Scandinavian Statistics, Some Early Lines of Development

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ABSTRACT. The history of Scandinavian statistics (pre 1920) is sketched. The exposition is journalistic. For 10 early Scandinavian statisticians some background material together with a rough summary of their contribution is presented. A drawn portrait and a selected bibliography is also presented for each.

Key words: History of statistics, least squares, cumulants, stochastic processes, survey sampling

Preface

The following is based on the exhibition with the above title, which was presented at the 11th European meeting of statisticians, Oslo, 1978. The aim of the exhibition was not to present a complete picture of the early history of statistics in Scandinavia, but rather to throw some light on some of our more outstanding predecessors. The following “journalistic” exposition is based on what I have come across in biographies, obituaries and the like and, on the study of some of the works mentioned in the bibliography. In addition to the coarse description of the work of the various authors I have tried to place their work against a cultural and personal background. I hope the reader will share my view that this is of interest in addition to the chronology and priority of the various contributions. My presentation is that of an amateur in the history of science, but despite its briefness, omissions, biases and other shortcomings, I hope it will be of some value.

Below follows a brief historical survey of early Scandinavian statistics, which except for Lindeberg is pre 1920. In the next section I have raised some questions and put forward some views. Then my 10 favourite early Scandinavian statisticians are presented in alphabetical order. The portraits are drawn by my father Carl Schweder. In the selected bibliographies I have included what seems to me to be of primary interest to statisticians. Secondary reading is also included, some of which gives complete bibliographies.

I want to thank the University library in Oslo and Tromsø for the help I received in carrying out this work. Anders Hald, Herman Wold, Lars Erik Öller and others deserve particular thanks for their help at the outset.

A brief historical survey

We can trace three distinct main lines of development in early Scandinavian statistics. The oldest one may be termed social statistics: population statistics and demography, statistical sociology and “official” statistics with survey sampling. The second line is that of mathematical statistics and its applications: regression, analysis of variance, series expansions, cumulants and other general theory. Modelbuilding and applications of theoretical concepts was undertaken in the fields of insurance, forestry, astronomy, biology and social sciences. The final main line was that of stochastic processes: Poisson process, risk theory and queuing theory, and its application to insurance and engineering.

Social statistics

The development of social statistics was basically nonmathematical. It started with Wargentin and the Swedish population statistics, produced regularly from 1749 — the world’s oldest.

Some earlier population numbers were for military reasons kept as state secrets and not published. The achievements of Wargentin and coworkers were not so much in the theoretical realm but more in making the value of population statistics understood, in establishing an administration for population enumeration and in publishing interesting and reliable population statistics. From the time of Wargentin, official statistics in the area of demography and also economics was steadily improved in Scandinavia.

The area of statistical sociology was opened by Eilert Sundt, a theologian who devoted his life to the study of the underprivileged in Norway. He collected statistics on a number of sociological variables previously neglected and he made impor-
important contributions to demography. His research was characterized by good design and analytical methodology. As his contemporary Karl Marx, Sundt wrote for the common man, often in a poetic style.

Throughout the 19th century statistical bureaus were organized to collect and present official statistics. It was felt that important questions, particularly of a social statistical nature, would be impossible to answer if only the method of total enumeration were used. Kier, head and originator of the Norwegian Central Bureau of Statistics had well documented stratified sample surveys carried out in the 1890’s. He argued strongly to have the method internationally accepted and further investigated.

Mathematical statistics with applications

The development of mathematical statistics in Scandinavia started in Copenhagen, actually in the Matematisk Forening (Mathematical Association). A strong Danish school of statistics evolved and it may be argued that it rivalled the London school as the leading center of statistics at the turn of the century. The background of the Copenhagen school was formed by Gauss and his theory of least squares and normal distribution.

Out of pure intellectual interest, a member of Parliament and professor of German, Oppermann, picked up the Gauss theory. The young Gram refined Oppermann’s ideas of linear space and orthogonalization aiming at a theory of approximation. In the hands of the astronomer Thiele the theory ripened into a mature and modern body of science featuring the basics of regression, analysis of variance, asymptotic expansions, semi invariants (cumulants) and elements of estimation and testing theory. In 1889 Thiele published the (first?) textbook of mathematical statistics with the well-chosen title: Almindelig Iagttagelseskunde (General theory of observation).

