Shuttling transport in nanostructures

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The properties of nanocomposite materials are affected by coupling between mechanical deformations of the materials and electronic charge transport. In the simplest system, which is a single electron transistor (SET) with deformable tunnel-barriers (Nano-Electro-Mechanical SET, NEM-SET), mechanically assisted charge transfer becomes possible. This can be viewed as “shuttling of single electrons” between metallic leads by a movable small-sized cluster. In this paper we review different theoretical and experimental achievements on shuttle transport, in both normal as well as superconducting systems.

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I. INTRODUCTION

During recent years, nanotechnology has advanced the ability to fabricate systems in which chemical self-assembly defines the functional and structural units of nanoelectronic devices (1). Since the elastic parameters of many compounds and devices currently utilized can be much “softer” than those of semiconductors and metals, mechanical degrees of freedom may play an important role in charge transfer. In particular, charge transfer via tunneling through a device can be dramatically enhanced by mechanical motion of some part of the device.

The role of nanoelectromechanical coupling in these systems follows immediately from the heteroelastic nature of the materials taken in combination with the advanced role of Coulomb correlations controlling electronic inter-cluster tunneling. Tunneling of a single electron between nanometer sized grains is accompanied by an increase in electrostatic energy which can be of the order of 10 percent of the energy responsible for the binding of the material. As a result significant deformations of the soft dielectric fraction appear which exponentially affect the tunneling of electrons. Such mutual influence of mechanical displacements and tunnel charge redistribution results indeed in a dynamical coupling between mechanical...
and electronic degrees of freedom. Especially, since the
typical times characterizing the dynamics of each of them
can be of the same order of magnitude for metal-organic
nanocomposites this coupling cannot be neglected.

Recently, nanomechanical oscillators have been com-
combined with single-electron tunneling (SET) (2) devices,
resulting in a new class of nanoelectromechanical sys-
tems (NEMS) (3). Transport experiments with single os-
cillating molecules (4–7), suspended semiconductor sys-
tems (8; 9), and suspended carbon nanotubes (10) clearly
demonstrate the influence of mechanical degrees of free-
don current in single-electron tunneling regime.

As a simple example of a device of this type consider
a metallic grain located between a source and drain elec-
trode and elastically coupled to the substrate as in Fig. 1.

![Diagram of a soft Coulomb blockade system](image)

**FIG. 1** (a) Simple model of a soft Coulomb blockade system in which a metallic grain (center) is linked to two electrodes by elastically deformable organic molecular links. (b) A dynamic instability occurs since in the presence of a sufficiently large bias voltage $V$ the grain is accelerated by the corresponding electrostatic force towards first one, then the other electrode. A cyclic change in direction is caused by the repeated “loading” of electrons near the negatively biased electrode and the subsequent “unloading” of the same at the positively biased electrode. As a result the sign of the net grain charge alternates leading to an oscillatory grain motion and a novel “electron shuttle” mechanism for charge transport. From Ref. (11). © (1998) by the American Physical Society.

If, due to fluctuation in it’s position, the grain ap-
proaches source (or drain) electrode the tunneling cou-
pling between them significantly increase and grain be-
comes charged negatively (positively). Then, accelerat-
ing by elastic and Coulomb electrostatic forces, the grain
moves back approaching drain (source) electrode, thus
transferring the charge. The described process is usu-
ally referred to as shuttling of electrons. In general, the
shuttle mechanism can be defined as a charge transfer
through a mechanical subsystem facilitated by its oscil-
latory center of mass motion. The key issue here is that
the charge of the grain, $q(t)$, is correlated with its veloc-
ity, $\dot{x}(t)$, in a way that time average $\langle \dot{x}(t) \rangle \neq 0$, and
as a consequence the average work performed by electro-
static force, doesn’t equal to zero even if $\dot{x}(t) = 0$. As a result the mechanical motion and charge transfer can be
unstable with respect to formation of periodic or quasi
periodic mechanical motion and electrical signal.

Another important feature of nanosystems is the
Coulomb blockade (12). A small system, which have
accepted an electron, becomes negatively charged, and
under certain conditions another electron cannot, due to
Coulomb repulsion, reach the grain. As a result, it has to
wait until the first electron has escaped. Until then fur-
ther transport is blocked. Thus, under Coulomb blockade
conditions the electrons can be transferred only one by
one. The smaller the system capacitance, the bigger is
the charging energy. Consequently, Coulomb blockade is
an intrinsic property of small devices, and its importance
increases with the progress in nanoscience and nanotech-
ology. Hence, due to Coulomb blockade, shuttling of
single electrons or single Cooper pairs can take place.
We give a very brief review of Coulomb blockade in Sec. A.

During the past five years, shuttles of different types
were theoretically and experimentally studied. In par-
cular, shuttling can result either from an internal in-
stability similar to the one discussed above, or it can be
achieved by externally driven motion by for instance
some ac electric or mechanical force. Another point is
that electron transport through the device can obey ei-
ther classical, or quantum mechanics. Also the mechani-
cal degrees of freedom can obey either classical or quan-
tum mechanics. Hence, shuttle charge transfer in soft
nanostructures involves rich and interesting physics.

Presently, many researchers are interested in shuttling
in nano-electro-mechanical systems aiming to determine
fundamental properties of electro-mechanical coupling
in nanostructures, electron and phonon transport, etc.
This knowledge will definitely give renewed impetus to
the design of new application, such as nano-generators,
nanoswitches, current standards, etc.

Shuttle electron transfer can take place not only
through a metallic grain, but also through a relatively
soft nanocluster. In this connection, it is important
to study vibrational modes of electromechanical systems
with several degrees of freedom. If the center-of-mass
motion can be clearly split from other modes, then the
shuttle electron transfer though the system is similar to
that in the case of a rigid grain. Otherwise, it is more
appropriate to speak of vibronic-assisted tunneling rather
than shuttling.

Further, electromechanical coupling and shuttle trans-
port is not only a feature of heteroelastic nano-compos-
itives. It also has, as will be seen below, relevance for
other nanoelectromechanical (NEMS) systems intention-
ally designed to work at the nanometer scale (13–15).
Our aim is here to discuss the issues listed above. We will start from the simplest case of a particle with one mechanical degree of freedom located between two electrodes and elastically coupled either to the substrate or to the leads. This situation is relevant to several experiments, e.g., to electron shuttling through an artificial structure (16), and to shuttling through an oscillating C-60 molecule (4). Both the classical and quantum electron transport will be considered, Sections II.A and II.C, respectively (see Fig. II). It will be shown that in both cases an electromechanic instability occurs which leads to formation of a periodic mechanical motion (11; 17; 18). Section II.E aims to review theoretical works regarding the systems where the mechanical degrees of freedom need to be treated quantum mechanically. A review of the available experimental results is given in Sec. III. Finally, the possibility of coherent shuttling of Cooper pairs between two superconductors through a movable superconducting grain will be considered in Sec. IV.

II. SINGLE-ELECTRON TRANSFER BY A NANO-SHUTTLE

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TABLE I Classification of shuttle transport. Shuttle transport of charge can be classified according to what type of physical description is needed for the mechanical and electronical subsystems.

As already been mentioned, the shuttle charge transfer consists of the tunneling coupling with the leads and “convective” motion together with the vibrating cluster. To specify the shuttling regime it is natural to compare the decay length \( \lambda \) of the tunneling coupling with the quantum mechanical zero-point vibration amplitude, \( x_0 = \sqrt{\hbar/2m\omega_0} \). Here \( \omega_0 \) is the natural angular vibration frequency. If \( x_0 \ll \lambda \), mechanical motion can be treated classically. However, the electron transfer through the grain can be either sequential, or coherent.

In the first case, the relevant physical picture is fully classical - electrons can be regarded as classical particles, their transport obeying the Master equation. We will call this situation the classical shuttling of particles. In the case of coherent tunneling electrons should be treated as wave packets keeping track of the properties of their wave functions. We will call this situation classical shuttling of waves (cf. Fig. II). The word “classical” here serves as a reminder that the mechanical motion is classical. In all recent experiments, including shuttling by a C-60 molecule (4), the condition \( x_0 \ll \lambda \) was met. However, scaling down of the vibrating system can, in principle, lead to violation of this inequality. In that case the mechanical motion also obeys quantum mechanics. We refer to this case as the quantum shuttling. In the following sections all three regimes will be considered (see Fig. II).

A. Classical shuttling of particles

We begin this section by considering the classical shuttling of electrons and specifying the conditions which has to be met for shuttling to fall into this category (section II.A.1). We then proceed to considering the classical shuttling of electrons by a harmonically bound cluster between two leads in section II.A.2. After that, having seen that low damping is in this case necessary for shuttle transport a system dominated by viscous forces is considered in II.A.3. Also in this case classical shuttling of particles take place.

The perspectives of the shuttle for applications depend strongly of several issues. Among them are: (i) conditions required for ideal shuttling, which is crucially important for standards of electric current or for sensors; (ii) gate-induced control of shuttling important for application as single-electron transistor; (iii) role of other than center-of-mass mechanical degrees of freedom. These issues will be consequently considered in the end of this Section in subsections II.A.4, II.A.5 and II.A.6.

1. Requirements for incoherent transport

A schematic picture of a single electronic device with a movable metallic cluster is presented in Fig. 1. Since electronic transport in the device is due to tunneling between the leads and the central small size conductor, it is strongly affected by the displacement of the latter. In this case center-of-mass mechanical vibrations of the grain will be present (consider the elastic springs, connecting the central electrode to the leads in Fig. 1).

A number of characteristic times determine the dynamical evolution of the system. Electronic degrees of freedom are represented by frequencies corresponding to such energies as the Fermi energy in each of the conductors and the applied voltage \( eV \). In addition one has an inverse relaxation time and inverse phase breaking time for electrons in the conductors and a tunneling charge...
relaxation time $\omega_R^{-1} = RC$. Here $R$ and $C$ respectively are the resistance and capacitance of the tunnel junction. Mechanical degrees of freedom are characterized by vibrational frequency $\omega_0$. The condition that $h\omega_R$ should be much smaller than the Fermi energy is the standard condition for a weak tunneling coupling and holds very well in usual tunnel structures. Since a finite voltage is supposed to be applied causing a non equilibrium evolution of the system the question of electronic relaxation becomes relevant. Two possible scenarios of electronic transfer through the metallic cluster can be identified depending on the ratio between the tunneling relaxation time $\omega_R^{-1}$ and the intra-grain electronic relaxation time $\tau_0$. In the case that $\tau_0$ is much shorter than $\omega_R^{-1}$, two sequential events of electronic tunneling, necessary to transfer an electron from one lead to another through the grain, cannot be considered as a quantum mechanical coherent process due to the relaxation and phase breaking occurring in between the events (which are separated in time by interval, which is approximately equal $\omega_R^{-1}$). In this case all electronic tunneling transitions between each of the leads and the grain can be considered as incoherent independent events while fast relaxation of electrons in all three conductors is supposed to be responsible for the formation of local (on each of the conductors) equilibrium electronic distribution functions. This is the approach which we will use in the present section. In the opposite limit, $\tau_0$ much larger than $\omega_R^{-1}$, quantum coherence plays a dominating role in the electronic charge transfer and all relaxation takes place in the leads far away from the central part of the device. This case will be considered in Sec. II.C

2. Shuttling of electrical charge by a movable Coulomb dot

The tunnel junctions between the leads and the grain in Fig. 1 are modelled by tunneling resistances $R_L(x)$ and $R_R(x)$ which are assumed to be exponential functions of the grain coordinate $x$. In order to avoid unimportant technical complications we study the symmetric case for which $R_L(x) = R(0)e^{x/\lambda}$. When the position of the grain is fixed, the electrical potential of the grain and its charge $q_{st}$ are given by current balance between the grain and the leads (12). As a consequence, at a given bias voltage $V$ the charge $q_{st}(x)$ is completely controlled by the ratio $R_L(x)/R_R(x)$ and $dq_{st}(x)/dx < 0$. In addition the bias voltage generates an electrostatic field $E = \alpha V$ in the space between the leads and, hence, a charged grain will be subjected to an electrostatic force $F_q = \alpha V q$.

The central point of our considerations is that the grain — because of the “softness” of the organic molecular links connecting it to the leads — may move and change its position. The grain motion disturbs the current balance and as a result the grain charge will vary in time in tact with the grain displacement. This variation affects the work $W = \alpha V \int q(t)dt$ performed on the grain during, say, one period of its oscillatory motion.

It is significant that the work is nonzero and positive, i.e., the electrostatic force, on the average, accelerates the grain. The nature of this acceleration is best understood by considering a grain oscillating with a frequency which is much less than the typical charge fluctuation frequency $\omega_R = 1/RC$ (Here $C$ is the capacitance of the metallic cluster which for room temperature Coulomb blockade systems is of the order $10^{-18} - 10^{-19}$ F). In this limit the charge deviation $\delta q \equiv (q - q_{st}(x))$ connected with retardation effects is given by the expression $\delta q = -\omega_R^{-1}\int dq_{st}(x)/dx$. Hence the extra charge $\delta q$ depends on the value and direction of the grain velocity and as a consequence, the grain acts as a shuttle that carries positive extra charge on its way from the positive to the negative electrode and negative one on its return trip. The electrostatic force $\delta F_q = \alpha V \delta q$ is thus at all times directed along the line of motion causing the grain to accelerate. To be more precise, it was shown (11; 17) that for small deviations from equilibrium ($x = 0, q = 0$) and provided $q(t)$ is defined as the linear response to the grain displacement, $q(t) = \int \chi(t - t')dx(t')dt'$, the work done on the grain is positive at any relation between charge fluctuation frequency $\omega_R$ and frequency of the grain vibration.

In any real systems a certain amount $Q$ of energy is dissipated due to mechanical damping forces which always exist. In order to get to the self-excitation regime more energy must be pumped into the system from the electrostatic field than can be dissipated; $W$ must exceed $Q$. Since the electrostatic force increases with the bias voltage this condition can be fulfilled if $V$ exceeds some critical value $V_c$.

If the electrostatic and damping forces are much smaller than the elastic restoring force self-excitation of vibrations with a frequency equal to the eigenfrequency of elastic oscillations arise. In this case $V_c$ can be implicitly defined by the relation $\omega_0^2 = \alpha V_c \Im\chi(\omega)$, where $\omega_0^2$ is the imaginary part of the complex dynamic modulus. In the general case, when the charge response is determined by Coulomb-blockade phenomena, $\chi$ is an increasing but rather complicated function of $V$ and there is no way to solve for $V_c$ analytically. However it was found (17) that the minimal value of $V_c$ corresponds to the situation when the charge exchange frequency $\omega_R$ is of the same order as eigenfrequency of the grain vibration $\omega$.

Above the threshold voltage the oscillation amplitude will increase exponentially until a balance between dissipated and absorbed energy is achieved and the system reaches a stable self-oscillating regime. The amplitude $A$ of the self-oscillations will therefore be determined by the criterion $W(A) = Q(A)$.

