Acoustoelectric current for composite fermions

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The acoustoelectric current for composite fermions in a two-dimensional electron gas (2DEG) close to the half-filled Landau level is calculated in the random phase approximation. The Boltzmann equation is used to find the nonequilibrium distribution of composite fermions to second order in the acoustic field. It is shown that the oscillating Chern-Simons field created by the induced density fluctuations in the 2DEG is important for the acoustoelectric current. This leads to a violation of the Weinreich relation between the acoustoelectric current and acoustic intensity. The deviations from the Weinreich relation can be detected by measuring the angle between the longitudinal and the Hall components of the acoustoelectric current. This departure from the Weinreich relation gives additional information on the properties of the composite fermion fluid.

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I. INTRODUCTION

Two-dimensional electron gases (2DEG’s) have been studied extensively both experimentally and theoretically. One important experimental technique is to investigate the interaction of the electron gas with surface acoustic waves (SAW) propagating along the sample. Because of the piezoelectric properties of the substrate materials (GaAs-Al$_{x}$Ga$_{1-x}$As), the acoustic wave is accompanied by an electric wave that interacts with the electron gas. A traveling wave of electric field can also be produced by placing a 2DEG sample on the surface of a piezoelectric crystal.

It is well known that both the attenuation and sound velocity are sensitive to changes in the properties of the electron gas. Less well explored experimentally is the acoustoelectric drag current induced by the acoustic wave. From the experiments so far, it seems that in some cases this offers the promise of greater sensitivity than the attenuation or velocity shift measurements, because one measures directly a small electric current or voltage instead of a small shift in a large quantity. In addition, measurement of the acoustoelectric current gives a direct measure of the intensity of the electric field created by the acoustic wave, thus enabling determination of the coupling between the SAW and the electron gas, whereas attenuation measurements give only relative intensities. The theoretical efforts in this direction have been modest and a better understanding is required to interpret measurements. Furthermore, there is some disagreement between the different methods that has not been clarified yet.

The properties of a 2DEG in a strong magnetic field have successfully been described by the composite fermion model. Especially near even-denominator filling fractions, like $\nu_f=1/2$, this seems to be a good description. In this paper, we will calculate the acoustoelectric current for composite fermions using the Boltzmann equation approach. This has previously been applied to the calculation of the conductivity tensor at finite wave vector and frequency. The paper is organized as follows. In Sec. II an interaction between composite fermions and a SAW will be discussed.

The Boltzmann equation is derived and solved in Sec. III, and the acoustoelectric current is calculated. The resulting expression is discussed in Sec. IV.

II. INTERACTION BETWEEN COMPOSITE FERMIONS AND A SURFACE ACOUSTIC WAVE

There are several fields involved in the problem. The real physical fields, those experienced by the electrons, are the external magnetic field $\mathbf{B}$ and a periodic electric field set up by the acoustic wave.

The materials used to create the 2DEG, GaAs-Al$_{x}$Ga$_{1-x}$As heterostructures, are piezoelectric. This means that an acoustic wave propagating on the sample will create a periodic electric wave that interacts with the composite fermions. To achieve stronger coupling, one sometimes uses a substrate with a larger piezoelectric constant, and places the heterostructure in close contact with (but acoustically isolated from) the substrate. The piezoelectric field induced by the SAW is then able to penetrate the 2DEG.

It is assumed that the wave is propagating in the $x$ direction, and that the piezoelectric field is in the same direction. The electric field is then given by the real part of $\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 e^{-i(\Omega t - \mathbf{q} \cdot \mathbf{r})} = -\nabla \Phi$ with $\Phi = \Phi_0 e^{-i(\Omega t - \mathbf{q} \cdot \mathbf{r})}$. Here $\Omega$ and $\mathbf{q}$ are the SAW frequency and wave vector, respectively; $\mathbf{E}_0|\mathbf{q}|\hat{x}$. $\Phi_0$ is the amplitude of the screened potential; it is related to the amplitude of the piezoelectric (external) potential $\Phi_{\text{ext}}$ by the equation (see Ref. 6) $\Phi_0 = [1 - \nu(q)K_{00}]\Phi_{\text{ext}}$, where $\nu(q) = 2\pi/\epsilon_{\text{eff}}q$ is the Fourier transform of the Coulomb potential and the response function is

$$K_{00}(q, \Omega) = q \sigma_m / v_\perp \left(1 - i\sigma_m / \sigma_x \right)$$

with $\sigma_x(q, \Omega)$ the longitudinal electron conductivity, $\sigma_m = v_\perp \epsilon_{\text{eff}}^2 2\pi$, $v_\perp = \Omega/q$ the SAW velocity, and $\epsilon_{\text{eff}}$ the effective dielectric constant. The amplitude of the piezoelectric field is related to the amplitude $A$ of the acoustic wave by the relation

$$A = \frac{v_\perp \epsilon_{\text{eff}}}{\Omega} q \sigma_m / v_\perp \left(1 - i\sigma_m / \sigma_x \right)$$
\[ \Phi_{\text{ext}} = A e \varepsilon F(qd) / \varepsilon, \]

where \( e \) is the piezoelectric stress constant and \( F(qd) \) is some dimensionless function calculated by Simon.6

