Demand Response Management With Multiple Utility Companies: A Two-Level Game Approach
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Abstract—Demand Response Management (DRM) is a key component of the future smart grid that helps to reduce power peak load and variation. Different from most existing studies that focus on the scenario with a single utility company, this paper studies DRM with multiple utility companies. First, the interaction between utility companies and residential users is modeled as a two-level game. That is, the competition among the utility companies is formulated as a non-cooperative game, while the interaction among the residential users is formulated as an evolutionary game. Then, we prove that the proposed strategies are able to make both games converge to their own equilibrium. In addition, the strategies for the utility companies and the residential users are implemented by distributed algorithms. Illustrative examples show that the proposed scheme is able to significantly reduce peak load and demand variation.

Index Terms—Demand response management, energy pricing, evolutionary game, nash equilibrium, smart grid.

I. INTRODUCTION

P
ower grid provides electricity to the global society through utilities and transmission lines. With the increasing challenges of rising demand, aging infrastructure, and integration of renewable green energy resources, a new power grid is needed to enhance energy efficiency and reduce cost. Smart grid is regarded as the next generation power system to meet these challenges [1]–[3].

Demand Response Management (DRM) is one of the main features in smart grid. DRM refers to the routines implemented to control the energy consumption at the customer side and aims to improve energy efficiency and reduce cost [4]–[7]. The main objective of DRM is to reduce peak-to-average ratio and balance power supply and demand [8], [9].

There are several existing works on DRM in the literature. [10] considered market models for designing demand response to match power supply and shape power demand. The authors in [11] proposed distributed algorithms for the utility company and the consumers to maximize the social welfare. [12] presented a novel algorithm for finding an optimum time-of-use electricity pricing in monopoly utility markets. Moreover, game theory has been applied to the DRM problem in smart grid since it is effective in dealing with complicated interaction. The authors in [8] formulated DRM as a N-person concave non-cooperative game, and a distributed DRM strategy is proposed to achieve the minimum energy cost. [13] proposed network congestion game-based load management strategy in smart grid. The authors in [14] studied residential power scheduling through Stackelberg game in which the energy management controller is the leader and the service providers are followers.

It is noteworthy that most of the aforementioned studies focus on the scenario in which there exists only one single utility company. With the development of open energy market and new renewable energy sources (e.g., solar and wind power), residential users can get easy access to power from multiple utility companies [15]–[17], which indeed makes the interaction between utility companies and residential users an extremely challenging problem. Hence, it is necessary to study the DRM problem with multiple utility companies. An early work is reported in [12], which, however, does not consider the welfare of utility companies.

In this paper, we study the DRM problem in the scenario with multiple utility companies and multiple residential users. In order to fully capture the interaction between the utility companies and the residential users, this problem is modeled as a two-level game. At the lower level, the residential users living in a neighborhood area are regarded as a population, and each user evolves gradually to adjust its power demand from the utility companies as a response to the power prices. Therefore, the evolutionary game is formulated for the residential users to characterize the evolution process. At the higher level, the competition among the utility companies is modeled as a non-cooperative game, in which each utility company chooses, according to the game, a proper strategies are imposed on both sides.

The contributions of this paper are summarized as follows:

- We propose a scenario with multiple utility companies and multiple residential users, and formulate the DRM problem as a two-level game, i.e., an evolutionary game for the residential users and a non-cooperative game for the utility companies.
For the evolutionary game of the residential users, rigorous proof has been presented to show the convergence of evolutionary equilibrium. Then, an iterative algorithm has been proposed for the residential users to achieve evolutionary equilibrium.

For the non-cooperative game of the utility companies, the existence of Nash equilibrium has been proven, then a distributed iterative algorithm has been proposed for the utility companies to reach Nash equilibrium.

The remainder of this paper is organized as follows: Section II describes the system model and formulates the DRM problem. In Section III, the preliminary knowledge of game theory is presented. In Section IV, we present the evolutionary game of the residential users. In Section V, a non-cooperative game is formulated for competition among the utility companies. Numerical results are presented to illustrate the performance of the proposed approach in Section VI.

II. SYSTEM MODEL

Fig. 1 shows the system architecture with multiple utility companies and multiple residential users. The residential users are equipped with smart meters which enable the residential users to schedule energy consumption and organize a Home Area Network (HAN). Moreover, the smart meters will connect to Data Aggregation Unit (DAU) and form a Neighborhood Area Networks (NAN) [18], [19]. The residential users in a certain area are supplied with power by a distribution station, and the distribution station connects to several utilities via separate power lines. The DAU will first receive the price information from different utilities and broadcast such information to the users. Then, each residential user will send the power demand from one particular utility back to the DAU, which gathers all demand information from the residential users and decides the overall demand from each utility [20], [21].