The transition from the static regime to the self-oscillating can be associated with either soft or hard excitation of self-oscillations depending on the relation between the charge exchange frequency $\omega_R$ and eigenfrequency of grain oscillation $\omega_0$ (17). The soft excitation takes place if $\omega_R/\omega_0 > 2\sqrt{3}$. In this case the amplitude of the stable self-oscillation regime increases smoothly (with
voltage increase) from zero at the transition voltage. In a case of hard excitation ($\omega_R/\omega_0 < 2\sqrt{3}$) the oscillation amplitude jumps to a finite value when voltage exceeds $V_c$. It was also found (17) that the hard excitation is accompanied by a hysteretic behavior of the current-voltage characteristics.

In the fully developed self-oscillating regime the oscillating grain, sequentially moving electrons from one lead to the other, provides a ‘shuttle mechanism’ for charge as shown in Fig. 1b. In each cycle $2n$ electrons are transferred, so the average contribution to the current from this shuttle mechanism is

$$I = 2enf, \quad n = \left[ \frac{CV}{e} + \frac{1}{2} \right].$$

where $f \equiv \omega_0/2\pi$ is the self-oscillation frequency. This current does not depend on the tunneling rate $\omega_R$. The reason is that when the charge jumps to or from a lead, the grain is so close that the tunneling rate is large compared to the elastic vibration frequency. Hence the shuttle frequency — not the tunneling rate — provides the ‘bottle neck’ for this process. We emphasize that the current due to this shuttle mechanism can be substantially larger than the conventional current via a fixed grain. This is the case when $\omega_0 \gg \omega_R$.

To support the qualitative arguments given above we have performed analytical and numerical analyses based on the simultaneous solution of Newton’s equation for the motion of a grain’s center of mass and a master equation for the charge redistribution.

Two different approaches were developed. The first one, presented in (11), gives the quantitative description of the shuttle instability for low tunnel-barrier resistances when the rate of charge redistribution is so large (in comparison with the vibrational frequency), that the stochastic fluctuations in the grain charge during a single-vibration period are unimportant. The second one, describing the opposite limit of low-charge redistribution frequencies characteristic of high-resistance tunnel barriers, is presented in (17).

In both cases it was shown that the electromechanical instability discussed above has dramatic consequences for the current-voltage characteristics of a single electron transistor configuration as shown in Fig. 2. Even for a symmetric double junction, where no Coulomb staircase appears in conventional designs, we predict that the shuttle mechanism for charge transport manifests itself as a current jump at $V = V_c$ and as a Coulomb staircase as the voltage is further increased.

A more precise calculation along the line $l$ sketched in Fig. 2 is shown in Fig. 3. The non-monotonic behavior of the current along this line is due to competition between the two charge transfer mechanisms present in the system, the ordinary tunnel current and the mechanically mediated current $I_{\text{mech}}(x_0,t) = \delta(x(t) - x_0)\dot{x}(t)q(t)$ through some cross section at $x_0$. We define the shuttle current as the time averaged mechanical current through the plane located at $x_0 = 0$. This current together with the tunnel current for the same cross section is shown in Fig. 3. As the damping in the system is reduced the oscillation amplitude grows and the shuttle current is enhanced while the ordinary tunneling current is suppressed. In the limit of low damping this leads to a quantization of the total current in terms of $2ef$. We will briefly sketch the calculation procedure leading to the above conclusions in Appendix B.

3. Shuttling in dissipative nanostructures

From the above analysis it is clear that a large damping is detrimental for the development of the shuttle instability and in the limit where $\gamma \gg \omega_0$, elastic shuttling of the charge becomes impossible. The mechanical lability of the system, however, is still a dominating feature of the charge transport even in the limit of strong dissipation. The consequences of such a lability are addressed in Ref. (19).

There the elastic restoring force is assumed absent or much weaker than viscous damping forces. According to that model, charge transport through the NEM-SET is affected both by the Coulomb blockade phenomenon and the mechanical motion of the cluster. These two phenomena are coupled since the threshold voltage for electron tunneling depends on the junction capacitances which, in turn, depend on the cluster position with respect to the leads. In general, the threshold voltage increases when the distance between the cluster and an electrode decreases.

To be specific, if a neutral cluster is located in its
The tunnel current is proportional to the fraction of the shuttle current approaches the quantized value \( \frac{I\pi}{e} \). This fraction is inversely proportional to the oscillation amplitude \( \omega \). The tunnel current is proportional to the fraction of the oscillation period spent in the middle region, \( |x| < \lambda \). This fraction is inversely proportional to the oscillation amplitude and hence the tunnel current decreases as \( \gamma^{-1} \) increases. The fine structure in the results is due to numerical noise. From Ref. (11). © (1998) by the American Physical Society.

### FIG. 3
Cross section along the line \( l \) in Fig. 2. The total time averaged current consist of two parts, the shuttle current and the tunneling current. The time averaged shuttle current is the mechanically transferred current through the center of the system \( \langle \delta \dot{x}(t) \rangle \). The remaining part comes from ordinary tunneling. As the inverse damping \( \gamma^{-1} \) increase the shuttle current approaches the quantized value \( \frac{I\pi}{e\omega} = 3 \). The tunnel current is proportional to the fraction of the oscillation period spent in the middle region, \( |x| < \lambda \). This fraction is inversely proportional to the oscillation amplitude and hence the tunnel current decreases as \( \gamma^{-1} \) increases. The fine structure in the results is due to numerical noise. From Ref. (11). © (1998) by the American Physical Society.

The conductance is now assisted by significant cluster displacements between the electrodes. This event changes the sign of the net charge on the grain. In this case the cluster can be pushed by the Coulomb force towards the more distant electrode where the above described process repeats itself. The conductance is now assisted by significant displacements of the grain and this scenario is qualitatively similar to the shuttle vibrations in fully elastic electromechanical structures (11). This process is also accompanied by a marked rise in the current through the system as shown in Fig. 4.

### FIG. 4
The solid line shows the current - voltage characteristics obtained by a Monte Carlo simulation of the charge transport through a dissipation dominated system. The calculated DC current is plotted as a function of the bias voltage \( V \) scaled by the Coulomb blockade threshold voltage \( V_0 \), that applies if the movable grain is equally far from both electrodes. The dashed line displays the current through a static symmetric double junction for the same parameters. For voltages above the threshold voltage \( V_t \) the current through the system increase drastically due to grain motion. The inset which shows a magnification of the voltage interval around \( V_t \). From Ref. (19). © (2002) by the American Physical Society.

4. **Accuracy of a mechanical single-electron shuttle.**

Several mechanisms contribute to deviation of the average current from the ideal shuttle value \( n_{eff} \):

1. “Shunting” sequential electron tunneling and cotunneling through the grain which leads to DC current without any grain motion.

2. Interplay between the contact time \( t_0 \) and the charge relaxation time \( e\gamma\omega^{-1} \) at the trajectory turning point. This relation determines if the grain can be fully loaded during a single contact event.

3. Thermal fluctuations which lift the Coulomb blockade limitation for the transferred charge to be integer.

The contribution of the shunting tunneling seems to be much less than the current conveyed by a shuffling grain. Indeed, the former is limited by the maximum tunnel resistance which is exponentially large. The second and
third limitations to the accuracy of the shuttling current were considered in Ref. (20) where a Master equation for the charge of the moving grain was analyzed. In this approach the shuttling was mapped on a sequence of contact events when charge transfer takes place. The results of analytical treatment of a simple model and of numerical treatment are shown in Figs. 5 and 6 taken from the paper (20). In Fig. 5 the average number of electrons transferred per period, as well as the root mean fluctuations, are shown for \( T = 0 \). In this figure \( t_0 \) represents the effective time the grain spends in contact with the leads while \( \tau \) denotes the RC-time at the point of closest approach to the leads. Assuming the grain is closest to the leads at a time \( t_{\text{max}} \) one has \( \tau = R(t_{\text{max}})C(t_{\text{max}}) \). Thermal smearing of the single-electron shuttling is demonstrated in Fig. 6. It is clear that for \( t_0 \gg \tau \) and \( T = 0 \) the Coulomb staircase is perfect. For increasing temperatures, the Coulomb staircase is washed out leading to an Ohmic behavior for high temperatures.

![Fig. 5](image-url) The average number of electrons transferred per period (left) and the root mean fluctuations (right) for \( T = 0 \). Coulomb blockade is clearly visible: up to a critical voltage \( V_{\text{C}}(t_{\text{max}})/e = \frac{1}{2} \) no electrons are transferred. From Ref. (20). © (1999) EDP Sciences.

![Fig. 6](image-url) Thermal smearing of single-electron shuttling. The energy scale is given by \( E_C = e^2/2C \). \( t_{\text{max}} \) is the instant when a particle is located at a turning point. From Ref. (20). © (1999) EDP Sciences.

5. Gate voltage control of shuttle mechanics

The electromechanical coupling also dramatically changes the transistor effect in NEM-SET as compared to an ordinary SET. We will discuss this problem following Ref. (21). Let us assume that the tunneling can take place only between the grain and the leads, while there is no tunneling exchange with the gate. The gate voltage controls the equilibrium position of the grain with respect to the leads since it determines the extra charge of the grain. Schematic configuration of a relevant NEM-SET is sketched in Fig. 7. The picture describes a situation when the grain has a net negative charge. The net negative charging of the grain occurs at certain relation between the bias, \( V_b \), and the gate, \( V_g \), voltages. The positively charged gate electrostatically induces a negative charge on the grain, which tends to be compensated by the tunneling of positive charge from the right lead. Here we assume that the tunneling from the negatively charged remote electrode is exponentially suppressed. Since the grain is shifted from the central position, the current through the device is exponentially small. An increase in the bias voltage decreases the total negative charge. At complete compensation the grain returns to the cen-
central position restoring the tunneling transport through the device. The “phase diagram” in the $V_g - V_b$ plane obtained in Ref. (21) is shown in Fig. 8. It is worth mentioning a qualitative difference of this “butterfly” phase-diagram from the conventional “diamond” phase diagram in conventional SET (15) where the mechanically blocked SET operation is absent.

6. Nanoparticle chains

We conclude this section concerning classical shuttling of electrons by considering theoretical work regarding nanoparticle chains (22). Nanoparticle chains consist of small metal grains stabilized by ligands, with electronic transport occurring via tunneling between the metal particles. Because of the relative softness of the ligand matrix, vibrations of the metal grains can significantly modify the electronic tunneling rates. In the systems with several degrees of freedom, the electro-mechanical instability at special values of system parameters can lead to excitation of more than one mode. As a result, the mechanical motion becomes in general non-periodic with a possibility of a telegraph-like switching between the modes. A crossover from a periodic to a quasiperiodic motion, as well as telegraph-like switching between these regimes was demonstrated in Refs. (22; 23). In this paper, electron shuttling through a system of two elastically and electrically coupled particles was numerically simulated, and a telegraph-like switching was observed at some value of bias voltage.

B. The charge shuttle as a nanomechanical ratchet

One of the advantages of self-oscillating shuttle structures is that they can generate very high frequency mechanical oscillation with static voltages. The fact that the system has an intrinsic and stable oscillating mode generated by a static voltage suggests that the application of an oscillating voltage may lead to new interesting effects, related to the interplay between the external AC drive and the internal frequency of the device. Moreover, as the dynamics of shuttle systems is essentially nonlinear, this interplay should emerge in a wide interval of the ratio of the two frequencies.

A shuttle driven by a time-dependent applied bias was considered in (24), where it was shown that an asymmetric structure can act as a ratchet (see, e.g., (25)) in which the forcing potential is generated in a self-consistent way. A sizeable ratchet effect is present down to bias frequencies much smaller than the mechanical resonating frequency $\omega_0$, due to the adiabatic change of the equilibrium position of the grain. In a very recent experiment (26) some results similar to the predictions of (24) were observed. However, the experiment is not in the single electron limit.

Assuming a harmonic bias voltage, $V = V_0 \sin \omega t$, the authors simulate the grain dynamics and stochastic electron transfer for an asymmetric system characterized by the resistances $R_L(0) \neq R_R(0)$. The asymmetric system is specifically interesting because no DC current would be generated in a symmetric one. Since the electron transition rates are proportional to $|V(t)|$, they turn out to be time-dependent. According to the simulation, after a transient time the system reaches a stationary behavior. Shown in Fig. 9 is the stationary DC current as a function of the frequency of the external bias.

The rich structure shown in the figure is generic, we observed a qualitatively identical behavior in a wide range of parameters. The existence of a direct current as a re-
is the energy level in the dot shifted due to the voltage dependent tunneling matrix element, \( H = \sum_{\alpha,k} (\epsilon_{ak} - \mu_\alpha) a_\alpha^k a_\alpha^k + \epsilon_d(t) c^\dagger c^\dagger + \sum_{\alpha,k} T_\alpha(t) (a_\alpha^k c + c^\dagger a_\alpha^k) \).

Here \( T_{L,R} = \tau_{L,R} \exp\{x(t)/2\lambda\} \) is the position-dependent tunneling matrix element, \( \epsilon_d(t) = \epsilon_0 - eE x(t) \) is the energy level in the dot shifted due to the voltage

result of an applied periodic modulation shows that the charge shuttle, whose stochastic dynamics has been defined above, behaves as a ratchet (25). Since the system is nonlinear, the external driving affects the dynamics also for values of \( \omega \) very different from the natural frequency \( \omega_0 \). Note that in this model the nonlinearities are intrinsic to the shuttle mechanism. They are due to specific time dependence of the grain charge, \( c(t) \), rather than to a nonlinear mechanical force. As it is evident from Fig. 9 the ratchet behaviour is present also in the adiabatic limit, \( \omega/\omega_0 \ll 1 \). In addition a series of resonances, due to frequency locking, see, e. g., (27), when \( \omega/\omega_0 = q/p \), with \( q \) and \( p \) integers. In this case the motion of the shuttle and the oscillating source become synchronized in such a way that every \( q \) periods of the oscillating field the shuttle performs \( p \) oscillations.

C. Classical shuttling of electron waves

In this section we follow the considerations of Ref. (18). There it is assumed that a vibrating grain has a single resonant level, both the position \( x(t) \) of this level and the coupling of the grain to the leads, \( T_{L,R}(t) \), are oscillatory functions of time, Fig. 10. The effective Hamiltonian of the problem is defined as

\[
H = \sum_{\alpha,k} (\epsilon_{ak} - \mu_\alpha) a_\alpha^k a_\alpha^k + \epsilon_d(t) c^\dagger c^\dagger + \sum_{\alpha,k} T_\alpha(t) (a_\alpha^k c + c^\dagger a_\alpha^k) \, .
\]

(2)

FIG. 9 Current as a function of the frequency for \( \epsilon = 0.5 \), \( \gamma/\omega_0 = 0.05 \) and \( \Gamma/\omega_0 = 1 \). The result of the simulation of the stochastic dynamics (points) is compared with the approximate current \( I \) (full line). In the small frequency region, enlarged in the inset, several resonances at fractional values of \( \omega \) appear. The dotted line is an analytical for the adiabatic limit. Lower inset: current noise from the simulation (points) and analytical result (dashed) for the static SET. From Ref. (24).

induced electric field \( E \), \( a_\alpha^\dagger \) creates an electron with momentum \( k \) in the corresponding lead, \( \alpha = L, R \) is the lead index, \( e_\chi \) creates an electron in the dot. The first term in the Hamiltonian describes the electrons in the leads, the second — the movable quantum dot and the last term — tunneling between the leads and the dot.