When we make the Chern-Simons (CS) transformation, additional, fictitious, fields are introduced. There will be a Chern-Simons magnetic field \( \mathbf{b} = -2 \hat{\phi}_0 n_{e,0} \hat{z}, \) where \( \hat{\phi}_0 = 2 \pi \hbar c / e \) is the flux quantum and \( n_{e,0} \) is the total electron density. Because of the interaction with the piezoelectric field, there will be an induced density modulation in the electron gas. Therefore, the electron density is conveniently split into two parts, an average and a fluctuating (ac) part: \( n_e^{\text{av}} = n_e + \delta n_e. \) Corresponding to these, there will be an average and an ac Chern-Simons magnetic field. The average field will partly cancel the external (real) magnetic field, leaving the effective field \( \mathbf{B}_e = \mathbf{B} (1 - 2 \nu_f) \), where \( \nu_f = \phi_0 n_e / B \) is the Landau level filling factor. For simplicity we assume two flux quanta to be attached to each electron, appropriate for \( \nu_f \) close to 1/2. The ac component of the Chern-Simons field is given by \( \mathbf{b}^{ac} = -2 \hat{\phi}_0 \delta n_e \hat{z}. \) In addition, the motion of the composite fermions (CF’s) will create an electric Chern-Simons field which is given by \( \mathbf{e} = \rho (\hat{z} \times \mathbf{j}), \) where \( \rho = 2 \phi_0 / \hbar c = 2 e \nu_f. \) The current is split in two parts, an ac part \( \mathbf{j}^{ac}, \) which is given by the linear response, and a dc part, the acoustoelectric current \( \mathbf{j}^{se}, \) which is given by the second order response. These give contributions \( \mathbf{e}^{ac} \) and \( \mathbf{e}^{se} \) to the CS electric field.

The strength of these fields relative to the perturbing field \( E = |\mathbf{E}| \) can be estimated from charge conservation. The equation for conservation of charge is \( e \partial(\delta n_e)/\partial t + \nabla \cdot \mathbf{j} = 0, \) or, Fourier transformed, \( -i \Omega e \delta n_e + i \sigma_x^{se} q E_x = 0. \) This gives

\[ -e \delta n_e = (\sigma_x^{se} / e_x) E_x. \]

Then the force from the fluctuating (ac) component of the magnetic Chern-Simons field is given by

\[ \frac{e \nu}{c} b^{ac} = -2 \frac{\nu}{c} \frac{\sigma_x^{se}}{e_x} E_x = \frac{4 \pi}{\alpha} \frac{\nu}{c} \frac{\sigma_x^{se}}{e_x} E_x, \]

where \( \alpha = e^2 / \hbar c = 1/137 \) is the fine structure constant and \( \nu \) is the composite fermion velocity. In order for the acoustic wave to be sensitive to changes in the properties of the electron gas, we see from Eq. (1) that the ratio of the conductivity to the sound velocity should be \( \left| \sigma_x^{se} / \nu \right| = \epsilon_{\text{eff}}^2 (2 \pi) / (\sigma_x^{se}) \) (or at least not differing by more than one order of magnitude; see Refs. 1, 9, and 12). With \( \epsilon_{\text{eff}}^2 = 6 \) (see Ref. 6), we have \( \left| \sigma_x^{se} / \nu \right| = 1. \) It is then seen that the relative strength of the force from the ac Chern-Simons field to the direct force from the piezoelectric field of the SAW is \( (4 \pi / \alpha) (\nu / c). \) With a Fermi velocity of \( 10^5 \) m/s, this factor is of order 1. That is, the piezoelectric and the CS fields are of comparable importance. For the electric CS field we find for the \( x \) component

\[ e_x^{ac} = -\rho \sigma_x^{se} (q, \Omega) E_x. \]

Since static \( \sigma_x^{se} \) is \( e^2 / 2h \) for \( \nu = 1/2, \) we see that the Chern-Simons field is of the same order as the external electric field. The \( y \) component is similarly

\[ e_y^{ac} = \rho |\sigma_x^{se}| E_x = \rho (4 \pi / \alpha) (v_x / c) E_x, \]

which is smaller than what is given by Eq. (4) by the factor \( v_x / c. \) Since the typical sound velocity is \( 3 \times 10^3 \) m/s, this will be small. However, it is acting along the trajectory of the composite fermions at the points of strong interaction (see below), and will therefore play an important role.

So far we have expressed the current as a response to the external field. However, as seen from the composite fermions it is not possible to separate the two fields, and it is more convenient to consider the response to the total effective electric field acting on the composite fermions, \( \mathbf{E} = \mathbf{E}^e + \mathbf{E}^{ac}. \) The latter is described by the composite fermion conductivity tensor \( \sigma_{jk}(q, \Omega). \) Because the field is not parallel to the wave vector, this is not a potential field, and we cannot write it as the gradient of a potential. However, we can consider the \( x \) component \( E_x = E_x + e_x^{ac} = -\nabla \cdot \mathbf{v}. \) In the following we will consider the response to the effective field \( E_x \) acting upon composite fermions. The only thing that needs to be changed in order to account for the Chern-Simons field is then that the induced density modulation must be expressed in terms of response to the total field,

\[ e \delta n_e = v_x^{-1} (\sigma_x^{se} E_x + \sigma_y^{se} E_y). \]

The \( y \) component of the total field is just the Chern-Simons field since \( E_y = 0, \) and one has

\[ E_y = \rho (\sigma_x^{se} E_x + \sigma_y^{se} E_y). \]

