Nanodevices for Quantum Computation

• Why do we need quantum computations?
• Building blocks of quantum computers – qubits and logical gates
• General requirements and some examples
• Josephson qubits: Main ideas behind single-Cooper-pair-box devices
• Decoherence and its role: Ways to decrease the decoherence
• Control-NOT gate: how it can be realized
• What I failed to discuss today
Classical bit

Quantum bit (or “qubit”)

Information as state of a two-level quantum system

values $|0\rangle$, $|1\rangle$ or

superposition: $\alpha|0\rangle + \beta|1\rangle$
What we need for realization of quantum algorithms?

A quantum processor consists of a collection of interacting quantum bits which can be independently manipulated and measured.

The coupling to the environment should be kept low enough to maintain quantum coherence.

Prediction: a 2,000 bit quantum computer = a conventional computer the size of universe.
Five criteria (Di Vinchenzo 1997)

1. A scalable physical system with well characterized qubits
2. The ability to initialize the state of the qubits
3. Long relevant decoherence times, much longer than the gate operation times (by factor of about $10^4$)
4. A universal set of quantum gates, i.e. logical operations involving two or more qubits
5. The ability to measure specific single qubits
Hardware

Atomic systems:
- atoms in an ion trap,
- atoms in an optical lattice,
- ensemble of nuclear spins in a liquid

Solid-state systems:
- spins of electrons in semiconductor quantum dots,
- nuclear spins of donor atoms in a semiconductor,
- superconducting microcircuits containing Josephson junctions.

Scalable, allow to preserve coherence
Quantum Computing

Nuclear Spins in Semiconductors

Coherent

Ion Traps
Liquid State NMR

Scalable

Controllable
Measurable

Cooper-pair box
SQUID’s
What exactly is the qubit?

Qubit is a typical quantum two-level system equivalent to $\frac{1}{2}$ spin

The qubit is described by effective Hamiltonian

$$\mathcal{H}_{\text{ctrl}} = -\frac{1}{2}B_z \hat{\sigma}_z - \frac{1}{2}B_x \hat{\sigma}_x .$$

with tunable $B_x$ and $B_z$ to perform single-qubit operations.

A controllable interaction in the form

$$\mathcal{H}_{\text{ctrl}}(t) = -\frac{1}{2} \sum_{i=1}^{N} B^i(t) \hat{\sigma}^i + \sum_{i \neq j} J^{ij}_{ab}(t) \hat{\sigma}^i_a \hat{\sigma}^j_b ,$$

(where a summation over spin indices $a, b = x, y, z$ is implied) to perform two-bit operations.
Some examples based on semiconductors: Proposal

The Loss-DiVincenzo proposal, 1998 – controlling spins of the electrons localized in quantum dots

Zeeman splitting is produced by magnetic field created by the current. The coupling is controlled by the back gates modulating g-factor. The exchange interaction is controlled by front gates.

It is demonstrated (also experimentally) that the quantum operations can be performed by proper manipulations of the magnetic field and gate voltages.

(see Burkard, cond-mat/0409626, for a review of solid state devices)
Experimental implementation

Harvard group, C.M. Marcus et al.
SEM image of a double-dot device
Stability diagram

Main problem – spin decoherence
At present time the Preskill criterion is not met

Diagnostics by QPCs
How one can make $\frac{1}{2}$ spin from a macroscopic system?

Use intrinsically coherent macroscopic systems – superconductors.