Let $\mathcal{J} = \{1, 2, \ldots, J\}$ denote the set of the utility companies, $\mathcal{I} = \{1, 2, \ldots, I\}$ denote the set of the residential users. We assume that the whole operation time in one day is divided into $H$ time slots, and let $\mathcal{H} = \{1, 2, \ldots, H\}$ denote the set of operation time slots.

A. Energy Cost Model of Utility Company

The cost function $f_j^h(t_j^h)$ is defined to specify the cost of utility company $j \in \mathcal{J}$, which produces $t_j^h$ power at time $h \in \mathcal{H}$. It is assumed that the cost function is increasing and strictly convex. Without loss of generality, quadratic functions [8]–[10] are considered here

$$f_j^h(t_j^h) = a_j^h (t_j^h)^2 + b_j^h t_j^h + c_j^h$$

(1)

where $a_j^h > 0$ and $b_j^h > 0$ are constant parameters.

Let $p_j^h$ and $s_j^h$ denote the power price and the amount of sold power of utility company $j$ at time $h$, respectively. Then, the welfare function is given by

$$U_j^h(p_j^h, s_j^h) = p_j^h s_j^h - a_j^h (t_j^h)^2 + b_j^h t_j^h + c_j^h$$

(2)

where $s_j^h = \min\{t_j^h, D_h^j\}$, and $D_h^j$ is the user demand from utility company $j$ to be defined later in (7).

B. Utility and Welfare of Residential User

The utility function quantifies the utility that residential user $i \ (i \in \mathcal{I})$ receives when it consumes $x_{i,h}$ power at time $h$. Specifically, the utility function of the residential user is the utility for the tasks rather than the utility for the appliances. In many recent DRM studies [8]–[10], [22], the power demand $x_{i,h}$ is deterministic or can be precisely predicted by exploring the consumption history. We also follow the similar assumption.

In addition, quadratic [8]–[10] and logarithmic [22] utility functions are frequently used, because they are non-decreasing and their marginal benefits are non-decreasing. Without loss of generality, this paper adopts the quadratic utility function as

$$u_{i,h}(x_{i,h}) = v_{i,h} x_{i,h} - \frac{\alpha_{i,h}}{2} x_{i,h}^2 \quad x_{i,h,\min} \leq x_{i,h} \leq x_{i,h,\max}$$

(3)

where $v_{i,h}$ and $\alpha_{i,h}$ are time-varying parameters, $x_{i,h,\min}$ and $x_{i,h,\max}$ denote the minimal and maximal power consumption capacity of residential user $i$ at time slot $h$, respectively. Clearly, different values of $v_{i,h}$ and $\alpha_{i,h}$ at different time slots of this utility function can capture the dynamics of user demand. The study [11] modeled the relationship among the inside temperature, outside temperature and power consumption. This study also shows that consumers will increase the demand if there exists a larger gap between the expected inside temperature and outside temperature. On the other hand, in our paper, we can set appropriate parameters, e.g., larger $v_{i,h}$ or smaller $\alpha_{i,h}$ to reflect
the bigger gap between the expected inside temperature and outside temperature. This will result in higher power demand from the consumers. As a consequence, our parameter setting can implicitly show the impacts of environmental variables on power demand.

The power consumption of the residential user, i.e., the accumulated power consumption of user’s appliances, exceeds a minimal threshold at each time slot to guarantee that all the tasks can be finished [4].

If a residential user selects utility company \( j \) as the energy provider, it has to pay \( p^j_h x_{i,h} \) when consuming \( x_{i,h} \) amount of power at price of \( p^j_h \). The welfare function can be described as

\[
w_{i,h}(x_{i,h}) = u_i(x_{i,h}) - p^j_h x_{i,h}.
\]

\[x_{i,h}^{\text{min}} \leq x_{i,h} \leq x_{i,h}^{\text{max}}.\] (4)

C. Interaction Between Utility Companies and Residential Users

It is noteworthy that most of the existing studies concentrate on the optimization of only one side, i.e., either the residential users side (e.g., [23]) or the utility companies side (e.g., [11]), but ignore the other side. In order to take the interests of both sides into consideration, we build a new analytical framework, which can capture the interactive characters of both utility companies and residential users. In this way, optimization can be carried out for both sides to achieve their own objectives.