From this we get

\[ E_y = \delta E_x, \]

where

\[ \delta = \rho \sigma_x^{se} \Omega (1 - \rho \sigma_y^{se}). \]

We then have

\[ b^{ac} = -(c / v_x) \delta E_x. \]

In the most interesting situation the acoustic wavelength \( 2 \pi / q \) is less than the typical diameter of CF orbits. That is, the parameter \( \kappa = q R_c, \) where \( R_c \) is the radius of the cyclotron orbit. Using the expression for \( \sigma_y \) obtained by Mirlin and Wolfle13 we find in this limit

\[ \rho \sigma_{xy} \simeq \frac{1}{\xi} \sin \pi (\omega + i \nu \tau)^{-1}, \quad \xi = \frac{h \Omega}{2 m v_x^2}. \]

Here we have introduced the dimensionless frequency \( \omega = \Omega / \omega_c \) and damping \( \nu = (\omega_c \tau)^{-1}, \) where \( \omega_c \times B^* \) is the CF cyclotron frequency and \( \tau \) is the relaxation time. Substituting reasonable values \( \Omega / 2 \pi = 3 \) GHz, \( v_x = 3 \times 10^3 \) m/s, \( m \approx 10^{-27} g \), and \( q = 1, \) we obtain a quantity of the order 1 out of cyclotron resonance, and an enhancement of the order \( \kappa \) at cyclotron resonance, where \( \omega = n \) is an integer. This means that the factor \( (1 - \rho \sigma_{xy})^{-1} \) may be arbitrary.

In the following we shall use the random phase approximation (RPA), according to which the composite fermions are considered as noninteracting particles. It is known15,16
that if we are to have a consistent Fermi liquid theory we must also include the Landau interaction parameters. This will affect the CF mass, and will possibly give some additional effects. This is an interesting possibility for further work.17

Even within the RPA, the problem is not equivalent to the corresponding problem for electrons in the effective magnetic field $B^\parallel$, because of the fluctuating Chern-Simons fields associated with the density modulation induced by the SAW.

III. ACOUSTOELECTRIC CURRENT

A. Solution of the Boltzmann equation

The acoustoelectric current will be expressed through the nonequilibrium distribution function for composite fermions, $f(r,t)$. In this section, we will calculate this function from the composite fermion Boltzmann equation. As long as the amplitude of the acoustic wave is sufficiently small we can treat the induced piezoelectric field as a perturbation. That means that the Boltzmann equation can be first linearized and then solved iteratively. Thus, we seek a solution to the equation

\[ \frac{\partial f}{\partial t} + v \nabla f + F^\parallel f(r,t) = C\{f\} \]  

of the form

\[ f = f_0[e^\Psi(r,t) + e^\Psi(-\delta f_0/\delta \epsilon)f_1(r,t) - e^{2|\Psi_0|^2q/2m}f_2(r,t), \]

where $f_0$ is the unperturbed solution (with $E=0$ but the constant magnetic field included). The force must be taken as

\[ F = -e\mathbf{E} - (e/c)[v \times (\mathbf{B}^\parallel + \mathbf{b}^\parallel)]. \]

The important scattering mechanism for composite fermions is scattering by the random magnetic (Chern-Simons) field which is created by density fluctuations in the electron gas because of electrostatic potentials from impurities in the doping layer.13 Further, it is known that to ensure particle number conservation we have to take the complex conjugate of the electric field and divide by 2. That is, the oscillating component of the product of two oscillating functions $A(r) = \Re\{A_1 e^{-i(t-q r)}\}$ and $B(r) = \Re\{B_1 e^{-i(t-q r)}\}$ is equal to $\Re\{A_1 B_1^*/2\}$.

Writing the second order approximation according to Eq. (14) we get the linear equation

\[ \hat{L}_0 f_2 = S(v,\phi) - \frac{2mp}{e|\Psi_0|^2 q \omega \epsilon} \frac{\partial f_0}{\partial \epsilon} \mathbf{v} \cdot (\hat{z} \times \hat{\mathbf{j}}). \]

Here $\hat{L}_0 = \partial/\partial \phi + v$, and $S(v,\phi)$ is the CS contributions.

The last term in the $S(v,\phi)$ are the CS contributions. The periodic solution for $f_2$ is

\[ f_2(v,\phi) = \frac{1}{e^{2|\Psi_0|^2q/2m - 1}} \int_{\phi}^{\phi + 2\pi} d\phi' S(v,\phi') e^{i\phi' - \phi} \]

\[ - \frac{2mp}{e|\Psi_0|^2 q \omega \epsilon} \frac{\partial f_0}{\partial \epsilon} \frac{1}{e^{2|\Psi_0|^2q/2m - 1}} \]

\[ \times \int_{\phi}^{\phi + 2\pi} d\phi' \mathbf{v} \cdot (\hat{z} \times \hat{\mathbf{j}}) e^{i\phi' - \phi}. \]

This is the solution of the Boltzmann equation to second order in the perturbation, from which we will now calculate the acoustoelectric current.