The evolution of the electronic subsystem is determined by the Liouville-von Neumann equation for the statistical operator \( \hat{\rho}(t) \),

\[
i\partial_t \hat{\rho}(t) = h^{-1}[\hat{H}, \hat{\rho}(t)] - \ , \quad (3)
\]

while the center of mass motion of the dot is governed by Newton’s equation,

\[
\ddot{x} + m_0^2 \dot{x} = F/m \ . \quad (4)
\]

Here \( \omega_0 = \sqrt{k_0/m} \), \( m \) is the mass of the grain, \( k_0 \) is a constant characterizing the strength of the harmonic potential, \( F(t) = -\text{Tr} \{ \dot{\rho}(t) \partial \hat{H}/\partial x \} \). The force \( F \) is computed from an exact solution of the tunneling problem, which exists for arbitrary \( T_\alpha(t) \) and \( \epsilon_d(t) \). Using the Keldysh Green’s function approach (28) in the so-called wide band approximation and follow Ref. (29) one obtains the following expression for the force \( F \):

\[
F(t) = \sum_{\alpha} g_\alpha \int d\epsilon f_\alpha(\epsilon) \left\{ \epsilon E |B_\alpha(\epsilon,t)|^2 + (-1)^\chi \lambda^{-1} T_\alpha(t) \text{Re}[B_\alpha(\epsilon,t)] \right\} \quad (5)
\]

where

\[
B_\alpha(\epsilon,t) = - i h^{-1} \int_{-\infty}^t dt_1 T_\alpha(t_1) \times \text{exp} \left\{ i h^{-1} \int_{t_1}^t dt_2 \left[ \epsilon - \epsilon_d(t_2) + i \Gamma(t_2) \right] \right\} \, ,
\]

\[
\Gamma(t) = 2\pi \sum_{\alpha} g_\alpha |T_\alpha(t)|^2, \quad g_\alpha \text{ is the density of states in the corresponding lead and } f_\alpha(\epsilon) = \{\text{exp}[\beta(\epsilon - \mu_\alpha)] + 1\}^{-1}.
\]
The first item in the expression (5) for $F(t)$ is the electric force due to accumulated charge; the second one is the “cohesive” force due to position-dependent hybridization of the electronic states of the grain and the leads. Equations (4) and (5) can be used to study stability of the equilibrium position of the cluster. Linearizing Eq. (4) in small displacement $\propto e^{-i\omega t}$ and solving proper equations of motion one can obtain complex vibration frequency, the mechanical instability can be found from the inequality $\text{Im} \omega \geq 0$. As shown in Ref. (18), the instability occurs if the driving voltage exceeds some critical value which for symmetrically applied voltage is $eV_c = 2(\epsilon_0 + h\omega_0)$. It was also shown that in the limit of weak electromechanical coupling, when $\Gamma/(4\hbar\omega_0^2\lambda^2)$, and $2e\mathcal{E}\lambda/(\sqrt{\hbar^2\omega_0^2 + \Gamma^2}) \ll 1$, the instability develops into a limiting cycle. This is in contrast with a classical shuttle, where stability of the system could be achieved only at finite mechanical dissipation. The reason of the difference is that in the present situation the dissipation is provided by coupling to electronic degrees of freedom. In the classical treatment this dissipation mechanism has been disregarded since phonon-assisted tunneling was ignored. One can see a qualitative agreement between the experimentally observed $I(V)$ curve for a fullerene based NEM-SET in Fig. 15 and Fig. 11 where results of the above calculation are presented. However, there are alternative interpretations of the experiment (4) (see (30; 31)), based on quantum treatment of mechanical motion. We will discuss these explanations below.

D. Spin-dependent transport of electrons in a shuttle structure

The possibility to place transition-metal atoms or ions inside organic molecules introduces a new “magnetic” degree of freedom that allows the electronic spins to be coupled to mechanical and charge degrees of freedom (32). By manipulating the interaction between the spin and external magnetic fields and/or the internal interaction in magnetic materials, spin-controlled nanoelectromechanics may be achieved. An inverse phenomenon — nanomechanical manipulation of nanomagnets — was suggested earlier in (33). A magnetic field, by inducing the spin of electrons to rotate (precess) at a certain frequency, provides a clock for studying the shuttle dynamics and a basis for a dc spectroscopy of the corresponding nanomechanical vibrations. Since spin effects are sensitive to an external magnetic field the electron transport through a shuttle structure becomes spin-dependent.

A particularly interesting situation arises when electrons are shuttled between electrodes that are so-called half-metals where all the electrons are in the same spin state — the material is fully spin-polarized. Examples of such materials can be found among the perovskite manganese oxides, a class of materials that show an intrinsic, so called “colossal magnetoresistance” effect at high magnetic fields (of order 10-100 kOe) (34).

A large magnetoresistance effect at lower magnetic fields has been observed in layered tunnel structures where two thin perovskite manganese oxide films are separated by a tunnel barrier (34–38). Here the spin polarization of electronic states crucially affects the tunnelling between the magnetic electrodes. Indeed, electrons extracted from the source electrode have a certain spin polarization while to be injected into a drain electrode they have to be polarized in a generally different direction. Clearly the tunnelling probability and hence the resistance must be strongly dependent on the relative orientation of the magnetization of the two electrodes. The relative magnetization can be tuned by an external magnetic field. A change in the resistance of trilayer devices by factors of order 2-5 have in this way been induced by magnetic fields of order 200 Oe (35–37). The required field strength is determined by the coercivities of the magnetic layers. This makes it difficult to use a tunnelling device of the described type for sensing very low magnetic fields.

A new functional principle — spin-dependent shuttling of electrons — for low-magnetic field sensing purposes was proposed in (39). This principle may lead to a giant magnetoresistance effect in external fields as low as 1-10 Oe. The key idea is to use the external magnetic field to manipulate the spin of shuttled electrons rather than the magnetization of the leads. Since an electron spends on a shuttle a relatively long time being decoupled from environment even a weak magnetic field can rotate its spin by a significant angle. Such a rotation allows the spin of an electron, loaded onto the shuttle from the spin-polarized source electrode, to be reoriented in order to allow the electron finally to tunnel from the shuttle to the spin-polarized drain lead. In this way the shuttle serves as a very sensitive giant magnetoresistance (GMR) device.

![Graph](image-url)
The model employed in (39) assumes that the source and drain are fully polarized in opposite directions. A mechanically movable quantum dot (described by a time-dependent displacement \( x(t) \)), where a single energy level is available for electrons, performs driven harmonic oscillations between the leads. The external magnetic field, \( \mathbf{H} \) is perpendicular both to the orientations of the magnetization in both leads and direction of mechanical motion.

The spin-dependent part of the Hamiltonian is specified as

\[
\hat{H}_s(t) = J(t)(a_1^\dagger a_1 - a_1^\dagger a_1) - \frac{g_\mu H}{2}(a_1^\dagger a_1 + a_1^\dagger a_1),
\]

where \( J(t) = J_L(t) - J_R(t), a_{1(2)}^\dagger(1) \) are the creation (annihilation) operators on the dot, \( J_{L(R)}(t) \equiv J_{L(R)}[x(t)] \) are the exchange interactions between the on-grain electron and the left (right) lead, \( g \) is the gyromagnetic ratio and \( \mu \) is the Bohr magneton. The proper Liouville-von Neumann equation for the density matrix is analyzed and an average electrical current is calculated for the case of large bias voltage.

The behavior of the current depends on an interplay between three frequency scales: (i) the frequency of spin rotation, determined by the tunnel exchange interaction, \( J_{L(R)} \), with the magnetic leads; (ii) the frequency of spin rotation in the external magnetic field, \( g_\mu H/\hbar \), and (iii) the frequency of shuttle vibrations, \( \omega_0 \).

In the limit of weak exchange interaction, \( J_{\text{max}} \ll \mu H \) one may neglect the influence of the magnetic leads on the on-dot electron spin dynamics.

\[
I = \frac{\omega_0 e}{\pi} \frac{\sin^2(\theta/2) \tanh(w/4)}{\sin^2(\theta/2) + \tanh^2(w/4)},
\]

where \( w \) is the total tunneling probability during the contact time \( t_0 \), while

\[
\theta = h^{-1}g_\mu H(\pi/\omega - t_0) \sim \pi g_\mu H/\hbar \omega_0
\]
is the rotation angle of the spin during the “free-motion” time. The \( H \)-dependence of magneto-transmittance in this limit is characterized by two different scales. The first one is associated with resonant magnetic field dependence through the angle \( \theta \) in the denominator of Eq. (6). This scale is

\[
\delta H = \hbar \omega_0/g_\mu,
\]

The second scale,

\[
\Delta H = \hbar \omega_0/g_\mu,
\]

comes from the periodic function \( \sin^2(\theta/2) \) in the numerator of Eq. (6). The magnetic-field dependence of the current is presented in Fig. 12.a. Dips in the transmittance of width \( \delta H \) appear periodically as the magnetic field is varied, the period being \( \Delta H \). This amount to a giant magneto-transmittance effect. It is interesting to notice that by measuring the period of the variations in \( I(H) \) one can in principle determine the shuttle vibration frequency. This amounts to a DC method for spectroscopy of the nanomechanical vibrations. Equation (8) gives a simple relation between the vibration frequency and the period of the current variations. The physical meaning of this relation is very simple: every time when \( \omega/\Omega = n + 1/2 \) (\( \Omega = g_\mu H/\hbar \) is the spin precession frequency in a magnetic field) the shuttled electron is able to fully flip its spin to remove the “spin-blockade” of tunneling between spin polarized leads having their magnetization in opposite directions.

A strong magnetic coupling to the leads, \( J_{\text{max}} \gg g_\mu H \), preserves the electron spin polarization, preventing spin-flips of shuttled electrons due to an external magnetic field. However, if the magnetization of the two leads are directed oppositely, the signs of the exchange coupling to the leads are different. Therefore, the exchange couplings to the two leads tend to cancel out when the dot is in the middle of the junction. Hence, the effective exchange Hamiltonian affecting a dot electron periodically changes sign, being small close to the time of sign reversal, see Fig. 13. Thus the effect of an external magnetic field is negligible almost everywhere, except in the vicinity of the level crossing, where the external magnetic field removes the degeneracy forming a gap is in the spectrum (dashed curve).

The electronic spin-flip in this case occurs via a Landau-Zener (40) reflection from the gap. Note that in this case a Landau-Zener transition across the gap is a mechanism for backscattering of the electron, since this is the channel where the electronic spin is preserved. The schematic \( I(H) \) dependence for this case is shown in Fig. 12. The width \( \delta H \) of the minimum in the \( I(H) \)
several models were discussed. Some of these works are
transport in the introduction). Several papers address
vided that the center of mass motion is correlated in
veloped a quantum mechanical model of electron tunnel-
under the influence of nanomechanic excitations has been
mode assistant charge transfer as a
be stronger coupled to the charge transfer than other
tunneling through nanostructures. However, in some
modes. In this case one can interpret center-of-mass
arrier is damped, the strength of the couplings between the
machanical degrees of freedom; (ii) dependence of tunneling
occurs due to (i) dependence of electronic levels, charges,
t without the position of the dot (see definition of shuttle
in Sec. II.A, II.C and Refs. (11; 18), it is assumed that
the vibrational frequency is several orders of magnitude
larger that the frequency associated with tunneling ef-
fects. The system is described by an effective Hamilton-
ian involving local bosonic degrees of freedom of the
molecule. One of the local modes can be interpreted as
center-of-mass motion. The quantum mechanical calcu-
lation leads to a set of horizontal plateaus in $I-V$ curve
due to excitation of different vibrational modes. It is
shown that in some regions of the parameters a nega-
tive differential resistance occurs. A similar calculation,
but with more detailed account of the dependence of the
charge-transfer matrix elements in the shuttle coordinate,
was reported by McCarthy et al (31).

The papers (30) and (31) provide a qualitative ex-
planation to the experiment (4). This explanation is ac-
tually alternative to that given in Ref. (18), where the
center-of-mass motion was treated classically. It is worth
mentioning that the latter model predicts a finite slope of
the $I-V$ curve at large voltages, which is more similar
to the experimental result.

A quantum oscillator consisting of a dot on springs
flanked by two stationary dots attached to semi-infinite
leads was considered by Mackinnon and Armour (41).
The authors concentrate on quantum aspects of the dot
and electron motion. It is shown that the $I-V$ charac-
teristics of the model shuttle can largely be understood
by analyzing the eigenspectrum of the isolated system of
three dots and the quantum oscillator. Tunnel coupling
of the dot states, to each other and to the position of the
oscillator, leads to repulsion of the eigenvalues and mix-
ing of the eigenstates associated with states localized
on individual dots. The mixed states consist of superposi-
tions of the states associated with the individual dots and
hence lead to delocalization of the electronic states be-
tween the dots. Analysis of the current which flows when
the shuttle is weakly coupled to leads, reveals strong res-
onances corresponding to the occurrence of the delocal-
ized states. The current through the shuttle is found to
depend sensitively on the amount by which the oscilla-
tor is damped, the strength of the couplings between the
dots and the background temperature. When the tun-
neling length $\lambda$ of the electrons is of order $x_0$, current far
from the electronic resonance is dominated by electrons
hopping on and off the central dot sequentially. As the
authors state, then the oscillator can be regarded as shut-
tling electrons across the system as it has been discussed
in Sec. II.A.

E. Charge transfer through a quantum oscillator

In the previous discussion we assumed that the me-
chanical degree of freedom was classical. However, the
center-of-mass displacement can be comparable with
the quantum mechanical zero-point vibration amplitude,
$x_0 = \sqrt{\hbar/m\omega_0}$. In this case electronic and mechanical
degrees of freedom behave as coupled parts of a quantum
system which should be considered according to quan-
tum mechanics. The coupling between the subsystems
occurs due to (i) dependence of electronic levels, charges,
their images, and, consequently, electric forces on the me-
chanical degrees of freedom; (ii) dependence of tunneling
barriers on the spatial system configuration.

