Mesoscopic superlattices
Periodic arrays of mesoscopic elements.

We will focus on lateral superlattices imposed onto a 2DEG by lithographic techniques.

One-dimensional superlattices

Patterned by holographic methods - interference of laser beams, a > 200 nm.

Shorter scales can be done using electron beam or scanning probe lithography, but accuracy is less.

The photo-resist is illuminated by the periodically modulated interference pattern of two laser beams.
The resist can be used as

a mask for lift-off process;

for modulation of distance between the gate and 2DEG;

as an etch mask

Interesting manifestation - Weiss oscillations, similar to Shubnikov-de Haas and also periodic in $1/B$.

They occur in weak magnetic fields and are anisotropic.
The density modulation is only 2%, but the effect is very large!

Why?

The reason is the drift of the cyclotron orbits induced by periodic electric field of the superlattice.

The local drift is proportional to $\mathbf{E} \times \mathbf{B}$. Let us assume $r_c \gg a$. Then for most parts of the trajectory the drift averages to zero.

The drift in x-direction is mostly accumulated near the turning points, and it is important whether the signs of the electric field are the same or opposite.
Two-dimensional superlattices

One, e. g., use the above procedure twice, second time after rotating by 90°.

Let us start with so-called antidot lattices, which are just 2D systems of holes in 2DEG.

One can find the discrete values of magnetic field at which the orbits do not hit the scatterers - so-called commensurate fields.

For a square lattice they enclose 1,2,4,9,21,... antidots
These values correspond to increased resistivity.

However:

• Resistance maxima are not exactly at the expected positions;
• An unexpected negative Hall resistance has been observed

How these features can be explained?
Consider results of modeling for a simple layout for measuring the Hall effect.

An example is the antidot lattice with lattice constants \( a = 240 \text{ nm} \) and \( b = 480 \text{ nm} \). The potential can be modeled as

\[
V(x, y) = V_0 \cos^{2\beta}(\pi x/a) \cos^{2\beta\gamma}(\pi y/b)
\]

In the modeling, it is assumed \( \beta = 2 \), while \( V_0 \) and \( \gamma \) are chosen such that the antidot potential peaks out of the Fermi sea with a radius of 43 nm.
The trajectories for $r_c = a/2$

Arrows - typical trajectories

Insets - positions of 1000 electrons after 135 ps in the area 24x24 lattice periods
Main features:

• Peaks in $\sigma_{xx}$ and smooth decrease of $\sigma_{yy}$ as magnetic field increases;

• At magnetic fields indicated by A and C the conductivity is approximately isotropic, while at the fields B and D they differ up to a factor 25.

These features can be understood from shapes of typical trajectories, see next slide.
At large magnetic field the electrons are trapped by antidots and conductance is very low. It is still easier to diffuse along $x$ since $a < b$.

At $\sigma_{xx}$-peaks antidots channel electron along $x$-direction.

There is no preferred directions of diffusion.

Similar trends are seen from the picture of diffusing electron clouds (insets).
How one can simulate conductance?

One can model diffusion instead of conductance using the Einstein relation.

\[ \mathbf{j} = \sigma \mathbf{E} - eD \nabla n = \sigma \nabla \phi - e \frac{\partial n}{\partial \mu} D \nabla \mu \]

On the other hand, the current must be proportional to the gradient of the electrochemical potential,

\[ \phi^* = \phi - \frac{\mu}{e} \]

Thus \[ \sigma_{ik} = e^2 \left( \frac{\partial n}{\partial \mu} \right) D_{ik} \]

Now, how one can model diffusion?
This is explained in the theory of Brownian motion, see courses in statistical physics.

In this theory a particle is assumed to be subjected random forces which makes velocity a random quantity.

Then

\[ D_{ik} = \int_0^\infty dt \frac{v_i(t') v_k(t + t')}{v_i(t + t') v_k(t')} \]

Here

\[ \frac{v_i(t + t') v_k(t')}{v_i(t + t') v_k(t')} \equiv \lim_{T \to \infty} \frac{1}{T} \int_0^T dt' v_i(t + t') v_k(t') \]
This expression is sometimes called the classical Kubo formula.

