Low-frequency shot noise in phonon-assisted resonant magnetotunneling

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(Received 4 September 1996)

A theory of low-frequency shot noise in a resonant tunneling double-barrier device has been worked out. The calculations have been carried out within the coherent tunneling model; only the electron-phonon interaction inside the quantum well is taken into account. The average current \( I_{dc} \) and the noise spectrum \( S(\omega) \) are expressed in terms of intrawell two-, three-, and four-electron Green’s functions. The expression is valid, in principle, for arbitrary temperatures and for any type of intrawell scattering. We use it to analyze excess noise in phonon-assisted resonant tunneling through a double-barrier device at zero temperature and to the lowest order in the electron-phonon interaction. Our results show that the suppression of excess noise due to a correlation in electron transport is expected for both elastic and inelastic tunneling. In particular, we note that the contribution of the elastic processes to the ratio \( S(\omega)/I_{dc} \) is very sensitive to asymmetry of the tunneling barrier heights. Such a sensitivity is reduced for phonon-assisted processes. [S0163-1829(97)04604-3]

I. INTRODUCTION

The discreteness of an electronic charge causes fluctuations in an electrical current flowing through a conductor. Such fluctuations, known as shot noise, contain additional information on the kinetic properties of the transport, which is not given by the average time-independent current \( I_{dc} \). This kind of noise is sensitive to a correlation between electrons passing through a device. The correlation suppresses the noise below its limiting value known as full shot noise, \( S(0) = 2|eI_{dc}| \) (at \( T = 0 \)). Here \( S(\omega) \) is the autocorrelation function of the fluctuations in the current at frequency \( \omega \), while \( e \) is the electron charge. In a mesoscopic conductor having several independent modes of transverse motion (channels), the noise is determined by the partial transmission probabilities \( T_m \) for \( m \)th mode as \( S(0) \propto \sum_m T_m (1 - T_m) \), while the conductance goes as \( G \propto \sum_m T_m \). Suppression of the shot noise is thus expected in a phase coherent system when the transmission probabilities are of order 1. The situation is of course more complicated in the presence of an electron-phonon interaction, which leads to inelastic processes and, in particular, to scattering between different transverse channels.

Our aim is to discuss the influence of the electron-phonon interaction on the noise in tunneling current through a double-barrier resonant-tunneling structure (DBRTS) (Fig. 1), which is an example of a two-terminal device with correlated electron transport. Such structures have been the focus of many experimental and theoretical investigations since their conception by Tsu and Esaki and the realization of negative differential resistance by Sollner et al. Many important characteristics of DBRTS’s have been intensely studied (for a review see Ref. 6 and references cited therein). In particular, several theoretical27–15 and experimental16–19 investigations of electric noises in DBRTS’s were performed.

We study the situation when an external magnetic field \( B \) is applied perpendicular to the interfaces and parallel to the tunneling current \( I \). In this configuration, due to the magnetic field, the tunneling problem in some important cases can be reduced to an effectively one-dimensional one. As a result, the fine structure of current- and noise-voltage curves becomes much more pronounced. In particular, phonon-assisted resonant tunneling manifests itself in a stronger and clearer way that allows one to use it as a tool to study the electron-phonon interaction. The main advantage of magnetotunneling in connection with the phonon-assisted transport is that one avoids the so-called electron recoil effect20 tending to suppress and smear out phonon replicas. Indeed, it turns out that the effective electron LO-phonon coupling increases with increasing magnetic field21,22 \( g^2 \propto B^{1/2} \).

A theory of phonon-assisted resonant tunneling through a DBRTS (Refs. 23 and 24) (formulated for the case of absence of an external magnetic field) relates the average current to the intrawell electron Green’s function. Below we develop a similar representation for excess noise.

FIG. 1. Schematic illustration of the double-barrier resonant magnetotunneling device.
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The paper is organized as follows. Section II describes the model employed. The basic expressions for the current and noise profiles are derived in Sec. III. In Sec. IV those expressions are used for the calculation of the shot noise in the case of phonon-assisted tunneling through DBRTS’s. The details of calculations are clarified in the Appendixes. In Appendix A a quantum-mechanical expression for the current is derived in a form that is then employed in the following calculations. In Appendix B we explain how the average current and noise are reduced to the intrawell Green’s functions. The lowest order in the electron-phonon interaction expansions for those Green’s functions are analyzed in Appendix C. Appendix D shows how the spectral functions, appearing in the noise expressions for certain special cases, can be expressed as appropriately normalized δ functions.

II. MODEL

As a model of a DBRTS we consider a GaAs$^{+}$/Al$_{0.3}$Ga$_{0.7}$As/GaAs/Al$_{0.3}$Ga$_{0.7}$As/GaAs$^{+}$ structure, with the barriers’ and the wall’s widths of the order of 40–60 Å and the barriers’ height about 300 meV. An external magnetic field is applied perpendicular to the interfaces in the positive z direction, B||z (Fig. 1). It is assumed that only one quasibound state χ(z) exists in the well with energy $ε_a$. The zero reference energy is set to the bottom of the conduction band of the quantum well. Consequently, the conduction-band minimum of the emitter is in the symmetric case raised by $E_{V j}$, while the collector band edge is lowered by $−E_{V j}/2$. We assume the same effective masses in the leads and in the well and also the scattering due to the interface roughness and impurities is ignored.

Under the Landau gauge $\mathbf{A}=(0, Bx, 0)$, the wave functions and corresponding energy levels can be specified by the set of quantum numbers $\alpha=(n, k_j)$ as

$$\phi_{\alpha}(r) = (L_x L_z)^{-1/2} \exp(ik_j y) \varphi_n(x + L^2 k_j) \chi(z),$$

$$E_{\alpha} = E_n = \epsilon_0 + h \omega_c (n + \frac{1}{2}).$$

(1)

Here $\varphi_n(x)$ denote harmonic-oscillator Landau states with Landau state index $n$, $\omega_c = eB/m^*$ is the cyclotron frequency, and $l=(h/eB)^{1/2}$ is the Landau magnetic length. Electron states in the leads are specified by the quantum numbers $\beta=(m, k_{jy}, k_{jz})$, where $j = e$ (c) refers to the emitter (collector), respectively. The corresponding wave functions and energy levels under the bias $eV$ are then given as

$$\phi_{j\beta}(r) = (L_x L_z)^{-1/2} \exp(ik_{jy} y + ik_{jz} z) \varphi_m(x + L^2 k_{jy}),$$

$$E_{j\beta} = \frac{(h k_{jz})^2}{2m^*} + h \omega_c (m + \frac{1}{2}) + a_j eV.$$

(2)

where $0 < a_j < 1$ and $a_c = a_e = 1$. For simplicity, in the following we assume $a_0 = 0.5$.

