Building blocks for nanodevices

- Two-dimensional electron gas (2DEG)
- Quantum wires and quantum point contacts
- Electron phase coherence
- Single-Electron tunneling devices
  - Coulomb blockage
- Quantum dots (introduction)
Single Electron Tunneling Devices
Coulomb blockade

$Q = -Ne$

$E = QV_g + \frac{Q^2}{2C}$

Repulsion at the dot

Attraction to the gate

At

the energy cost vanishes!

Single-electron transistor (SET)
Coulomb blockade in a tunnel barrier

Why R matters?
- time delay $\delta t = eR/V$
- duration $\tau \sim \hbar/eV$

$\delta t \gg \tau \rightarrow R \gg \hbar/e^2$

Energy stored is $q^2/2C$

At $|q| < e/2$ the electron tunneling will increase the energy stored in the barrier - one has to pay for the tunneling by the bias voltage.

Because of environment capacitances it is difficult to observe CB in single junctions.
Resulting I-V curve

Experiment: Al-Al\(_2\)O\(_3\)-Al, 10 nm x10 nm (superconductivity was destroyed by magnetic field)
SET: Basic circuit and devices
Basic tunneling circuits

Isolated island:

\[
\Delta E = \frac{[(n + 1)e + q_0]^2}{2C} - \frac{[ne + q_0]^2}{2C}
\]

\[
= \frac{e}{C} \left[ \left(n + \frac{1}{2}\right) + q_0 \right]
\]

At \( q_0 = -(n + 1/2)e \) the energy cost vanishes!
Double barrier structure: Circuitry

Residual capacitance

Tunneling barriers

Island between the barriers
The charge conservation requires that

\[-ne = Q_e + Q_c + Q_g = C_e(V_e - U) + C_c(V_c - U) + C_g(V_g - U),\]

where \(U\) is the potential of the grain. The effective charge of the grain is hence

\[Q = CU = ne + \sum_{i=e,c,g} C_i V_i, \quad C = \sum_i C_i.\]

This charge consists of 4 contributions, the charge of excess electrons and the charges induced by the electrodes. Thus, the electrostatic energy of the grain is

\[E_n = \frac{Q^2}{2C} = \frac{(ne)^2}{2C} + \frac{ne}{C} \sum_i C_i V_i + \frac{1}{2C} \left( \sum_i C_i V_i \right)^2.\]

Single electron tunneling
Electrostatics: 4 different charge transfer events are relevant

\[ \Delta E = \frac{e}{C_{1S} + C_{1D}} \left[ \frac{e}{2} \pm (ne - q_0 + C_{1D}V) \right] \]

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induced charge
Symmetric system: \[ C_{1S} = C_{1D} \equiv C/2 \]

For \( n=0 \) and \( q_0=0 \), all transfers are suppressed until
\[-\frac{e}{C} \leq V \leq \frac{e}{C}\]

Electrons can tunnel both from drain onto island and from island to source

Coulomb blockade of transport
What happens when an electron reaches the island?

\[ V = \frac{e}{C} \]

During each cycle a single electron is transferred!
First observation:

Giæver and Zeller, 1968 - granular Sn film, superconductivity was suppressed by magnetic filed

Coulomb gap manifests itself as increased low-bias differential resistance

The background charges, $q_0$, influence Coulomb blockade and can even lift it.
I-V curves: Coulomb staircase

How one can calculate I-V curve?
For simplicity, we will do it only for a stationary case.

The current through the emitter-grain transition we get

\[ I = e \sum_n p_n [\Gamma_{e\rightarrow g} - \Gamma_{g\rightarrow e}] . \]

Here \( p_n \) is the stationary probability to find \( n \) excess electrons at the grain. It can be determined from the balance equation,

\[ p_{n-1} \Gamma_{n-1}^n + p_{n+1} \Gamma_{n+1}^n - (\Gamma_{n}^{n-1} + \Gamma_{n}^{n+1}) p_n = 0 . \]

Here

\[ \Gamma_{n-1}^n = \Gamma_{e\rightarrow g}(n - 1) + \Gamma_{c\rightarrow g}(n - 1) ; \]
\[ \Gamma_{n+1}^n = \Gamma_{g\rightarrow e}(n + 1) + \Gamma_{g\rightarrow c}(n + 1) . \]
The partial probabilities, $\Gamma$, can be calculated from the Fermi Golden Rule: the probability is

$$W_{i \to f} = \frac{2\pi}{\hbar} |\langle i | \hat{H} | f \rangle|^2 \delta(E_f - E_i - \Delta E)$$

To get the rate we have to multiply the probability by

$$g_ig_f F(i)[1 - F(f)]$$

and then sum over initial and final states.

Since only the vicinity of the Fermi level matters we can take the densities of states and matrix elements at the Fermi level and express the results through tunneling conductance, $G$, of the junctions.
We use the tunneling Hamiltonian

\[ \mathcal{H}_{e \leftrightarrow g} = \sum_{k,q,\sigma} T_{kq} c_{k\sigma}^\dagger c_{q\sigma} + \text{h.c.} \]

that gives the following expression for the tunneling conductance

\[ G_e = \left( \frac{4\pi e^2}{\hbar} \right) g_e(\epsilon_F) g_g(\epsilon_F) V_e V_g \langle |T_{kq}|^2 \rangle \]