Parallel to their theoretical work, the Copenhagen statisticians were eagerly applying mathematical and statistical methods to problems of practical importance. Oppermann and Gram were interested in forestry, Thiele of course in astronomy, and they were all deeply involved in insurance.

Copenhagen remained the center of Scandinavian statistics. After 1900 mathematical statistics also dispersed to Sweden and later to Finland and Norway. In Lund the professor of astronomy Charlér worked to spread statistical ideas to various fields of applications. So did the Finnish mathematician Lindeberg, who is known for his work on the Central Limit Theorem.

Stochastic processes

Important areas of the young field of stochastic processes were developed in Sweden and Denmark. In 1903 the actuary Lundberg published a remarkable thesis in which he worked out a theory for Markov jump processes. His aim was to establish a collective risk theory of insurance. Independently, in 1909, the Copenhagen mathematician Erlang was led to the Poisson process and its relatives. His area was that of communication engineering and he is well known as the founder of queueing theory.

Both the two men had a sound mathematical background, and they were early engaged in practical work. Lundberg started his long insurance career at the age of 21, and Erlang was hired by the Copenhagen Telephone Company. In addition to the mathematical fascination, their ground-breaking work was inspired by the practical problems they were faced with.

Erlang’s works were immediately recognized and put to use. Lundberg was also judged as brilliant—but impossible to understand. It took 30 years before the statistical community really started to digest the conceptual methodological achievements of Lundberg.

Historical perspective

Our Scandinavian history is, of course, part of the broader European history. The cultural contacts with continental Europe and Britain were strong throughout the period of our interest and this is reflected in the Scandinavian history of statistics. The flux of ideas from Europe is quite evident in the work of Wargentin—he corresponded with Süssmilch, Deparcieux, Price; in that of Sundt and Kier—the latter was a very active member of the newly founded ISI; and in the Copenhagen school with their dependence on Gauss and the German mathematical tradition. Both Lundberg and Erlang relied on the mathematical tradition, but as to their original contributions in stochastic processes it is not straightforward to trace any direct influences from abroad.

How about the flux of ideas the other way? What was the impact of Thiele’s 1903 English translation: Theory of observations? Did the young Fisher read Thiele and thereby get inspiration for his ANOVA and his cumulants which was a re-invention of Thieles semiinvariants? What about Lundberg? He probably did not have much direct impact; why was it necessary with a Cramér to make the contribution of Lundberg appreciated? There are many questions to which I have no answers, but
I think it would be of great interest if someone would trace the history of these statistical ideas.

Besides this subject we have the equally important subject of the sociology of science. What were the forces behind the scientific developments — was there any technical, economical or bureaucratic need for the results obtained? Did the ideas mainly spring out from an academic tradition or was the contact with other sectors of society of importance? Without exception the actors on the statistical scene presented in this paper had a solid University training. However, the majority had their occupation outside the University, and only Lindeberg worked in a mathematical department. Wargentin and Kiaer worked mainly in the interest of an enlightened state and bureaucracy, Sundt worked for the enlightenment and the interest of the underprivileged out of own moral and basic political reasons. The initiating forces behind Oppermann, Gram and Thiele seem to have been basically mathematical and intellectual, but their interest and engagement in insurance, forestry and astronomy no doubt represented inspirations and contributed to the molding of their work. Also Lundberg and Erlang were mathematicians at the outset who brought their knowledge and abilities out in the field (insurance and communication engineering) and had it exposed to practical needs. This contact not only influenced their choice of area of study, but also had a strong impact on the way they carried out their research and the results they obtained.

It may also be noted that all, maybe except Sundt, were regarded as prominent and successful men of their time. Sundt's research was too radical to be sufficiently appreciated, but for the rest it seems that particularly their applied mathematical work was highly appreciated by their society.

In summary, early Scandinavian statistics emerged as a basically applied science rooted in mathematics and the broader European intellectual tradition. Its practitioners were respected members of society and the achievements were of a high, some times history-making standard.
Charlier, Carl Wilhelm Ludwig (1861–1934)

Charlier was professor of astronomy in Lund. Early he was mainly occupied with theoretical astronomical questions. Later he worked with stellar statistics and, in particular, with the spatial distribution of helium stars.