In the presence of electron-phonon interaction, our system is described by the model Hamiltonian

$$\mathcal{H} = \mathcal{H}_e + \mathcal{H}_{ph} + \mathcal{H}_{eph}.$$

(3)

The electronic part is given by

$$\mathcal{H}_e = \sum_{j, \beta} E_{j\beta} c_{j\beta}^\dagger c_{j\beta} + \sum_{\alpha} E_{\alpha} c_{\alpha}^\dagger c_{\alpha} + \sum_{j, \alpha, \beta} [V_{j\beta\alpha} c_{j\beta}^\dagger c_{\alpha} + H.c.],$$

(4)

where the tunnel matrix elements $V_{j\beta\alpha}$ have to be calculated using the eigenstates listed above. Usually, the energy distances between the resonant level in the well and the tops of the barriers are much greater than $|V_{j\beta\alpha}|$. In this case the tunneling matrix element can be considered as a smooth function of the energy in comparison with the energy dependence of the quantity

$$g_j(n, c) = \sum_{k_{jy}} \delta(c - E_{j\beta}).$$

(5)

proportional to the density of states in the lead j. In the present calculations $V_{j\beta\alpha}$ always appear as products by $g_j(c, c)$ and consequently $V_{j\beta\alpha}$ are assumed to be constants characterizing each tunneling barrier. Moreover, since the interfaces are assumed to be perfect, the quantum numbers $n$ and $k_j$ are conserved during the tunneling process and consequently we write

$$V_{j\beta\alpha} = V_j \delta_{m\alpha} \delta(k_j - k_{jy}).$$

(6)

In the following we assume that the electrons interact only with longitudinal optical (LO) phonons. Neglecting weak phonon dispersion, one can write the resulting phonon Hamiltonian as

$$\mathcal{H}_{ph} = h \omega_0 \sum_q b_q^\dagger b_q,$$

(7)

where $h \omega_0$ is the phonon energy ($\sim 36$ meV). Following the conventional assumption, we take into account only the electron-phonon interaction inside the well. This assumption is usually valid because the electron must spend a long enough time inside the well to manifest pronounced resonant-tunneling features. In terms of the Fröhlich Hamiltonian, the electron-phonon interaction can then be expressed as

$$\mathcal{H}_{eph} = \sum_{\alpha_1, \alpha_2, q} M_{\alpha_1, \alpha_2} (q) (b^\dagger_q + b_q) c_{\alpha_2}^\dagger c_{\alpha_1},$$

(8)

where $\alpha_1=(n_1, k_{jy_1})$, $\alpha_2=(n_2, k_{jy_2} + q_{jy})$, and

$$M_{\alpha_1, \alpha_2} (q) = \frac{M}{q \sqrt{\nu_0}} \int \phi_{\alpha_2}^*(r) e^{i q \cdot r} \phi_{\alpha_1}(r) dr,$$

$$M^2 = 4 \pi e^2 h \omega_0 \left( \frac{1}{\varepsilon_0} - \frac{1}{\varepsilon_z} \right).$$

(9)

III. BASIC EXPRESSIONS FOR THE AVERAGE CURRENT AND NOISE

The current density operator can be expressed through the Heisenberg field operator $\hat{\Psi}_j(r, t)$ as

$$\hat{j}_j(r, t) = \frac{\hbar}{2m^*} \left( \hat{\Psi}_j^\dagger \nabla \hat{\Psi}_j - \nabla \hat{\Psi}_j^\dagger \hat{\Psi}_j \right).$$

(10)
In the scattering representation\textsuperscript{26–28} it is common to express the field operators in Eq. (10) in terms of creation and annihilation operators of incoming and outgoing states,

\[
\hat{\Psi}_j(r,t) = \sum_{\beta} \left[ \phi_{j\beta}^{in}(r)c_{j\beta}(t) + \phi_{j\beta}^{out}(r)b_{j\beta}(t) \right].
\] (11)

The operator \(c_{j\beta}\) annihilates an incident electron in the state

\[
\phi_{j\beta}^{in} = \phi_{j\beta}, \quad \beta = (m,k_j, \eta_j|j_{z_j})
\]

where \(\eta_j = 1\) and \(\eta_j = -1\). Similarly, outgoing modes

\[
\phi_{j\beta}^{out} = \phi_{j\beta}, \quad \beta = (m,k_j, \eta_j|j_{z_j})
\]

are annihilated by \(b_{j\beta}\), which is related to \(c_{j\beta}\) through the scattering matrix

\[
b_{j_1\beta_1} = \sum_{j_2\beta_2} S_{j_1\beta_1 j_2\beta_2} c_{j_2\beta_2}. \tag{14}
\]

In a DBRTS, electrons tunnel from the leads inside the well. Then they can either tunnel out of the well directly or experience inelastic scattering by phonons before tunneling out. To relate these processes to the intrawell Green’s functions it is convenient to characterize the leads' modes by the total energy \(E = E_{j\beta}\) rather than by the longitudinal momentum \(k_{jz}\). Such a replacement is equivalent to the renormalization of the annihilation/creation operators as\textsuperscript{28}

\[
c_{j\beta} \rightarrow c_{ja}(e) = \left( \frac{L_z}{2\pi U_{ja}(e)} \right)^{1/2} c_{j\beta},
\]

where \(U_{ja}(e)\) is the longitudinal velocity of an electron in the lead \(j\), moving in the quantum channel \(\alpha = (n,k_{jy})\) with energy \(e\). The annihilation/creation operators of the outgoing states are transformed similarly and we have

\[
b_{j,\alpha_1}(e_1) = \sum_{j_2,\alpha_2} \int d\varepsilon_2 S_{j\alpha_1 j_2\alpha_2}(e_1,\varepsilon_2) c_{j_2\alpha_2}(\varepsilon_2). \tag{16}
\]

The expression (11) for the field operators is now converted into an integral over \(e\) as

\[
\hat{\Psi}_j(r,t) = \sum_{\alpha} \int d\varepsilon \left[ \phi_{j\beta}^{in}(r)c_{ja}(\varepsilon,t) + \phi_{j\beta}^{out}(r)b_{ja}(\varepsilon,t) \right]. \tag{17}
\]

