districts). These central nodes experience a majority of vehicle traffic and charging load.

3) Service range: We now study the EV demand of a central node as its service radius is being extended. In the NY dataset, zip code 10021 (in New York city) is selected as the central node. In the CA dataset, zip code 93907 (in Salinas city) is selected as the central node. The service radius of each central node is progressively increased to extend its coverage. This service radius is determined by actual trip distance. As the service radius grows, more and more nodes will connect to the central node, and the corresponding V2G network becomes bigger and bigger.

As shown in Fig. 7(a), the number of incoming vehicles stops increasing as the service radius keeps growing. This is because only a few of the vehicles travel long distances, as shown Fig. 5(a) and Fig. 6(a). When a node is too far from the central node, there are no vehicles traveling between them. The corresponding link in the V2G network becomes lost, as indicated in Fig. 7(b), which shows that the number of nodes connected to the central node stops increasing after a certain point of the service radius. Fig. 7(c) shows the equilibrium value of the battery pool demand, which indicates the charging load in the central node. It is shown that the charging load stops increasing even if the service radius keeps growing. The results in Fig. 7 show that the actual service range of a district usually stops growing even if the size of the V2G network keeps growing.
VI. CONCLUSION

Based on the methodology of dynamical complex networks, this paper develops a DR model of V2G mobile energy network, in which mobile EVs transport energy and storage capacity among the network nodes. Each node runs the proposed DR algorithm separately. The dynamics of nodes participating in the DR are described by a difference equation system that explicates the coupling among nodes through mobile EV fleets. The stability of the DR algorithm is analyzed in the V2G network, and it is shown to be robust to the mobility-induced coupling. Driven by real-world vehicle traces from the NY and CA datasets, simulation results illustrate possible impact of the EV fleets on the DR performance in the V2G network. Main conclusions from the simulation results are summarized as follows.

- For a network node, incoming EVs that bring in prior unknown quantities of external energy and storage capacity will influence the node’s local DR dynamics. Appropriate control on the EV charging or discharging is needed to achieve stability.
- Typical driving behaviors of the household vehicles cause a majority of vehicle traffic and charging load to appear in a few central nodes. The V2G mobile energy network exhibits a scale-free feature.
- Household vehicle users typically make trips of limited distances. Nodes that are long distances apart will not be connected with each other. Hence, the effective service range of a node is limited.

APPENDIX

Derivation 1: SOC of an isolated battery pool.

\[ SOC(t + 1) = \frac{1}{S_0} \left[ Q_0 + \sum_{\tau=1}^{t+1} D_b(\tau) \right] + \frac{1}{S_0} D_b(t + 1) \]

\[ = SOC(t) + \frac{1}{S_0} \alpha S_0 \{ \beta [1 - SOC(t)] - P^* \} \]

\[ = (1 - \alpha \beta) SOC(t) + \alpha (\beta - P^*) \]

Derivation 2: Demand of an isolated battery pool.

\[ D_b(t + 1) = \alpha S_0 \left( \beta \left[ \frac{1}{S_0} Q_0 + \sum_{\tau=1}^{t} D_b(\tau) \right] \right) + P^* \]

\[ = \alpha S_0 \left( \beta \left[ \frac{1}{S_0} Q_0 + \sum_{\tau=1}^{t} D_b(\tau) \right] \right) + P^* - \alpha \beta D_b(t) \]

\[ = D_b(t) - \alpha \beta D_b(t) \]

\[ = (1 - \alpha \beta) D_b(t) \]

Derivation 3: SOC of a battery pool in the case of coupling nodes.

\[ SOC(t + 1) = \frac{1}{S_0 + (t + 1)S_m} \left\{ Q_0 + \sum_{\tau=1}^{t+1} D_b(\tau) + S_{out} SOC(\tau) + Q_{in} \right\} \]

\[ = \frac{1}{S_0 + tS_m} \cdot \frac{1}{S_0 + s(t + 1)S_m} \left\{ Q_0 + \sum_{\tau=1}^{t} D_b(\tau) + S_{out} SOC(\tau) + Q_{in} \right\} + \frac{1}{S_0 + (t + 1)S_m} \left[ D_b(t + 1) + S_{out} SOC(t + 1) + Q_{in} \right] \]

\[ = \frac{1}{S_0 + (t + 1)S_m} SOC(t) + \frac{1}{S_0 + (t + 1)S_m} \left[ \alpha (S_0 + tS_m) \cdot \{ \beta [1 - SOC(t)] - P^* \} + S_{out} SOC(t + 1) + Q_{in} \right] \]

\[ = \frac{SOC(\tau) + S_{out} SOC(\tau) + Q_{in}}{S_0 + (t + 1)S_m + 1} \alpha (\beta - P^*) + \frac{Q_{in}}{S_0 + tS_m + S_m} \]

Derivation 4: Demand of a battery pool in the case of coupling nodes.

\[ D_b(t + 1) = \alpha S_0 \left\{ \beta [1 - SOC(t)] - P^* \right\} \]

\[ = - \alpha \beta (S_0 + tS_m) SOC(t) + \alpha (S_0 + tS_m) (\beta - P^*) \]

\[ = - \alpha \beta (S_0 + tS_m) \cdot \frac{1}{S_0 + tS_m} \left\{ Q_0 + \sum_{\tau=1}^{t} D_b(\tau) + S_{out} SOC(\tau) + Q_{in} \right\} \]

\[ - \alpha \beta \cdot \left[ 1 + \frac{1}{S_0 + tS_m + S_m} \left( \alpha (S_0 + tS_m) \beta - P^* \right) \right] \]

\[ = (1 - \alpha \beta) D_b(t) - \alpha \beta Q_{in} + \alpha S_m (\beta - P^*) - \alpha S_{out} \]

\[ = (1 - \alpha \beta) D_b(t) - \frac{1}{\alpha \beta} \left[ \frac{S_{out}}{S_0 + tS_m + S_m} D_b(t + 1) + 1 - \frac{1}{\beta} P^* \right] \]

\[ = (1 + \frac{S_{out}}{S_0 + tS_m - S_{out}}) (1 - \alpha \beta) D_b(t) \]

\[ + \left( 1 + \frac{S_{out}}{S_0 + tS_m - S_{out}} \right) [\alpha S_m (\beta - P^*) - \alpha S_{out}] \]

REFERENCES