In general, the charge transfer in the systems un-
der consideration is assisted by excitation of vibrational
degrees of freedom which is similar to phonon-assisted
tunneling through nanostructures. However, in some
cases the center-of-mass mechanical mode turns out to
be stronger coupled to the charge transfer than other
modes. In this case one can interpret center-of-mass
mode assistant charge transfer as a quantum shuttle pro-
vided that the center of mass motion is correlated in
time with the charge on the dot (see definition of shuttle
transport in the introduction). Several papers address
the charge transport through a quantum oscillator, and
several models were discussed. Some of these works are
reviewed below.

Single-electron tunneling through molecular structures
under the influence of nanomechanic excitations has been
considered by Boese and Schoeller (30). The authors de-
veloped a quantum mechanical model of electron tunnel-
through a vibrating molecule and applied it to the
experiment (4). Contrary to the situation described in
Secs. II.A, II.C and Refs. (11; 18), it is assumed that
the vibrational frequency is several orders of magnitude
larger that the frequency associated with tunneling ef-
fects. The system is described by an effective Hamilton-
ian involving local bosonic degrees of freedom of the
molecule. One of the local modes can be interpreted as
center-of-mass motion. The quantum mechanical calcu-
lation leads to a set of horizontal plateaus in $I-V$ curve
due to excitation of different vibrational modes. It is
shown that in some regions of the parameters a nega-
tive differential resistance occurs. A similar calculation,
but with more detailed account of the dependence of the
charge-transfer matrix elements in the shuttle coordinate,
was reported by McCarthy et al (31).

The papers (30) and (31) provide a qualitative ex-
planation to the experiment (4). This explanation is ac-
tually alternative to that given in Ref. (18), where the
center-of-mass motion was treated classically. It is worth
mentioning that the latter model predicts a finite slope of
the $I-V$ curve at large voltages, which is more similar
to the experimental result.

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and electron motion. It is shown that the $I-V$ charac-
teristics of the model shuttle can largely be understood
by analyzing the eigenspectrum of the isolated system of
three dots and the quantum oscillator. Tunnel coupling
of the dot states, to each other and to the position of the
oscillator, leads to repulsion of the eigenvalues and mix-
ing of the eigenstates associated with states localized
on individual dots. The mixed states consist of superposi-
tions of the states associated with the individual dots and
hence lead to delocalization of the electronic states be-
tween the dots. Analysis of the current which flows when
the shuttle is weakly coupled to leads, reveals strong res-
onances corresponding to the occurrence of the delocal-
ized states. The current through the shuttle is found to
depend sensitively on the amount by which the oscilla-
tor is damped, the strength of the couplings between the
dots and the background temperature. When the tun-
neling length $\lambda$ of the electrons is of order $x_0$, current far
from the electronic resonance is dominated by electrons
hopping on and off the central dot sequentially. As the
authors state, then the oscillator can be regarded as shut-
tling electrons across the system as it has been discussed
in Sec. II.A.

III. EXPERIMENTS ON ELECTRON SHUTTLING

The first experimental realization of the shuttle insta-
bility resulting in a classical shuttle-electron transfer was
reported by Thuminen et al (16). The experimental
setup is shown in Fig. 14. The measured current-voltage
characteristics display distinctive jumps and hysteresis
which reflect the influence of the vibrational environment (the metal beam the figure) on the shuttle dynamics.

Systems in which electron transport between two contacts is mediated by a vibrational mode of a self-assembled structure have also been investigated (4; 42). The most striking example of such a system is the C\textsubscript{60} single electron transistor, fabricated by Park et al. (4). In this device, a single C\textsubscript{60} molecule was deposited in a narrow gap between gold electrodes. The current flowing through the device was found to increase sharply whenever the applied voltage was sufficient to excite vibrations of the molecule about the minima of the van der Waals potential in which the molecule resides, or an internal mode of the molecule itself.

These transport measurements provided clear evidence for coupling between the center-of-mass motion of the C\textsubscript{60} molecules and single-electron hopping. This new conduction mechanism had not been observed previously in quantum dot studies. The coupling is manifested as quantized nano-mechanical oscillations of the C\textsubscript{60} molecule against the gold surface, with a frequency of about 1.2 THz. This value is in good agreement with a simple theoretical estimate based on van der Waals and electrostatic interactions between C\textsubscript{60} molecules and gold electrodes. The observed current-voltage curves are shown in Fig. 15.

The device fabricated by Park et al. is an example of a molecular electronic device (43) in which electrical conduction occurs through single molecules connected to conventional leads. The junctions between molecular components and leads will be much more flexible than those in conventional solid-state nanostructures and fluctuations in their width may modify their current characteristics significantly. Furthermore, vibrational modes of the molecular components themselves may play an important role in determining the transport properties (44).

An interesting possibility of a nanomechanical double barrier tunneling structure involving shuttling has been realized by Nagano et al. (45; 46). The system consists of scanning vibrating probe/colloidal Au particles/vacuum/PtPd substrate, see Fig. 16. The colloidal particles act as Coulomb islands, due to probe vibration they are brought to motion. What is important is the phase shift between the probe vibrations and the ac current in the system which allows to single out the displacement current. The latter shows clear features of the Coulomb blockade. Comparison of the experimen-
tal results with theoretical calculation drove the authors to the conclusion that about 280 Au particles vibrate in accordance with each other.

An externally driven nanomechanical shuttle has been designed by Erbe et al. (13; 14). In these experiments a nanomechanical pendulum was fabricated on Si-on-insulator substrate using electron and optical lithography, and a metallic island was placed on the clapper which could vibrate between source and drain electrodes, see Fig. 17. The pendulum was excited by ac power by two gates on the left- and right-hand sides of the clapper. The observed tunneling source-drain current was strongly dependent on the frequency of the exciting signal having pronounced maxima at the frequencies of mechanical modes. This fact signalizes shuttling mechanism of electron transfer at typical shuttle frequencies about 100 MHz. The measured average DC current at 4.2 K corresponded to $0.11 \pm 0.001$ electrons per cycle of mechanical motion. Theoretical analysis (20) and numerical simulations showed that a large portion of the voltage also acts on the island. The authors of Refs. (13; 14) expect that the resolution of the transport through the shuttle should also resolve Coulomb blockade after minimizing the effects of the driving ac voltage. According to their estimates, Coulomb blockade should be observable below 600 mK. A very important modification of the setup in Fig. 17 was recently presented by D. V. Scheible et al. (15). There a silicon cantilever is part of a mechanical system of coupled resonators - a construction that makes it possible to drive the shuttle mechanically with a minimal destructive influence from the actuation dynamics on the shuttle itself. This is achieved by a clever design that minimizes the electrical coupling between the driving part of the device (either a magneto-motively driven, doubly clamped beam resonator, or a capacitively coupled remote cantilever) and the driven part (the cantilever which carries the shuttle on its tip). Systems of the above mentioned type can, in principle, be used for studies of shuttle transport through superconducting and magnetic systems.

Interesting results on mechanically-assisted charge transfer were obtained in a device is fabricated as a silicon nanopillar located between source and drain contacts (26), see Fig. 18. The device is manufactured in a two-step process: First, nanoscale lithography using a scanning electron microscope (SEM), and, second, dry etching in a fluorine reactive ion etcher (RIE). The lithographically defined gold structure acts as both electrical current leads and etch mask for the RIE. A simple geometry defined by the SEM consequently results in the free-standing isolating nanopillar of intrinsic silicon with a conducting metal (Au) island at its top, see Fig. 18.a. This island serves as the charge shuttle. The metal island and the nanopillar are placed in the center of two facing electrodes, denoted by source $S$ and drain $D$. The system is biased by an AC voltage at source, rather than a sole DC bias, to avoid the DC-self excitation. Moreover, ap-
FIG. 18 (a) SEM micrograph and experimental circuitry of the silicon nanopillar: At source $S$, an AC signal, $V_{ac}$, is applied with a superimposed DC bias $V$. The net current, $I_D$, is detected at drain $D$ with a current amplifier. The third electrode, $G$, is floating. (b) Finite element simulation of the base oscillation mode which compiles for the nanopillar to $f_0 = 5367$ MHz. (c) When the island is deflected toward one electrode, the instantaneous voltage bias determines the preferred tunneling direction. Cotunneling is absent in this case, due to an increased distance to the opposite electrode. From Ref. (26).

Application of an ac-signal allows excitation of the nanopillar resonantly in one of its eigenmodes. The device itself is mounted in a probe station, providing vacuum condition for a reproducible and controlled environment of the pillar. This also removes water and solvents which may have condensed at the surface of the NEMS. The devices operated at room temperature and the capacitance was not sufficiently small to realize the single-electron regime.

Experimentally, dependences of the current through the system on bias frequency, as well as on additional DC bias allowing to tune resonances, were measured. The results are qualitatively explained on the basic of numerical simulations.

Coupling of electron transport to mechanical degrees of freedom can lead to other interesting phenomena. In particular, Kubatkin et al (47) have observed a current-induced Jahn-Teller deformation of a Bi nanocluster. They have shown that such a transformation influences the electron transport through a change in the geometrical shape of the cluster. We do not review here in detail other experiments, e.g., (4–10) involving nano-electromechanical phenomena since they are not directly connected to shuttle charge transport.

IV. COHERENT TRANSFER OF COOPER PAIRS BY A MOVABLE GRAIN

In this section we study a superconducting weak link where the coupling between two bulk superconductors is due to Cooper pairs tunneling through a small movable superconducting grain. The system is depicted in Fig. 19. We begin this section by looking at the requirements one has to put on the system for the analysis below to be valid. Then we briefly review the Parity effect and the single-Cooper-pair box in Section IV.B. Following this, in Sec. IV.C, we consider the two basic processes involved in shuttling of Cooper pairs; (i) Scattering of a single grain with a lead, thereby creating entanglement; and (ii) free motion of a grain whose charge state is a quantum mechanical superposition.

Two possible experimental configurations can be imagined for the system in Fig. 19

1. The pair of remote superconductors might be coupled by an external superconducting circuit, forming a loop. In this case the superconducting phase difference is a given number (and is for example determined by a magnetic flux through the loop). Shuttling of Cooper pairs is in this case a mechanism allowing for supercurrent flow through the loop. This scenario is analysed in Sec. IV.D.

2. A qualitatively different situation is when the two leads are disconnected from each other. Cooper pair exchange between two remote and isolated superconductors (leads) is then allowed only through tunneling via the single Cooper pair box, performing oscillatory motion between leads. In this case the relevant question is whether or not phase coherence between the leads can be established. This situation is considered in Sec. IV.E.

A. Requirements for shuttling of Cooper pairs

The main question we will focus on is how mechanical vibration of the cluster affects coherent tunneling of Cooper pairs. To put the question in a more dramatical way: Could one have coupling between remote superconductors mediated mechanically through inter-lead transportation of Cooper pairs, performed by a small movable superconductor? A positive answer to this question which will be given here was partly presented in (48) and (49). This follows from the possibility to preserve phase coherence of Cooper pairs despite of the non-stationary and non-equilibrium dynamics of the electronic system originating from a time dependent displacement of a small superconducting mediator. Such a possibility occurs if only a few electronic degrees of freedom are involved in the quantum dynamics. The latter is guaranteed if two conditions are fulfilled:

1. $\omega_0$, the frequency of vibrations is much smaller than the superconducting gap $\Delta$. 

2. 


FIG. 19 Superconducting shuttle junction. Two superconducting leads are placed too far away from each other to allow for direct tunneling between them. Using a movable grain they may still exchange Cooper pairs indirectly via lead-grain tunneling. If the gates are appropriately biased a single-Cooper-pair box situation occurs close to either lead allowing for a coherent superposition of two different charge states on the grain. Hence, for a grain making repeated alternate contacts with the leads a coherent exchange of Cooper pairs between them is possible.

2. The charging energy $E_C$ is much larger than the superconducting coupling energy $E_J$ and the temperature $k_B T$.

Here $E_J$ is the maximal Josephson energy characterizing the superconducting coupling between the grain and the leads. The condition (i) prevents the creation of elementary electronic excitations due to the grain motion and therefore guarantees the disconnection of the system evolution from contributions relating to the continuous spectrum of quasiparticles on the superconductors. The condition (ii) guarantees a Coulomb Blockade for Cooper pair tunneling preventing a significant charge fluctuation on the dot. The appearance of such fluctuations corresponds to the existence of a large number of channels for the Cooper pair transportation and results in strong decoherence due to destructive interference between these different channels. In what follows we will consider the conditions (i) and (ii) as being fulfilled.

B. Parity effect and the Single-Cooper-pair box

The ground state energy of a superconducting grain depends essentially on the number of electrons on it. Two contributions to such dependence are given by the electrostatic Coulomb energy $E_C(N)$ connected with the extra charge accumulated on the superconducting grain and by the so called parity term $\Delta_N$ (50–52). The latter originates from the fact that only even number of electrons can form a BCS ground state of a superconductor (which is a condensate of paired electrons) and therefore in case of an odd number of electrons $N$ one unpaired electron should occupy one of the quasiparticles states. The energy expense to occupy a quasiparticle state, which is equal to the superconducting gap $\Delta$, brings a new scale to selection of a number of electrons at small superconducting grain. Taking the above into account one presents the ground state energy $E_0(N)$ in the form (see (52))

$$E_0(N) = E_C(N - \alpha V_g)^2 + \begin{cases} 0 & \text{even } N \\ \Delta & \text{odd } N. \end{cases}$$

One can see from (10) that if $\Delta > E_C$ only an even numbers of electrons can be accumulated in the ground state of the superconducting grain. Moreover, at special values of the gate voltage $V_g$, such as $\alpha V_g = 2n + 1$, a degeneracy of the ground state occurs with respect to changing the total number of electrons by one single Cooper pair. An energy diagram illustrating this case is presented in Fig. 20. The occurrence of such a degeneracy brings about an important opportunity to create a quantum hybrid state at low temperatures which will be a coherent mixture of two ground states, differing by a single Cooper pair:

$$|\Psi\rangle = \gamma_1 |n\rangle + \gamma_2 |n + 1\rangle.$$ 

An example of system where this is possible is the so called single-Cooper-pair box (51). The possibility to create a single-Cooper-pair box experimentally was demonstrated by Nakamura at al (53) and was recently confirmed by Devoret at al (54). The idea of the experiment is presented in Fig. 21 where the superconducting dot is shown to be in tunnel coupling with a bulk superconductor. A gate electrode is responsible for the lifting of the Coulomb Blockade of Cooper pair tunneling (creation of the above discussed ground state degeneracy) which allows for delocalization of a Single Cooper pair between two superconductors. Such hybridization results in a certain charge transfer between the bulk superconductor and the grain.
The vicinity of each of the superconducting electrodes. The two gate electrodes ensure the lifting of the Coulomb blockade of Cooper-pair tunneling at turning points in the vicinity of each lead. This allows for coherent tunneling of Cooper pairs between them. For a nanoscale system, such quantum fluctuations of the charge on the island are generally suppressed due to the strong charging energy associated with a small grain capacitance. However, by appropriate biasing of the gate electrode it is possible to make the two states $|n\rangle$ and $|n+1\rangle$, differing by one Cooper pair, have the same energy (degeneracy of ground state). This allows for the creation of a hybrid state $|\Psi\rangle = \gamma_1 |n\rangle + \gamma_2 |n+1\rangle$.

\[ |\Psi(t_1)\rangle = \alpha |n=0\rangle \otimes |\psi_{\text{Leads}}(t_1)\rangle + \beta |n=1\rangle \otimes |\psi_{\text{Leads}}(t_1)\rangle \]

\[ |\Psi(t_2)\rangle = |n=0\rangle \otimes |\psi_{\text{Leads}}\rangle \]

![Fig. 21](image1)

**FIG. 21 A schematic diagram of a single Cooper pair box.** An island of superconducting material is connected to a larger superconducting lead via a weak link. This allows for coherent tunneling of Cooper pairs between them. For a nanoscale system, such quantum fluctuations of the charge on the island are generally suppressed due to the strong charging energy associated with a small grain capacitance. However, by appropriate biasing of the gate electrode it is possible to make the two states $|n\rangle$ and $|n+1\rangle$, differing by one Cooper pair, have the same energy (degeneracy of ground state). This allows for the creation of a hybrid state $|\Psi\rangle = \gamma_1 |n\rangle + \gamma_2 |n+1\rangle$.