Substituting Eq. (17) into Eq. (10) and integrating over the \(x\)-\(y\) cross section, one obtains the following expression for the current operator\textsuperscript{28} (Appendix A):

\[
\hat{I}_j(t) = \frac{e}{\hbar} \sum_{\alpha} \int d\varepsilon_1 d\varepsilon_2 \left[ c_{ja}(\varepsilon_1,t)c_{ja}(\varepsilon_2,t) - b_{ja}(\varepsilon_1,t)b_{ja}(\varepsilon_2,t) \right]. \tag{18}
\]

The average current (which is the same in both leads) can be found by taking the quantum and thermal average of Eq. (18) in the presence of an electron-phonon interaction. Expressing \(b\) operators in the products \(b^\dagger b\) in terms of \(c\) operators according to Eq. (16), we thus arrive at an expression that includes the so-called intrawell two-particle transmission Green’s function\textsuperscript{24} \(K^A\) (Appendix B)

\[
I_{dc} = \frac{2e}{\hbar} \sum_{\mu_1,\mu_2} K^A_{\mu_1\mu_2} \left[ f(c_{\mu_1}(\varepsilon_1)\gamma_c(\mu_1)\gamma_c(\mu_2) - f(c_{\mu_1}(\varepsilon_1)\gamma_c(\mu_1)\gamma_c(\mu_2)) \right], \tag{19}
\]

where we have introduced the notation

\[
\mu_i = (\alpha_i,\varepsilon_i), \quad \sum_{\mu_i} = \sum_{\alpha_i} \int d\varepsilon_i. \tag{20}
\]

The escape rate from the intrawell Landau state \(n\) to the lead \(j\), \(\gamma_j(\mu) = \gamma_j(n,\varepsilon)\), is in the case of a dissipation-free three-dimensional emitter characterized by the tunneling strength \(Y_j\) as

\[
\gamma_j(\mu) = 2\pi |V_j|^2 g_j(n,\varepsilon) = \frac{Y_j}{\sqrt{E_a - a_e V - \hbar \omega_0(n + 1/2)}}, \tag{21}
\]

while

\[
K^A_{\mu_1\mu_2} = \int \frac{dt dt_1 dt_2}{2\pi \hbar} e^{i[\varepsilon_1 t - \varepsilon_1 t_1 + \varepsilon_2 t_2 - \varepsilon_2 t]} \Theta(s_1) \Theta(s_2) \times \langle c_{\alpha_1}(t - s_1)c_{\alpha_2}^\dagger(t)c_{\alpha_2}(s_2)c_{\alpha_1}(0) \rangle. \tag{22}
\]

Spin-flip effects have been ignored in our analysis and hence a spin degeneracy factor of 2 has been introduced in Eq. (19). Since both leads are assumed to be in a state of local thermal equilibrium, \(f_{\varepsilon_e}(\varepsilon)\) is simply the Fermi function for the emitter (collector), \(f_\varepsilon(\varepsilon) = \left[ e^{E - \varepsilon_e - a_e V}/k_B T + 1 \right]^{-1}\).

The noise spectrum is defined as the Fourier transform of the current-current auto-correlation function\textsuperscript{29}

\[
S(\omega) = \frac{1}{2} \int_{-\infty}^{\infty} S(t)e^{i\omega t} dt = 4 \int_0^\infty S(t)\cos(\omega t) dt, \tag{23}
\]

where \(S(t)\) is the quantum and thermal average of the current-current anticommutator,

\[
S(t) = \frac{1}{2} \langle \{ \hat{I}(t), \hat{I}(0) \} \rangle = \frac{1}{2} \langle [\hat{I}(t), \hat{I}(0)] \rangle - I_{dc}^2. \tag{24}
\]

In noise calculations, the time dependence of the tunneling current is important and hence the role of junction capacitances should, in principle, be taken into account. In the capacitively symmetric case, the measured current is simply given as the average of the emitter and collector tunneling currents as\textsuperscript{30} \(I(t) = \left[ I_e(t) + I_c(t) \right]/2\), and this is the model used below. Another prescription \(I(t) = I_e(t) = I_c(t)\), which ignores the charge accumulation, was used in Ref. 8. Those two approaches lead to different frequency dependences of the shot noise. However, at low frequency \(\omega \rightarrow 0\), they are in the elastic case equivalent.

The shot noise is determined by the square of the current operator. Consequently, instead of the two-electron Green’s function (as in the case of the average current) we now arrive at a more complicated picture with contributions from different two-, three-, and four-electron Green’s functions. Ex-
panding Eq. (23) with the substitution of the current operators (18) and with the inclusion of a spin degeneracy factor of 2, we arrive at (Appendix B)

\[ S(0) = \frac{e^2}{\hbar} \sum_{\mu_1, \mu_2} \left( F^1_{\mu_1, \mu_2} K^A_{\mu_1, \mu_2} - F^2_{\mu_1, \mu_2} \text{Re} \{ K^B_{\mu_1, \mu_2} \} \right) 
- \sum_{\mu_3} \left( F^3_{\mu_1, \mu_2, \mu_3} \text{Im} \{ L^A_{\mu_1, \mu_2, \mu_3} \} + F^4_{\mu_1, \mu_2, \mu_3} \text{Im} \{ L^B_{\mu_1, \mu_2, \mu_3} \} \right) 
+ \sum_{\mu_3, \mu_4} F^5_{\mu_1, \mu_2, \mu_3, \mu_4} \text{Re} \{ M^A_{\mu_1, \mu_2, \mu_3, \mu_4} \} \right), \tag{25} \]

This result includes the two-particle Green’s function \( K^A_{\mu_1, \mu_2} \) defined by Eq. (22) as well as the function

\[ K^B_{\mu_1, \mu_2} = -\int \frac{dt \, ds \, ds_2}{2 \pi \hbar} e^{i(\varepsilon_2 - \varepsilon_1) t + \varepsilon_2 s_1 + \varepsilon_1 s_2 + \varepsilon_1 \varepsilon_2 / 2} \Theta(s_1) \Theta(s_2) \times \langle c_{a_2} (t + s_1) c_{a_1}^\dagger (t) c_{a_2} (s_2) c_{a_1}^\dagger (0) \rangle. \tag{26} \]