B. Calculation of the direct current

Neither the equilibrium distribution nor the first order perturbation $f_1$ will give any contribution to the direct current, so the lowest order contribution is found from the second order perturbation. Let us for the moment forget the last term in the expression (21), which comes from the Chern-Simons
The first term of $S(v, \phi')$, Eq. (20), can be integrated by parts over $v$ to get

$$
\frac{e^3|\Psi_0|^2 q}{2m\omega_c} \int_0^\infty dv \frac{\partial f_0}{\partial \epsilon} \int_0^{2\pi} d\phi \left\{ \cos \phi \right\} \langle \sin \phi \rangle \frac{F(\phi)}{e^{2\pi v - 1}}
$$

(23)

where

$$
F(\phi) = \int_0^{\phi + 2\pi} d\phi' \left[ 2 \cos \phi' \mathrm{Im} f_1 + \sin \phi' \frac{\partial \mathrm{Im} f_1}{\partial \phi'} + 2\rho \sigma \sin \phi' \frac{\partial \mathrm{Im} \{e^{i\phi f_1}\}}{\partial \phi'} - \rho \sigma \cos \phi' \frac{\partial \mathrm{Im} \{e^{-i\phi f_1}\}}{\partial \phi'} + \beta \rho \sigma \frac{\partial \mathrm{Im} \{e^{i\phi f_1}\}}{\partial \phi'} e^{i(\phi' - \phi)}. \right]
$$

(24)

Since $\mathrm{Im} f_1$ and $\mathrm{Im} \{e^{i\phi f_1}\}$ are periodic functions of $\phi$ they can be expanded in Fourier series,

$$
\mathrm{Im} f_1 = \sum_{n=-\infty}^{\infty} A_n e^{i n \phi}, \quad \mathrm{Im} \{e^{i\phi f_1}\} = \sum_{n=-\infty}^{\infty} \tilde{A}_n e^{i n \phi},
$$

where the $A_n$ and $\tilde{A}_n$ will be functions of $v$. The angular integrals are then reduced to the six cases

$$
\int_0^{2\pi} d\phi \left\{ \cos \phi \right\} \langle \sin \phi \rangle e^{-\phi} \int_\phi^{\phi + 2\pi} d\phi' \left\{ \cos \phi' \right\} \langle 1 \rangle e^{(v + i n)\phi'},
$$

where all combinations of the expressions in the braces are implied. These can be evaluated directly, and it is found that only the zeroth, $A_0 = \mathrm{Im} f_1$, and first Fourier components will give contributions. The current is then given by

$$
\mathbf{j}_0^{ae} = \frac{\pi e^3|\Psi_0|^2 q}{m\omega_c (v^2 + 1)} \int_0^\infty dv \frac{\partial f_0}{\partial \epsilon} \sum_{n=1}^{\infty} \mathbf{j}_n(v)
$$

(25)

where

$$
\mathbf{j}_0^{ae} = \begin{pmatrix} j_0^{ae} \\ j_0^{ae} \end{pmatrix}, \quad \mathbf{j}_1 = A_0 \begin{pmatrix} v \\ 1 \end{pmatrix},
$$

(26)

At low temperatures we may assume $\partial f_0/\partial \epsilon = -(m/4\pi\hbar^2)\delta(\epsilon - \epsilon_f)$, and we can perform the integral over $v$, which will fix $v$ at $v_F$ (and $\beta$ at $v_F/v_x$). The final result is then

$$
\mathbf{j}_0^{ae} = \frac{j_0}{v^2 + 1} \sum_{n=1}^{\infty} \mathbf{j}_n(v_F), \quad j_0 = \frac{e|\Psi_0|^2 q}{\hbar \omega_c \rho}.
$$

(30)

We must now return to the last item in Eq. (21). Since $\mathbf{j}_n^{ae}$ is a constant, the integral may be evaluated directly and the resulting equation solved for $\dot{\mathbf{j}}_0^{ae}$. There is, however, a simpler and physically more transparent way of obtaining the same result. We calculate $\dot{\mathbf{j}}_0^{ae}$ as before. Then we write $\dot{\mathbf{j}}_0^{ae} = \dot{\mathbf{j}}_0^{ae} + \delta\dot{\mathbf{j}}_0^{ae}$, where $\delta\dot{\mathbf{j}}_0^{ae}$ is the response to the CS electric field created by the acoustoelectric current,

$$
\delta\dot{\mathbf{j}}_0^{ae} = \dot{\sigma}^{ae} - \dot{\sigma}^{ae}_{CS} = -\dot{\sigma}^{ae}_{CS} \mathbf{j}_0^{ae},
$$

where

$$
\dot{\sigma}^{ae}_{CS} = \rho \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.
$$

Using the Drude conductivity for $\sigma$ we have

$$
\dot{\sigma} = \sigma_0 \nu \begin{pmatrix} 1 & -1 \\ 1 & \nu \end{pmatrix}, \quad \sigma_0 = \frac{n e^2 \tau}{m},
$$

(31)

which gives

$$
\dot{\mathbf{j}}_0^{ae} = (1 + \dot{M})^{-1} \mathbf{j}_0^{ae},
$$

(32)

where

$$
\dot{M} = \dot{\hat{\rho}}_{CS} = \frac{2 \nu^*}{1 + \nu^*} \begin{pmatrix} 1 & \nu \\ -\nu & 1 \end{pmatrix}, \quad \epsilon_f = \nu^*.
$$

Here $\nu^*$ is the filling factor of composite fermions (the filling factor in the effective field $B_{0*}^e$).