For the residential users, the main objective is to purchase larger amount of power at lower price to achieve higher welfare. On the other hand, from the perspective of the utility companies, they aim at selling power at higher price to achieve higher welfare. Therefore, appropriate strategies should be designed for both utilities side and users side to maintain the balance between supply and demand.

Besides the interaction between two sides shown in Fig. 2, there are also two levels of competition in the scenario. The higher level is among the utility companies, and they compete with each other to sell power to the residential users. Each utility company shall carefully choose the generation amount and power price, so that its welfare can be maximized. The lower level is the competition among the residential users, and each user has to decide one utility company to buy power. As shown in later sections, each level of competition can be modeled as a game.

It is noteworthy that this paper mainly focuses on the optimization in a time slot for both residential users and utility companies. In the proposed game-theoretical framework, the residential users and the utility companies are able to exchange their information, hence each side knows the information at slot \( h \) of the other side, i.e., \( p_h \) or \( x_{i,h} \). However, it is interesting to extend this framework to a more comprehensive case, by taking into account the optimal planning of generation and consumption over several slots [24]. In that case, the prediction of power price and demand is necessary. As the prediction may not be perfect, we need to deal with prediction error, which can be regarded as random uncertainties and solved in a stochastic manner [4], [7], [25].

III. PRELIMINARIES OF GAME THEORY

Game theory is the study of conflicts and cooperaions among intelligent rational decision-makers [26], which has been used in smart grid [8], [13], [14]. In this paper, the DRM problem with multiple utility companies and multiple residential users is formulated as a two-level game model. In the following, some preliminary knowledge about non-cooperative game and evolutionary game will be given.

A. Non-Cooperative Game

A non-cooperative game \( G = \{N, \{S\}, \{U\}\} \) consists of three components [27], [28].

- Player set \( N = \{1, 2, \ldots, i, \ldots, n\} \); where \( i \) is the identification of a player.
- Strategy space \( S_i \); player \( i \) in a game selects strategy \( s_i \) from its strategy set \( S_i \). \( S = \times^n_i S_i \) represents strategy space of the game. Denote \( s = (s_1, s_{-i}) \) as the strategy vector, where \( s_i \) is the player \( i \)'s strategy, and \( s_{-i} \) represents all the other players’ strategies.
- Utility set \( u : u = \{u_1, u_2, \ldots, u_i, \ldots, u_n\} \) for all players. Player \( i \)'s utility is determined by the strategy vector \( s \).

Nash equilibrium is the most important concept in game theory, which is a static stable strategy vector that no player has any incentive to unilaterally change its strategy from it. The definition of Nash equilibrium can be described as follows [28]

Definition 1: An strategy vector \( s^* = (s^*_i, s^*_{-i}) \) is a Nash equilibrium if and only if \( \forall i \in N \) and \( \forall s_i \in S_i \),

\[u_i(s_i, s_{-i}) \geq u_i(s_i^*, s_{-i}^*).\] (5)

In particular, the existence of Nash equilibrium can be obtained according to the following lemma [28].

Lemma 1: If the following conditions are satisfied, there exist Nash equilibriums in the game.

- The player set is finite.
- The strategy sets are closed, bounded, and convex.
- The utility functions are continuous and quasi-concave in the strategy space.
B. Evolutionary Game

The evolutionary game extends the formulation of non-cooperative game by introducing the concept of population, which refers to a set of players [29]. However, it is different from a non-cooperative game that the evolutionary game mainly focuses on the overall behavior of population. The evolutionary equilibrium in evolutionary game is analogous to the Nash equilibrium in a non-cooperative game. Moreover, it is a solution of an evolutionary game and the population will not gain by deviating from this point. Another important concept is the replicator, which depicts the selection dynamics of population and can be modeled as a set of ordinary differential equations. Therefore, by designing an appropriate replicator, the population is able to gradually achieve its evolutionary equilibrium [30].

Although evolutionary game was originally from biology, it has wide applications in engineering research due to its efficiency [30], especially for the multiple buyers-multiple sellers scenario [31], [32]. Clearly, the DRM scenario with multiple utility companies and multiple residential users matches well with the evolutionary game model.

Since multiple residential users can be regarded as a population because they are connected and form a NAN, the evolutionary game for our DRM problem can be described as follows,

- **Players:** residential user \( i \in I \).
- **Population:** the set of users in a NAN.
- **Strategy:** the selection on utility companies
- **Utility:** the welfare function of a residential user given in (4).