It furthermore includes two different three particle Green’s functions \( L^A \) and \( L^B \) defined as

\[ L^A_{\mu_1, \mu_2, \mu_3} = \int \frac{dt_1 \, dt_2 \, ds \, ds_3}{4 \pi \hbar^2} \Theta(s_1) \Theta(s_2) \Theta(s_3) \times \langle c_{a_1} (t_1 - s_1) c_{a_2}^\dagger (t_1) c_{a_2} (t_2 + s_2) c_{a_2}^\dagger (t_2) c_{a_2} (s_3) c_{a_1}^\dagger (0) \rangle, \tag{27} \]

\[ L^B_{\mu_1, \mu_2, \mu_3} = \int \frac{dt_1 \, dt_2 \, ds \, ds_3}{4 \pi \hbar^2} \Theta(s_1) \Theta(s_2) \Theta(s_3) \times \langle c_{a_2} (t_1 - s_1) c_{a_1}^\dagger (t_1) c_{a_2} (t_2 - s_2) c_{a_2}^\dagger (t_2) c_{a_2} (s_3) c_{a_1}^\dagger (0) \rangle, \tag{28} \]

and finally the four-particle Green’s function

\[ M^A_{\mu_1, \mu_2, \mu_3, \mu_4} = \int \frac{dt_1 \, dt_2 \, ds \, ds_3 \, ds_4}{8 \pi \hbar^4} \Theta(s_1) \Theta(s_2) \Theta(s_3) \Theta(s_4) e^{i \phi} \times \langle c_{a_3} (t_1 - s_1) c_{a_3}^\dagger (t_1) c_{a_1} (t_2 - s_2) c_{a_1}^\dagger (t_2) c_{a_3} (t_3 - s_3) c_{a_3}^\dagger (t_3) c_{a_3} (s_4) c_{a_1}^\dagger (0) \rangle, \tag{29} \]

where \( \phi = (\varepsilon_3 - \varepsilon_1) t_1 + (\varepsilon_2 - \varepsilon_1) t_2 + (\varepsilon_4 - \varepsilon_2) t_3 + \varepsilon_3 s_1 + \varepsilon_1 s_2 + \varepsilon_3 s_3 + \varepsilon_2 s_4 \). Also, Eq. (25) includes thermal factors as defined below using the shorthand notations \( \gamma_{j_n} = \gamma_{j_n}(\mu_n) \) and \( f_{j_n} = f_{j_n}(\mu_n) \)

\[ F^1_{\mu_1, \mu_2} = \sum_{\langle ij \rangle} \left( \eta_{j_1} \eta_{j_2} - \eta_{j_2} \eta_{j_1} \right) \gamma_{j_1} \gamma_{j_2} f_{j_1} (1 - f_{j_2}), \]

\[ F^2_{\mu_1, \mu_2} = \sum_{\langle ij \rangle} \eta_{j_1} \eta_{j_2} \gamma_{j_1} \gamma_{j_2} [ \gamma_{j_1} f_{j_1} (1 - f_{j_2}) + f_{j_2} (1 - f_{j_1})], \]

\[ F^3_{\mu_1, \mu_2, \mu_3} = 2 \sum_{\langle ij \rangle} \eta_{j_1} \eta_{j_2} \gamma_{j_1} \gamma_{j_2} \gamma_{j_3} f_{j_1} f_{j_2} (1 - f_{j_3}), \]

\[ F^4_{\mu_1, \mu_2, \mu_3} = 2 \sum_{\langle ij \rangle} \eta_{j_1} \eta_{j_2} \gamma_{j_1} \gamma_{j_2} \gamma_{j_3} f_{j_1} (1 - f_{j_3}), \]

\[ F^5_{\mu_1, \mu_2, \mu_3, \mu_4} = \sum_{\langle ij \rangle} \eta_{j_1} \eta_{j_2} \gamma_{j_1} \gamma_{j_2} \gamma_{j_3} \gamma_{j_4} f_{j_1} (1 - f_{j_4}). \tag{30} \]

IV. THE SOLUTION IN THE CASE OF ZERO-TEMPERATURE AND LOWEST-ORDER ELECTRON-PHONON COUPLING

To solve our problem, the task is to expand the Green’s functions (22) and (26)–(29). Here we analyze for simplicity only the zero-temperature case. Also, we restrict ourselves to the lowest order in the electron-phonon interaction. In this case the three- and four-particle Green’s functions reduce to products of one- and two-particle Green’s functions according to Fig. 2. While expanding the Green’s functions, we make use of the fact that after summation over the phonon wave vectors, as well as over the “internal” electron wave vector, the electron-phonon coupling factor can be written as

\[ g^2 = M^2 / 4 \pi^2 l, \tag{31} \]

where \( g^2 = M^2 / 4 \pi^2 l \), while

\[ A_{n_{1n_2}} = \sqrt{2} \pi \frac{P_1 P_2}{P_1} \int_0^\infty d \xi \delta^{2 (p_1 - p_2)} e^{-\xi^2} \left[ L^p_2 (\xi^2) \right]^2, \tag{32} \]
A nis what makes magnetotunneling a convenient tool to study

\[ \Sigma_R(n,e) = \sum_{\nu_1} g^2 A_{\nu_1} [(1 + N(h\omega_0)] G_R^0(n_1, e - h\omega_0) \]

\[ + N(h\omega_0) G_R^0(n_1, e + h\omega_0) \].

Here \( N(h\omega) \) is the Planck distribution of phonons and \( G_R^0 \) is the zeroth-order (\( \Sigma_R = 0 \)) electron Green’s function dressed by the total tunneling escape rate \( \gamma(n,e) = \gamma_c(n,e) + \gamma_r(n,e) \). In our model, we focus on single-electron tunneling processes neglecting electron-electron interactions in the well. Consequently, Fermi effects of the self-energies have been ignored and occupation probabilities of the resonant levels are neglected, an approximation that is certainly good in the vicinities of phonon replicas. According to the above results, the \( k_r \) summation of our two-, three-, and four-electron Green’s functions are easily performed and result in the multiplication of the result by the magnetic degeneracy factor \( g_B = L_z L / 2 \pi l^2 \):

\[ \sum_{\{k_r\}} K_{\mu_1\mu_2}^{A/B} = g_B K_{\nu_1\nu_2}^{A/B}, \quad \sum_{\{k_r\}} L_{\mu_1\mu_2\mu_3}^{A/B} = g_B L_{\nu_1\nu_2\nu_3}^{A/B} \]

\[ \sum_{\{k_r\}} M_{\mu_1\mu_2\mu_3\nu} = g_B M_{\nu_1\nu_2\nu_3\nu}. \]

