PRODUCT DIFFERENTIATION
Lecture notes 08.03.04

Introduction

Homogenous products
- only price determines from which supplier to buy
- market-wide equilibrium price

Differentiated products
- equilibrium prices may vary between producers
- suppliers compete in other dimensions also

Differentiation
- vertical (‘quality’)
- horizontal (‘taste’)

We consider static models in which
- product type is chosen first (long-term decision) and
- thereafter suppliers compete in prices (short-term decision).

‘Principle of differentiation’
- product differentiation softens price competition

The Hotelling line

Consumers are located uniformly along the line [0,1].

Firms 1 and 2 are located at a and 1-b, respectively, where 0 ≤ a ≤ 1−b ≤ 1 (i.e. Firm 1 is located to the left of Firm 2):
- a = b = 0: maximum distance between firms (i.e. maximum product differentiation);
- a + b = 1: identical location (i.e. homogenous products).

Firms incur identical marginal production costs c (constant).

Each consumer wants at most one unit of the good and has a maximum willingness to pay equal to s.
A consumer located at a distance $y$ from its supplier incurs transportation costs $ty^2$.

(d’Aspremont et al (1979): with linear transportation costs there typically does not exist an equilibrium in pure strategies when firms are located closely.)

We consider the case in which an interior equilibrium solution exists
- each firm supplies a positive amount of output;
- the market is covered ($s$ large enough that all consumers buy).

**Price competition**

Note that all consumers to the left (right) of Firm 1 (2) buys from this firm (or else no-one would buy from that firm). Furthermore, there exists a consumer located at $x$ somewhere between Firms 1 and 2 who is just indifferent between buying from either of these firms:

$$s - p_1 - t[a - x]^2 = s - p_2 - t[1 - b - x]^2$$

Solving for $x$, we obtain the demand facing each firm:

$$D_1(p_1, p_2) = x = a + \frac{1 - a - b}{2} + \frac{p_2 - p_1}{2t[1 - a - b]}$$

$$D_2(p_1, p_2) = 1 - x = b + \frac{1 - a - b}{2} + \frac{p_1 - p_2}{2t[1 - a - b]}$$

Profits are

$$\Pi^1(p_1, p_2) = [p_1 - c]D_1(p_1, p_2)$$

$$\Pi^2(p_1, p_2) = [p_2 - c]D_2(p_1, p_2)$$

The Nash equilibrium for the price game then becomes:

$$p_1(a, b) = c + t[1 - a - b][1 - \frac{a - b}{3}]$$

$$p_2(a, b) = c + t[1 - a - b][1 - \frac{a - b}{3}]$$

Examples:
- homogenous products: $a + b = 1 \Rightarrow p_1 = p_2 = c$
- maximum differentiation: $a = b = 0 \Rightarrow p_1 = p_2 = c + t$

We have
Quality competition

We are looking for a Nash equilibrium for the location game, given equilibrium behaviour in the following price game (i.e. we require subgame perfection). Define \( \Pi_i(a,b) = [p(a,b) - c]D_i(a,b,p_i(a,b),p_2(a,b)) \).

Applying the Envelope Theorem, we find
\[
\frac{d\Pi}{da} = [p_i - c] \left[ \frac{\partial D_i}{\partial a} + \frac{\partial D_i}{\partial p_2} \frac{dp_2}{da} \right]
\]

Furthermore, since
\[
\frac{\partial D_i}{\partial a} = \frac{3 - 5a - b}{6[1 - a - b]}
\]
\[
\frac{\partial D_i}{\partial p_2} \frac{dp_2}{da} = \frac{-2 + a}{3[1 - a - b]}
\]

we find \( d\Pi_i/da < 0 \), so that Firm 1 will always want to increase \( a \), that is, move to the left. Correspondingly, Firm 2 will want to move as far as possible to the right. We conclude that the Nash equilibrium involves maximum product differentiation (i.e. \( a = b = 0 \)).

Welfare optimum
- since demand is inelastic transportation costs determine optimal location,
- transportation costs are minimised, and hence welfare is maximised, at \( a = b = \frac{1}{2} \)
- so, the market outcome involves too much differentiation.