IV. EVOLUTIONARY GAME OF RESIDENTIAL USERS

In this section, we shall formulate the interaction among multiple residential users as an evolutionary game. Then, we prove that the evolutionary equilibrium can be reached by the proposed replicator. Finally, an iterative algorithm is proposed for the residential users to implement the replicator dynamics.

A. Formulation of Evolutionary Game

With the assistance of bi-directional communication structure, multiple residential users can connect together to form a NAN. Without loss of any generality, this paper shall focus on one population scenario. After receiving the power prices announced by different utility companies, each residential user has to decide the company to purchase power from. To characterize the selection process, the residential users possess the following behaviors.

- Each residential user has to choose one utility company \( j \) to purchase power.
- Each residential user can gradually adjust its strategy on the selection of utility company.
- Each residential user behaves independently.

In an evolutionary game each player can observe and replicate the strategies of others in the same population [31]. That is, the strategies of different users are identical in one population. Let \( y^j_h \) denote the probability of a residential user choosing utility company \( j \) at time slot \( h \), where \( 0 \leq y^j_h \leq 1 \) and \( \sum_{j=1}^{J} y^j_h = 1 \). Further, we denote the population state as \( Y_h = [y^1_h, y^2_h, \ldots, y^j_h, \ldots y^J_h] \).

B. Population Behavior and Replicator Dynamics

The optimal power demand of residential user \( i \) buying power from utility company \( j \) at the time slot \( h \) can be achieved according to (4)

\[
x^j_{i,h} = \arg \max_{x^j_{i,h}} u_i(x^j_{i,h})
\]

\[
= \left\{ \begin{array}{ll}
  x^j_{i,h \text{ min}} < x^j_{i,h} \leq x^j_{i,h \text{ max}} < x^j_{i,h \text{ max}} & \\
  x^j_{i,h \text{ min}} & < x^j_{i,h} \leq x^j_{i,h \text{ max}} & \text{otherwise}
\end{array} \right.
\]

(6)

Let \( D^j_h \) denote the total power demand from utility company \( j \) at time \( h \)

\[
D^j_h = y^j_h \sum_{i=1}^{I} x^j_{i,h}.
\]

Then the generation-demand ratio \( r^j_h \) can be defined as

\[
r^j_h = \frac{\sum_{i=1}^{I} x^j_{i,h}}{y^j_h} = \frac{Q^j_h}{y^j_h}
\]

(8)

where \( Q^j_h = (l^j_h)/(\sum_{i=1}^{I} x^j_{i,h}) \). After a utility company announces \( p^j_h \) and \( l^j_h \), they will keep constant in the evolutionary process of the residential users. Moreover, \( x^j_{i,h} \) is a constant in the evolutionary process from (6). Therefore, \( Q^j_h \) is a constant in the evolutionary process.

Then, we will define net utility associated with utility company \( j \) as the accumulated residential users’ welfare obtained from utility company \( j \). There are two possibilities.

Case 1: \( r^j_h \geq 1 \), the demand can be satisfied and the net utility is

\[
\pi^j_h = \frac{1}{2} \sum_{i=1}^{I} \left[ (x^j_{i,h} - r^j_h x^j_{i,h} - \frac{\alpha_i}{2} (x^j_{i,h})^2) \right]
\]

\[= \frac{1}{2} \sum_{i=1}^{I} \left[ \alpha_i (x^j_{i,h})^2 - \frac{\alpha_i}{2} (x^j_{i,h})^2 \right]
\]

\[= \frac{1}{2} \sum_{i=1}^{I} \alpha_i (x^j_{i,h})^2
\]

(9)

Case 2: \( r^j_h < 1 \), the demand cannot be satisfied, i.e., the consumer \( i \) can only get \( r^j_h x^j_{i,h} \) amount of power. \( \pi^j_h \) is then given by

\[
\pi^j_h = \frac{1}{2} \sum_{i=1}^{I} \left[ (v_{i,h} - y^j_h) x^j_{i,h} - \frac{\alpha_i}{2} (x^j_{i,h})^2) \right]
\]

\[= \frac{1}{2} \sum_{i=1}^{I} \left[ \alpha_i (x^j_{i,h})^2 - \frac{\alpha_i}{2} (x^j_{i,h})^2 \right]
\]