Here and below we use the notations \( \nu_i = (n_i, e_i) \), \( \Sigma_r = \Sigma_f \int d\epsilon_i \). The two-electron Green’s functions are expanded according to Fig. 2(a). This expansion is done according to the standard perturbation theory,25,24 where the electron occupation probability of the intrawell levels is negligible. Consequently, the averages are taken over phonon states and we arrive at

\[ K_{\nu_1\nu_2}^{A} = K_{\nu_1\nu_2}^{A(0)} + K_{\nu_1\nu_2}^{A(1)} = |G_R(\nu_1)|^2 \left[ \delta_{n_1,n_2} \delta(\epsilon_1 - \epsilon_2) \right. \]

\[ + g^2 A_{n_1n_2} \left. \left[ 1 + N(\epsilon_1 - \epsilon_2) \right] |G_R(\nu_2)|^2 \right] \right] D(\epsilon_1 - \epsilon_2), \]

\[ K_{\nu_1\nu_2}^{B} = K_{\nu_1\nu_2}^{B(0)} + K_{\nu_1\nu_2}^{B(1)} = G_R(\nu_1 \nu_2) \left[ \delta_{n_1,n_2} \delta(\epsilon_1 - \epsilon_2) \right. \]

\[ + g^2 A_{n_1n_2} |N(\epsilon_1 - \epsilon_2) G_R(\nu_2)|^2 D(\epsilon_1 - \epsilon_2), \]

where \( D(\epsilon) = 2 \pi \left[ \delta(\epsilon - \omega_0) - \delta(\epsilon + \omega_0) \right] \) is the spectral function of LO phonons. With the substitution of Eq. (36) into Eq. (19) at \( T = 0, \)

\[ L_{p_1}^{p_1} \] is the Laguerre polynomial, and \( p_1 \) (or \( p_2 \)) is the larger (or smaller) of \( (n_1, n_2) \). The effective electron-phonon coupling thus increases with the magnetic field \( g^2 \ll B \). This is what makes magnetotunneling a convenient tool to study electron-phonon interaction. The Landau level factor \( A_{n_1n_2} \) is a dimensionless factor of order 1.
\[ I_{dc} = \frac{2e^2g_B}{h} \sum_{\nu_1, \nu_2} |G_R(\nu_1)|^2 \{ \delta_{\nu_1, \nu_2} \gamma_e(\nu_1) \gamma_e(\nu_1) \}
\times [f_e(\nu_1) - f_c(\nu_1)] + \delta(\nu_1 - \nu_2 - \hbar \omega_0) \]
\[ \times g^2 A_{\nu_1 \nu_2} |G_R(\nu_2)|^2 [f_e(\nu_1) \gamma_e(\nu_1) \gamma_e(\nu_2)]
- f_c(\nu_1) \gamma_e(\nu_1) \gamma_e(\nu_2) \]. \]

(37)

Let us now turn to the shot noise and expand Eq. (25) with the substitution of the Green’s functions according to Fig. 2 (Appendix C). While doing this, the noise is regrouped into three parts

\[ S(0) = S_A(0) + S_B(0) + S_C(0). \]

(38)

\( S_A \) is the contribution from the first group of diagrams in Fig. 2 when no phonons are interchanged between the single-electron lines. Consequently, \( S_A \) reduces to a product of single-electron Green’s functions

\[ S_A(0) = \frac{e^2g_B}{h} \sum_{\nu} [f_e(\nu) - f_c(\nu)]^2 \gamma_e(\nu) \gamma_e(\nu) [4|G_R(\nu)|^2]
- A(\nu)^2 + [\gamma_e(\nu) - \gamma_c(\nu)]^2 |G_R(\nu)|^2, \]

(39)

where \( A(\nu) = -2 \text{Im} G_R(\nu) \) is the electron spectral function. When the electron-phonon interaction is omitted \( (g = 0) \) \( S_A \) is the only contribution to noise and our formulas reduce to the familiar ones \(^1-3\)

\[ I_{dc} = \frac{2e^2g_B}{h} \sum_{\nu} [f_e(\nu) - f_c(\nu)] T_0^n, \]

(40)

\[ S(0) = \frac{4e^2g_B}{h} \sum_{\nu} [f_e(\nu) - f_c(\nu)]^2 T_0^n [1 - T_0^n] \]

(41)

where \( T_0^n = \gamma_e(\nu) \gamma_c(\nu)|G_R^0(\nu)|^2 \) is the transmission probability for the \( n \)-th Landau channel. In the presence of an electron-phonon interaction these expressions are modified due to the dressing of single-particle Green’s functions and corresponding renormalization of the spectral function \( A(\nu) \).

\( S_B \) is the contribution from two-particle Green’s functions that originates from the phonon interchange between the electron lines within the same \( I(t) \) factor in Eq. (24) (the second group of diagrams in Fig. 2),

\[ S_B(0) = \frac{2e^2g_B}{h} \sum_{\nu_1, \nu_2} \delta(\nu_1 - \nu_2 - \hbar \omega_0) g^2 A_{\nu_1 \nu_2} [f_e(\nu_1) - f_c(\nu_1)] |G_R(\nu_1)|^2 |G_R(\nu_2)|^2 [\gamma_e(\nu_1) \gamma_e(\nu_2) \gamma_e(\nu_1) - \gamma_e(\nu_1)]
\times [\gamma_e(\nu_2) - \gamma_c(\nu_2)]. \]

(42)

This part is proportional to \( g^2 \). It contains factors of the type \( \gamma_e - \gamma_c \) and will contribute only when the asymmetry in the emitter and collector escape rates is significant.

Finally, \( S_C \) is the contribution from diagrams where phonons are interchanged between the two \( I(t) \) factors corresponding to the last group of diagrams in Fig. 2,

\[ S_C(0) = \frac{e^2g_B}{h} \sum_{\nu_1, \nu_2} g^2 A_{\nu_1 \nu_2} (\delta(\nu_1 - \nu_2 - \hbar \omega_0) \Phi_{\nu_1 \nu_2} |G_R(\nu_1)|^2 |G_R(\nu_2)|^2 + \delta(\nu_1 - \nu_2 + \hbar \omega_0) \{- F_{\nu_1 \nu_2}^2 \text{Re} \{G_R(\nu_1)^2 G_R(\nu_2)^2\}
+ F_{\nu_1 \nu_2}^3 G_R(\nu_1)^2 \text{Im} \{G_R(\nu_1) G_R(\nu_2)^2\} + F_{\nu_1 \nu_2}^4 G_R(\nu_2)^2 \text{Im} \{G_R(\nu_1)^2 G_R(\nu_2)\}
+ 2 F_{\nu_1 \nu_2}^5 G_R(\nu_1)^2 \text{Re} \{G_R(\nu_1) G_R(\nu_2)\} \}), \]