Too little or too much differentiation?

When transportation costs increase quickly with distance (i.e. consumers are willing to pay relatively much more for a product that is close to their most-preferred variety) there is a tendency for too much product differentiation
- with transportation costs \( ty^2 \) : \( \alpha \geq 1.67 \) implies maximum differentiation, while \( \alpha < 1.67 \) leads to less differentiation (Economides, 1986).

Price-elastic demand
- high prices also reduce aggregate demand
Institutional, or other conditions, may lead firms to locate closely together (access to harbour, raw materials etc.).

Customer search costs may also make it profitable for firms to cluster.

If prices are exogenous (maybe fixed by government regulation) competition for market shares may lead firms to locate closely (i.e. at the centre of the market).

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**The Circular City**

In the ‘line’ model locations are not ‘homogenous’; in particular, end-point locations are different from interior-point locations.

In the ‘circle’ model all locations are identical *ex ante*.

Assume consumers are located uniformly along a circle with circumference 1. Each consumer has maximum willingness to pay $s$ for one unit of the good.

Firms are located around the circle also (each firm may have at most one location).

Transportation costs are linear and equal $t$ per unit of distance.

There is a fixed cost $f$ for establishing a firm, while unit costs are $c$.

We consider Subgame Perfect equilibria of the following two-stage game:

1) Firms simultaneously decide whether to enter the market. The $n$ entering firms are located equidistantly around the circle, at positions $0, \frac{1}{n}, \ldots, \frac{1-n}{n}$.

2) Firms simultaneously set prices.

**Price competition**

Assume that sufficiently many firms enter that ‘neighbours’ compete effectively (i.e. the market is covered).

We are looking for a symmetric equilibrium in which firms choose price $p$.

Call the firm located at 0 Firm $i$.

A consumer $x \in (0, \frac{1}{n})$ is indifferent between buying from the firm $i$ at price $p_i$ and from the firm located at $\frac{1}{n}$ at price $p$ if
\[ p_i + tx = p + t \left( \frac{1}{n} - x \right) \]

implying that demand facing Firm \( i \) is
\[ D_i(p, p) = 2x = \frac{1}{t} \left[ p + \frac{t}{n} - p_i \right] \]

The first-order condition for profit maximum, given that \( p_i = p \), reduces to
\[ p = c + \frac{t}{n} \cdot \]

**Entry – location**

Firms enter up to the point where (treating \( n \) as a continuous variable) \( \Pi^i = 0 \), assuming equilibrium behaviour at Stage 2. Consequently, the equilibrium number of firms is (the integer value of)
\[ n^c = \sqrt{\frac{t}{f}} \cdot \]

Note
- firms price above marginal costs, but do not earn profits;
- higher fixed costs implies fewer firms and higher price-cost margins;
- in the limit, as fixed costs approach 0, the market approaches perfect competition.

**Welfare optimum**
- optimum is achieved by minimising the sum of transportation costs and fixed costs of production, i.e. \( \frac{t}{4n} + nf \);
- optimum number of firms is \( n^* = \frac{1}{2} \sqrt{\frac{t}{f}} = \frac{1}{2} n^c \); that is, there is excess entry at equilibrium;
- the optimum number of firms balances the gain from variety (lower transportation costs) against the duplication of fixed costs;
- the private incentive for entry is affected by the ‘business stealing effect’.

**Extensions**

Endogenous location choices
- extending the model by introducing a location game between the entry stage and the pricing stage, and assuming quadratic transportation costs,
one can show that there exists a symmetric Nash equilibrium (Economides, 1984).

Sequential entry
- may lead to asymmetric market shares

Multi-product firms (i.e. firms with multiple locations)
- strategic positioning.

**Advertising**

A. Informative advertising
- information about
  - existence of product
  - product characteristics
  - price
- typically (but not always)
  - reduces product differentiation and intensifies price competition
  - eases entry
  - stimulates production of high quality products

B. Persuasive advertising
- may distort information
- thereby creating/enhancing product differentiation.

**Informative advertising - the Hotelling model**

Two firms producing identical products are located at either end of [0,1].