C. Results

The Fourier coefficients $A_n$,

$$
A_n = \frac{1}{2\pi} \int_0^{2\pi} d\phi e^{-in\phi} \mathrm{Im} f_1,
$$

(33)

and $\tilde{A}_n$, given by a similar expression, cannot be evaluated in closed form for arbitrary $\omega$ and $\kappa$. However, in the most interesting situation the acoustic wavelength appears much smaller than the CF cyclotron radius, and $\kappa \gg 1$. In this paper
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we restrict ourselves to this case. It will be shown later that
our calculation method remains valid provided \( \nu \equiv (1/\pi) \ln \kappa \). Then the coefficients \( A_n \) and \( \bar{A}_n \) can be evaluated using the method of stationary phase. The critical points are approximately \( \phi = \pm \pi/2 \), which is consistent with the physical picture that the CF will interact significantly with the acoustic wave only when the electron momentum is normal to the direction of propagation of the wave. For other directions of the momentum, the CF will be subject to a rapidly oscillating force giving no net contribution (see Fig. 1). It is not convenient to calculate these coefficients directly, because the expression for \( \text{Im} f_1 \) is complicated. A simpler way is to calculate the coefficients

\[
B_n = \frac{1}{2 \pi} \int_0^{2\pi} d\phi e^{-i n \phi} f_1
\]

and then calculate \( A_n \) as \( (B_n - \bar{B}_n)/2i \), and similarly for \( \bar{A}_n \). The result is (see the Appendix)

\[
A_0 = \text{Re} \left[ 1 + z^2 + 2z (\sin 2\kappa - i \beta \delta \cos 2\kappa) / \beta(z^2 - 1) \right], \tag{34}
\]

\[
\bar{A}_0 = \cos \eta \text{Re} B_0 + \sin \eta \text{Im} B_0, \tag{35}
\]

\[
\bar{A}_1 = -\frac{2i}{\beta} \cos 2\kappa \text{Im} \left[ \frac{ze^{-i\eta}}{z^2 - 1} - \frac{1}{\beta} \sin \eta - i \rho \bar{\sigma} \text{Re} \left[ \frac{z^2 + 1}{z^2 - 1} \right] \right] + 2i \rho \bar{\sigma} \sin 2\kappa \text{Re} \left[ \frac{z^2}{z^2 - 1} \right], \tag{36}
\]

where \( z = e^{\pi(\nu - i\omega)} \). These expressions are then to be inserted into the formula (30) for the acoustoelectric current.

In the last expressions we have neglected terms suppressed by factors of \( \beta^{-1} \) or \( (\beta \omega)^{-1} = \kappa^{-1} \), even if these were of larger power in \( e^{\pi \nu} \). This is justified as long as \( e^{\pi \nu} \) is smaller than \( \kappa \), or \( \nu \equiv (1/\pi) \ln \kappa \). This inequality sets the limit of applicability of the stationary phase approximation.

### IV. DISCUSSION

The final expressions (34)–(36) are not very simple, and we will try to understand how they behave as the external magnetic field is changed. Also, we will see how this affects the acoustoelectric current (30).

The Fourier coefficients show two kinds of oscillation:

- geometric oscillations and cyclotron resonance. We analyze the behavior of these as functions of increasing dimensionless parameter \( \omega \). This corresponds to decreasing effective magnetic field at fixed acoustic frequency \( \Omega \). As the magnetic field changes, the value of \( \nu = (1/\omega \kappa)^{-1} = \bar{\nu} \omega \) will also change. Here \( \bar{\nu} = (\Omega \tau)^{-1} \). Consider, for example, \( A_0 \), Eq. (34). First, there are the terms \( \sin 2\kappa \) and \( \cos 2\kappa \), which give geometric oscillations as for metals.

- oscillations arise as the difference in the phase of the sound wave at two points of the cyclotron orbit where the CF’s interact efficiently with the sound wave is changing. There is a complete oscillation as the diameter of the cyclotron orbit increases by one wavelength (2\( \kappa \) increases by 2\( \pi \)). On top of this come oscillations from the \( e^{i \pi \omega} \) terms. Since \( \beta \approx 30 \gg 1 \), these oscillations are much slower.

This behavior is seen in Fig. 2, which shows the expression

\[
\cos 2\kappa \text{Re} \left[ \frac{z}{z^2 - 1} \right] = \cos 2\kappa \text{Re} \left[ \frac{e^{\pi(\nu - i\omega)}}{e^{2\pi \nu} - 1} \right],
\]

which comes from the last term in the numerator of \( A_0 \), as a function of \( \omega \) for the case \( \beta = 30 \) and \( \bar{\nu} = 0.5 \). Our approximations are valid for \( \nu \leq (1/\pi) \ln \kappa \) and \( \kappa \gg 1 \). For these values of the parameters, this corresponds to the range 0.04 < \( \omega < 2.8 \). We will now concentrate on the most realistic limit of large damping \( \nu \gg 1 \) [remember, however, that we must have \( \nu < (1/\pi) \ln \kappa \) for our stationary phase approximation to be valid]. In this limit we have \( 1 < e^{\pi \nu} < e^{2\pi \nu} \), and the Fourier components can be expanded in powers of \( |z|^{-1} = e^{-\pi \nu} \). To lowest order we get