According to quantum mechanics, velocities are operators which do not commute at different times. Then calculation is much more difficult.

In the absence of magnetic field and driving force,

\[ v_i(t' + t)v_k(t') = e^{-t/\tau}v_i(t')v_k(t') = e^{-t/\tau} \delta_{ik}v_i^2 \]

Thus

\[ D_{ik} = (1/d)v_F^2\tau \delta_{ik} \]

where \( d \) is the dimensionality of the problem.
The motion in the antidot lattice in magnetic field is partly chaotic – it contains stability islands and chaotic seas.

**Summary**

Electron dynamics in superlattices and other periodic arrays is a “laboratory” for studying both coherent and incoherent electron transport in nanodevices.

They allow checking fundamental concepts, testing new numerical methods, and characterizing novel nanostructures.

Superlattices are very promising for various applications.
A quantum-cascade laser is a sliver of semiconductor material about the size of a tick. Inside, electrons are constrained within layers of gallium and aluminum compounds, called quantum wells are nanometers thick -- much smaller than the thickness of a hair.

Adapted from the Bell Labs web-site
Inter-band transitions in conventional semiconductor lasers emit a single photon.

In quantum cascade structures, electrons undergo inter-subband transitions and photons are emitted. The electrons tunnel to the next period of the structure and the process repeats.

This diagram is oversimplified. To optimize lasing one has to invent much more complicated design of the active region.
The scattering rate between two subbands is heavily dependent upon the overlap of the wave functions and energy spacing between the subbands.

Energy diagram of a quantum cascade laser with diagonal transition also showing the moduli squared of the wave functions.

Schematic representation of the dispersion of the $n = 1; 2$ and 3 states parallel to the layers; $k$ is the corresponding wave vector. The wavy lines represent the laser transition; the straight arrows identify the inter-subband scattering process by optical phonons.
Schematic energy diagram of a portion of the $\text{Ga}_{0.47}\text{In}_{0.53}\text{As}$–$\text{Al}_{0.48}\text{In}_{0.52}\text{As}$ quantum cascade laser with vertical transition.
Distributed feedback QC laser - schematic

grating selects well defined single wavelength
tunable by temperature
“The quantum cascade (QC) laser is an excellent example of how quantum engineering can be used to design new laser materials and related light emitters in the mid-IR.

The population inversion occurs between excited subbands of coupled quantum wells and is designed by tailoring the electron inter-subband scattering times.

This tailoring adds an important dimension to the quantum engineering of heterostructures.

The pumping mechanism is provided by injecting electrons into the upper state of the laser transition by resonant tunneling through a potential barrier.”

From Sirtori et al., 1998
Photonic crystals are periodic optical nanostructures that are designed to affect the motion of photons in a similar way that periodicity of a semiconductor crystal affects the motion of electrons.

SEM micrographs of a photonic crystal fiber produced at US Naval Research Laboratory.

The diameter of the solid core at the center of the fiber is 5 µm, while (right) the diameter of the holes is 4 µm.

To create a biosensor, a Photonic Crystal may be optimized to provide an extremely narrow resonant mode whose wavelength is particularly sensitive to modulations induced by the deposition of biochemical material on its surface.
Cyanophrys remus

Natural photonic crystals

Very important new research area ...

Macroporous Si

and many other books
• Photonic crystals are attractive optical materials for controlling and manipulating the flow of light.

• One-dimensional photonic crystals are already in widespread use in the form of thin-film optics with applications ranging from low and high reflection coatings on lenses and mirrors to color changing paints and inks.

• Higher dimensional photonic crystals are of great interest for both fundamental and applied research, and the two dimensional ones are beginning to find commercial applications. The first commercial products involving two-dimensionally periodic photonic crystals are already available in the form of photonic-crystal fibers, which use a microscale structure to confine light with radically different characteristics compared to conventional optical fiber for applications in nonlinear devices and guiding exotic wavelengths.

• The three-dimensional counterparts are still far from commercialization but offer additional features possibly leading to new device concepts (e.g. optical computers), when some technological aspects such as manufacturability and principal difficulties such as disorder are under control.