At the age of 43 Charlier turned to statistics and after 1904 he functioned more or less as a professor of statistics. A chair of statistics was established in Lund in 1924, mainly on Charlier’s initiative.

Charlier had regular contacts with Copenhagen and must have known the works of Thiele who also was a professor of astronomy. In Thiele’s textbook the expansion of Gram was discussed at length. Without referring neither to Gram nor Thiele, Charlier published the same expansion in 1905. He based his exposition on Fourier series and (improperly) on the inversion formula.

Charlier worked to disperse statistical ideas to other academic disciplines, mainly biology, and he fitted probability models to a number of data sets.
With a solid background in mathematics and physical science, Erlang was in 1908 hired by J. L. W. V. Jensen (Jensen's inequality) to apply probability methods to the problems of the Copenhagen Telephone Company. The two met in the Mathematical Association. At that time the manager of the Company had already published two papers employing probability arguments to telephony. Erlang continued working with the Company for the rest of his life.

In order to find the right number of circuits of an automatic telephone exchange with a given traffic load and to solve other problems in communication engineering, Erlang introduced the Poisson processes and other related stochastic processes. He established a conceptual and methodological framework of queueing theory. Prior to 1908 he was consulted by Jensen, and he was able to publish this major achievement as early as in 1909. In solving for the waiting time distribution and other properties of his models, Erlang considered the stationary aspects of the system. He called this the method of “statistical equilibrium”, and used it efficiently. In this manner Erlang was able to present quite brief and elegant arguments for his results.

The time was ripe for Erlang's contributions. His results were promptly put to use both in Denmark and abroad. His 1917 paper was translated into English and German in 1918 and into French in 1922. The engineering community was so eagerly waiting for such results that a French and an American engineer studied Danish to be able to read Erlang's works as they appeared.

On his mother's side Erlang was of the great Krarup family. Since the time of Tycho Brahe, members of the family held chairs at the University of Copenhagen. Traditionally the male members of the family were clergymen and the females were wives of clergymen. Erlang's father was however a schoolteacher. As Galton did in 1873, Erlang published in 1929 the problem of finding the probability of a surname going extinct. Unlike the readership in London 55 years earlier, that in Copenhagen was able to deal with the problem and the journal received three solutions, two of which were complete. The same year Erlang died, leaving no children.
At the outset, Gram was a Copenhagen mathematician. Later he became interested in forestry and insurance, but he maintained his mathematical interest. Gram was managing director of a non life insurance company which he founded and he was involved in the life insurance company Hafnia together with Oppermann and Thiele.

In his thesis of 1879, later published in Crelle's journal, Gram worked out a theory of function spaces. Following Oppermann (and Gauss), he developed stepwise regression by successive orthogonal projections. By letting the number of observations increase to infinity, he was led to the concepts of $L_2$ distance, inner product and orthogonality for the space of continuous functions. He also obtained the Pythagoras theorem for the projection on a space spanned by a set of orthogonal functions. Gram considered this a theory of approximations. Because of a lack of measure theory, his characterization of the space spanned by an orthogonal basis is not completely satisfactory. But except for the notion and results on completeness he laid out the basics of Hilbert space theory.

Among the three systems of orthogonal functions which Gram considered we find the derivatives of the normal curve $D'(e^{-x^2}) n = 1, 2, \ldots$. Linear approximation by these functions give the Gram-(Charlier) expansion.
The name of Anders Kjær is intimately connected with the Central Bureau of Statistics of Norway—and with random survey sampling. As a young lawyer he came to the Statistical Office of the Department of the Interior. On his initiative this office was reorganized in 1873 as the Central Bureau of Statistics with Kjær as director, a position which he held for 46 years.

Kjær personally led a number of the statistical investigations of the Bureau in the fields of economics, sociology and demography, and he normally wrote the introductions to the various statistical publications.

