APPLIED GAME THEORY - OLIGOPOLY

Introduction

Strategic interaction
Strategic variables
Static vs. dynamic models

Static oligopoly models – Cournot

Number of firms \( n \)

Individual firm output \( q_i, i = 1,2,\ldots,n \)

Firms produce homogenous products, for which the (inverse) demand function is \( P(Q) \), where \( Q = \sum_{i=1}^{n} q_i \) is market supply.

Constant marginal costs \( c \)

Firms choose output so as to maximise profits

\[
\pi(q_i;Q_{-i}) = \left[ P(Q) - c \right] q_i
\]

Firms set outputs simultaneously.

We consider first the case of duopoly, i.e. \( n = 2 \).

Residual demand

Consider first Firm 1.

If the competitor’s output is given at \( \bar{q}_2 \), Firm 1 effectively controls the market price. In particular, if Firm 1 chooses output \( q_1 \), the market price becomes \( P(q_1 + \bar{q}_2) \). We may think of this schedule as the ‘residual demand curve’.
Figure (residual demand - \( P(0 + \bar{q}_2) \) and \( P(q_1 + \bar{q}_2) = c \))

**Nash equilibrium**

The Nash equilibrium has each firm choosing output optimally given the (equilibrium) output of all competitors.

First-order conditions

\[
P'(q_1 + q_2)q_1 + P(q_1 + q_2) - c = 0
\]
\[
P'(q_1 + q_2)q_2 + P(q_1 + q_2) - c = 0
\]

At equilibrium, both conditions are satisfied.

Figure (marginal revenue equals marginal costs at optimum)

Rearranging, we find

\[
\alpha = \frac{q_iQ_i}{PQ}
\]

where \( \alpha = q_i/Q \) is Firm i’s market share and \( \varepsilon = [P'Q/P]^{-1} \) is the price elasticity of demand.

**Best response**

The first-order conditions implicitly define firms’ “best-response” functions:

\[
q_i^* = q_i^* (q_j)
\]

Totally differentiating the first-order conditions, we find that there firm i’s best response is decreasing in the output of its competitor.

Figure (best response curves, intersections at monopoly and perfectly competitive outputs)

**Comparison**

Duopoly market supply lies between monopoly and perfectly competitive supply.

Figure (best response curves and diagonals between monopoly and perfectly competitive points).

**Example**

Suppose \( P(Q) = 1 - Q \).
Then best-response functions become \( q_i^* (q_j) = \frac{1-q_j - c}{2} \)

Equilibrium outputs become: \( q_i^* = \frac{1-c}{3}, \quad Q^* = 2q_i^* = \frac{2}{3} \left[ 1 - c \right]. \)

Compare with monopoly and perfectly competitive outcomes: \( Q^m = \frac{1}{2} \left[ 1 - c \right], \quad Q^c = 1 - c. \)

**Oligopoly**

First-order conditions

\[
P' \left( q_i + Q_{-i} \right) q_i + P \left( q_i + Q_{-i} \right) - c = 0,
\]

where \( Q_{-i} = \sum_{j \neq i} q_j. \)

Price-cost margin at the symmetric equilibrium (\( \alpha_i = \alpha = \frac{1}{n} \)):

\[
\frac{P - c}{P} = H = \frac{1}{n\varepsilon}.
\]

where \( H = \sum_{i=1}^{n} \left[ \alpha_i \right] \) is the Hirschman-Herfindahl index of market concentration.

Relationship between market performance and market concentration

Linear example: \( q_i^* = \frac{1-c}{n+1}, \quad Q^* = \frac{n}{n+1} \left[ 1 - c \right]. \)

**Static oligopoly models: Bertrand**

Suppose instead that firms set prices simultaneously.

Consider the case of duopoly.

**Residual demand**

Residual demand is given as:

\[
D_i \left( p_i, p_j \right) = \begin{cases} 
0 & \text{if } p_i > p_j \\
\frac{1}{2} D(p_i) & \text{if } p_i = p_j \\
D(p_i) & \text{if } p_i < p_j
\end{cases}
\]
where $D(p)$ is market demand at price $p$.