(43)

where

\[ \Phi_{\nu_1 \nu_2} = \left[ F_{\nu_1 \nu_2}^1 - \frac{1}{2} F_{\nu_1 \nu_2}^2 \text{Re} \{A(\nu_1)\} - \frac{1}{2} F_{\nu_1 \nu_2}^2 \text{Re} \{A(\nu_2)\}
+ F_{\nu_1 \nu_2}^3 G_R(\nu_1)^2 \text{Re} \{G_R(\nu_1) G_R(\nu_2)\} \right]. \]

(44)

It can be shown that when integrating out the energy, the last four terms of Eq. (43) make a negligible contribution. Omitting those terms, writing out the thermal factors, and keeping only the lowest-order terms, we arrive at

\[ S_C(0) = \frac{e^2g_B}{h} \sum_{\nu_1, \nu_2} \delta(\nu_1 - \nu_2 - \hbar \omega_0) g^2 A_{\nu_1 \nu_2} |G_R(\nu_1)|^2 |G_R(\nu_2)|^2 \left( \gamma_e(\nu_1) f_e(\nu_1) + \gamma_c(\nu_1) f_c(\nu_1) \right) \}
\times \left[ f_c(\nu_1) - f_e(\nu_1) \right] \left[ f_e(\nu_2) - f_c(\nu_2) \right] \left[ f_e(\nu_1) f_c(\nu_1) + f_c(\nu_1) f_e(\nu_1) \right]. \]
As we will see in our numerical calculations, this part is responsible for the major part of the shot noise under phonon-assisted tunneling. Figure 3 shows the results of a calculation where we have assumed for simplicity that only the two first Landau channels contribute. The calculations are performed for a device with symmetric barriers $Y_e = Y_c = 0.65 \text{ (meV)}^{3/2}$, $h\omega_c = 12 \text{ meV}$, $\varepsilon_0 = 60 \text{ meV}$, and $E_F = 25 \text{ meV}$. With the electron-phonon coupling constant $g = 4.5 \text{ meV}$ (corresponding to the polaron constant $\alpha = 0.07$). Figure 3(a) shows the average current and shot noise as functions of bias voltage. The average current increases stepwise as a new Landau channel enters the resonant-tunneling region, while tilted plateaus reflect the $\varepsilon^{-1/2}$ dependence of the emitter’s density of states. Without interlevel scattering, the phonon replica’s profile would be more or less a copy of the main peak. However, we see a triple-step structure that is a direct consequence of the interlevel scattering. In Fig. 3(b) we have plotted the noise ratio $S(0)/eI_{dc}$. As we see, the shot noise is suppressed below its uncorrelated value $S = 2eI_{dc}$ for both the main peak and phonon replica. We also note from the figure that the term $S_C$ is responsible for the major part of noise in phonon-assisted tunneling. This means that the major part of the noise in the region of phonon replica comes from the electron-phonon processes during the correlation time. The strong voltage dependence of the noise ratio in the region of the main peak is due to strong energy dependences of the escape rates. Indeed, in the vicinity of the main peak, the integration of Eq. 41 yields an expression of the type
\[ S(0) = 2eI_{dc} \left[ 1 - \frac{\gamma_e \gamma_c}{\gamma^2} \right], \] (46)
where $\gamma_j = Y_j / \sqrt{\varepsilon_0 - eVq_j}$. The noise ratio thus increases with increasing asymmetry in emitter and collector escape rates, explaining why the noise ratio increases with increasing voltage. We see from the figure that this is not the case in the phonon replica region where the noise seems to be suppressed by the factor 1/2 nearly independent of bias voltage and consequently of escape rate asymmetry. To check this point closer, without being confused by the voltage dependence of the escape rates, we also plot in Fig. 4 the result of a calculation within the wideband approximation where escape rates are constant. The results are presented both for symmetric ($\gamma_e = \gamma_c = 0.3 \text{ meV}$) and asymmetric ($\gamma_e = 0.5 \text{ meV}$, $\gamma_c = 0.2 \text{ meV}$) cases. We see that the increase of shot
noise by the barrier’s asymmetry is much more important for the main peak than for the phonon replica.

Our expressions for the noise are rather complicated and not very transparent because of the following facts: (i) we are operating outside the wideband approximation, (ii) both elastic and inelastic tunneling processes are present and they are open at different energies, and (iii) the problem is complicated by the interlevel scattering. To obtain a more simple expression let us discuss a simplified situation. First, we assume that only one Landau channel \((n=0)\) contributes and thus the interlevel scattering is not important. Second, we suppose that the voltage bias in the resonant-tunneling region is large enough to make the backward current from collector to emitter negligible \(f_\varepsilon(,\varepsilon) = 0\). Third, we use the wideband approximation where the escape rates are constant. Finally, we assume that Fermi energy is large enough and that the system is biased such that both the elastic and inelastic tunneling channels are open. In this case, the current and the corresponding noise can be written as

\[
I_\text{dc} = \frac{2e\varepsilon g_b}{\hbar} \int d\varepsilon [T^{(0)}(\varepsilon) + T^{(1)}(\varepsilon)],
\]

\[
S(0) = \frac{4e^2\varepsilon g_b}{\hbar} \int d\varepsilon [T^{(0)}(\varepsilon) [1 - T^{(0)}(\varepsilon)]
\]

\[
+ T^{(1)}(\varepsilon) \left[ \frac{1}{2} - 2T^{(0)}(\varepsilon) - \frac{1}{2} (\gamma_e - \gamma_c)
\right]
\]

\[
\times \left[ \frac{1}{\gamma_c} T^{(0)}(\varepsilon) + \frac{1}{\gamma_e} T^{(0)}(\varepsilon - \hbar \omega_0) \right].
\]

(47)

where

\[
T^{(0)}(\varepsilon) = \frac{\gamma_e \gamma_c}{\gamma} A^{(0)}(0,\varepsilon) = \gamma_e \gamma_c |G_R(0,\varepsilon)|^2
\]

is the elastic tunneling probability, while the inelastic tunneling probability is denoted as

\[
T^{(1)}(\varepsilon) = \frac{\gamma_e \gamma_c}{\gamma} A^{(1)}(0,\varepsilon)
\]

\[
= g^2 A_{00} \gamma_e \gamma_c |G_R(0,\varepsilon)|^2 |G_R(0,\varepsilon - \hbar \omega_0)|^2.
\]

(49)

In contrast to the average current, we observe that even in this (over)simplified case the noise cannot be found simply by the replacement \(T^{(0)} \rightarrow T^{(0)} + T^{(i)}\) in the corresponding expression for the elastic case.