The standard method of the time was that of total enumeration. It was however clear to Kjær and his coworker Jacob Neuman Mohn that many questions, particularly of a social statistical nature, would remain impossible to answer if only this method was to be used. They found survey sampling to be necessary to get information of sufficient depth. Mohns investigation *Om husmandsklassen* (On the class of crofters), *Norsk rettstidende* 1–2, 1880, and later studies were forerunners of Kjær’s 1891 investigation of personal income and capital, based on quasi random sampling. In 1895 Kjær used a method of proportional stratified random sampling.

Kjær was not a mathematician (nor was Mohn) and he did not formally develop his ideas of the “representative method of sample surveys”. However, he described in detail how the sample was drawn and noted in one case that “the sampling mechanism is so haphazard and random relative to the population that one may assume that it does function in the same way as a lottery”.

Kjær found it hard to convince the 1895 ISI-meeting that the “representative method” was useful. It was argued that although the method now and then had given interesting information, its principle was so much in opposition to “la statistique serieuse” that the “imperfect” method should not be granted equal status with the statistical ideal.

The ISI was founded in 1885. Kjær was an active member of the organization and he was the host for the fifth ISI meeting in Oslo (then called Kristiania) in 1899.
Lindeberg studied mathematics in Helsingfors and Paris. From 1900 he published works on differential equations and the calculus of variations and in 1905 he was appointed adjunkt in mathematics at the University of Helsingfors. Because of his scientific merits he could easily have been promoted to professor, but he refused such proposals. He was afraid of having more responsibilities, and being unable to spend the whole summer at his beloved resort place.

At the age of 40 Lindeberg became interested in probability and statistics. In 1920 he published his first work in this area, the well known "Lindeberg-Lévy theorem". Here he introduced the technique of adding a normal variate to the sum of independent variates to smooth the distribution of the sum and make the Taylor expansion work.

In 1926 Lindeberg defined his correlation percentage which is nothing but Kendall's $\tau$. He calculated the first two moments, but did not discuss its distribution any further. He proposed to use normal scores when calculating the product moment correlation.

Lindeberg took part in the discussion of the line transect method in forestry. When deriving the number of transects necessary to obtain a sufficiently narrow confidence intervall, he in 1926 apparently rediscovered the Student distribution.

Lindeberg was an active member of the actuarial society and he sat on the board of an insurance company. He was frequently consulted on statistical questions by researchers in areas like forestry, biology, medicine and linguistics.
Natural sciences and chemistry was what interested Lundberg the most. Heading for a career as a school teacher, but being a poor student, Lundberg chose to study mathematics instead of chemistry to cut down on study time.

At the age of 22 Lundberg began to work for a life insurance company. The business policy of the company was not a lucky one and it was next to ruin after a few years of operation. Lundberg quit his job for this reason. His next job was in the life insurance company De Förenade with which he worked for half a century, most of the time as managing director. Lundberg's new company was however also in difficulties and got into financial trouble. He decided to take up his studies to become a teacher because he figured the school to be a safer business than insurance.

For his thesis work, Lundberg looked for an "area of limited literature and in which it was not required to master the methodology of mathematical research too profoundly. To me the mathematical risk theory appeared to be such an area." Another reason for choosing this area of research was probably his personal experience with the risky business.

Lundberg's fundamental Uppsala thesis appeared in 1903. Instead of following the individual cases he considered the risk process of the whole collective: the stochastic claims coming in at random points in time. For this marked Poisson process Lundberg developed a substantial amount of theory including the forward equation, the concept of operational time, asymptotics and Berry-Esseen type bounds for the cumulative risk process.

For a degenerate claim distribution, the Poisson distribution was found to be a simple solution to the forward equation.

In later papers various modifications of the risk process were introduced, and problems of level crossing, extreme value and ruin were made subject to more detailed study.

Because of a condensed style and a difficult subject. Lundberg's works got the reputation of being impossible to understand. Unlike Erlang who was immediately accepted, Lundberg had to work alone and wait some thirty years before his fundamental ideas were picked up by Cramér and others.

_Lundberg, Ernst Filip Oscar (1876–1965)_
Oppermann studied language, was a schoolmaster and later a professor of German. He also studied forestry aiming at a career as “skovridder” (forester) as his father. He was a member of the Danish Parliament for 12 years.