Consumers are uniformly located along the line, each wanting at most one unit of the good for which he or she has maximum willingness to pay equal to \( s \) (\( s \) is assumed large enough that the market is covered).

Transportation costs are linear and equal \( t \) per unit of distance.

Let \( \phi_i \) be the share of consumers receiving advertising messages from firm \( i \), \( i = 1,2 \). Costs of advertising are \( A(\phi_i) = \frac{a}{2} [\phi_i]^2, i = 1,2 \).

Demand for Firm 1’s product becomes
\[ D_1 = \phi_1 \left[ 1 - \phi_2 + \phi_2 \frac{p_2 - p_1 + t}{2t} \right] \]

Note

- only the share \( \phi_1 \) of consumers who have received advertising messages from Firm 1 are potential buyers;
- a share \( 1 - \phi_2 \) of these have not received an advertising message from Firm 2 and hence surely buys from Firm 1;
- the rest (a share \( \phi_2 \)) has received a message from Firm 2 also – and are hence fully informed – and therefore buys from the firm that offers the greater surplus.

In the symmetric case \((p_1 = p_2 = p)\) and \(\phi_1 = \phi_2 = \phi\), the price elasticity of demand becomes

\[ \varepsilon = \frac{\phi p}{[2 - \phi] t} \]

which is increasing in \( \phi \).

We are looking for a Nash equilibrium of a game in which firms set price and advertising levels simultaneously.

Firm 1 maximises

\[ (p_1 - c) \phi_1 \left[ 1 - \phi_2 + \phi_2 \frac{p_2 - p_1 + t}{2t} \right] - \frac{a}{2} \phi_1^2 \]

From the first-order conditions we find the reaction functions

\[ p_1 = \frac{p_2 + t + c}{2} + \frac{1 - \phi_2}{\phi_2} t \]

\[ \phi_1 = \frac{1}{a} \left( p_1 - c \right) \left[ 1 - \phi_2 + \phi_2 \frac{p_2 - p_1 + t}{2t} \right] \]

Note

- that the price is increased due to imperfect consumer information that makes a market segment less price sensitive;
- the incentive to advertise is related to the size of the price-cost margin.

At the symmetric equilibrium (assuming that \( a \geq t/2 \) ) we have

\[ p^c = c + \frac{2 - \phi^c}{\phi^c} t = c + \sqrt{2at} \]
\[ \phi^c = \frac{2}{1 + \sqrt{\frac{a}{t}}} \]

\[ \Pi^c = \frac{2a}{\left[1 + \sqrt{\frac{a}{t}}\right]^2} \]

Note
- the equilibrium price is higher than under full information;
- the equilibrium price is increasing in \( a \) and \( t \) (but the rate at which price increases with \( t \) is less than when consumers are perfectly informed);
- the equilibrium advertising level is increasing in \( t \) and decreasing in \( a \);
- equilibrium profits are increasing in \( a \); the direct effect on costs reduces profits but this is more than outweighed by the strategic effect, whereby less advertising and increased (perceived) product differentiation increases prices and hence profits.

Welfare maximisation involves choosing the (symmetric) level of advertising that maximises (expected) total surplus:

\[ \max_{\phi} \phi^2 \left[ s - c - \frac{t}{4} \right] + 2\phi \left[ 1 - \phi \right] \left[ s - c - \frac{t}{2} \right] - 2 \left[ \frac{a}{2} \phi^2 \right] \]

Note
- fully informed consumers travel an average distance of \( \frac{1}{4} \);
- partly informed consumers travel an average distance of \( \frac{1}{2} \).

We find

\[ \phi^* = \frac{2[s - c] - t}{2[s - c] - \frac{3}{2} t + 2a} \]

Consequently,
- \( \phi^c > \phi^* \) if \( a \) is close to \( t/2 \) (i.e. \( a \) is small)
- \( \phi^c < \phi^* \) if \( a \) and \( t \) are both small.

The differences between market and optimal outcomes are associated with firms’ inability to capture all of consumer surplus (that reduces the incentive to advertise) and a business stealing effect (that increases the incentive).