Superconductor is a macroscopically-coherent state with the two-component order parameter $\Psi = |\Psi| e^{i\chi(\vec{r},t)}$

Since it can be considered as wave function for the Cooper pairs condensate, a superconductor carries persistent non-dissipative current

$$\vec{j} = e|\Psi|^2 \vec{v}_s, \quad \vec{v}_s = \frac{\hbar}{2m} \nabla \chi$$

In the presence of a transport current a phase difference

$$\Theta = \chi_2 - \chi_1$$

across the superconductor is created.
Josephson junction: Reminder

Sketch

Electrical symbol

Hamiltonian: Phase representation: $\hat{H}_J = -E_J \cos \Theta$

According to quantum mechanics, the phase should be considered as an operator with eigenstates $\Theta$, $\hat{\Theta}|\Theta\rangle = \Theta |\Theta\rangle$

Since the wave function must be periodic in phase one can introduce a new basis

$$|N\rangle = \frac{1}{\sqrt{2\pi}} \int_0^{2\pi} d\Theta e^{-iN\Theta} |\Theta\rangle$$
The inverse transform is

$$|\Theta\rangle = \frac{1}{\sqrt{2\pi}} \sum_N e^{iN\Theta} |N\rangle$$

Quantum mechanics:

$|N\rangle$ can be considered as (discrete) eigenstates of the operator, $\hat{N}$, conjugated to the operator $\hat{\Theta} \rightarrow [\hat{\Theta}, \hat{N}] = i$.

In the phase representation, $\hat{N} = -i\partial/\partial \Theta$

The Josephson Hamiltonian in the new basis reads as

$$\hat{H}_J = -\frac{E_J}{2} \sum_N (|N\rangle\langle N + 1| + |N + 1\rangle\langle N|)$$

$N$ has a meaning of the number of CPs passed through the junction.

Josephson effect is a coherent transfer of Cooper pairs!
Single Cooper pair box: Reminder

How much we pay to transfer $N$ electrons to the box?

Coulomb energy:

$$E = \frac{Q^2}{2C} + QV_g$$
$$Q = -eN$$

Parity effect:

$$E(N) = E_C(N - \alpha V_g)^2 + \Delta_N$$

$$\Delta_N = \begin{cases} 
0, & N = 2n \\
\Delta, & N = 2n + 1 
\end{cases}$$

$$E_C = \frac{(2e)^2}{2C}, \quad \Delta - \text{gap}$$
At $\alpha V_g = 2n + 1$ ground state is degenerate with respect to addition of 1 CP

Temperature is low!

We can think about a degenerate state in the space of Cooper pair numbers

Thus, the classical Hamiltonian is:

$$\mathcal{H} = E_C(N - \alpha V_g)^2 - E_J \cos \Theta$$

Quantization: $N \rightarrow \hat{N} = -i \frac{\partial}{\partial \Theta}$
In the case of a small Cooper pair box, $E_C \gg E_J$, it is convenient to introduce the basic of excess Cooper pair numbers, $N$.

The Hamiltonian reads as:

$$\mathcal{H} = \sum_{N} \left\{ E_C (N - N_g)^2 |N\rangle \langle N| - \frac{E_J}{2} (|N\rangle \langle N + 1| + |N + 1\rangle \langle N|) \right\}$$

Near half-integer $N_g \equiv \alpha V_g$ we arrive at two-level quantum systems behaving as $\frac{1}{2}$ quasi-spins.
In the phase representation, one arrives at the Schrödinger equation with the Hamiltonian

\[ \hat{H}(N_g) = E_C \left( -i \frac{\partial}{\partial \Theta} - N_g \right)^2 - E_J \cos \Theta \]

and periodic boundary conditions: \[ \psi(\Theta) = \psi(\Theta + 2\pi) \]

Just like a Bloch electron in a periodic field!

In general, its solution can be expressed through Mathieu functions.

We will look at approximate solutions near the degeneracy points, where the device can be represented as \( \frac{1}{2} \) spin.
Consider the case $|N_g - 1/2| \ll 1$. Then the change states $N=0$ and $N=1$ can be mapped on the spin states

$$|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Thus we have made a quasi-$1/2$ spin with Hamiltonian

$$\mathcal{H}_{\text{ctrl}} = -\frac{1}{2} B_z \hat{\sigma}_z - \frac{1}{2} B_x \hat{\sigma}_x$$

At this stage we can control - by the gate voltage – only $B_z$, while $B_x$ has a constant value set by the Josephson energy. No chance to realize quantum logics.
Solution: Josephson interferometer

$$E_{J}^{\text{eff}} = 2E_{J}\cos \pi \frac{\Phi}{\Phi_0}$$

Realization: the split Cooper pair box
Now we have a real qubit able to perform quantum operations. However, first one should test whether the artificial ½ spin is able to be coherent during a sufficiently long time.