**Best-response curves**

Marginal cost, epsilon below 45 degree line, monopoly price

**Equilibrium**

Price equal to marginal cost: otherwise

- $\min p_i < c$ would imply that the firm with the lowest price was running a deficit;
- $c < p_i \leq p_j$ would imply that firm $j$ could increase profits by (slightly) undercutting firm $i$
- $c \leq p_i < p_j$ would imply that firm $i$ could increase profits by increasing price towards that of firm $j$

**The Bertrand Paradox**

Equilibrium outcome is perfectly competitive (price equal to marginal cost), even in the case of duopoly.

How alternative assumptions might affect equilibrium outcome:

- Product differentiation: firms would have market power, i.e. would not lose all customers even if price exceeds those of competitors.
- Capacity constraints: limit competitive response and drive prices above marginal cost.
- Dynamic competition: may sustain collusive-like equilibria.

**Dynamic competition – collusion**

Repeated version of the Bertrand duopoly game.

With a given number of repetitions, again unique (competitive) equilibrium.

With infinite repetition, however, different equilibria may be sustained.

Suppose each firm plays the following strategy:

- In the first period, charge the monopoly price $p^m$. 

Continue charging $p^m$ so long as both firms have charged $p^m$ in all previous periods. Otherwise, charge a price equal to marginal cost.

We want to see if this set of strategies can constitute a Subgame Perfect equilibrium.

Along the equilibrium path, each firm earns half the monopoly profit in each period:

$$\pi_i = \frac{1}{2} \pi^m = \left[ p^m - c \right] \frac{1}{2} D(p^m).$$

Total discounted profit is

$$\Pi_i = \sum_{t=0}^{\infty} \delta^t \frac{1}{2} \pi^m = \frac{1}{1 - \delta} \frac{1}{2} \pi^m,$$

where $\delta$ is the per period discount factor.

The maximum payoff a firm could gain by charging a different price would be from charging a price just below the monopoly price, in which case the firm would capture the entire market:

$$\pi_i = \pi^m$$

For all subsequent periods, firms would earn zero profits.

Consequently, the above strategies constitute an equilibrium only if

$$\frac{1}{2} \pi^m \geq \pi^m$$

or

$$\delta \geq \frac{1}{2}.$$

Consequently, the monopoly (collusive) outcome may be sustained as an equilibrium if the discount factor is sufficiently high.

One may show (see Cabrals textbook) that the discount factor may be written

$$\delta = \frac{1}{1 + r/f} h [1 + g]$$

where $r$ is the annual interest rate, $f$ is the frequency with which (i.e. number of times) firms change their prices over the year, $h$ is the probability that the market is not interrupted over the next period (due, for example, to the entry
of a superior product that eliminates firms’ sale) and $g$ is the market growth rate.

So the (effective) discount factor is greater, and (tacit) collusion more likely,

- the more frequently firms interact ($f$);
- the greater the probability of continuation of the market ($h$); and
- the higher the growth rate of the market.

Note that (tacit) collusion may be the result in a non-cooperative, competitive game.

How reasonable is the assumption that a price war lasts forever once some firm deviates from the collusive price?

- Note: the shorter the punishment period, the more difficult it is to sustain collusion.

What about the assumption that deviations are immediately observed and reacted to?

- Note: a longer detection lag makes it more difficult to sustain collusion.

(Tacit) collusion may also refer to other types of decisions, such as market splitting, quality regulations, restrictions on advertising and no-use of bonuses and discounts.

**Oligopoly**

In the case of oligopoly, the relevant criterion for the sustainability of a collusive-like equilibrium becomes

\[
\frac{1}{1 - \delta} \frac{1}{n} \pi^m \geq \pi^m
\]

or

\[
\delta \geq 1 - \frac{1}{n}
\]

Consequently, sustaining collusion becomes more difficult (i.e. requires a higher degree of patience) the larger the number of firms.

So, again, relationship between concentration and market performance.