(10)
Therefore, the net utility $\pi_h^i$ can be rewritten into the following expression:

$$\pi_h^i = \psi_h^i P_i$$  \hspace{1cm} (11)$$

where $P_i = \sum_j \alpha_{i, j} (x_{i, j})^2$ will not change in the evolution process and is regarded as a constant, and

$$\psi_h^i = \begin{cases}  \frac{1}{2} & \text{if } r_h^i \geq 1; \\ r_h^i - \frac{\left( r_h^i \right)^2}{2} & \text{otherwise}. \end{cases}$$  \hspace{1cm} (12)$$

Now, we can design the replicator dynamics, which depicts the selection dynamics of the population as follows

$$\frac{\partial y_h^j}{\partial t} = y_h^j \left( \pi_h^j - \bar{\pi}_h \right)$$  \hspace{1cm} (13)$$

where $\bar{\pi}_h$ denotes the average net utility, and is given by

$$\bar{\pi}_h = \sum_{j=1}^J y_h^j \pi_h^j = \sum_{j=1}^J y_h^j \psi_h^j \sum_{i=1}^I \alpha_{i, j} (x_{i, j})^2.$$  \hspace{1cm} (14)$$

Remark 1: It is clear from (13) that the proportion of the residential users buying power from utility company $j$ will increase when the net utility of company $j$ is larger than average net utility of all companies. This situation coincides with our common sense.

C. Evolutionary Equilibrium

The evolutionary equilibrium refers to a stable condition that the population will not change its selection. Since the selection is determined by the difference between the net utility of utility company $j$ and the average net utility, the evolutionary equilibrium is achieved when the net utility of utility company $j$ equals to the average net utility, i.e.,

$$\frac{\partial y_h^j}{\partial t} = \dot{y}_h^j = 0$$  \hspace{1cm} (15)$$

Note that (15) can also be rewritten into

$$\pi_h^1 - \cdots - \pi_h^j - \bar{\pi}_h$$  \hspace{1cm} (16)$$

We consider the dynamics of $\sum_{j=1}^J y_h^j$ and have

$$\frac{\partial \sum_{j=1}^J y_h^j}{\partial t} = \sum_{j=1}^J y_h^j \left( \pi_h^j - \bar{\pi}_h \right)$$

$$= \sum_{j=1}^J \pi_h^j y_h^j - \bar{\pi}_h \sum_{j=1}^J y_h^j$$

$$= \bar{\pi}_h - \bar{\pi}_h$$

$$= 0$$  \hspace{1cm} (17)$$

Therefore, $\sum_{j=1}^J y_h^j = 1$ will always hold in the evolution process. We denote the evolutionary equilibrium by $Y_h^* = \{ y_h^1, y_h^2, \ldots, y_h^J, \ldots, y_h^J \}$. Then, the convergence to the evolutionary equilibrium (15) with the replicator dynamics (13) can be established via Lyapunov method [33].

Lemma 2: Assume that there exists a scalar function $V$ of the state $x$, with continuous first order derivatives such that

- $V(x)$ is positive definite
- $\dot{V}(x)$ is negative definite
- $V(x) \to \infty$ as $\|x\| \to \infty$

Then the equilibrium at the origin is globally asymptotically stable.

Theorem 1: The replicator dynamics will converge to the evolutionary equilibrium.

Proof: Define the tracking error function $e_h^j = y_h^j - y_h^j^*$, and the Lyapunov function $V_h^j(t) = ((e_h^j)^2)/(2)$, where the Lyapunov function is positive definite since $V_h^j(t) \geq 0$ all the time.

The time derivative of $V_h^j(t)$ can be described as

$$\dot{V}_h^j(t) = \frac{\partial \left( e_h^j \right)^2}{\partial t} = e_h^j \frac{\partial \left( e_h^j \right)}{\partial t} = -y_h^j \left( y_h^j - y_h^j^* \right) \left( \pi_h^j - \bar{\pi}_h \right)$$

$$- y_h^j \left( y_h^j - y_h^j^* \right) \left( \pi_h^j - \bar{\pi}_h \right) \sum_{j=1}^J y_h^j \pi_h^j.$$  \hspace{1cm} (18)$$