We also need is a device for measurement of the quantum state, say, SET electrometer.
How spin moves in a magnetic field?

Magnetic field causes magnetization $M$ to rotate (or *precess*) about the direction of $B$ at a frequency proportional to the size of $B$ — 42 million times per second (42 MHz), per Tesla of $B$. 
A way to manipulate spin is to apply *AC field* perpendicular to the DC magnetic field.

When excitation is turned off, M is left pointed off at some angle to $B_0$.

Precessing part of $M$, $M_{xy}$, is like having a magnet rotating around at very high speed (at AC frequencies).

It will generate an oscillating voltage in a coil of wires placed around the subject — this is *magnetic induction*. It decays due to relaxation.
Measurements of free induction is not very good to find spin properties.

If there are few close eigenfrequencies, then the signal consists of beatings.

How to remove beats, which have nothing to do with true decoherence?

Hahn spin echo!
Experimental realization

The electrodes were fabricated by electron-beam lithography and shadow evaporation of Al on a SiN$_x$ insulating layer (400-nm thick) above a gold ground plane (100-nm thick) on the oxidized Si substrate.

The `box' electrode is an Al strip containing $10^8$ conduction electrons.

The reservoir electrode contains two parallel low-resistive tunnel junctions with Josephson energy $E_J$, which can be tuned through magnetic flux penetrating through the loop.

Two gate electrodes (d.c. and pulse) are capacitively coupled to the box electrode.

Josephson charge qubit
Nakamura et al., 1999
Small, Cold and Fast

Millikelvins

Dilution refrigerator
$T = 15 \text{ mK}$

Microwaves

1 $\mu$m

Nanometers
After switching off

Rapid switch on of pulse

Initial state

Decay due to quasiparticle tunneling (measurement)

Coherent evolution, forming of anti-crossing

Since the pulse amplitude was beyond the control the probe current was measured as a function of the induced charge.

Broad peak without the pulse corresponds to initially degenerate states
Josephson energy was determined from the oscillation frequency and measured independently using spectroscopic methods.

Comparison is shown in the inset.
Charge echo, Nakamura et al., 2002

\[ Q_0 = 0.45e \]
\[ |\delta E(Q_0)| \approx 270 \, \mu eV \gg E_J \]

\[ \pi \text{ pulse} \quad \Delta Q_p = 0.55e \quad \Delta t = 80 \, \text{ps} \]

Second \( \pi \)-pulse projects the phase information onto \( \langle \sigma_z \rangle \) (preparing for readout).
Without the second pulse with $t_d = 0$

Echo-signal current $I$ vs $\delta t_2$

Normalized echo signal

Amplitude of the echo-signal
The echo signal decays because of decoherence

The model is based on the account of charge hopping between traps and parts of the qubit. The calculations are based on the analysis of the qubit's density matrix.
Decoherence and energy relaxation: Spin-Fluctuator Model

**Fluctuators:** structural defects, charge traps, which can exist in dielectric parts of the device

The fluctuators randomly switch between their states due to interaction with extended modes of environment – phonons or electrons.

Switching $\Rightarrow$ random fields $\Rightarrow$ decoherence

Modulation of induced charge

$$-\frac{1}{2} B_z \sigma_z - \frac{1}{2} B_x \sigma_x$$

Modulation of critical Josephson current
How one can decrease the decoherence?

Decoherence is mainly due to charge noise, which causes fluctuations in the effective magnetic field $B_z$.