![FIG. 1. The path of the composite fermions relative to the acoustic wave. The interaction is efficient only at the turning points labeled 1 and 2.](image1)

![FIG. 2. The graph shows geometrical oscillations. The first cyclotron resonance is seen as a broad increase in the amplitude around \( \omega = 1 \).](image2)
\[A_0 = \beta^{-1} + 2\beta^{-1}e^{-\pi r}[\cos \omega \cos 2\kappa + \beta|\delta|\cos 2\kappa \sin(\omega + \eta)]\]
\[\tilde{A}_0 = \beta^{-1}\cos \eta + 2\beta^{-1}e^{-\pi r}[\cos(\omega - \eta)\sin 2\kappa + \beta|\delta|\cos 2\kappa \sin \omega]\]
\[\tilde{A}_1 = -\beta^{-1}\sin \eta - i\rho\tilde{\sigma} + 2ie^{-\pi r}|\delta|\sin 2\kappa \cos \omega - \beta^{-1}\cos 2\kappa \sin(\omega - \eta)].\]

We observe that the factor \(\rho\tilde{\sigma}\) repeatedly occurs in these expressions, and also in the expression (30) for the acoustoelectric current. We will therefore analyze this expression. To this end we recall the definition (10) of the quantity \(\delta\) and use the expressions obtained by Mirin and Wölfle for \(\sigma_{xz}\) and \(\sigma_{xy}\) [Eq. (8) of Ref. 13]. In the limit \(\kappa \gg 1\) we can expand the Bessel functions in asymptotic series in \(1/\kappa\) and obtain \(\rho\sigma_{xz} = -i/\zeta\), and \(\rho\sigma_{xy}\) is given by Eq. (12). This gives
\[\delta = -i\left[\zeta - \cos 2\kappa / \sin(\omega + i\nu)\right]^{-1}.\] (37)

Substituting reasonable values \(\omega/2\pi = 3\) GHz, \(\nu_i = 3 \times 10^5\) cm/s, \(m = 10^{-27}\) g, we obtain \(\zeta = 1/10\). We consider now two cases.

1. **At cyclotron resonance**, where \(\omega = n\) for an integer \(n\). In this case we have \(\sin \pi(\omega + i\nu) = \pm i\sinh \nu\), the sign being + or − when \(n\) is even or odd, respectively. In the denominator of Eq. (37) we then have a real contribution from \(\zeta\) and a purely imaginary contribution \(\pm i\cos 2\kappa/\sinh \nu\). Thus, we get
\[\rho\tilde{\sigma} = |\delta| \approx \zeta^{-1} \approx 10.\]
At the same time, since \(\sinh \pi \approx 10\) we will have
\[\rho\tilde{\sigma} \approx 1/\zeta \sqrt{2} \approx 7.\]
We see that \(\rho\tilde{\sigma}\) is not oscillating very much. Observe also that for moderately large \(\nu \approx 1\) the angle \(\eta\) may show considerable oscillations.

2. **Midway between cyclotron resonances**, \(\omega = n + 1/2\). Now \(\sin \pi(\omega + i\nu) = \pm \cosh \nu\), still with + for \(n\) even and − for \(n\) odd. In this case, the two terms in the denominator are both real, which means that for moderate \(\nu\) we can get a cancellation between the terms, which will make \(\rho\tilde{\sigma}\) large, and show large oscillations. Since we have neglected terms of order \(1/\kappa\) we expect these oscillations to be limited by \(\rho\tilde{\sigma} < \kappa\). Observe also that in this case we have \(\eta = -\pi/2\) not oscillating.

We remark that in both cases we have \(\rho\tilde{\sigma} \gg 1\). This means that as long as the damping factor \(e^{-\pi r}\) is larger than \(1/\beta\) we can neglect the \(1/\beta\) terms, and simplify the Fourier coefficients further. For \(\omega \approx 1\) this is the same condition as the range of validity of the stationary phase approximation.

We can now use Eq. (32) to calculate the total acoustoelectric current. The factor \(n^*\), being the ratio of the Fermi energy to the cyclotron energy in the effective magnetic field, will be large. We can then approximate \(j_{ue} = M^{-1}j_{0e}^*\). We have
\[\dot{M}^{-1} = \frac{1}{2}\nu_i^2 \left(\frac{1}{\nu} - 1\right)\]
which gives
\[\dot{M}^{-1} = -\frac{j_0}{2\nu_i^2} \left(\frac{|\rho\tilde{\sigma}|}{1}\right)\Im \tilde{A}_1.\] (38)
We see that all the current is in the \(y\) direction.

Taking into account all terms we find the \(\sim\)Hall angle\(\) given by
\[\tan \theta_H = \frac{j_y}{j_x} = -\frac{A_0 + \beta|\delta|\Im \tilde{A}_1}{|\delta|A_0 + \beta|\delta|\Re \tilde{A}_1} = \frac{|\beta|\delta|}{2|\delta|e^{-\pi r} \cos 2\kappa \sin \pi \omega - \sin \eta}.\] (39)

The two terms in the denominator are of the same magnitude.