Notice that net utility function $\pi_h^j$ of company $j$ is a non-increasing function with respect to $y_h^j$. First consider the utility company $n$ with the largest net utility, i.e., $n = \arg \max_{i} \pi_h^i \{ \pi_h^1, \ldots, \pi_h^J \}$. Clearly, we have $y_h^n < y_h^n^*$, and $\pi_h^n = \sum_{j=1}^J y_h^j \pi_h^j - \sum_{j=1}^J y_h^j \pi_h^j - \sum_{j=1}^J y_h^j \pi_h^j > 0$. Therefore, the time derivative of $V_h^n(t)$ is negative, i.e., $\dot{V}_h^n(t) \leq 0$. Then, the dynamics of residential users choosing utility company with the maximal net utility will converge to the equilibrium. Similarly, the replicator dynamics of residential users choosing other utility companies will also converge to the equilibrium. Therefore, the dynamic system is stable, i.e., the strategy will converge to the evolutionary equilibrium of this game.

D. Iterative Algorithm

The replicator dynamics given in (13) is time continuous and can be approximated by the following discrete replicator:

$$y_h^j(m+1) = y_h^j(m) + \sigma_1 y_h^j(m) \left( \pi_h^j(m) - \bar{\pi}_h(m) \right)$$  \hspace{1cm} (19)$$

where $m$ denotes the iteration number and $\sigma_1$ is the step size. The continuous terminal criterion (16) has been accordingly revised as

$$|\pi_h^j(m) - \bar{\pi}_h(m)| < \varepsilon_1$$  \hspace{1cm} (20)$$

where $\varepsilon_1$ is a small positive constant. Then, the iterative algorithm is given as follows.

Algorithm 1 Executed by residential user $i \in \mathcal{I}$$

1: Randomly choose one utility company $j$ to buy power;
2: $m = 1$;
3: repeat
4: Calculate the net utility \( \pi^j_h(m) \) of buying power from utility company \( j \) according to (11);
5: Receive all of the net utility, and then calculate the average net utility \( \bar{\pi}_h(m) \) according to (14);
6: Change utility company to buy power with the probability \( y^j_h(m) \) according to (19);
7: \( m = m + 1 \);
8: until (20) is satisfied;

V. NON-COOPERATIVE GAME AMONG UTILITY COMPANIES

In general, each utility company behaves in a selfish and rational way, and aims at maximizing its own welfare. Therefore, the competition among the utility companies can be modeled as a non-cooperative game, whose solution is the well-known Nash equilibrium.

A. Analysis of the Non-Cooperative Game

The non-cooperative game for the utility companies can be described as follows,

- **Players**: utility company \( j \in J \).
- **Strategy**: the price \( p^j_h \) of any utility company \( j \) at any time slot \( h \).
- **Utility**: the welfare function of a utility company given in (21).

In this non-cooperative game, the welfare function of a utility company given in (21) can be written as

\[
U^j_h = p^j_h s^j_h - \left( a^j_h \left( \frac{r^j_h}{t^j_h} \right)^2 + b^j_h t^j_h + c^j_h \right)
- \left\{ \begin{array}{ll}
p^j_h s^j_h - \left( a^j_h \left( \frac{r^j_h}{t^j_h} \right)^2 + b^j_h t^j_h + c^j_h \right) & \text{if } r^j_h \leq 1 \\
p^j_h D^j_h - \left( a^j_h \left( \frac{r^j_h}{t^j_h} \right)^2 + b^j_h t^j_h + c^j_h \right) & \text{otherwise} \end{array} \right.
\]

(21)

Then, we have the following theorem.

**Theorem 2**: There exist Nash equilibriums in the non-cooperative game among the utility companies.

**Proof**: When a utility company announces its power price, it cannot obtain the generation-demand ratio and welfare at the same time. Thus, the company shall assume that the generated power can be sold out [9], [11], i.e., \( r^j_h \leq 1 \). This assumption will not influence the existence and convergence of Nash equilibrium, because the non-cooperative game will finally stop when \( r^j_h = 1 \). It guarantees the balance between power generation and demand. Thus, if the power price is set as \( p^j_h \), the optimal choice of power load \( t^j_h \) can be derived as [9]

\[
U^*_h = \arg \max_{t^j_h} U^j_h \left( p^j_h, t^j_h \right) = \frac{p^j_h - b^j_h}{2a^j_h} \quad (22)
\]

There are two cases. If \( r^j_h \leq 1 \), then the welfare function can be described as

\[
U^j_h = \frac{(p^j_h - b^j_h)^2}{4a^j_h} - c^j_h \quad (23)
\]