**Main idea** is to keep the working point very close to the degenerate state.

**Single Cooper pair qubit:**

$$ H = \frac{1}{2}(\Delta + v)\sigma_z - \frac{1}{2}E_J \sigma_x $$

$$ \Delta = E_c C_g (V_g - V_g^{\text{opt}}) / e $$

$\Delta, v \ll E_J$

$$ E_\pm = \pm \frac{1}{2} \sqrt{(\Delta + v)^2 + E_J^2} $$

$E_\pm$ can be adjusted by gate voltage

$$ E_0 = \frac{1}{2}E_J + \frac{\Delta^2}{4E_J} $$

At $\Delta=0$ inclusion of quadratic coupling
Manipulating the Quantum State of an Electrical Circuit

D. Vion,* A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina,† D. Esteve, M. H. Devoret‡

We have designed and operated a superconducting tunnel junction circuit that behaves as a two-level atom: the "quantronium." An arbitrary evolution of its quantum state can be programmed with a series of microwave pulses, and a projective measurement of the state can be performed by a pulsed readout subcircuit. The measured quality factor of quantum coherence $Q_\phi \approx 25,000$ is sufficiently high that a solid-state quantum processor based on this type of circuit can be envisioned.

Two inventions:

• Readout via extra Josephson junction, robust against shot noise back-action

• Using the degenerate operating point, were 1st derivatives of both components of the effective field vanish. That makes the system much more robust against flicker noise

At present time, quantronium has the longest decoherence time among superconducting devices
Another bright idea – **multi-junction flux qubits**, allowing controlled operation near degeneracy point, J. E. Mooij et al., 1999

The positions of saddle points are controlled by the currents $I_{c1}$ and $I_{c2}$, which change phase drops on the Josephson junctions.
We have developed a theory of charge fluctuations near the optimal point.

Result:

Far from the optimal point

At the optimal point

But
Control-NOT (CNOT) gate has two inputs and two outputs (its classical counterpart has only one output).

The CNOT has the following truth table:

<table>
<thead>
<tr>
<th>inputs</th>
<th>outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td></td>
<td>0&gt;</td>
</tr>
<tr>
<td></td>
<td>0&gt;</td>
</tr>
<tr>
<td></td>
<td>1&gt;</td>
</tr>
<tr>
<td></td>
<td>1&gt;</td>
</tr>
</tbody>
</table>

The symbols |0> and |1> represent two orthogonal states.

Notice that output B is the NOTed (inverted) whenever input A is |1>; in other words A is controlling the operation of a NOT on B. On the other hand, A's output are unchanged.

Is it possible to make a CNOT gate using single-Cooper-pair boxes?
The demonstration was given by NEC group in 2003.
The qubits were fabricated by electron-beam lithography and three-angle evaporation of Al on a SiN$_x$ insulating layer above a gold ground plane on the oxidized Si substrate.

Temperature – 40 mK
Energies

- $E_{c1} = 484 \mu\text{eV} (117 \text{ GHz})$
- $E_{c2} = 628 \mu\text{eV} (152 \text{ GHz})$
- $E_m = 65 \mu\text{eV} (15.7 \text{ GHz})$
- $E_{J1} = 55 \mu\text{eV} (13.4 \text{ GHz})$
- $E_{J2} = 38 \mu\text{eV} (9.1 \text{ GHz})$

Superconducting gap is 210 $\mu\text{eV}$.

Right qubit has a SQUID geometry.
Two-qubit charge basis

Hamiltonian

Tuned by $V_{g1/2}$

\[ E_{n_1n_2} = E_{c1}(n_{g1} - n_1)^2 + E_{c2}(n_{g2} - n_2)^2 + E_m(n_{g1} - n_1) \times (n_{g2} - n_2) \]

$E_{J1}(E_{J2})$ is the Josephson coupling energy of the first (second) box and the reservoir,

$E_{c1,2} = \frac{4e^2}{C_{\Sigma 1,2}} \times 2(C_{\Sigma 1,2} - C_m^2)$, Cooper-pair charging energies,

$C_m$ coupling capacitance,

$E_m = 4e^2/C_m/(C_{\Sigma 1} C_{\Sigma 2} - C_m^2)$.