As a result, for, e.g., for the transition between the emitter and the grain

\[ \Gamma_{e \rightarrow g} = \frac{G_e}{e^2} \int d\epsilon_k \int d\epsilon_q F_e(\epsilon_k)[1 - F_g(\epsilon_q)] \times \delta(\epsilon_q - \epsilon_k + E_{n+1} - E_n - eV_e) \]
Finally,

\[
\begin{align*}
\Gamma_{e\rightarrow g}(n, V_e) &= \Gamma_{g\rightarrow e}(-n, -V_e) = \frac{2G_e}{e^2} \mathcal{F}(\Delta_+, \epsilon); \\
\Gamma_{g\rightarrow e}(n, V_e) &= \Gamma_{e\rightarrow g}(-n, -V_e) = \frac{2G_e}{e^2} \mathcal{F}(\Delta_-, \epsilon).
\end{align*}
\]

Here

\[
\mathcal{F}(\epsilon) = \frac{\epsilon}{1 + \exp(-\epsilon/kT)} \to \epsilon \Theta(\epsilon) \text{ at } \Theta \to 0,
\]

while

\[
\Delta_{\pm, \mu}(n) = E_n - E_{n\pm 1} \pm eV_\mu = \frac{1}{C} \left[ \frac{e^2}{2} \mp e\Delta \right] \pm eV_\mu
\]

is the energy cost of transition. The temperature-dependent factor arise from the Fermi occupation factor for the initial and final states, physically they describe thermal activation over Coulomb barrier.

With induced charges, SET
Thermal smearing

Coulomb staircase

Calculations for different background charges

Experiment: STM of surface clusters
The SET transistor

An extra electrode (gate) defined in a way to have very large resistance between it and the island.

That allows to tune induced charges by the gate voltage.

Fulton & Dolan, 1987

The so-called "orthodox" theory discussed before is valid; we have just to remember that the energy cost is

\[ \frac{1}{C} \left[ \frac{e^2}{2} \pm en \mp e \sum_i C_i V_i \right] \pm eV_\mu \]
In this way we arrive at the so-called **stability diagram** of Single Electron Transistor (SET).

**Coulomb diamonds:** all transfer energies inside are positive.

**Conductance oscillates as a function of gate voltage** - **Coulomb blockade oscillations**
Experimental test: Al-Al$_2$O$_3$ SET, temperature 30 mK

Coulomb blockade oscillations

V=10 µV
The single electron pump

Two islands - each one can be tuned by a nearby gate electrode.

The structure is symmetric.

Six electron transfers are important.

Equalities between direct and reverse processes define lines at the stability diagram.

It define the regions of stable configurations characterized specific charges of the island.

The inter-island capacitance $C_{12}$ is responsible for the interplay of the contributions from $V_A$ and $V_B$. 
At $C_{12}=0$, the diagram is a set of squares, unaffected by the transitions of $3^d$ type - the corresponding lines just touch corners.

In the general situation there are “triple points”, where an electron can transfer the whole system for free.

We will show that it allows one to “pump” electrons one by one.
Experiment:

SETs 3 and 4 work as electrometers to measure charges at the islands 1 and 2.
How one can pump the electrons?

Bias voltage moves lines at the stability diagram.

It creates triangles where the energy is relatively large, and CB is impossible.

Let us adjust the gate voltages to start within a triangle, and then apply to the gates AC voltages shifted in phase. Then the path in the phase space is

\[(n_1, n_2) \rightarrow (n_1+1, n_2) \rightarrow (n_1, n_2+1) \rightarrow (n_1, n_1)\]

Exactly one electron has passed through the device!
The current is then, $I = -ef$

Opposite phase shifts

This is an excellent current standard!

Accuracy check: $10^{-6}$
Clocking single electrons through electrical circuits one-by-one:

**Single-Electron Tunneling (SET) Pump**

An animation made by
Hansjörg Scherer
Physikalisch-Technische Bundesanstalt Braunschweig
Germany
Another current standard: Quantized electron drag in quantum wires

Only integer number of electrons can be trapped in the potential wells – drag current is quantized in units of $e\hbar$, depending on the gate voltage.

Talyanskii et al., 1996
Single electron tunneling
How accurate are the Coulomb-blockade devices? Are there principle limitations?

We discussed only sequential tunneling through a grain, which is exponentially suppressed by the Coulomb blockade.

In addition, there is a coherent transfer. Consider the initial and final states in different leads. Then the transition rate in the second order of the perturbation theory is

$$W_{i \rightarrow f} = \frac{2\pi}{\hbar} \left| \sum_{\psi} \frac{\langle i | \mathcal{H}_{\text{int}} | \psi \rangle \langle \psi | \mathcal{H}_{\text{int}} | i \rangle}{E_{\psi} - E_i} \right|^2 \delta(E_i - E_f)$$

The grain states are involved only in virtual transitions!
This process is called the **quantum co-tunneling**. Its rate is

\[
\Gamma_{\text{cot}} = \frac{\hbar G_e G'_c}{2\pi e^4} \int_{\epsilon_k} d\epsilon_k \int_{\epsilon_q} d\epsilon_q \int_{\epsilon_{q'}} d\epsilon_{q'} \int_{\epsilon_{k'}} d\epsilon_{k'} f(\epsilon_k)[1 - f(\epsilon_q)] f(\epsilon_{q'})[1 - f(\epsilon_{k'})] \times \left[ \frac{1}{\Delta_{-,e}(n + 1)} + \frac{1}{\Delta_{+,e}(n - 1)} \right]^2 \delta(eV + \epsilon_k - \epsilon_q + \epsilon_{q'} - \epsilon_{k'}). 
\]

We pay by additional small tunneling transparency (one more factor containing conductance).

However, the energy costs enter as **powers** rather than exponents.

Due to its importance, the quantum co-tunneling has been thoroughly studied. It can lead to the contributions to the current proportional to $V^3$ and $V$. 

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Single electron tunneling

31
In the previous lecture we discussed electrons in terms of waves. However, in this lecture we spoke about particles, their charge, etc.

Are we running two horses at the same time?

How single-electron effects interplay with quantum interference?

These problems are solved to some extent and we will discuss them later.