Since the welfare function should be larger than 0, the minimal available price \( p^j_h \) satisfies, \( p^j_h > b^j_h \). Thus, we have

\[
\frac{dU^j_h}{dp^j_h} - \frac{p^j_h - b^j_h}{2a^j_h} > 0 \quad (24)
\]

If \( r^j_h > 1 \), the differential function of the welfare with \( p^j_h \) can be described as follows

\[
\frac{dU^j_h}{dp^j_h} = p^j_h \frac{dD^j_h}{dp^j_h} + D^j_h - \left( 2a^j_h t^j_h \frac{d^j_h}{dp^j_h} + b^j_h \right)
= p^j_h \frac{dD^j_h}{dp^j_h} + D^j_h - \left( 2a^j_h t^j_h \frac{1}{2a^j_h} + b^j_h \right)
= p^j_h \frac{dD^j_h}{dp^j_h} - b^j_h + D^j_h - t^j_h \quad (25)
\]

We can notice that \( D^j_h \) is a decreasing function with \( p^j_h \) according to (6) and (7), thus \( p^j_h (D^j_h)/(dp^j_h) < 0 \). Since \( r^j_h > 1 \), \( D^j_h < t^j_h \). We obtain

\[
\frac{dU^j_h}{dp^j_h} < 0 \quad (26)
\]

In summary, we have the following expression

\[
\frac{dU^j_h}{dp^j_h} \begin{cases} > 0 & r^j_h \leq 1 \\ < 0 & r^j_h > 1 \end{cases} \quad (27)
\]

That is, the welfare function is always quasi-concave over the strategy set for both cases. In addition, there are \( J \) utility companies involved in the game and the strategy sets of players are closed, bounded and convex. According to **Lemma 1**, there exist Nash equilibriums in the game.

Both under-generation and over-generation will decrease the welfare of a utility company. The residential users will respond to the optimal price and then compute their optimal demand. However, in most cases the power demand is not always equal to the generation; and the utility companies and the users need to interact with each other within the game-theoretical framework and finally reach balance.

B. Iterative Distributed Algorithm for Utility Companies

An iterative distributed algorithm is proposed for the utility companies to reach the Nash equilibrium. According to (27), utility company \( j \) should increase price when \( r^j_h \leq 1 \) and reduce price when \( r^j_h > 1 \) in order to increase its own welfare. Therefore, for utility company \( j \), the updating strategies for power price and generation amount are designed as follows.
where $\sigma_2$ is a parameter representing the adjustment ratio, and $m$ denotes the iteration number.

In addition, (27) implies that utility company $j$ reaches its maximal welfare when $\tau^j_h = 1$, i.e., the power generation equals to the demand, which is the so-called “market clearance” and perfectly coincides our common sense. Therefore, the criterion of the iterative algorithm can be designed as

$$\left| \tau^j_h(m) - 1 \right| < \varepsilon_2$$  \hspace{1cm} (29)

where $\varepsilon_2$ is a small positive constant.

After adjusting generation amount and power price, the residential users will evolve to achieve a new evolutionary equilibrium. Then, the utility companies will adjust generation amount and price again. This iterative process can be characterized by the following algorithm.

\textbf{Algorithm 2 Executed by utility company $j \in \mathcal{J}$}

1: \textbf{for} each $h \in \mathcal{H}$ \textbf{do}
2: \quad Keep initial power price $p^j_h(1) = p_{h-1}^j$ and amount of power generation $l^j_h(1) = l_{h-1}^j$
3: \quad $m = 0$;
4: \quad \textbf{repeat}
5: \quad \quad $m = m + 1$;
6: \quad \quad Calculate generation-to-demand ratio $\tau^j_h(m)$ according to (8);
7: \quad \quad Update power price $p^j_h(m)$ and amount of power generation $l^j_h(m)$ according to (28);
8: \quad \textbf{until} (29) is satisfied;
9: \quad Execute Algorithm 1 by the residential users;
10: \textbf{end for}

Specifically, the interaction between the evolutionary game and the non-cooperative game can be expressed as Fig. 3. One iteration for a utility company refers to the process from line 5 to line 7 in Algorithm 2. Furthermore, one iteration of evolution for residential users refers to the process from line 4 to line 7 in Algorithm 1.

In the non-cooperative game, the residential users will be involved in the evolutionary game and finally reach the evolutionary equilibrium. Then, the utility companies adjust the amount of generation and price to converge to the Nash equilibrium. By applying Algorithm 2, the utility companies and the residential users will finally reach Nash equilibrium and evolutionary equilibrium, respectively.