1. Scattering and free motion.

To illustrate the process, consider first the simple case when an initially uncharged grain, $n = 0$, arrives into contact with the left lead. Before contact the state of the system is then

\[ |\Psi(t_0)\rangle = |n=0\rangle \otimes |\psi_{\text{Leads}}\rangle , \]

where $|\psi_{\text{Leads}}\rangle$ is the state of the leads. During the time spent in tunneling contact with the lead the Cooper pair number on the grain may change. When the grain quits contact with the lead the general state of the system is thus

\[ |\Psi(t_1)\rangle = \alpha |n=0\rangle \otimes |\psi_{\text{Leads}}(t_1)\rangle + \beta |n=1\rangle \otimes |\psi_{\text{Leads}}(t_1)\rangle . \]

This process is depicted in Fig. 22. The coefficients $\alpha$ and $\beta$ are complex and will depend both on the time spent in contact with the lead as well as the initial state $|\psi_{\text{Leads}}\rangle$. It is to note here, that in general, the grain will become entangled with the leads.

As the grain traverses the region between the leads there is no tunneling and the magnitudes $|\alpha|$ and $|\beta|$ will remain constant. However, the relative phase between them may change. Thus, when the grain arrives at the right lead at a time $t_2$ the state of the system will have acquired an additional phase labelled $\chi_+$

\[ |\Psi(t_2)\rangle = \alpha |0\rangle \otimes |\psi'_{\text{Leads}}(t_2)\rangle + e^{-i\chi_+} \beta |1\rangle \otimes |\psi'_{\text{Leads}}(t_2)\rangle . \]

As the grain comes into contact with the right lead charge exchange is again possible and the coefficients $\alpha$ and $\beta$ will change. Then, in the same fashion as during the motion from left to right, the only effect on the state as the grain moves towards the left lead again is the acquiring of another relative phase denoted by $\chi_-$. The whole process is schematically depicted in Fig. 23.

Both scattering events and “free motion” are thus characterized by quantum phases accumulated by the system, which we here call Josephson phase, $\vartheta$, and electrostatic phase, $\chi_{\pm}$

\[ \vartheta = \hbar^{-1} \int dt E_J(t) . \]  \hspace{1cm} (12)

\[ \chi_{\pm} = (i/\hbar) \int dt \delta E_c[x(t)] \]  \hspace{1cm} (13)

2. Hamiltonian.

Under condition of adiabatic variation of the grain position $x$ no quasiparticle degrees of freedom are involved and one considers only quantum dynamics of the coupled ground states on each of the superconductors. The corresponding Hamiltonian is expressed through the operator $\hat{n}$ of Cooper pair number on the grain and the
The effect of this is to replace the operators $e^{\pm i\Phi_{L,R}}$ in the Hamiltonian above with c-numbers making it a two-level system. This leaves us with a reduced Hamiltonian where the phases $\Phi_{L,R}$ enter only as parameters, 

$$H_{\text{red}}(x(t), \Phi_L, \Phi_R) \equiv \langle \Phi_L | H(x(t)) | \Phi_R \rangle | \Phi_L \rangle.$$  

The dynamics of the system is described by the Liouville-von Neumann equation for the density matrix $\hat{\rho}$ (48),

$$\frac{d}{dt} \hat{\rho} = -i [H_{\text{red}}, \hat{\rho}] - \nu(t) [\hat{\rho} - \rho_0(t)].$$  

Since we are not interested here in transient processes, connected with the initial switching on of the mechanical motion, solutions which do not depend on any initial conditions will be in focus of our analysis. To prevent any memory of initial conditions one needs to include a dissipation term (the last term on the right hand side of (16)) into the dynamics. If this dissipation is weak enough it does not affect the system dynamics on a time scale comparable to the period of vibrations. However in a time scale longer than the period of rotation such relaxation causes a solution to (16) to be independent of any initial conditions. We choose the simplest possible form for relaxation ($\tau$-approximation) with $\rho_0$ being an equilibrium density matrix for the system described by Hamiltonian $H$. Relaxation is due to quasiparticle exchange with the leads and depends on the tunneling transparencies

$$\nu(x) = \nu_0 \exp(-\delta x/\lambda).$$  

The Cooper-pair exchange, being an exponential function of the grain position, is mainly localized in the vicinity of the turning points. In this region Coulomb blockade for the Cooper-pair tunneling is suppressed and can be neglected while considering dynamics of the formation (or transformation ) of the single-Cooper-pair hybrid, see Eq. (11). In contrast to this, the dynamics of the system in the time intervals, when grain is far away from the superconducting leads, is not significantly affected by Cooper-pair tunneling and essentially depends on electrostatic energy $E_{\text{C}} - E_{\text{C}}(n+1) - E_{\text{C}}(n)$ appearing due to the lifting of the Coulomb degeneracy in the regions far from the turning points. This circumstance allows for the simplification of the analysis and considering the quantum evolution of the system as a sequence of scattering events (taking place due to tunneling of Cooper pairs in the vicinity of the turning points) which are separated by intervals of "free motion" of the system, where tunneling coupling between the grain and leads is neglected. A schematic picture of the above regimes of evolution is presented in Fig. 23.

Direct calculations give the following simple expression for the average current (48) (for details of the derivation see also (56; 57))

$$I = 2\epsilon f \frac{\cos \vartheta \sin^3 \vartheta \sin \Phi (\cos \chi + \cos \Phi)}{1 - (\cos^2 \vartheta \sin \chi - \sin^2 \vartheta \cos \Phi)^2}. $$  

Here $\Phi = (\Phi_R - \Phi_L) + (\chi_+ - \chi_-)$ and $\chi = \chi_+ + \chi_-$. The following features of Eq.(18) should be mentioned:
1. The mechanically assisted supercurrent is an oscillatory function of the phase differences of the superconducting leads similarly to other types of superconducting weak links.

2. The current can be electrostatically controlled if an asymmetrical \((\chi_+ \neq \chi_-)\) phase accumulation is provided by an external electric field varying in tacit with the grain rotation. The same effect appears if the grain trajectory embeds a finite magnetic flux. Then \(\chi\) is proportional to the flux given in units of the superconducting flux quantum.

3. Depending on the value of the electrostatic phase \(\chi\) one can have any direction of the supercurrent flow at a given superconducting phase difference. Also a nonzero supercurrent at \(\Phi_L - \Phi_R = 0\) is possible in contrast to ordinary superconducting weak links.

4. The supercurrent is a non-monotonic function of the Josephson coupling strength \(\vartheta\). This fact reflects the well known Rabi oscillations in the population of quantum states with different numbers of Cooper-pairs when a single-Cooper-pair box is formed due to sudden switching of pairs tunneling at the turning points of the trajectory.

In the weak coupling limit the current is proportional to the third power of the maximal Josephson coupling strength. One needs to stress that this strength might be several orders of magnitude bigger than the one corresponding to the real spatial separation between the superconducting leads. Cooper-pair transportation serves as an alternative to direct Cooper-pair tunneling between the leads thereby providing a mechanism for supercurrent flow between remote superconductors. In Fig. 24 a diagram of the supercurrent as a function of both superconducting and electrostatic phases is presented. Signs + and - represent the direction of the supercurrent while the black lines corresponds to configurations with a zero value of the current.

E. Shuttling Cooper pairs between disconnected leads

We are now are turning to the question whether or not the superconducting coupling between remote and isolated superconductors can be established by mechanical shuttling of Cooper-pairs between them (49). Here we are interested in the situation when at an initial moment of time the shuttle is absent and no superconducting phases can be introduced on the leads (due to large quantum fluctuations of the phases on conductors with a fixed number of Cooper-pairs). At times \(t\) bigger than zero, a superconducting grain between the leads starts to rotate, and the number of Cooper-pairs on each lead is no longer conserved. The moving single-Cooper-pair box provides an exchange of Cooper-pairs between the leads. We are interested to know if this exchange is able to establish superconducting phase coherence between the leads.

In the case with strongly coupled leads the limitation that the grain could carry only zero or one excess Cooper pair reduced the problem to a two state problem. Since the problem was essentially a two conductor problem (grain + one lead) only this variable was needed to determine the state of the system. For the case with decoupled leads one has to keep the operator nature of \(\hat{\Phi}_{L,R}\) in the Hamiltonian (14). The dimensionality of the Hilbert space depends on the maximum number of Cooper pairs that can be accommodated on the leads. The factors that put a limit to this number are the capacitances of the leads which are not present in the Hamiltonian (14). Instead of including these charging energy terms another approach was been used in (49). The Hilbert space was there reduced in such a way that each lead can only accommodate a maximum (minimum) of \(\pm N\) extra Cooper pairs.

The time evolution of the system is determined by the Liouville-von Neumann equation for the density matrix. If the total number of particles in the system is conserved and one assumes the whole system to be charge neutral, the state of the system can be written in terms of the state of the grain and one of the leads. For instance the
density matrix can be written
\[
\rho(t) = \sum_{\eta, \sigma = 0,1} \sum_{N_{L}, N'_{L} = -N} \rho_{\eta \sigma}^{N_{L} N'_{L}}(t)
\times |\eta\rangle_{\text{grain}} \langle \sigma| \otimes |N_{L}\rangle_{\text{left lead}} \langle N'_{L}| \otimes |N_{L} - \eta\rangle_{\text{right lead}} \langle -N'_{L} - \sigma|.
\]

Here, it has been explicitly indicated to which part of the system the various operators belong. The simple relaxation time approximation used for the case with connected leads is not possible to use in this case. That approximation assumed the leads to be in BCS states with definite phases towards which the phase of the grain relaxed. Instead, to account for loss of initial conditions the influence of the fluctuations of the gate potential on the bath degrees of freedom.

The fluctuations are modelled by a harmonic oscillator bath with a spectral density determined by the impedance of the island charge has been accounted for. The fluctuations are introduced by a harmonic oscillator bath with a spectral density determined by the impedance of the island charge has been accounted for. The fluctuations are introduced by a harmonic oscillator bath with a spectral density determined by the impedance of the island charge has been accounted for.

The built up phase difference due to the shuttle junction will determine the current through an ordinary junction connecting the leads if the latter is weak. In Fig. 26 this current is shown as a function of the dynamical phases for fixed \( \vartheta \) (contact time). Here an auxiliary, weak, probe Josephson junction is assumed to be connected after a large number of rotations. The current is given by the usual Josephson relation weighted over the distribution function \( f(\Delta \Phi) \)

\[
I = I_{c} \int_{0}^{2\pi} d(\Delta \phi) \sin(\Delta \phi) f(\Delta \Phi),
\]

where \( I_{c} \) is the critical current of the probe junction. The direction of the current is indicated by the signs in the graph. Black areas correspond to no current.

In conclusion, phase coherence can be established by mechanical transfer of Cooper pairs and that this mechanism can also carry a non dissipative current.

Role of environment-induced decoherence in the shuttling of Cooper pairs was considered in (58). To allow for the decoherence a finite model damping of the off-diagonal part of the density matrix is introduced in the Liouville-von Neumann equation (16). The damping is assumed to be different in the contact regions \( (\gamma_{J}) \) and in the region between the contacts \( (\gamma_{C}) \) and computed in the Born-Markov approximation. Strong decoherence exponentially suppresses the supercurrent, the leading term being proportional to \( \exp(-\gamma_{J}t_{0} - \gamma_{C}t_{C}) \) where \( t_{C} = t - t_{0} \). The supercurrent is also suppressed in the case of weak coupling and \( \gamma_{J}t_{0} \ll \gamma_{C}t_{C} \). Consequently, there exists an intermediate region where the decoherence leads to an enhancement of the supercurrent. In addition, the decoherence may result in a change of its sign (\( \pi \) junction). It remains to be seen whether these conclusions are sensitive to damping of diagonal elements of the density matrix.
V. NOISE IN SHUTTLE TRANSPORT

Noise properties are crucial for performance of nanomechanical systems and they were extensively studied both experimentally and theoretically. In this review we address only the works related to shuttle electron transport.

While many papers were aiming at studies of the conditions for the realization of shuttle instability or at the dependence of the current on the external parameters, at the initial stage only few papers investigated current fluctuations. However, during last years the interest in current fluctuations significantly increased, and it was shown that the current noise contains much more information about the nature of shuttle instability than the average current. Indeed, even near the equilibrium noise spectrum allows one to study ac response of the system without ac excitation. Out of equilibrium, the noise spectrum is specifically sensitive to coherence properties of the electron transport, as well as to electron-electron correlations. These two issues lead to several interesting works aimed at various aspects of noise in electron shuttling. Below we will discuss these works following the scheme of Table II.