It is customary to relate the acoustoelectric current to the absorption of acoustic energy per area. This is given by the expression
\[P = \langle \hat{H}_{f1} \rangle - \frac{1}{2} \Re \{\hat{H}^* f_1\},\]
where \(\langle \cdots \rangle\) denotes an average over the period of the acoustic wave, and the replacement indicated by the arrow is because of the use of complex notation.

\[H = \left[\frac{P + (e/c)A}{2m}\right]^2 - e\Psi\]
is the Hamiltonian, and the vector potential is given by $A_x = A_z = 0$, $A_y = -\delta(c/v_s)\Psi$. This gives rise to the CS magnetic field, and the solenoidal part ($y$ component) of the CS electric field. We then get

$$P = (g e^2/2)|\Psi|^2 \Omega(A_x - \beta \rho \tilde{\sigma} \text{Im} \tilde{A}_1),$$

where $g = m/2\pi\hbar^2$ is the density of states. Comparing with Eq. (38) in the limit $\nu > 1$ and within the range of applicability of the stationary phase approximation, we then have the relation between the acoustoelectric current, acoustic attenuation $\Gamma$, and sound intensity $I$,

$$I^{\text{ac}} = \frac{\mu_{xy}}{\mu_{xy}} = \frac{\Gamma I}{V_s}.$$ 

Here we have introduced the so-called traveling-wave mobility $\mu_{xy}^{\text{ac}}$ defined as $\mu_{xy}^{\text{ac}} = j_y^0/v_s/I \Gamma I$. In the lowest approximation $\mu_{xy} = 1/\rho e n_e$, and $n_e = g e \varepsilon$ is the composite fermion concentration. Note, however, that the Weinreich relation is only approximate as written here. It is valid only as long as we consider all the acoustoelectric current to be in the $y$ direction. By measuring the Hall angle one can determine how large a proportion of the current is turned by the magnetic field, and then reconstruct the Weinreich relation to take this into account.

It is instructive to compare the present expression for the Weinreich relation with that which would be expected in a normal metal, which can be expressed through the dc electron conductivity

$$j^{\text{ac}} = \left[ \begin{array}{c} \mu_{y}^{\text{ac}} \\ \mu_{xy}^{\text{ac}} \end{array} \right] \frac{\Gamma I}{V_s}. \tag{40}$$

The mobility $\mu_{xy}^{\text{ac}}$ can be found from the electron conductivity which, in turn, can be expressed in terms of the composite fermion dc conductivity (31) (see, e.g., Ref. 14). As a result,

$$\mu_{xy}^{\text{ac}} = \frac{1}{\rho e n_e} \left( \begin{array}{c} \nu / \nu_f^{*} \\ -1/\nu_f \end{array} \right) \frac{\Gamma I}{V_s}. \tag{41}$$

Here $\nu_f$ is the electron filling factor, which will be close to 1/2, and $\nu_f^{*}$ is the composite fermion filling factor (the filling factor in the effective field $B_0^{\text{CS}}$) which is much greater than 1. Looking at the $y$ components, we see that the two predictions agree at exactly $\nu_f = 1/2$. Since $\Gamma$ is some function of the magnetic field, it is natural to focus on the traveling-wave mobility $\mu_{xy}^{\text{ac}} = j_y^0/I \Gamma$. The normal metal expression would predict that this quantity should increase linearly with increasing magnetic field. Our result gives instead a constant value in the region around $\nu_f = 1/2$. In other words, within our approximations, the mobility $\mu_{xy}^{\text{ac}}$ is constant close to $\nu_f = 1/2$. We can do a similar comparison for the Hall angle. According to Eq. (39) this will be close to $\pi/2$, and we therefore expand around this point. The normal metal prediction would be

$$|\theta_H - \pi/2| = \sigma_{xy}^e / \sigma_{yy}^e = \nu \nu_f / \nu_f^{*} = (\omega_c / \tau)^{-1},$$

which decreases monotonically with increasing magnetic field. Our result gives

$$|\theta_H - \pi/2| = (\beta |\delta|)^{-1} (2 |\delta| \epsilon^{-\pi e} \cos 2 \sin \pi \omega - \sin \eta),$$

which will show both geometric oscillations and cyclotron resonance. In the limit $\nu_f \rightarrow 1/2$ we have $\omega, \nu \gg 1$, which gives $|\delta| = 1/\zeta$ and $\sin \eta = -1$. The Hall angle simplifies then to

$$|\theta_H - \pi/2| = (\beta |\delta|)^{-1} = \hbar q / 2 p_F .$$

This limit is outside the range of applicability of our approximation. The result is, however, correct, as can be checked by a direct calculation for $\nu_f = 1/2$.

By measuring the acoustoelectric current one can use Eq. (30) to determine the amplitude $\psi_0$ of the effective potential acting upon the composite fermions. As a result, using the theory which relates the effective potential to the coupling constant, one can determine the coupling constant $C$ between the piezoelectric field and the composite fermions. Further, we note that, whereas the non-CS part of the acoustoelectric current can be expressed in terms of the complex, longitudinal conductivity for electrons through the Weinreich relation, this is not possible for the CS part. This means that more information on the system is available if one can measure the acoustoelectric current compared to the situation where one measures only attenuation and velocity shifts. For example, if one measures attenuation and velocity shift, one can determine the complex longitudinal electronic conductivity $\sigma_{xx}^e$, but in general it is impossible to extract the composite fermion conductivities from this. Measurement of the acoustoelectric current will give one additional relation which enables one to determine one more parameter in the composite fermion theory.