Besides his daily work, Oppermann had a living interest in mathematics. He favoured Archimedes, Newton and Gauss. At the age of 28 he published a textbook on geometry which was found too difficult to be put in use. This text was later “modified” by Professor Steen, but the necessity of this modification remained impossible for Oppermann to understand. He never felt part of “the official mathematics”.

Oppermann studied Gauss very carefully, in particular his method of least squares which he complained was not developed any further. In 1863 he published two remarkable papers signed “En dilettant” (A dilettant). The first one “En forespergsel” was a 9 line piece requesting the distribution of the empirical variance of a normal sample.

The other paper, “Minimumsproblemer” was an impressively concise presentation of the least squares method. Oppermann used in essence the fundamental notion of linear space of vectors \( u, v \ldots \) with inner product \( [uv] \), orthogonality between vectors and linear subspaces. And he turned the Gauss elimination procedure into the Gram–Schmidt orthogonalization procedure. This paper was apparently hard to read when it appeared, but together with subsequent verbal communication it had a great impact on Gram and Thiele and their school of statisticians.

As most of the subsequent Danish statisticians, Oppermann was engaged in life insurance. He was Mathematical director of a life insurance company. In the 1860’s Oppermann employed least squares methods to smooth mortality tables. He kept his mortality functions a secret for many years, but after Thiele had guessed the functional form from the graphs, he turned more communicative.

The empirical cumulative distribution function was used by Oppermann to check the assumption of normality.
As a young theology student, Eilert Sundt met Henrik Wergeland, our Norwegian poet of freedom. He was inspired by Wergeland and felt a desire to be challenged by a "high and worthy aim" in his spirit. This he found in sociological research—in describing and making understood the nature and causes of the misery of various underprivileged groups, but also to bring forth their pride and worth.

As a Sunday school teacher in a prison Sundt met a number of gypsies. Among them old men who were not put in for any crime but were kept for years to be taught religion and prepared for confirmation. This moved Sundt so much that he gave up his promising university career and decided to investigate the conditions of this out group. He made a two month trip on foot across the country but did not meet a single gypsy. He saw however so much misery that he decided to devote his life to the study of the common man. His books, written in a poetic style but based on solid statistics covered areas like gypsies, the state of morality, temperance conditions, housing and sanitary conditions, the conditions of labour-, peasant-, fishery- and mining populations, demography and more anthropological areas like the home industry of women.

In addition to the scanty official statistics available, Sundt made use of statistics he collected by having clergymen and school teachers report on their districts. He also conducted an interview survey of all the families with school children of the community Piperviken and Ruselekabken, a labour class suburb of Christiania (Oslo).

For several months each year for a period of ten years Sundt walked through valleys and along shores to gather first-hand information on the people he studied. On his travels he collected much original and interesting data and he was inspired for further studies.

Sundt's research was characterized by good design and logic. He collected his data with some quite clear hypotheses in mind, hypotheses which grew out of painstaking ground work and which he stuck with throughout the analysis. The hypotheses were informally tested by comparing distributions, not only mean values.

By cross-classification Sundt was able to expose a variety of possible biases of his reporters (teachers, clergymen). When using an interviewer he kept his hypotheses secret to avoid bias. Spurious correlations, and possible external factors were sought and Sundt was often led to consider two- or three-way tables.

It is regrettable that none of Sundt's many works have been translated. He must be regarded as one of the first modern sociologists. After twenty years of research the stingy Storting (Parliament) was no longer willing to fund further research. Jaabæk (with the nickname Neibekk) argued that since such research was not undertaken in any other country it could not be of any value.

But the Storting did make Sundt's research possible by direct yearly funding for almost twenty years. One of the reasons was the fear in the ruling class of the rising socialism around 1848. One hoped that the theologian Sundt could help dam up for this movement. Even though Sundt himself was strongly against Marcus Thrane and his socialism, the long-term effects of Sundt's writing have been positive for the left in Norway. Sundt himself founded and led the Christiania (Oslo) Workers Association, and must be regarded not only as an outstanding statistician and a father of sociology but also a father of Norwegian social democracy.
Thiele studied astronomy and became professor in Copenhagen. Because of mathematical interests—and weak eyes, he worked mainly with theoretical astronomy.