A. General concepts

Before doing that let us introduce basic definitions. The instant current through the device, $I(t)$, differs from its time-averaged value, $\overline{I}$, and the difference $\Delta I(t) \equiv I(t) - \overline{I}$ is called the current fluctuation. A natural object to study fluctuations is the correlation function

$$\Delta I(t) \Delta I(t') = \overline{I(t)I(t')} - \overline{I}^2.$$  

In the absence of external time-dependent fields this correlation function depends only on the time difference, $t - t'$. In addition, for ergodic systems we are interested in, the time average does not differ from the ensemble average, which we will denote as $\langle \cdots \rangle$. The ensemble average is just the average over the realizations of the random quantity $I(t)$. As a result, the correlation function is conventionally defined as

$$S(\tau) \equiv \langle \Delta I(\tau) \Delta I(0) \rangle.$$  

The noise spectrum is then defined as a doubled Fourier image of the correlation function, see (59) for a review,

$$S(\omega) \equiv 2 \int_{-\infty}^{\infty} d\tau e^{-i\omega \tau} S(\tau).$$  

This is a pure classical definition which assumes that the current operators commute at different times. In the case of quantum transport the current is an operator, and in general case $I(t)$ and $I(t')$ do not commute. Then the definition of the correlation function is generalized as

$$S(\tau) = \frac{1}{2} \left\langle \left[ \hat{I}(\tau), \hat{I}(0) \right] \right\rangle - \overline{I}^2,$$  

where $[\hat{A}, \hat{B}]_\pm \equiv \hat{A}\hat{B} \pm \hat{B}\hat{A}$. At low applied voltage, $V \to 0$, the current is proportional to the applied electric filed, $I(\omega) = G(\omega)E(\omega)$, where $G(\omega)$ is the complex conductance of the structure. In this regime the noise spectrum can be expressed through the real part of the conductance as

$$S(\omega) = \hbar \omega \Re G(\omega) \coth \frac{\hbar \omega}{2k_B T} \approx 2k_B T \Re G(\omega)$$  

at $\hbar \omega \ll k_B T$. Here $k_B$ is the Boltzmann constant. The relation (22) is just the well know fluctuation-dissipation theorem (60). As follows from Eq. (22), in the linear response regime the noise spectrum provides exactly the same information as the linear conductance. Even in this case studies of noise can be informative since the noise spectrum allows to determine the frequency dependence of the conductance without a direct ac excitation.

Nonequilibrium noise ($V \neq 0$) is more interesting, because it gives information on the temporal correlation of the electrons, which is not contained in the conductance. The contribution proportional to the first power of the applied bias voltage is often called the shot noise. Such noise has been thoroughly studied in many systems. In the devices such as tunnel junctions, Schottky barrier diodes, $p-n$ junctions, and thermionic vacuum diodes (61), the electrons are transmitted randomly and independently of each other. Hence, the transfer of electrons can be described by Poisson statistics, which is used to analyze events that are uncorrelated in time. For these
Incoherent electron transport and classical mechanical motion

The case of incoherent electron shuttling by a classical mechanical motion was first addressed by Isacsson and Nord (64). The considered a model one-dimensional shuttle structure similar to the one shown in Fig. 1.a and described in detail in Ref. (65). Namely, it was assumed that a metallic grain of mass $M$ and radius $r$ is placed between two leads via elastic, insulating materials. Applying a bias voltage $V = V_L - V_R$, electron transport occurs by sequential, incoherent, tunneling between the leads and the grain. The tunneling rates, $Γ_{L,R}^±(x, q)$, depend on the $x$ and $q$ through the tunneling matrix elements and the differences in the (Gibbs) free energy, $Δg_{L,R}^±(x, q)$, between the configurations with $\{q, q_{L,R}\}$ and $\{q \pm e, q_{L,R} \pm e\}$. The electric potentials and charges are determined using a conventional electric circuit, where the voltage sources $V_L$ and $V_R$ are connected in series with the leads and the grain as shown in Fig. 27. As a result, the quantities $Δg_{L,R}^±(x, q)$ are expressed through position-dependent capacitances specified as $C_{L,R}(x) = \frac{C_{L,R}^{(1)}}{1+\frac{x}{a_{L,R}}}$, where $a_{L,R}$ are characteristic length scales. The tunneling matrix elements are expressed through the position-dependent resistances specified as $R_{L,R}(x) = \frac{R_{L,R}^{(0)}}{1+\frac{x}{a_{L,R}}}$, in this way the motion-induced feedback to the stochastic electron hopping is taken into account. Another relationship between the grain charge and displacement is given by the Newtonian equation of motion, $m\ddot{x} = F(x, \dot{x}, q)$. The force $F$ in this equation contains both elastic and electric components, as well as the friction force $\propto \dot{x}$. A new feature is an additional account of van der Waals force which turns out to be important (65) in general, $F(x, \dot{x}, q)$ is a nonlinear function of $x$ and $q$.

The results for noise were obtained by direct numerical integration of the set including the Master equation for the grain population with $\{x, q\}$-hopping rates and anharmonic mechanical equation of motion. A typical noise spectrum is shown in Fig. 28.

FIG. 27

FIG. 28 Power spectrum $S(ω)$ in the shuttle regime. For frequencies above the vibrational frequency, the Fano factor is close to 1/2 as for a static Coulomb-blockade junction. The peaks are located at the frequency of vibration and at the first harmonic. For frequencies below the vibrational frequency, the in-time correlation due to the periodic grain motion leads to a slight suppression of the noise level. At still lower frequencies, the noise is increased due to slow fluctuations in mechanical energy. From Ref. (64). © (2004) EDP Sciences.
and decharging of the oscillating grain. Directly below
the peaks, region II, the noise is suppressed below the
shot noise level of a static double junction, due to the
additional in time correlations between successive tunnel
events induced by the oscillating grain. This is a clear
hallmark of classical shuttling.

The most interesting part of the spectrum, however,
is the low-frequency part in region I, where the Fano
factor increases with the frequency decrease. The authors
attribute this increase to low-frequency fluctuations in
mechanical energy, which, in turn, lead to low-frequency
fluctuations in the current.

The authors have also performed an anaytical sta-

tability analysis valid in the case of weak electromechanical
coupling, \(\epsilon \equiv F(x = 0, q = e)/m\omega_0^2\lambda \ll 1\). The in-

stability increment is determined by the difference be-

tween the the energy, \(W(E)\), pumped into mechanical motion
during one period and the average dissipation per pe-

riod, \(D(E)\): \(p(E) = W(E) - D(E)\). Hence, the station-

ary oscillation amplitude is determined by the equation
\(p(E_0) = 0\). Since \(p(E)\) depends both on the bias voltage
and damping of the mechanical motion, this equation ac-
tually determines the the dependence of the oscillation
amplitude on the bias voltage. It has a finite solution
only above the instability threshold.

The analysis, following the conventional pro-
dure (66), takes into account the fluctuations in the me-

chanical energy around the value \(E_0\). These fluctua-
tions induce electrical noise with Lorentzian spectrum,
\(S(\omega) \propto (\nu^2 + \omega^2)^{-1}\). The width \(\nu\) of the spectrum is
proportional to \(|p'(E_0)|\), where \(p'(E_0) \equiv \partial p/\partial E|_{E=E_0}\).
Since at the instability threshold \(p'(E_0) \to 0\) (17),
the noise spectrum diverges while approaching the instability
threshold from the “shuttling” side.

The results of numerical studies corresponding to a
nanometer-sized Au grain commonly used in experiments
with self-assembled Coulomb-blockade double junctions
are shown in Fig. 29.

Although, as explained in (65), the non-parabolic con-
fining potential smears any step-structure in the current-
voltage characteristics, the transition between static- and
shuttle-operation is clearly visible in the noise spectrum.
In accordance with the above analytical treatment, on
approaching the threshold from above (higher to lower voltages) the \(S(\omega)\) shows divergent behavior. Below the
threshold voltage the Fano factor is of the order 1/2.

A rather unusual prediction is that in the shuttle
regime, well above the threshold voltage, the Fano factor
is increased. This fact is attributed to anharmonicity of
the potential. For a harmonic potential used in the
analytical treatment the lowered noise level in region II is
continued into region I.

C. Noise in quantum shuttle

Below we will review a theory (67) for shot noise in
a quantum shuttle. The theory extends the previous

paper (68), in which the average current has been con-
sidered. It is assumed that the shuttling grain has two
electron states, \(|0\rangle\), and \(|1\rangle\), and the situation when only
diagonal in \(|i\rangle\) states of the density matrix are important.
To calculate the noise, the number-resolved diagonal den-
sity matrices, \(\rho_{ii}(n)\delta_{hk}\), are introduced. Here \(n\) is the
number of electrons which tunneled to the right electrode
by time \(t\). These matrices obey the generalized master equation (GME) where tunneling into the leads is de-
scribed by position-dependent transition rates \(\Gamma_{L,R}\).
In addition, damping of the oscillator motion due to in-
teracting with a thermal bath is taken into account.

Knowing \(\rho_{ii}(n)\), one finds the probability for \(n\) elec-
trons to be shuttled as
\[
P_n(t) = \text{Tr}_{\text{osc}} \sum_{i=0,1} \rho_{ii}(n)\delta_{hk}.
\]

The calculation of the average current and noise is then
straightforward:
\[
I = e \lim_{t \to \infty} \sum_n n P_n(t),
\]
\[
S(0) = 2e^2 \lim_{t \to \infty} \frac{d}{dt} \left[ \sum_n n^2 P_n(t) - \left( \sum_n n P_n(t) \right)^2 \right].
\]

The relevant elements of the density matrix were found
using the generating functional concept, and both the
average current and Fano factor were calculated for dif-
ferent relationships between the characteristic tunneling
length, \(\lambda\), and quantum length, \(x_0\), as well as for dif-
ferent ratios \(\gamma/\omega_0\). The numerical results agree with an

FIG. 29 Current-voltage characteristics plotted together with
(\(\omega \to 0\)). The current is the solid line with the scale on the left
ordinate while the Fano factor is shown for a discrete set of
points with the scale on the right ordinate (lines connecting
the points are a guide to the eye). Below the critical voltage
where there is no sustained grain motion, the Fano factor
is that of a Coulomb-blockade double junction. Above the
critical voltage, the grain is oscillating and the Fano factor
is increased and shows a divergent behavior at the critical
analytical treatment valid for small injection rates. The results are summarized in Fig. 30. The $I$ versus $\gamma$ curve

![FIG. 30 Current $I$ (upper panel) and Fano factor $F$ (lower panel; log scale) versus damping $\gamma$ for different transfer rates $\Gamma = \Gamma_L = \Gamma_R$ and tunneling lengths $\lambda$, $\gamma$ and $\Gamma$ are measured in units of $\omega_0$, $\lambda$ is measured in units of $x_0$. Other parameters are $eE = m/\omega_0^2 = 0.5x_0$, $T = 0$. The asterisk defines the parameters of Wigner distributions in Fig. 31. From Ref. (67). © (2004) American Institute of Physics.](image)

shows the tunneling to shuttling crossover as damping is decreased, in agreement with previous results. The crossover spans a narrower range of $\gamma$ in the case of $\lambda/x_0 = 2$ compared to the $\lambda/x_0 = 1$ case. Thus, already for $\lambda/x_0 = 2$ the shuttle behaves almost semiclassically, where a relatively sharp transition between the two regimes is expected. There is no sharp transition from tunneling to shuttling, and near the transition these regimes can coexist. To demonstrate this phenomenon, following (68) the Wigner distribution functions

$$W_{ii}(x,p) = \int_{-\infty}^{\infty} \frac{dy}{2\pi\hbar} \left| x - \frac{y}{2} \right| \rho_n(t \to \infty) \left| x - \frac{y}{2} \right| e^{ipy/\hbar}$$

(24)

are calculated, see Fig. 31. In this figure pure classical motion would correspond to a sharp classical phase trajectory for $W_{tot}$, which for an oscillator is a ring. The smearing around this ring corresponds to quantum fluctuations. In addition, one clearly sees a spot in the center, which correspond to a tunneling through a static grain. Thus the motion has a complex character showing the features corresponding both to classical and quantum regimes. The semiclassical transition is accompanied by the nearly singular behavior of the Fano factor reaching the value $\approx 600$ at the peak. This is in agreement with the classical study (64) discussed above.

What disagrees with the classical study is that the predicted in (64) enhancement of the Fano factor in the shuttling regime comparing to the tunneling regime is not reproduced. It still remains to be clarified whether this discrepancy is due to difference between the models being considered, or whether it is due to the simplifying approximations used in the classical treatment.

D. Driven charge shuttle

Noise properties of a driven charge shuttle (14) are much simpler than those of a self-oscillating one. Here mechanical energy is conserved and does not fluctuate, so the only source of noise is random electron transfer. An average number of electrons transferred by a driven shuttle per one cycle, as well as its variance were considered in (20) in connection with accuracy of a mechanical single-electron shuttle, see Sec. II.A.4. The variance, $\Delta n \equiv \langle n^2 - \langle n \rangle^2 \rangle^{1/2}$ was found using a conventional ME with the tunneling probabilities given by the orthodox theory, see (69) for a review. A typical plot of $\Delta n$ versus $V$ is shown in Fig. 5 (lower panel), and the main conclusion is that the variance is small at low temperatures, $k_B T < e^2/2C$, and for relatively long contact times, $t_0 \gg RC$.

However, the variance $\Delta N$ differs from the noise actually measured since typical measurement times are much longer than one period of oscillation. This case was addressed in (70) where both zero-frequency noise and full counting statistics of transferred charge were considered.

To find the statistics of the transferred charge one needs the probabilities for $n$ electron to be transferred, $P_n(t)$, for all $n$. They can be calculated using an elegant formalism of generating functional (63). The generating functional is defined as

$$e^{-G(x)} = \sum_{n=0}^{\infty} P_n(n) e^{i\chi_n}$$

(25)

where $\chi$ is called the counting field. Then all cumulants of the transferred charge can be calculated as

$$\pi = \langle n \rangle = \left. \frac{\partial G}{\partial \chi} \right|_{\chi=0}, \quad \langle n^2 - \pi^2 \rangle = \left. \frac{\partial^2 G}{\partial \chi^2} \right|_{\chi=0}.$$

(26)
etc. The concrete calculation follows the method developed in (71) with a proper generalization for a dynamic case. It is assumed that the shuttle has two states with 0 or 1 excess electron. The probability to find the shuttle in one of these states can be expressed as a vector \(|p\rangle\) with components \(|p_0, p_1\rangle\). The dynamics of \(|p(t)\rangle\) is governed by the Liouville equation
\[
\frac{\partial}{\partial t}|p(t)\rangle = \hat{L}|p(t)\rangle,
\]
where \(\hat{L}\) is constructed from the matrix \(\hat{L}(t)\) by multiplying the lower off-diagonal matrix element by the factor \(e^{i\chi}\).

Then generating functional can be expressed as
\[
e^{-\hat{G}(x)} = \left\langle q \right| T \exp \left\{ - \int_0^t \hat{L}_x(t') \, dt' \right\} |p(0)\rangle
\]
where \(|p(0)\rangle\) is the probability at time \(t = 0\), \(|q\rangle \equiv \{1, 1\}\), and \(T\exp\) is the time ordered exponential. The matrix \(\hat{L}_x(t)\) is constructed from the matrix \(\hat{L}(t)\) by multiplying the lower off-diagonal matrix element by the factor \(e^{i\chi}\).

Time ordering is very important since the matrices \(\hat{L}_x(t)\) for different times do not commute.

The generating functional (28) was analyzed numerically, as well as analytically for the limiting cases of small and large oscillation amplitude \(\alpha\). In the static case, \(\alpha = 0\), and for a symmetric shuttle with \(\Gamma_L = \Gamma_R = \Gamma_0 = \Gamma_0\exp(\pm \alpha \sin \omega_0 t)\) the known result (72) for a static tunneling system is reproduced:
\[
\hat{G}_t(x) = -i\Gamma_0(t) \left( e^{i\chi/2} - 1 \right) .
\]
In this case \(I = S(0) = \pi \omega_0 \Gamma_0^{(0)}\) and the Fano factor \(F = 1/2\).