Finally, we will give a rough estimate for the noise. As above, we assume only one Landau channel, as well as the absence of the interlevel scattering and backward current. However, we now assume that the elastic and the phonon-assisted tunneling channels open at different bias voltages (as in the previous numerical calculations). We keep only the lowest-order electron-phonon corrections (proportional to \(g^2\)). The barriers are assumed to be thick enough to treat the spectral functions as \(\delta\) functions (Appendix D) and to neglect contributions of the order \(\nu/\hbar \omega_0\). For the main peak (MP), we then arrive at

\[
S(0)_{\text{MP}} \sim 1 - 2 \frac{\gamma_e \gamma_c}{\gamma^2} \left( \frac{g}{\hbar \omega_0} \right)^2 \left( \gamma_e + 5 \gamma_e^2 \gamma_c^2 - \gamma_e \gamma_c^2 \right).
\]

(50)

The first two (elastic) terms dominate and the noise near the main peak increases towards its uncorrelated value as asymmetry in the barriers increases. The sign of the electron-phonon correction to this result is dependent on the properties of both the barriers. In particular, it is asymmetric with respect to the barriers’ heights. This is a direct consequence of the fact that electron-phonon coupling is inelastic. An electron tunneling into the well through the emitter barrier can lose energy. Consequently, the possibility of backscattering to the emitter is closed and the electron will certainly have to leave through the collector barrier. Under our present rough estimates, at the bias where only the phonon-assisted tunneling channel is open [phonon replica (PR)], we arrive at \(S(0)_{\text{PR}} \sim 2eI_\text{dc}\). Shot noise near the phonon replica is thus essentially suppressed to half of its uncorrelated value and the suppression is nearly independent of the barrier heights’ symmetry. Choosing instead the model of Ref. 8, \(I_1 = I_2 = I_R\), we obtain in the phonon replica approximately the uncorrelated value \(S(0)_{\text{PR}} \sim 2eI_\text{dc}\). This result was obtained from our general formulas by the replacements \(\eta_c \rightarrow 2\) and \(\eta_e \rightarrow 0\) in the thermal factors (30). The phonon-assisted resonant-tunneling device can thus be used to discriminate between the two models.

V. CONCLUSION

In conclusion we have worked out a rather general theory that relates the average current and low-frequency shot noise in the resonant magnetotunneling device to the intra-well electron Green’s functions. We have applied the theory to the case of LO-phonon-assisted tunneling at zero temperature and have demonstrated suppression of the shot noise for both elastic and inelastic tunneling processes. For the elastic processes, the noise ratio is very sensitive to the barriers’ asymmetry, while for the phonon-assisted processes this sensitivity is lost.

ACKNOWLEDGMENTS

The present work has been financially supported by the Norwegian Research Council, Grant No. 100267/410.

APPENDIX A:

DETAILS OF CURRENT OPERATOR EVALUATION

Following Ref. 28, we review some details of the evaluation of the current operator (18). Integrating the current density operator (10) over the \(x\)-\(y\) cross section with the substitution of the field operator (17) using the notation of Eq. (20), one directly gets

\[
\hat{I}_j(t) = \sum_{\mu_1, \mu_2} \left[ I_{j, \mu_1 \mu_2}^{\mu_1 \mu_2} \hat{c}_{\mu_1}(t) \hat{c}_{\mu_2}(t) + I_{j, \mu_1 \mu_2}^{\mu_1 \mu_2} \hat{c}_{\mu_1}(t) \hat{b}_{\mu_2}(t) + I_{j, \mu_1 \mu_2}^{\mu_1 \mu_2} \hat{b}_{\mu_1}(t) \hat{c}_{\mu_2}(t) + I_{j, \mu_1 \mu_2}^{\mu_1 \mu_2} \hat{b}_{\mu_1}(t) \hat{b}_{\mu_2}(t) \right].
\]

(A1)

Here the matrix elements are defined as
In the case of the average current, we will always have \( \varepsilon_2 = \varepsilon_1 \). This also turns out to be the case for the shot noise at zero frequency, as well as at finite low frequency provided \( \hbar \omega \ll E_F \). The matrix elements (A2) then simplify to

\[
\Gamma_{\jmath,\alpha_1,\alpha_2}(\varepsilon_1,\varepsilon_2) = \delta_{\alpha_1,\alpha_2} \frac{e\eta_j}{2\hbar} \sigma_1. \tag{A3}
\]

Consequently, only the first and last terms of Eq. (A1) contribute and we arrive at Eq. (18), which is appropriate for our problem.

**APPENDIX B: TRANSFORMATION FROM AN S-MATRIX REPRESENTATION TO THE GREEN’S FUNCTIONS APPROACH**

In the Dirac notation the elements of the S matrix are written \( S_{j_1\mu_1,j_2\mu_2}(\varepsilon) \equiv \langle j_1\mu_1 |S| j_2\mu_2 \rangle \). The S matrix can be expanded as

\[
S = 1 - i \int \frac{d\tau}{\hbar} e^{iH_0 \tau_1/\hbar} \hat{H}_t e^{-iH_0 \tau_1/\hbar} + i \int \frac{d\tau}{\hbar} e^{iH_0 \tau_2/\hbar} \hat{G}_R(t_2 - t_1) \hat{H}_t e^{-iH_0 \tau_2/\hbar}. \tag{B1}
\]

Here \( \hat{H}_t \) is the electronic part of the total Hamiltonian, \( \hat{H}_t \) is the sum of tunneling and electron-phonon interaction terms, and \( \hat{G}_R(t) = -i \Theta(t) e^{-i\hbar t/\hbar} \) is the Green’s-function operator. The second term in Eq. (B1) does not contribute since it takes at least two factors of the tunneling Hamiltonian to hop from one of the leads into the quantum well and out again. Substituting the tunneling Hamiltonian for \( \hat{H}_t \) into Eq. (B1) with the transformation (15) yields

\[
\langle \hat{I}_e \rangle = \frac{e}{\hbar} \sum_{\mu} \{ \gamma_\mu(\mu) \gamma_\mu(\mu) |G_R(\mu)|^2 [f_\varepsilon(\varepsilon) - f_f(\varepsilon)] + f_f(\varepsilon) \gamma_\mu(\mu) [A(\mu) - \gamma(\mu) |G_R(\mu)|^2] \}
\]