At the age of 34 he took part in establishing the life insurance company Hafnia in which both Oppermann and Gram were engaged. He later founded the Danish Actuarial Society.

Thiele's interests in mathematics, astronomy and insurance led him naturally to statistics. In 1889 he published his text *Almindelig Iagttagelseslere* which in revised form appeared as *Theory of observations* (1903) in a "barbarous English translation (Hilary Seal). The theory of Gauss–Oppermann–Gram was here developed to a mature body of science.

Cumulants were invented by Thiele, he called them half invariants. They were used both for estimation purposes and for theoretical arguments. How Thiele happened to understand that the cumulants $\lambda_t$ defined by

$$
\sigma_{t+1} = \sum_{i=0}^t \binom{t}{i} \sigma_{t-i} \lambda_{t+1} \quad t = 1, \ldots ,
$$

$\sigma_k$ being the $k$th moment, had the property

$$
\lambda_t(\Sigma a_k X_k) = \Sigma a_k^t \lambda_t(X_k)
$$

for independent $X_1, \ldots, X_n$ is a mystery. But in 1889 he used this as an argument for the Central Limit theorem.

Thiele made a clear distinction between empirical and theoretical distributions (he called them laws of actual versus presumptive errors). He discussed the sampling properties of his estimates and adjusted them for bias. For least squares analysis he stressed the assumptions of normality, independence and equal (but known) variance.

By orthogonalization it was known how to partition the sum of squares. This Thiele used for "summary criticism", in effect hypothesis testing about the whole or a part of a linear model. Only the first two moments of the appropriate $\chi^2$ distribution was used, tail probabilities were not considered.

In his 1889 text, a thoroughly worked-out example from astronomy of a two way ANOVA was included. This was however replaced by more "pedagogic" examples in the English edition.

In addition to the ANOVA-summary criticism, Thiele proposed residual analysis including residual plots, runs tests and sign tests. By inspecting the residual plot, left-out-regressors may be revealed, but it is warned against overfitting the model. Also here the 1889 edition is more interesting.
At the age of 11 Wargentin experienced a lunar eclipse. That it was possible to predict this event to the hour, puzzles him so much that he decided to study astronomy. In Uppsala Celsius was his teacher and Linné his friend. He did such remarkable work on the satellites of Jupiter that, at the age of 24, he was offered the chair after Celsius. He balked because he would rather be a “respectable school teacher than a mediocre professor”.

In 1749 Wargentin was appointed secretary to the then 10 years old Swedish Royal Academy of Science. He kept this position to his death. The responsibilities included: to edit the quarterly proceedings, to edit various almanacs, to establish a library and to be the librarian, to take care of the correspondence (he received more than 4 000 letters), daily to take astronomical and meteorological observations, to establish an astronomical observatory —and to raise much needed money.

In addition to his regular duties as secretary, Wargentin kept working on the Jupiter problem. He became an active member in various commissions dealing with: constructing the Trollhättan channel, making maps of the coastal waters of Sweden, revising the Bible—and with establishing national population statistics.

From 1686 the clergymen were to keep records of the inhabitants of their parishes. In 1749 a central statistical office was established, and from that year regular national population statistics were produced in Sweden. Fearing that enemies should discover how small the population was, the numbers were kept a state secret until 1757.

The records produced by the clergymen turned out to be of varying quality. To improve the motivation for this work, Wargentin proposed in 1753 to have Süßmilchs Die göttliche Ordnung of 1741 translated to Swedish. Wargentin did in fact know the works of the leading demographers of the time: Graunt, Halley, Kersseboom, Süßmilch, Deparcieux, Price, and he corresponded with the latter three.

To open the eyes of the public for the value of population statistics in areas like medicine and pension funds, and for the national goal of increasing the population, Wargentin wrote a series of papers. Another motive was to persuade the authorities to make the population numbers public.

Wargentin covered most of the demographic field. To smooth his mortality tables he averaged the number of deaths over three year periods. He studied the monthly variation of deaths and births and he studied migration. He envisaged the population as an age pyramid “in which the children form the base and the old ones the top”.

_Wargentin, Pehr Wilhelm (1717–1783)_}

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