In the opposite limit of large oscillation amplitude of the shuttle, since for most of the time the ratio \(\Gamma_L/\Gamma_R\) is either very large or very small, it was assumed that (i) for \(0 < \omega_0 t < \pi\) the quantity \(\Gamma_L\) vanishes identically and (ii) for \(\pi < \omega_0 t < 2\pi\) the opposite holds: \(\Gamma_L = 0\). The approximation becomes exact for \(\Gamma \ll \omega_0\), since in that case electrons can tunnel only when the shuttle is near one of the two leads. Under this assumption the problem can be treated analytically, the result being
\[
I = e^{\frac{1 - \alpha}{1 + \alpha}}, \quad S(0) = 4 e^{\alpha} \frac{1 - \alpha}{(1 + \alpha)^3} \rightarrow F = \frac{2\alpha}{(1 + \alpha)^2}.
\]

Here the quantity \(1 - \alpha\) with
\[
\alpha = \exp \left( -\frac{\Gamma_0^{(0)}}{\omega_0} \int_0^\pi d\phi \, e^{i\phi} \sin \phi \right)
\]
is the probability of transferring one electron during half cycle.

For \(\alpha \ll 1\) the probability of transferring the electron during the half cycle is nearly 1, and the generating function
\[
e^{-\hat{G}_t(x)} = \left[ 2\alpha + (1 - 2\alpha) \left( e^{i\chi} \right) \right]^{\omega_0 t/2\pi}\]
corresponds to a binomial distribution
\[
P_n(N) = \binom{N}{n} \left( 1 - 2\alpha \right)^n \left( 2\alpha \right)^{N-n}
\]
where \(N(t) = \lfloor \omega_0 t / 2\pi \rfloor\) is the number of oscillation cycles during the measurement time \(t\). This is a very clear result since at each cycle one electron is transmitted with probability \(1 - 2\alpha\), and since \(\alpha \ll 1\) the cycles are independent. Indeed, at each cycle the system is reset to the stationary state within accuracy \(\alpha^2\), regardless of the initial state. This limiting case agrees with the results of (20) where the variance of the charge transfer during one cycle was analyzed.

For \(\alpha \to 1\) the probability for one electron to tunnel during a cycle is very small. The result for this case reads as
\[
e^{-\hat{G}_t(x)} = \left[ \alpha + (1 - \alpha) e^{i\chi/2} \right]^{\omega_0 t/2\pi}\]

One can notice that the periodicity of the generating function has changed. Equation (31) describes a system of \(e/2\) charges that at each cycle have a probability \(1 - \alpha\) of being transmitted. Thus the system can be mapped on a fictitious system of charges \(e/2\) saying that every time that one electron succeeds in jumping on or off the central island, one charge \(e/2\) is transmitted in the fictitious system. This is possible, since it is extremely unlikely that one electron can perform the full shuttling in one cycle. Thus after many cycles \((N \gg 1)\) the counting statistics of these two systems coincide. The cycles are no more independent like in the case for \(\alpha \ll 1\), but the problem can be mapped onto an independent tunnelling one. For \(\alpha\) intermediate it is more difficult to give a simple interpretation of the charge transfer statistics, since different cycles are correlated in a nontrivial way.

Generally, both noise and full counting statistics demonstrate very rich and interesting behaviors. That permits us to understand more deeply the dynamics of charge transfer and to indicate the threshold of the shuttling instability with greater accuracy.

### E. Noise in Cooper pair shuttling

Noise in Cooper pair shuttling between two superconductors was considered is specifically interesting since in allows one to better understand coherent properties of superconductor devices. At present, only driven superconducting systems were considered, and the noise was first considered in (58).

The main purpose of this paper was analysis of the environment-induced decoherence, and the noise is just a by-product of a general analysis of the Cooper pair shuttling dynamics. The initial model is similar to that of (48; 49). In addition, a finite coupling to a thermal bath was taken into account along the Caldeira-Leggett model, see for a review (73). In the Born-Markov approximation the coupling result in a damping of the den-
sity matrix, which is assumed to be different in the tun-
ingeling region and the region of free motion - \( \gamma_J \) and \( \gamma_C \), respectively.

The results both for average current and noise are strongly dependent on the products \( \gamma_J t_0 \) and \( \gamma_C t_C \), where time of free motion between the leads. Strong decoherence occurs at \( \gamma_J t_0 \gg 1 \) or \( \gamma_C t_C \gg 1 \), the details being dependent on the ratio \( \gamma_J t_0/\gamma_C t_C \). Naturally, at strong decoherence the phase dependent contribution to \( S(0) \) is exponentially suppressed since it comes from correlations over times larger than the period. In the main approximation \( S(0) = \omega_0 e^{2\pi/\pi} \).

In the case of strong decoherence, \( \gamma_J t_0 \ll \gamma_C t_C \ll 1 \),
\[
I = \frac{e\omega_0}{\pi} \tanh \left( \frac{E_f}{k_B T} \right) (\cos \Phi + \cos 2\chi) \tanh \vartheta \sin \Phi \frac{1}{1 + \cos \Phi \cos 2\chi},
\]
\[
S = \frac{e^2 \omega_0}{\pi} \frac{1}{\gamma_C t_C} \tanh^2 \vartheta \sin^2 \Phi \frac{1 + \cos \Phi \cos 2\chi}{1 + \cos \Phi \cos 2\chi},
\]
(32)
which show rich structures as a function of the phases \( \vartheta \) and \( \chi \).

Full counting statistics of Cooper pair transfer was considered in (74). The authors focus on the incoherent regime, where coherence suppressed by classical fluctuations in the gate voltage and no net supercurrent is shuttled. However, the charge transfers occur, and the current is zero only in average Thus the FCS presents a convenient way to reveal this circumstance.

The initial model for a superconducting shuttle is similar to that of (48; 49). The fluctuation of the gate voltage are allowed for an assumption of stochastic \( \delta \)-correlated fluctuations,
\[
\langle V_g(t) \rangle = V_g \quad \text{and} \quad \langle V_g(t)V_g(t') - V_g^2 \rangle = \frac{\gamma h^2}{4e^2} \delta(t-t').
\]
Thus defined \( \gamma \) has a meaning of inverse decoherence rate of the two charge states. It leads to a damping of the off-diagonal elements of the density matrix, while diagonal elements are assumed to be undamped. The FCS is computed using the method of generation function.

The physics of charge transfer could be clearly understood for the limiting cases of long and short cycles comparing to the decoherence time. If the period of the shuttling is sufficiently long for decoherence to be accomplished,
\[
t_C^{-1}, t_0^{-1} \ll \gamma E_J/\hbar,
\]
the FCS can be interpreted in terms of classical elementary events: Cooper pair transfers. During the shuttling cycle, either no transfer takes place or a pair is transferred in either direction. There is an apparent similarity with the FCS of the pumping in normal systems studied in (63; 75; 76). In this case
\[
p_0 = 1/2, \quad p_{\pm 2} = 1/4,
\]
so that, each shuttling between the superconductors transfers either one Cooper pair or none, this occurs with equal probabilities. The pair is transferred with equal probabilities in either direction. This simple result is quite general and relays on neither the periodicity of shuttling nor the concrete time dependence of \( E_J(t) \) provided the adiabaticity is preserved. Leading corrections to adiabatic FCS are exponentially small, \( \sim e^{-2Tt_0} \).

Adiabaticity is also preserved at small Josephson couplings where
\[
t_C^{-1}, t_0^{-1} \ll \gamma E_J/\hbar.
\]
In this case the factor \( f \equiv e^{-t_0E_J/\hbar} \) can be arbitrary, and the FCS becomes more complicated (74). At finite \( f \) all \( p_n \neq 0 \), but remain positively defined. In the adiabatic limit the FCS does not depend on the superconducting phase \( \Phi \) and dynamical phases \( \vartheta \) and \( \chi \).

Beyond the adiabatic limit, the FCS does depend on \( \Phi \), and classical interpretation in this case can fail since \( p_n \) can be negative of even complex. A relatively simple treatment is possible in the case of very short periods, \( \gamma/\omega_0 \ll 2\pi \). The FCS in this case corresponds to a supercurrent random switching at the time scale \( 1/\gamma \) between the values \( \pm I_s \). The quantities \( \tilde{g} \) and \( I_s \) depend on the phases \( \Phi, \vartheta, \) and \( \chi \), concrete form of the dependences being given in (74), see also (77).

To summarize, in limiting cases of long and short periods the FCS allows relative simple classical interpretations. In an intermediate situation, the FCS can not be interpreted in classical terms - the charge transfer probabilities per cycle may be negative or complex. This is a clear signature of the fact that the superconducting coherence survives strong dephasing although this coherence does not manifest itself in net superconducting current.

One can conclude that both the noise spectrum (second cumulant) and the FCS provide valuable information on shuttle transport, which complimentary to the information extracted from the average current. It is a combination of the features of the average current and noise that can assure that shuttling is the underlying transport mechanism.

VI. DISCUSSION AND CONCLUSION

While designing nanometer-sized devices one inevitably faces Coulomb correlation in the electron transport. The most peculiar feature of such correlation is a single-electron operation which determines transport properties of many interesting nanodevices. Furthermore, in nanosystems electric charges produce not only large potential differences, but also large mechanical forces which can be comparable with atomic forces. These forces tend to produce mechanical displacements which, in turn, lead to a feedback to the distribution of electric charges. As result, coupling of electrical and mechanical degrees of freedom is a hallmark of nanodevices. The aim of this review is to demonstrate one fundamental manifestation of such coupled motion – the shuttle
transfer of the charge due to conveying of electrons by a movable part of the nanosystem. Shuttling of charge can occur either due to an intrinsic instability, or can be driven by an external ac source. From an “applied” point of view, the role of shuttling can be either positive, or negative. Indeed, it can hinder a proper operation of a single-electron transistor at the nanometer scale. On the other hand, an intended periodic mechanical motion resulting from the instability can be used to create building blocks for new applications. In particular, new principal possibilities for generators and sensors at nanometer scale appear.

As we have tried to show, the area centered around shuttle instability involves a few new principles and possibilities. Thus, there exists a rich physical picture containing both coherent and incoherent electron transport facilitated by either classical or quantum mechanical motion. In particular, one can expect very interesting physics regarding the coherent shuttling of Cooper pairs between relatively large distances, as well as to the creation of quantum coherence between remote objects by movable superconducting grains. This system, if realized experimentally, would allow for the determination of the decoherence rate of the superconducting devices due to their interaction with environment.

One can imagine several concrete systems where electromechanical coupling is very important. Among them are nanoclusters or single molecules which can vibrate between the leads, metal-organic composites showing pronounced heteroelastic properties, colloidal particles, etc. A possibility of a similar physical picture involving coupling of magnetic and mechanical degrees of freedom has also been considered (78). In the latter case the coupling is due to exchange forces, and it can lead to shuttling of magnetization.

There are a few experiments where electromechanical coupling has been observed and some evidence in favor of single-electron shuttle instability remains still a challenging problem. To solve this problem in a convincing way it seems to be a good idea to study anomalous structure of Coulomb blockade in nanomechanical structures with and without gates, as well as to detect periodic ac current.

To summarize, movable nanoclusters can serve as new weak links between various normal, superconductor and magnetic systems.

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APPENDIX A: Coulomb blockade: Review of orthodox theory

Here we give a brief review of the so-called orthodox theory of Coulomb blockade (12). Consider a dot coupled to two leads via tunnel barriers. If one transfers the charge $q$ from the source to the grain the change in the energy of the system is

$$\Delta U = qV_g + \frac{q^2}{2C}.$$

Here the first item is the work by the source of the gate voltage while the second one is the energy of Coulomb repulsion at the grain. We describe it by the effective capacitance $C$ to take into account polarization of the electrodes. The graph of this function is the parabola with the minimum at

$$q = q_0 = -CV_g.$$

So it can be tuned by the gate voltage $V_G$. Now let us remember that the charge is transferred by the electrons with the charge $e$. Then, the energy as a function of the number $n$ of electrons at the grain is

$$\Delta U(n) = neV_g + \frac{n^2e^2}{2C}.$$

Now let us estimate the difference

$$\Delta U(n + 1) - \Delta U(n) = eV_g + \frac{e^2}{C}.$$

We observe that at certain values of $V_g$, 

$$V_{gn} = -(2n + 1)\frac{e}{2C}, \quad (A1)$$

the difference vanishes. It means that only at that values of the gate voltage resonant transfer is possible. Otherwise one has to pay for the transfer that means that only inelastic processes can contribute. As a result, at

$$k_BT \leq \frac{e^2}{2C}$$

the linear conductance is exponentially small if the condition (A1) is met. This phenomenon is called the Coulomb blockade of conductance. As a result of the Coulomb blockade, electron tunnel one-by-one, and the conductance vs. gate voltage dependence is a set of sharp peaks. That fact allows one to create a so-called single-electron transistor (SET) which is now the most sensitive electrometer. Such a device (as was recently demonstrated) can work at room temperature provided the capacitance (size!) is sufficiently small. Below we shall review the simplest variant of the theory, so called the orthodox model.

For simplicity, let us ignore discrete character of energy spectrum of the grain and assume that its state is fully characterized by the number $n$ of excess electrons with
We observe that there is a threshold voltage which is necessary to exceed to organize transport. This is a manifestation of Coulomb blockade. It is important that the threshold linearly depends on the gate voltage which makes it possible to create a transistor. Of course, the above considerations are applicable at zero temperature.

The current through the emitter-grain transition can be expressed through the probability, \( p_n \), to find \( n \) excess electrons at the grain and transition rates, \( \Gamma_{g \rightarrow e} \). We have

\[
I = e \sum_n p_n (\Gamma_{e \rightarrow g} - \Gamma_{g \rightarrow e}) .
\]  

The tunneling rates can be calculated from the golden rule expressions using tunnel transmittance as perturbations. For a symmetric case, the expressions for these rates are given in Sec. B, Eq. (B7). At low temperatures and low bias voltages, \( V_0 C/e < 1 \), only two charge states play a role. At larger bias voltage, more charge states are involved. As a result, the current-voltage curve shows steps called the Coulomb staircase.

end

APPENDIX B: Shuttle instability at weak electromechanical coupling

1. Model and basic equations

Consider a simple model of a charge particle with charge \( q = e n \) and mass \( m \) placed in a one-dimensional harmonic confining potential \( k_0 x^2/2 \). Assuming that the particle motion is damped by a viscous friction described by the force \(- m \gamma \dot{x}\) one arrives at the following equation of motion:

\[
m \ddot{x} = -k_0 x - m \gamma \dot{x} + e E x .
\]  

For small oscillations the electrical field \( E \) can be considered as position-independent and proportional to the bias voltage \( V \). We put \( E = V/L \), and in this way define the effective length, \( L \), of the system. In the absence of electromechanical coupling this equation describes harmonic oscillations with the frequency \( \omega_0 = \sqrt{k_0/m} \) and amplitude damping \( \gamma/2 \). Hereby we assume that the damping is weak, \( \gamma \ll \omega_0 \).