V. CONCLUSIONS

We have shown that, in the cases where the acoustic wave is sensitive to the properties of the electron gas, the ac magnetic Chern-Simons field created by the induced density fluctuations in the electron gas creates forces that are of strength comparable to the direct perturbation, and that the same is true for the $x$ component of the electric Chern-Simons field. The $y$ component of this field is smaller but plays an important role because it acts along the composite fermion trajectory at the points of strong interaction. At the second order, necessary for the calculation of the acoustoelectric current, the Chern-Simons field will be relevant, and we expect a violation of the Weinreich relation. This violation can be detected in measurements of the Hall angle. This gives additional information on the composite fermion system not attainable by linear response measurements. Further, the acoustoelectric current gives a direct measure of the strength of the piezoelectric field as experienced by the composite fermions, and may therefore be used to extract the value of the coupling constant between the acoustic wave and the composite fermions.
APPENDIX: ASYMPTOTIC EXPANSIONS

The stationary phase approximation is used to extract the leading term in the asymptotic expansions for both the conductivity and the acoustoelectric current. In general, if we have a function

$$F(\kappa) = \int_a^b f(x)e^{i\kappa g(x)}dx,$$

(A1)

the leading terms in the asymptotic expansion will be

$$F(\kappa) = F^{(1)}(\kappa) + F^{(2)}(\kappa),$$

$$F^{(1)}(\kappa) = \sum_{j=1}^{\infty} \frac{e^{i\kappa g(x_j)}}{\sqrt{|g''(x_j)|}} f(x_j) e^{i\kappa g(x_j)},$$

where the sum is over all the stationary points of the exponent $[g'(x_j)=0]$, and the sign is $\pm$ according as $g''(x_j) \geq 0$.

$$F^{(2)}(\kappa) = \frac{1}{i\kappa} \left[ \frac{f(b)}{g'(b)} - \frac{f(a)}{g'(a)} \right]$$

is the contribution from the end points (see, e.g., Ref. 24). For example to find $I$ we have to evaluate

$$I = \int_0^{2\pi} d\phi e^{-i\phi} f_1(\phi).$$

(A2)

Inserting the expression (17) for $f_1$ we get among other terms the integral

$$I = M \int_0^{2\pi} d\phi \cos \phi \int_0^{\phi+2\pi} d\phi' e^{i(\phi'-\phi) + i\kappa [g(\phi)-g(\phi')]}.$$

$$M = i\omega e^{-2\pi(\nu-i\omega)}$$

$$g(\phi) = \sin \phi - \phi/\beta.$$

(A3)

Performing first the integral over $\phi'$ we get

$$I' = \int_0^{\phi+2\pi} d\phi e^{i(\phi'+\kappa[\sin \phi'-\phi'])/\beta}$$

$$- \sqrt{\frac{2\pi}{\kappa}} \sum_{j} e^{i\phi_j' + i\kappa g(\phi_j') - i\pi/4 \text{sgn}(\sin \phi')}$$

$$- \frac{i}{\kappa \cos(\phi) - i/\beta} e^{i\phi + i\kappa g(\phi)} [e^{2\pi(\nu-i\omega)} - 1].$$

(A4)

Here $\phi_j'$ are the stationary points of $g(\phi'), \cos \phi_j' = 1/\beta$, which are in the range $\phi \leq \phi_j' \leq \phi + 2\pi$. The integral over $\phi$ can then be evaluated as

$$I = M \int_0^{2\pi} d\phi \cos \phi e^{-i\phi - i\kappa g(\phi)}$$

$$\times \sum_{j} e^{i\phi_j' + i\kappa g(\phi_j') - i\pi/4 \text{sgn}(\sin \phi')} + \frac{1}{\beta} \int_0^{2\pi} d\phi \cos \phi - i/\beta$$

$$= 2\pi M \kappa \beta$$

$$\times \sum_{i,j} e^{i\phi_i' - i\kappa g(\phi_i') - i\pi/4 \text{sgn}(\sin \phi_i')} - i\pi/4 \text{sgn}(\sin \phi_i') - i\pi/4 \text{sgn}(\sin \phi_j')$$

$$+ \frac{1}{\beta} \int_0^{2\pi} d\phi \cos \phi - i/\beta,$$

where the critical points $\phi_j$ are the same as before. In the sum over the critical points we can then set $\phi_i, \phi_j' = \pi/2 + 2\pi n$ in the exponent since $\beta > 1$. Also, critical points that lie on the boundary of the domain of integration are given half their normal value. In the second integration, the contributions from the end points have been neglected, as they will be of higher order. The sum then is

$$\sum_{i,j} \cdot \cdot \cdot = 1 + e^{2\pi(\nu-i\omega)} + 2e^{\pi(\nu-i\omega)} \sin 2\kappa.$$

The last integral must be expanded in powers of $1/\beta$, yielding

$$\int_0^{2\pi} d\phi \cos \phi - i/\beta = \int_0^{2\pi} d\phi \left( 1 + \frac{1}{\beta} \cos \phi + \cdot \cdot \cdot \right) = 2\pi,$$

since $P \int_0^{2\pi} d\phi / \cos \phi = 0$, where $P$ is the principal value. Combining these results we get one contribution to $B_1$, the rest of the $B_n$ being calculated in a similar way.

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17 The importance of the Fermi liquid effects for the acoustoelectric effect was pointed out to us by Ady Stern. We are grateful to him for a useful discussion.