\[
- \frac{e}{\hbar} \sum_{\mu_1,\mu_2} K^{(1)}_{\mu_1,\mu_2} \gamma_\mu(\mu_1) f_f(\varepsilon_1) \gamma_\mu(\mu_1) + f_f(\varepsilon_1) \gamma_\mu(\mu_1),
\]

\[
\langle \hat{I}_c \rangle = \frac{e}{\hbar} \sum_{\mu} \{ \gamma_\mu(\mu) \gamma_\mu(\mu) |G_R(\mu)|^2 [f_\varepsilon(\varepsilon) - f_f(\varepsilon)] - f_f(\varepsilon) \gamma_\mu(\mu) [A(\mu) - \gamma(\mu) |G_R(\mu)|^2] \}
\]

\[
+ \frac{e}{\hbar} \sum_{\mu_1,\mu_2} K^{(1)}_{\mu_1,\mu_2} \gamma_\mu(\mu_1) f_f(\varepsilon_1) \gamma_\mu(\mu_1) + f_f(\varepsilon_1) \gamma_\mu(\mu_1). \tag{B5}
\]
Using the relation \( \langle \hat{I}_e \rangle = \langle \hat{I}_c \rangle \) and the fact that \( f_e \) and \( f_e \) enter the equations independently, we obtain

\[
\sum_{\mu} f_e(\varepsilon) \gamma_e(\mu)[\bar{A}(\mu) - \gamma(\mu)]G_R(\mu)^2 = 2 \sum_{\mu_1, \mu_2} K^{(1)}_{\mu_1, \mu_2} \gamma_e(\mu_2) \gamma_e(\mu_1) f_e(\varepsilon_1). \tag{B6}
\]

With the substitution of this result into Eq. (B5), we arrive at Eq. (19) for the average current.

The shot noise (23) is expanded in a similar manner, using the current operators (18) and (22) for the \( S \)-matrix elements. We also apply the result that the quantum statistical expectation value of a product of four \( e \) operators away from its average is given by

\[
\langle c_{j_1 \mu_1}^\dagger c_{j_2 \mu_2} c_{j_3 \mu_3} c_{j_4 \mu_4} \rangle - \langle c_{j_1 \mu_1}^\dagger c_{j_2 \mu_2} \rangle \langle c_{j_3 \mu_3} c_{j_4 \mu_4} \rangle = \delta_{j_1 j_3} \delta_{j_2 j_4} \delta_{\mu_1 \mu_3} \delta_{\mu_2 \mu_4} f_{j_1}(\varepsilon_1)[1 - f_{j_2}(\varepsilon_2)]. \tag{B7}
\]

The subtraction of the average current squared in Eq. (24) is performed by the application of the above formula. Consequently, we directly arrive at Eq. (25) for the shot noise.

**APPENDIX C: EXPANDING THE SHOT NOISE TO LOWEST ORDER IN THE ELECTRON-PHONON INTERACTION**

We want to expand the general expression (25) to the lowest order in the electron-phonon interaction at \( T = 0 \). As an example, we look at the expansion of one single term, choosing the very last diagram in Fig. 2(c). The contribution from this term is denoted \( S^{C(M^2)}(0) \) since it results from the seventh and the last term in the expansion of the four-particle Green’s function \( M \):

\[
S^{C(M^2)}(0) = \sum_{\{\mu\}} f_{\mu_1 \mu_2 \mu_3 \mu_4}^S \text{Re}[M^{(7)}_{\mu_1 \mu_2 \mu_3 \mu_4}]. \tag{C1}
\]

\( M^{(7)} \) is, according to Fig. 2(c), given by

\[
M^{(7)} = \delta_{\mu_1 \mu_3} \delta_{\mu_2 \mu_4} G_R^R(\mu_1) G_R^R(\mu_2) K^{B}_{\mu_2 \mu_1}. \tag{C2}
\]

Similarly, all the other three- and four-particle Green’s functions are also reduced to products of single- and two-particle Green’s functions. Summing up all terms, we then arrive at Eqs. (39), (42), and (43).

**APPENDIX D: SIMPLIFICATION OF SPECTRAL FUNCTIONS**

The integrals (37), (39), (42), and (45) involve certain combinations of spectral functions or Green’s functions that can be expressed through spectral functions. For our rough estimates, we are interested in the case of a single quantum channel \( n = 0 \) and no interlevel scattering. The barriers are assumed to be thick enough that contributions of relative size \( \gamma/\hbar \omega_0 \) can be neglected. We also assume that the bias is adjusted such that the system is driven in resonance (main peak or phonon replica) with an eV margin much greater than \( \gamma \). The spectral function \( A(\varepsilon) \) is split into the two parts \( A^{(0)} \) and \( A^{(1)} \), each of which can be approximated by appropriate normalized \( \delta \) functions:

\[
A^{(0)}(\varepsilon) = 2 \pi \left[ 1 - \left( \frac{\gamma}{\hbar \omega_0} \right)^2 \right] \delta(\varepsilon - \varepsilon_0), \tag{D1}
\]

\[
A^{(1)}(\varepsilon) = 2 \pi \left[ \frac{\gamma}{\hbar \omega_0} \delta(\varepsilon - \varepsilon_0) + \delta(\varepsilon - \varepsilon_0 - \hbar \omega_0) \right], \tag{D2}
\]

where \( \gamma = \gamma(\varepsilon) \) and \( \gamma_\varepsilon \) means \( \gamma(\varepsilon - \hbar \omega_0) \). In the expressions for noise, also products of two spectral functions appear. The actual products can then be expressed with \( \delta \) functions as (neglecting contributions of order \( \gamma/\hbar \omega_0 \))

\[
A^{(0)}(\varepsilon)^2 = \frac{4 \pi}{\gamma} \left[ 1 - \left( \frac{\gamma}{\hbar \omega_0} \right)^2 \right] \delta(\varepsilon - \varepsilon_0), \tag{D3}
\]

\[
A^{(0)}(\varepsilon)A^{(1)}(\varepsilon) = \frac{4 \pi \gamma}{\gamma_\varepsilon} \left[ 1 - \left( \frac{\gamma}{\hbar \omega_0} \right)^2 \right] \delta(\varepsilon - \varepsilon_0), \tag{D4}
\]

\[
A^{(0)}(\varepsilon - \hbar \omega_0)A^{(1)}(\varepsilon) = \frac{4 \pi \gamma}{\gamma_\varepsilon} \left[ 1 - \left( \frac{\gamma}{\hbar \omega_0} \right)^2 \right] \delta(\varepsilon - \varepsilon_0 - \hbar \omega_0). \tag{D5}
\]