Let us multiply Eq. (B1) by \( \dot{x} \) and average over a long time \( \Theta \) equal to a large integer number of periods, \( \Theta = 2 \pi N/\omega_0 \), which is also much greater than the electron transfer time \( t_R = RC \). In this way one obtains the energy balance for the mechanical energy \( U = m \ddot{x}^2/2 + k_0 x^2/2 \) as

\[
\dot{U} = e E n \dot{x} - m \gamma \ddot{x}^2 .
\]  

The first item in the right hand side, \( W = e E n \dot{x} \), is the work produced by the electric field while the second one is the viscous dissipation. This is actually a stochastic
equation since the electron transfer occurs via random electron hops.

Obviously, an instability would correspond to $\dot{U} > 0$.

The analysis can be substantially simplified for a weak electromechanical coupling. Then, one can assume that the mechanical energy grows much slower than both the grain and the electron move. In this case the averages in the r.h.s. can be calculated assuming that the grain performs small harmonic oscillations with conserved the mechanical energy. This assumption is valid if the averaging time is still less than the typical instability growth time, $t_g \equiv U/\dot{U}$, which we will estimate later. Thus we chose

$$1, \omega_0 RC \ll N \ll \omega_0 t_g \quad \text{(B3)}$$

There are two consequences of the coupling weakness. First, the average kinetic and potential energies are equal, $m \langle \dot{x}^2 \rangle / 2 = k_0 \langle x^2 \rangle = U/2$. Secondly, one can split the time average $n(t)x(t)$ into an average over a period of oscillating motion and a subsequent average over different periods:

$$\frac{n(t)\dot{x}(t)}{t} = \frac{\omega_0}{2 \pi N} \int_{0}^{t+2\pi N/\omega_0} dt' n(t')\dot{x}(t') = \frac{1}{N} \sum_{k=1}^{N} \frac{\omega_0}{2 \pi} \int_{0}^{t+2\pi k/\omega_0} dt' n(t')x(t').$$

This expression only weakly depends on $t$ through a weak time dependence of the oscillation amplitude, and under conditions of Eq. B3 this dependence can be ignored.

Since $n(t)$ is a random quantity and $N \gg 1$ one can replace the average over the periods by an ensemble average, $\langle n \rangle_t$, introducing a probability $p_n(t)$ to find $n$ excess electrons at the grain at time $t$. Thus,

$$n(t)\dot{x}(t) = \langle \langle n \rangle_t \dot{x}(t) \rangle_{osc}, \quad \langle n \rangle_t \equiv \sum_n n p_n(t), \quad \langle b \rangle_{osc} \equiv \frac{\omega_0}{2 \pi} \int_{0}^{2\pi \omega_0} dt' b(t').$$

As a result, we arrive at the following energy balance equation:

$$\dot{U} = eEa \langle \langle n \rangle_t \dot{x}(t) \rangle_{osc} - \gamma U \quad \text{(B5)}$$

where it is assumed that $x(t) = a \sin \omega_0 t$ and the amplitude $a$ is essentially time independent at the scale of $\omega_0^{-1}$.

To calculate the distribution function $p_n(t)$ one has to specify the transition rates $\Gamma_\pm(n, x)$ for the processes $\{n \rightarrow n \pm 1\}$ occurring at a position $x$. Knowing these probabilities one can find $p_n(t)$ from the ME

$$\frac{\partial p_n}{\partial t} = [\Gamma^-(n + 1, x)p_{n+1} + \Gamma^+(n - 1, x)p_{n-1} - p_n \sum_{\pm} \Gamma^\pm(n, x) x = \pm a \sin \omega_0 t]. \quad \text{(B6)}$$

To specify the probability one has to take into account that an electron can arrive from and escape to any of the electrodes. Thus

$$\Gamma^\pm(n, x) = \Gamma^\pm_L(n, x) + \Gamma^\pm_R(n, x)$$

where $\Gamma^\pm_L(n, x)$ is the probability for an electron to hop on the grain from the left electrode while $\Gamma^\pm_R(n, x)$ is the probability to hop from the right electrode. These partial probabilities are strongly influenced to the Coulomb blockade, and can be expressed as

$$\Gamma^\pm_L = \frac{1}{RC} e^{-x/\lambda} f\left(\pm \frac{v}{2} \pm n - \frac{1}{2}\right),$$

$$\Gamma^\pm_R = \frac{1}{RC} e^{-x/\lambda} f\left(\pm \frac{v}{2} \pm n - \frac{1}{2}\right),$$

$$f(z) \equiv \frac{z}{1 - \exp(-\beta E_G z)}. \quad \text{(B7)}$$

Here we assume that the system is symmetric, the capacitances between the grain and both leads are position-independent, $C_R = C_L = C$, while tunneling resistances depend on $x$ exponentially, $R_{L,R}(x) = R e^{x/\lambda}$. The dimensionless voltage $v$ is defined as $V C/e$, while $\beta = 1$ is the temperature. At low temperatures, $\beta E_G \gg 1$ the function $f(z) \approx z \Theta(z)$ where $\Theta(z)$ is the Heaviside unit step function. This implies quantization of the grain charge ($12$).

Here will discuss the simplest case of low temperature and when the voltage is chosen at the point where a new channel is about to switch on, $v_0 = 2n + 1$. In this case one can directly calculate the ensemble average $\langle n \rangle_t \equiv \sum_n n p_n(t)$ from the differential equation (79)

$$RC \frac{\partial \langle n \rangle_t}{\partial t} = -2 \langle n \rangle_t \cos \left(\frac{U}{U_0} \sin \omega_0 t\right) + (1 - v) \sin \left(\frac{U}{U_0} \sin \omega_0 t\right) \quad \text{(B8)}$$

Here $U_0 \equiv m\omega_0^2 \lambda^2 / 2$ is a typical scale for the mechanical energy. Thus, the produced work depends on the mechanical energy itself and on the bias voltage, $v$. To estimate a typical scale of this work let us recall that the transition probabilities (B7) introduce a natural scale $e/C$, for the bias voltage, and a typical scale $\lambda$ for the displacement $x$. Thus, a typical scale for the quantity $eE$ can be written as $e^2/C$. Since both $v$ and $n$ are of the order of $1$, one has to compare $eE$ with the mechanical force, $m\omega_0^2 x \sim m\omega_0^2 \lambda$. As a result, one arrives at the following expression for the dimensionless electromechanical coupling constant,

$$\epsilon = e^2 / mC L \omega_0^2 \lambda. \quad \text{(B9)}$$

For a nanoscale grain and typical organic junctions $\epsilon \sim 10^{-2}$. We conclude that the mechanical energy should indeed vary slowly in time, the characteristic scale being
The parameters of the system, see below. Using Eq. (B13) while the sign of $\nu$ even though there are peaks in the differential conduc-
tance due to the switching on of new channels at voltages $v_n = 2n + 1$ (12).

As a result, at low temperatures and for the threshold values of the voltage one has to analyze the set of equations,

$$
\dot{E} = \nu \omega_0 W(E) - \gamma E, \quad (B10)
$$

$$
W(E) = \frac{1}{2} \int_0^{2\pi} d\varphi \langle n \rangle \varphi \cos \varphi, \quad (B11)
$$

$$
\tau_R \frac{\partial \langle n \rangle}{\partial \varphi} = -2 \langle n \rangle \varphi \cosh (\sqrt{E} \sin \varphi) + (1 - \nu) \sinh (\sqrt{E} \sin \varphi). \quad (B12)
$$

Here $E \equiv U/U_0$, $\varphi \equiv \omega_0 t$, $\langle n \rangle \varphi = \langle n \rangle t = \varphi$, $\tau_R = \omega_0 RC$.

### 2. Analysis of shuttle instability

The stationary regime of the system is obviously given by equation

$$
\nu \omega_0 W(E) - \gamma E = 0. \quad (B13)
$$

This equation has a trivial solution $E = 0$, and when it is stable the system resides in the equilibrium position. Let us call this state the stationary regime. For our symmetric system the current-voltage curve in this regime shows no pronounced Coulomb blockade structure even though there are peaks in the differential conductance due to the switching on of new channels at voltages $v_n = 2n + 1$ (12).

To analyze stability of the stationary regime it is convenient to expand the work $W(E)$ in powers of the dimensionless mechanical energy, $E$,

$$
W(E) = \alpha(v) E + \beta(v) E^2 + \ldots. \quad (B14)
$$

It can be shown that $\alpha(v) = (\partial W/\partial E)_{E=0} > 0$ (79), while the sign of $\beta = (1/2)(\partial^2 W/\partial E^2)_{E=0}$ depends on the parameters of the system, see below. Using Eq. (B13) we can define the dimensionless critical voltage $v_c$ from the equation

$$
\nu \alpha(v_c) = \tilde{\gamma}, \quad \tilde{\gamma} \equiv \gamma/\omega_0. \quad (B15)
$$

Near the critical voltage, $|v - v_c| \ll v_c$, one can expand the mechanical energy as

$$
W(E) = [\alpha(v_c) + \alpha'(v_c)(v - v_c)]E + \beta E^2. \quad (B16)
$$

The energy of mechanical vibrations at a given voltage $v$ is then determined by the equation

$$
v\alpha(v) + \beta E = \tilde{\gamma}. \quad (B17)
$$

Using Eq. (B15) we find for the energy

$$
E = -\frac{1}{\beta} \frac{v - v_c}{\nu_c} \left[ \alpha(v_c) + \nu_c \alpha'(v_c) \right].
$$

This result is meaningful at $\beta < 0$. It shows that at $v > v_c$ the stationary regime is unstable and the stationary amplitude of mechanical oscillations grows as $\sqrt{E} \sim \sqrt{v - v_c}$. Adopting nomenclature from the theory of oscillations (80) we are dealing here with soft excitation of self oscillation where the amplitude is a smooth function of the difference $v - v_c$. The hard excitation is associated with a hysteretic behavior of the amplitude versus $v - v_c$. In the language of phase transitions – taking the oscillation amplitude to be the order parameter – the ‘soft’ case corresponds to a second order transition and the ‘hard’ case to a first order transition.

When $\beta > 0$ the shuttle instability develops in a completely different way. Consider the graphs in Fig. 33 showing $W(E)$ for a fixed set of parameters for four different voltages along with the line $\tilde{\gamma} E$. Consider now the system being located in $O$ at a voltage $v < v_{c1}$. In this case the system is in the static regime and exhibits the same behavior as an ordinary double junction. As the

![FIG. 33 Schematic energy diagram for the case $\beta > 0$. The graph shows $W(E)$ for a fixed set of parameters for four different voltages along with the line $\tilde{\gamma} E$. When $v < v_{c1}$ only $O$ will be a stable stationary point. At $v = v_{c2}$ a second unstable stationary point $A$ appears. For $v_{c1} < v = v' < v_{c2}$ we have two coexisting stable points $O$ and $B$ leading to the hysteretic behavior of the system discussed in the text. At $v = v_{c2}$ $A$ will become unstable and the system is determined to be in the limit cycle with amplitude corresponding to energy $E_2$ at the intersection $C$. As $v$ is increased above $v_{c2}$, the only stable stationary point left is $C$ corresponding to a limit cycle with amplitude $\propto \sqrt{E}$. Voltage is increased above $v_{c1}$ a second stable stationary](attachment:fig33.png)
point $B$ appears but the system cannot reach this point since $O$ is still stable. At $v = v_{c2}$, $O$ becomes unstable and the system “jumps” from $O$ to $C$. This instability we refer to as hard since the amplitude changes abruptly from $E = 0$ to $E = E_2$ as the voltage is raised above $v_{c2}$. Now consider the case of $v > v_{c2}$ when the system is originally is at some stationary point and the voltage is lowered. At $v < v_{c2}$, $O$ becomes stable but cannot be reached by the system until $v$ has dropped to $v = v_{c1}$. At $v_{c1}$ the point $A$ becomes unstable and the system will “jump” to $O$. This transition is characterized by an abrupt drop in amplitude from $E_1$ to $E = 0$ at $v = v_{c1}$. Since $v_{c1} < v_{c2}$ the system will obviously exhibit a hysteretic behavior in the transition region.

For a simplified case of low temperatures and $v = v_0$, one can solve Eq. (B12) approximately for small $E$, then substitute the solution in Eq. (B11) and compute $\beta$. One obtains (79)

$$\beta = \frac{v - 1}{16} \frac{\tau_R (\tau_R^2 - 12)}{(\tau_R + 4)^2}. \tag{B18}$$

Thus the border between the soft and hard excitation corresponds to $\tau_R = 2\sqrt{3}$.

It is worth mentioning that numerical analysis (79) predicts significant difference in the $I - V$ curve in the static and shuttling regime, see Fig. 34.

![Current through the Coulomb blockade system](image)

**FIG. 34** Current through the Coulomb blockade system in the static regime ($v < v_c$) and in the shuttle regime ($v > v_c$). As the system enters the shuttle regime the current (solid line) deviates from the current for a static double junction (dashed line). After the transition to the shuttle regime distinct steps can be seen in the current. The current is normalized to the frequency of harmonic oscillations, $\omega_0$, to demonstrate that the step height in the shuttle regime is proportional to $\omega_0$. In order to make a comparison with the current in the static regime this current has been scaled by the same factor. From Ref. (79).

**APPENDIX C: Abbreviations**

AC – (alternating current) non-stationary; DC – (direct current) stationary; FCS – full counting statistics; GMR – giant magnetoresistance; ME – master equation; NEMS – nano-electromechanical system; RIE – reactive ion etching; SEM – scanning electron microscope; SET – single-electron transistor.

**APPENDIX D: Notations**

$e$ – Elementary (electron) charge
$n$ – Number of electrons on island
$x$ – Oscillator displacement from equilibrium position
$x_0 = \sqrt{n} \hbar \omega_0/2m$ – de Broglie wave length of the oscillator
$q$ – Oscillator charge
$m$ – Oscillator mass
$k_0$ – Effective spring constant
$\omega_0 = \sqrt{k_0/m}$ – Natural angular vibration frequency
$\omega$ – Natural vibration frequency [Hz]
$\omega_1$ – Natural angular vibration frequency
$\omega_2$ – Natural angular vibration frequency
$a$ – Amplitude of mechanical oscillations
$t_0$ – Contact time
$f_0 = \omega_0/2\pi$ – Period of vibration
$\lambda$ – Characteristic tunneling length
$R$ – Charge relaxation rate
$V$ – Applied DC bias voltage
$I$ – Direct current
$T$ – Temperature
$p_n(t)$ – Probability for $n$ electrons to be transferred during time $t$
$p_n(t)$ – Probability to find $n$ excess electrons on the shuttle

**References**