\section{VI. NUMERICAL RESULTS}

We evaluate the performance of the proposed algorithms in this section. In our simulation, there are $I = 10$ residential users and $J = 2$ utility companies. The operating time $\mathcal{H}$ is divided into 24 time slots representing 24 hours in a whole day. In the residential users side, $w_i, \alpha_i$ of each residential user is randomly selected from $[4, 10]$, and $\alpha_{\sigma, h}$ of each residential user is set at 0.5. In the utility companies side, $b^j_h$ and $c^j_h$ are set at 0.1 and 0 respectively [9], and $a^j_h$ is randomly selected from $[0.2, 0.3]$.

\subsection{A. Evolutionary Game Among Residential Users}

To evaluate the performance of the iterative algorithm for the residential users, we perform the first group of simulations, in which the residential users conduct Algorithm 1 to achieve the evolutionary equilibrium. Fig. 4 shows the convergence process of the residential users and it depicts that the residential users can converge to the evolutionary equilibrium in a few iterations. The dynamic process of the average net utility is presented in Fig. 5. It is clear that the residential users are able to obtain better welfare.

The scalability of Algorithm 1 is presented in Fig. 6. The proposed algorithm achieves good scalability with respect to the increasing number of residential number. It is observed that the iteration number gradually decreases. This is because, according to (19), the net utility increases with the increasing number of the residential users. Thus, $(\bar{w}^j_h - \bar{w}^j_h)$ as well as the adjustment
of $y^i_k$ at each step is proportionally increased and then the iteration gradually decreases.

B. Non-Cooperative Game Among Utility Companies

To evaluate the performance of Algorithm 2, we investigate the competition among the utility companies and convergence of Nash equilibrium in the non-cooperative game. Fig. 7 shows the convergence process of the utility companies in terms of the amount of generated power and power price. It is clear that both the power price and the amount of power generation converge to a value in only few iterations. Similarly, Fig. 8 shows the convergence of generation-demand ratio. It is clear that after several iterations power supply and demand approach balance.

Fig. 9 shows that the welfare of the utility companies will gradually increase and converge. It also indicates that the welfare has been significantly improved, compared with the initial values. At the beginning of the process, the utility companies collect demand response information of the residential users and realize that the generation is much lower than the demand. Then, they adjust the amount of generation and power price to reach balance between supply and demand. Finally, Nash equilibrium is achieved and the welfare functions of the utility companies are maximized.

C. Comparison With No-Demand Response Scheme

To evaluate the performance of the proposed algorithm in shaping power demand, we perform the third group of simulations. Firstly, to have a benchmark being compared with, we present another demand response scheme. This scheme is referred to as No-demand response scheme, in which the utility companies keep a fixed power price in the entire operating process, as in a traditional power grid. Therefore, the residential users have no incentive to change their power demand.

Fig. 10 shows the power demand response under two schemes in a whole day, and it also depicts the corresponding power price under the proposed algorithm. It is clear that the proposed scheme is very effective in shaping power demand, i.e., reducing peak load and the demand variation. According to (6), the demand of each user will decrease/increase when electricity price increases/decreases. As a result, the residential users are able to
shift their demand from peak price period to off-peak price period to achieve higher welfare. In addition, the residential users can also benefit from the proposed algorithm in saving payment. Table I compares the total payment, demand and welfare of all residential users under the two different demand response schemes. Compared with the no-demand response scheme, the proposed algorithm leads to less payment, but higher welfare and more power.

VII. CONCLUSION AND DISCUSSION

We have studied the DRM problem with multiple utility companies and multiple residential users. In order to address the interaction among utility companies and residential users, a two-level game framework has been built. The interaction among the residential users is formulated as an evolutionary game, while the interaction among the utility companies is formulated as a non-cooperative game. Then, we prove that the proposed strategies can drive both games converge to the equilibriums. Moreover, iterative algorithms are designed to implement the strategies. Simulation results are presented to illustrate that the proposed scheme can significantly reduce both the peak load and the variation of power demand.

Time-coupled constraints (i.e., a constraint that insures the total power consumption of multiple time slots exceeds a minimum value) and parameter setting of the utility function (i.e., $c_{t,h}$ and $a_{t,h}$ in (3)) are not considered in this work. Based on the proposed framework, it is interesting for us to extend and explore the time-coupled constraint in a comprehensive manner. In addition, we will study how to properly capture the dynamics of residential loads and the influence of environmental variables [34], [35].

REFERENCES

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