$E_{J1,2} \approx E_m < E_{c1,2}$. 
Superposition of 4 states at $n_{g1} = n_{g2} = 0.5$.

Other states are not accessible.
Stability diagram in the absence of Josephson coupling

- Degenerate in the left qubit
- Degenerate in the right qubit
- Doubly degenerate

Arrows show how the pulses shift the system in the experiment

Calibration of the device

Energy diagram along the line $n_{g1} = n_{g2}$

Quantum beatings close to the doubly degenerate state
two-qubit charge basis $|00\rangle$, $|10\rangle$, $|01\rangle$ and $|11\rangle$

$$H = 
\sum_{n_1,n_2=0,1} E_{n_1n_2} |n_1,n_2\rangle \langle n_1,n_2| - \frac{E_1}{2} \sum_{n_2=0,1} (|0\rangle \langle 1| + |1\rangle \langle 0|)$$

$$\quad + |1\rangle \langle 0| \otimes |n_2\rangle \langle n_2| - \frac{E_2}{2} \sum_{n_1=0,1} |n_1\rangle \langle n_1| \otimes (|0\rangle \langle 1| + |1\rangle \langle 0|).$$

Energy spectrum of the system shows that far from the co-resonant point there are two more-or-less independent qubits.

To perform the operation we will drive the system by pulses applied to the gates

$n_{g,i}$ are induced by gates
Let us start from the point A and apply a rectangular pulse to gate 2 in order to drive the system to the degeneracy point. During pulse duration it evolves as

$$\cos(\Omega \Delta t/2) |00\rangle + \sin(\Omega \Delta t/2) |01\rangle$$

By adjusting $\Delta t$ so that $\Omega \Delta t = \pi$ (π-pulse) we stop the evolution in the $|01\rangle$ state.

After termination of the pulse, the system resides at the point C.

On the other hand, if we start from point B and apply the same pulse, the system does not reach the degeneracy point. Thus the system comes back to B after termination of the pulse.

Similarly, we can realize the transition from the $|01\rangle$ state to the $|00\rangle$ state by the same pulse, and suppress the transition out of the $|11\rangle$ state.

The target bit is flipped only when the control bit is in the state $|0\rangle$. 

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Input preparation

Superimposes $|00\rangle$ and $|10\rangle$

Superimposes $|00\rangle$ and $|10\rangle$

Brings the system to point C
Calibration

Pulse-induced current

Sequence (i)

Sequence (ii)

Truth table

Ideal

\[
\begin{pmatrix}
0.37 & 0.62 & 0.004 & 0.003 \\
0.62 & 0.37 & 0.004 & 0.007 \\
0.004 & 0.004 & 0.97 & 0.018 \\
0.003 & 0.007 & 0.018 & 0.97
\end{pmatrix}
\]

\[
\begin{pmatrix}
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]
Further reading


- General concepts, elements of theory of quantum computation, information processing, and measurement.

Guido Burkard, *Theory of solid state quantum information processing*, cond-mat/0409626

- Comprehensive review of recent achievements based on various solid-state devices.


- A review of superconducting devices based on superconducting circuits.
What I failed to tell you?

• A comprehensive review of other solid-state implementations based on
  - nuclear spins on implanted atoms,
  - orbital and spin degrees of freedom of the electrons localized at quantum dots,
  - superconducting and hybrid devices based on other principles;

Conclusions

• Quantum computation is an exciting research area, both for mathematicians and physicists

• Even being far from commercial applications, the quantum-computing-relevant research will certainly lead to progress in coherent nano-electronics, nano-optics and other areas.
Thank you for your attention